J Transp Secur (2013) 6:151–174
DOI 10.1007/s12198-013-0109-z

Multi-item interval valued solid transportation problem with safety measure under fuzzy-stochastic environment

A. Baidya · U. K. Bera · M. Maiti

Received: 20 February 2013 / Accepted: 28 February 2013 / Published online: 4 April 2013

Abstract In this paper we introduce "safety factor" in transportation problem. Here we solve Multi Item Interval Valued Solid Transportation Problem (MIIVSTP) with safety factor under Desire Safety Measure (DSM) fuzzy-stochastic and stochastic. When items are transported from origins to destinations through different conveyances, there are some difficulties/risks to transport the items due to bad road, insurgency etc. in some routes specially in developing countries. Due to this reason desired total safety factor is being introduced. Also our goal is to evaluate the solution of MIIVSTP using Global Criteria Method. Here we developed five model with taking DSM as fuzzy-stochastic and stochastic and safety factor as crisp, fuzzy, interval, stochastic, fuzzy-stochastic. Here the transportation costs are intervals, the corresponding multi-objective transportation problem is formulated using "mean and width" technique. Then the problem is converted to a single objective transportation problem taking convex combination of the objectives according to their weights. Finally all the models are solved by Generalized Reduced Gradient (GRG) method using LINGO software. Numerical examples are used to illustrate the model and methodologies.

Keywords Solid transportation problem · Safety factor · Global criteria method · Convex combination

A. Baidya · U. K. Bera (✉)
Department of Mathematics, National Institute of Technology, Agartala,
Jirania 799055 West Tripura, India
e-mail: uttamkr.math@nita.ac.in

U. K. Bera
e-mail: bera_uttam@yahoo.co.in

A. Baidya
e-mail: abhijitnita@yahoo.in

M. Maiti
Department of Applied Mathematics, Vidyasagar University, Midnapore 721102 WB, India
e-mail: mmaiti2005@yahoo.co.in

Introduction

As a generalization of traditional Transportation Problem, the Solid Transportation Problem (STP) was stated by Shell in 1955, which he considered the three item properties in the constraint set instead of two items namely source and destination. He also suggested the situations where the STP would arise, and four cases of STP were discussed according to the data given on the item properties and developed its solution procedure. Basu et al. (1994) developed an algorithm for finding the optimum solution for the solid fixed charge linear transportation problem. Although STP was forgotten for long time, because of existing advanced solution methodologies, recently it is receiving the attention of many researchers of this field. Models and algorithms have been developed by many authors (Bit et al. 1993; Gen et al. 1995; Jimenez and Verdegay 1998, 1999; Li et al. 1997a, b; Yang and Liu 2007). The fuzzy set theory concept was first introduced by Zadeh (1965). Linear programming problems with several objective functions was solved by using fuzzy membership functions by Zimmerman (1978) and he showed that the results obtained from fuzzy are always efficient. A special type of non-linear membership function was used for the vector maximum linear programming problem (Liberling 1981).

In the real world, sometimes data cannot be measured/ collected precisely. This impreciseness may occur in stochastic or non – stochastic (i.e., fuzzy) sense or both stochastic and fuzzy sense together i.e., fuzzy – stochastic sense. In some real – life transportation problems, it is difficult to obtain in advance the exact value amounts of resources, demands, direct unit costs and fixed charges for transportation for traditional (2-dimentional) TP and transportation capacities and transportation times for transport conveyances in addition to the above mentioned parameters in the case STP (3-dimentional TP). These parameters are sometimes treated as random variables according to the statistical experience when enough sample data are available.

Stochastic programming deals with situations where the input data are imprecise in stochastic sense and described by random variables with known probability distribution. Probability theory provides the theoretical foundation for stochastic programming models. The probability and mathematical expectation have often been used during the formulation of stochastic models to deal quantitatively with random data.

Again, data/ parameters imprecise in both fuzzy and stochastic senses are called fuzzy – random or hybrid parameters. The concept of fuzzy – random variable was introduced by Kwakernaak (1978, 1979) and Puri and Ralescu(1986). The occurrence of fuzzy – random variable/ parameter makes the combination of randomness and fuzziness more persuasive. Though in the literature, these are some decision making problems formulated and solved with fuzzy – random parameters/ variables, till now, to the best of our knowledge, no T.P. has been formulated and solved with fuzzy – random costs/ resources.

Scope of an interval objective function in the light of maximization/minimization problem

The objective of a conventional linear programming problem (LPP) is to maximize or minimize the value of its (one only, single-valued) objective function satisfying a given set of restriction. However, a single-objective interval linear programming problem (ILPP) contains an interval-valued objective function (IOF). Let us consider the following problem:

$$\begin{array}{l} \text{Maximize/Minimize } Z = \sum_{j=1}^{N} \left[C_{Lj}, C_{Rj}\right] x_j \\ \text{Subject to, } \{\text{Set of feasibility constraints}\} \end{array} \tag{1}$$

As an interval can be represented by any two of its four attributes(viz., left limit, right limit, mid-value and width), then by using attributes mid-value and width(say), the ILPP(1) can be reduced into a bi-objective LPP as follows:

$$\text{Max/Min}\{\text{mid–value of the IOF}\} \tag{2.1}$$

$$\text{Min}\{\text{Width of the IOF}\} \tag{2.2}$$

$$\text{Subject to}\{\text{Set of feasible Constraints}\} \tag{2.3}$$

From this problem, naturally one may get two conflicting optimal solutions:

$$\begin{array}{l} x' = \left\{x_j^{'}\right\}, \text{ from(2.1)and(2.3)}, \\ x'' = \left\{x_j^{''}\right\}, \text{ from(2.2)and(2.3)} \end{array}$$

and hence we get two optimal values Z' and Z'' of Z respectively.

If $x' = x''$ then there does not exists any conflict and x' is the solution of the problem. But if $x' \neq x''$, for the maximization problem, $m(Z') > m(Z'')$ and $w(Z') > w(Z'')$, (because, Z' is obtained through maximizing m(Z) and Z'' is obtain through another goal, by minimizing w(Z)).

Similarly, for minimization problem, if $x' \neq x''$ then $m(Z') < m(Z'')$ and $w(Z') > w(Z'')$, (because, Z' here is obtained by minimizing m(Z) and Z'' by minimizing w(Z)). Therefore, if $x' \neq x''$, then Z' and Z'' become the non-dominated extreme alternative(Sengupta and Pal 2000).

On the other hand, the principle of $\mathcal{A}$ -index indicates that for the maximization (minimization) problem, an interval with a higher mid-value is superior (inferior) to an interval with a lower mid-value. Therefore, though Z' and Z'' are two non-dominated alternative extremes from the viewpoint of a bi-objective problem, they can ranked though $\mathcal{A}$ -index.

Hence, in order to obtain maximum/minimum of the interval objective function, considering the mid-value of an interval-valued objective function is our primary concern. Therefore, if we reduce the interval objective function in its central value and use conventional LP technique for its solution, the solution will give the best-expected optimum for the problem concerned. Further, we also need to consider the width but as a secondary attribute, only to determine best reachable certainty level and to confirm whether the best-expected optimum is within the acceptable limit of the DM for the problem concerned. If it is not, one has to go for smaller extend of width (uncertainty) according to his satisfaction and thus to obtain a less wide interval from among the non-dominated alternative accordingly (Sengupta and Pal 2004).

Henceforth we develop a composite goal to define the IOF as follows:
For the maximization problem:

$$\text{Maximize } \lambda * \mathrm{m(Z)} - (1-\lambda) * \mathrm{w(Z)} \tag{3}$$

$$\text{w.r.t } (2.1,\ 2.2,\ 2.3) \text{ and } 0 \le \lambda \le 1 \tag{4}$$

For minimization problem:

$$\text{Minimize ZZ} = \lambda * \mathrm{m(Z)} + (1-\lambda) * \mathrm{w(Z)} \tag{5}$$

$$\text{w.r.t } (2.1,\ 2.2,\ 2.3) \text{ and } 0 \le \lambda \le 1 \tag{6}$$

The lemda (λ) factor defines the DM's pessimistic or optimistic bias. If λ=1, (3) and (5) show DM's absolute optimistic bias and if λ=0, (3) and (5) indicate, on the contrary, the pessimistic DM's attitude (Sengupta and Pal 2000). With λ=0.5 or with similar other value, a similar proportional balance between DM's optimistic and pessimistic preference may be thought of.

Order relations between intervals

Here, the order relations which represent the decision-makers preference between interval costs are defined for minimization problems. Let the uncertainty costs for two alternatives be represented by interval A and B respectively. It is assumed that the cost of each alternative is known only to lie to the corresponding interval. The order relation by the left and right limits of interval is defined in definition below.

Definition

The order relation $\le_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \le_{LR} B \; iff \; a_L \le b_L \; and \; a_R \le b_R \tag{f}$$

$$A <_{LR} B \; iff \; A \le_{LR} B \; and \; a_R \ne b_R$$

The order relation $\le_{LR}$ represents the DM's performance for the alternative with the lower minimum cost, that is, if $A \le_{LR} B$, then A preferred to B.

Note If $\widehat{x}_i, \mathrm{i} = 1, 2, \ldots, \mathrm{m}$ are random variables and a_i's are all constants then $\sum_{i=1}^{m} a_i \widehat{x}_i$ is also random.

Note If $\widehat{\widehat{x}}_i, \mathrm{i} = 1, 2, \ldots, \mathrm{m}$ are fuzzy-random variables and a_i's are all constants then $\sum_{i=1}^{m} a_i \widehat{\widehat{x}}_i$ is also fuzzy-random.

Approximate value of Triangular Fuzzy Number (TFN)

According to Kaufmann and Gupta (1991), the approximate value of TFN TFN $\widetilde{a} = (a_1, a_2, a_3)$ is given by $\widetilde{a} = \frac{a_1+2a_2+a_3}{4}$.

Chance constraint programming

As the name indicates, the chance-constrained programming technique can be used to solve problems involving chance constraints, i.e., constraints having finite probability of being violated. This technique was originally developed by Charnes and Cooper.

1. If ε are the probabilities of non-violation of the constraint $a \geq \widehat{\widetilde{b}}$ then the constraint can be written as

$$\mathrm{Prob}\left[\mathrm{a} \geq \widehat{\widetilde{\mathrm{b}}}\right] \geq \varepsilon$$

$$\mathrm{Or,}\ \ \mathrm{Prob}\left[\frac{\mathrm{a}-\mathrm{E}\left(\widehat{\widetilde{\mathrm{b}}}\right)}{\mathrm{var}\left(\widehat{\widetilde{\mathrm{b}}}\right)} \geq \frac{\widehat{\widetilde{\mathrm{b}}}-\mathrm{E}\left(\widehat{\widetilde{\mathrm{b}}}\right)}{\mathrm{var}\left(\widehat{\widetilde{\mathrm{b}}}\right)}\right] \geq \varepsilon$$

At first defuzzify the expectation and variance as:

$$m_b = E\left(\widehat{\widetilde{b}}\right) = \widehat{\widetilde{b}} = \left(\overline{b}_1, \overline{b}_2, \overline{b}_3\right)$$

$$\mathrm{Or,}\ \ \mathrm{m_b} = \frac{\overline{\mathrm{b}}_1 + \overline{\mathrm{b}}_2 + \overline{\mathrm{b}}_3}{3}$$

$$\mathrm{Similarly,}\ \sigma_{\mathrm{b}}^2 = \mathrm{var}\left(\widehat{\widetilde{\mathrm{b}}}\right) = \widetilde{\sigma}_{\mathrm{b}}^2 = \left(\sigma_{1\mathrm{b}}^2, \sigma_{2\mathrm{b}}^2, \sigma_{3\mathrm{b}}^2\right)$$

$$\mathrm{Or,}\ \sigma_{\mathrm{b}}^2 = \frac{\sigma_{1\mathrm{b}}^2 + \sigma_{2\mathrm{b}}^2 + \sigma_{3\mathrm{b}}^2}{3}$$

Then the constraint reduces to a standard chance constraint as:

$$\mathrm{Prob}\left[\mathrm{h} \geq \widehat{\mathrm{d}}\right] \geq \varepsilon$$

Where, $\widehat{d} = \frac{\widehat{\widetilde{b}} - m_b}{\sigma_b^2}$ are the standard normal variate and $h = \frac{a - m_b}{\sigma_b^2}$

Or, $a \geq m_b + \lambda \sigma_b^2$.

Where λ be the real number such that $\mathrm{Prob}\left[\lambda \geq \widehat{\mathrm{d}}\right] = \varepsilon$.

2. If ε are the probabilities of non-violation of the constraint $\widehat{a} \geq \widehat{\widetilde{b}}$ then the constraint can be written as

$$\mathrm{Prob}\left[\widehat{\mathrm{a}} \geq \widehat{\widetilde{\mathrm{b}}}\right] \geq \boldsymbol{\varepsilon}$$

$$\begin{aligned} &\mathrm{Or,}\ \ \mathrm{Prob}\left[\widehat{\mathrm{a}} - \widehat{\widetilde{\mathrm{b}}} \geq 0\right] \geq \boldsymbol{\varepsilon} \\ &\mathrm{Or,}\ \ \mathrm{Prob}[\mathrm{R} \geq 0] \geq \boldsymbol{\varepsilon} \\ &\mathrm{Or,}\ \ \mathrm{Prob}\left[\frac{\mathrm{R-E(R)}}{\mathrm{var(R)}} \geq \frac{\mathrm{-E(R)}}{\mathrm{var(R)}}\right] \geq \boldsymbol{\varepsilon}. \end{aligned}$$

At first defuzzify the expectation and variance as:

$$m_R = E(R) = E\left(\widehat{a} - \widehat{\widetilde{b}}\right) = E(\widehat{a}) - E\left(\widehat{\widetilde{b}}\right) = \overline{a} - \overline{\widetilde{b}} = \overline{a} - m_b.$$

Then the constraint reduces to standard chance constraint as:

$$Prob\left[\widehat{T} \geq -K\right] \geq \boldsymbol{\varepsilon}$$

$$\mathrm{Or,}\ \ \mathrm{Prob}\left[\widehat{\mathrm{T}} \leq \mathrm{K}\right] \geq \boldsymbol{\varepsilon}$$

Where, $\widehat{T} = \frac{R-m_R}{\sigma_R^2}$ *are the standard normal variate and* $K = \frac{m_R}{\sigma_R^2}$.

$$m_R \geq \lambda\sigma_R^2$$

Where λ is the real number such that $Prob\left[\widehat{T} \geq \lambda\right] = \boldsymbol{\varepsilon}$.

3. If ε are the probabilities of non-violation of the constraint $\widehat{\widetilde{a}} \geq \widehat{\widetilde{b}}$ then the constraint can be written as

$$Prob\left[\widehat{\widetilde{a}} \geq \widehat{\widetilde{b}}\right] \geq \boldsymbol{\varepsilon}$$

$$\begin{aligned} &\mathrm{Or,}\ \ \mathrm{Prob}\left[\widehat{\widetilde{\mathrm{a}}} - \widehat{\widetilde{\mathrm{b}}} \geq 0\right] \geq \boldsymbol{\varepsilon} \\ &\mathrm{Or,}\ \ \mathrm{Prob}\left[\widehat{\widetilde{\mathrm{Q}}} \geq 0\right] \geq \boldsymbol{\varepsilon} \\ &\mathrm{Or,}\ \ \mathrm{Prob}\left[\frac{\widehat{\widetilde{\mathrm{Q}}} - \mathrm{E}\left(\widehat{\widetilde{\mathrm{Q}}}\right)}{\mathrm{var}\left(\widehat{\widetilde{\mathrm{Q}}}\right)} \geq \frac{-\mathrm{E}\left(\widehat{\widetilde{\mathrm{Q}}}\right)}{\mathrm{var}\left(\widehat{\widetilde{\mathrm{Q}}}\right)}\right] \geq \boldsymbol{\varepsilon}. \end{aligned}$$

At first defuzzify the expectation and variance as:

$$m_Q = E\left(\widehat{\widetilde{Q}}\right) = E\left(\widehat{\widetilde{a}}\right) - E\left(\widehat{\widetilde{b}}\right)$$

Or, $m_Q = \overline{\widetilde{Q}} = \overline{\widetilde{a}} - \overline{\widetilde{b}}$
Where, $\overline{Q}_1 = \overline{a}_1 - \overline{b}_1, \overline{Q}_2 = \overline{a}_2 - \overline{b}_2, \overline{Q}_3 = \overline{a}_3 - \overline{b}_3$.

$$\text{Or, } \mathrm{m_Q} = \frac{\overline{\mathrm{Q}}_1 + \overline{\mathrm{Q}}_2 + \overline{\mathrm{Q}}_3}{3}.$$

Similarly, $\sigma_Q^2 = var\left(\widehat{\widetilde{Q}}\right) = \widetilde{\sigma}_Q^2 = \left(\sigma_{Q_1}^2, \sigma_{Q_2}^2, \sigma_{Q_3}^2\right)$

$$Or, \sigma_Q^2 = \frac{\sigma_{Q_1}^2 + \sigma_{Q_2}^2 + \sigma_{Q_3}^2}{3}.$$

Then the constraint reduces to standard chance constraint as:

$$Prob\left[\widehat{T} \geq -K\right] \geq \varepsilon$$

$$\text{Or, } \mathrm{Prob}\left[\widehat{\mathrm{T}} \leq \mathrm{K}\right] \geq \varepsilon$$

Where, $\widehat{T} = \frac{\widehat{\widetilde{Q}} - m_Q}{\sigma_Q^2}$ are the standard normal variate and $K = \frac{m_Q}{\sigma_Q^2}$

$$m_Q \geq \lambda \sigma_Q^2$$

Where λ be the real number such that $Prob\left[\widehat{T} \geq \lambda\right] = \varepsilon$.

4. If ε are the probabilities of non-violation of the constraint $a \geq \widehat{b_1}$ then the constraint can be written as

$$\mathrm{Prob}\left[\mathrm{a} \geq \widehat{\mathrm{b_1}}\right] \geq \varepsilon.$$

$$\text{Or, } \mathrm{Prob}\left[\frac{\mathrm{a} - \mathrm{E}\left(\widehat{\mathrm{b_1}}\right)}{\mathrm{var}\left(\widehat{\mathrm{b_1}}\right)} \geq \frac{\widehat{\mathrm{b}} - \mathrm{E}\left(\widehat{\mathrm{b_1}}\right)}{\mathrm{var}\left(\widehat{\mathrm{b_1}}\right)}\right] \geq \varepsilon.$$
$$\text{Or, } \mathrm{Prob}\left[\mathrm{h} \geq \widehat{\mathrm{d}}\right] \geq \varepsilon.$$

Where, $\widehat{d} = \frac{\widehat{b_1} - m_{b_1}}{\sigma_{b_1}^2}$ are the standard normal variate and $h = \frac{a - m_{b_1}}{\sigma_{b_1}^2}$.

$$\text{Or, } \mathrm{a} \geq \mathrm{m_{b_1}} + \lambda \sigma_{\mathrm{b_1}}^2.$$

5. If ε are the probabilities of non-violation of the constraint $\widehat{a} \geq \widehat{b_1}$ then the constraint can be written as

$$Prob\left[\widehat{a} \geq \widehat{b_1}\right] \geq \varepsilon$$

$$\begin{aligned}
&\text{Or, } \mathrm{Prob}\left[\widehat{\mathrm{a}} - \widehat{\mathrm{b}_1} \geq 0\right] \geq \varepsilon \\
&\text{Or, } \mathrm{Prob}[\widehat{\mathrm{q}} \geq 0] \geq \varepsilon \\
&\text{Or, } \mathrm{Prob}\left[\frac{\widehat{\mathrm{q}} - \mathrm{E}\left(\widehat{\mathrm{q}}\right)}{\mathrm{var}\left(\widehat{\mathrm{q}}\right)} \geq \frac{-\mathrm{E}\left(\widehat{\mathrm{q}}\right)}{\mathrm{var}\left(\widehat{\mathrm{q}}\right)}\right] \geq \varepsilon. \\
&\text{Or, } \mathrm{Prob}\left[\widehat{\mathrm{T}} \geq -\mathrm{K}\right] \geq \varepsilon \\
&\text{Or, } \mathrm{Prob}\left[\widehat{\mathrm{T}} \leq \mathrm{K}\right] \geq \varepsilon
\end{aligned}$$

Where, $\widehat{T} = \frac{\widehat{q} - m_q}{\sigma_q^2}$ are the standard normal variate and $K = \frac{m_q}{\sigma_q^2}$

$$m_q \geq \lambda \sigma_q^2$$

Where λ be the real number such that $Prob\left[\widehat{T} \geq \lambda\right] = \varepsilon$.

Formulation of the solid transportation problem

Notation and assumptions

(i) M : number of origins/sources of the transportation problem.
(ii) N : number of destinations/demands of the transportation problem.
(iii) K : number of conveyances i.e. different modes of transporting units from sources to destinations.
(iv) E_k: amount of product which can be carried by the k-th conveyance.
(v) O_i^q : amount of homogeneous product available at the i-th origin.
(vi) D_j^q : demand at the j-th destination.
(vii) C_{ijk}^q : per unit transportation cost from i-th origin to j-th destination by k-th conveyance of q-th item.
(viii) x_{ijk}^q : the amount transported from i-th origin to j-th destination by k-th conveyance.
(ix) s_{ijk}^q : the safety factor when an item is transformed from i-th origin to j-th destination by k-th conveyance of q-th item. If q-th item is transported from source i to destination j by conveyance k, then the safety factor s_{ijk}^q is considered. This implies that if $x_{ijk}^q > 0$, then we consider the safety factor for this route as a part of the safety constraint. Thus for the convenience of modeling, the following notation is introduced:

$$y_{ijk}^q = \begin{cases} 1 & for \quad x_{ijk}^q > 0 \\ 0 & otherwise \end{cases}$$

Model formulation

According to the above assumptions and notations, in a solid transportation problem we formulate the interval valued Solid Transportation Problem to evaluate the Optimized solution (Minimized) of the Total Transportation Cost. Here goal of a decision maker is to optimize these objective functions under Interval availability, Interval requirement and Interval conveyance capacities. So mathematically problem can be expressed as:

Model 1: Formulation of MIIVSTP without safety factor

$$Minimize\ Z = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left[C_{ijkL}^{q}, C_{ijkU}^{q}\right] x_{ijk}^{q}$$

Subject to

$$\left.\begin{array}{l} \sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk}^{q} = O_{i}^{q}, i = 1, 2, \ldots\ldots, M \\ \sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk}^{q} = D_{i}^{q}, j = 1, 2, \ldots\ldots, N \\ \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ijk}^{q} = E_{k}, k = 1, 2, \ldots\ldots, K \\ x_{ijk}^{q} \geq 0 \text{ for all i, j, k, q} \end{array}\right\} \tag{7}$$

The problem is feasible if and only if $O \cap D \cap E \neq \phi$, where

$$\mathrm{O} = \sum_{q=1}^{Q} \sum_{i=1}^{M} \mathrm{O}_{i}^{q} = \left[\sum_{q=1}^{Q} \sum_{i=1}^{M} o_{i^{1}}^{q}, \sum_{q=1}^{Q} \sum_{i=1}^{M} o_{i^{2}}^{q}\right]$$
$$\mathrm{D} = \sum_{q=1}^{Q} \sum_{j=1}^{N} \mathrm{D}_{j}^{q} = \left[\sum_{q=1}^{Q} \sum_{i=1}^{N} d_{j^{1}}^{q}, \sum_{q=1}^{Q} \sum_{j=1}^{N} d_{j^{2}}^{q}\right], \mathrm{E} = \sum_{k=1}^{K} \mathrm{E}_{k} = \left[\sum_{k=1}^{K} e_{k^{1}}, \sum_{k=1}^{K} e_{k^{2}}\right].$$

Model 2: Formulation of MIIVSTP with safety factor

Model 2a: Formulation of MIIVSTP with safety factor as a crisp number

$$\left.\begin{array}{l} \text{Minimize } \mathrm{Z} = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left[\mathrm{C}_{ijkL}^{q}, \mathrm{C}_{ijkU}^{q}\right] x_{ijk}^{q} \\ \text{Subject to the constraints } (7b) - (7e) \end{array}\right\} \tag{8}$$

$$\mathrm{S} = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} s_{ijk}^{q} y_{ijk}^{q} > \mathrm{B} \tag{8a}$$

Where s_{ijk}^{q} are crisp numbers, B is the desired safety measure for the whole transportation system.

Model 2b: Formulation of MIIVSTP with safety factor as a crisp number and Desired Safety Measure (DSM) as a fuzzy-random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} S_{ijk}^{q} y_{ijk}^{q} \geq \widehat{\widetilde{B}}.$$

Model 2c: Formulation of MIIVSTP with safety factor as a fuzzy number and Desired Safety Measure (DSM) as a fuzzy-random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widetilde{S}_{ijk}^{q}y_{ijk}^{q}\geq\widehat{\widetilde{B}}$$

Model 2d: Formulation of MIIVSTP with safety factor as a random number and Desired Safety Measure (DSM) as a fuzzy-random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum\nolimits_{q=1}^{Q}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{N}\sum\nolimits_{k=1}^{K}\widehat{S}_{ijk}^{q}y_{ijk}^{q}\geq\widehat{\widetilde{B}}.$$

Model 2e: Formulation of MIIVSTP with safety factor as a interval number and Desired Safety Measure (DSM) as a fuzzy-random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum\nolimits_{q=1}^{Q}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{N}\sum\nolimits_{k=1}^{K}\left[S_{ijkL}^{q},S_{ijkR}^{q}\right]y_{ijk}^{q}\geq\widehat{\widetilde{B}}.$$

Model 2f: Formulation of MIIVSTP with safety factor as a fuzzy-random number and Desired Safety Measure (DSM) as a fuzzy-random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum\nolimits_{q=1}^{Q}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{N}\sum\nolimits_{k=1}^{K}\widehat{\widetilde{S}}_{ijk}^{q}y_{ijk}^{q}\geq\widehat{\widetilde{B}}.$$

Particular case

Model 3a: Formulation of MIIVSTP with safety factor as a crisp number and Desired Safety Measure (DSM) as a random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum\nolimits_{q=1}^{Q}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{N}\sum\nolimits_{k=1}^{K}S_{ijk}^{q}y_{ijk}^{q}\geq\widehat{B}.$$

Model 3b: Formulation of MIIVSTP with safety factor as a fuzzy number and Desired Safety Measure (DSM) as a random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S=\sum\nolimits_{q=1}^{Q}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{N}\sum\nolimits_{k=1}^{K}\widetilde{S}_{ijk}^{q}y_{ijk}^{q}\geq\widehat{B}.$$

Model 3c: Formulation of MIIVSTP with safety factor as a random number and Desired Safety Measure (DSM) as a random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \widehat{S}_{ijk}^{q} y_{ijk}^{q} \geq \widehat{B}.$$

Model 3d: Formulation of MIIVSTP with safety factor as an interval number And Desired Safety Measure (DSM) as a random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \left[S_{ijkL}^{q}, S_{ijkR}^{q}\right] y_{ijk}^{q} \geq \widehat{B}.$$

Model 3e: Formulation of MIIVSTP with safety factor as a fuzzy-random number and Desired Safety Measure (DSM) as a random number It is same as problem (7) and safety constraint (8a) is replaced by,

$$S = \sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \widehat{\widetilde{S}}_{ijk}^{q} y_{ijk}^{q} \geq \widehat{B}.$$

Global criteria method

The Multi – objective Non – Linear Programming (MONLP) problems may be solved by Global Criteria Method converting it to a single objective optimization problem. The solution procedure is as follows:

Step – 1: Solve the multi – objective programming problem (8) as a single objective problem using one objective at a time ignoring the others.

Step – 2: From the results of Step – 1, determine the ideal objective vector, say $\left(f_1^{min}, f_2^{min}, f_3^{min}, \ldots\ldots\ldots, f_k^{min}\right)$ and the corresponding values of $\left(f_1^{max}, f_2^{max}, f_3^{max}, \ldots\ldots\ldots, f_k^{max}\right)$. Here the ideal objective vector is used as a reference point. The problem is then to solve the following auxiliary problem:

Find $x = (x_1, x_2, , \ldots\ldots, x_n)^T$
Which minimizes GC
Subject to

$$\begin{aligned} & g_j(x) \leq 0, j = 1, 2, 3, \ldots\ldots, m \\ & x_i \geq 0, \qquad i = 1, 2, \ldots\ldots, n \end{aligned}$$

where $\mathrm{GC} = \mathrm{Minimize}\left\{\sum_{\mathrm{i}=1}^{\mathrm{k}} \left(\frac{\mathrm{f_i(x)} - \mathrm{f_i^{min}}}{\mathrm{f_i^{max}} - \mathrm{f_i^{min}}}\right)^{\mathrm{p}}\right\}^{\frac{1}{\mathrm{p}}}$,

Or, $\mathrm{GC} = \mathrm{Minimize}\left\{\sum_{\mathrm{i}=1}^{\mathrm{k}} \left(\frac{\mathrm{f_i(x)} - \mathrm{f_i^{min}}}{\mathrm{f_i^{min}}}\right)^{\mathrm{p}}\right\}^{\frac{1}{\mathrm{p}}}$,

Where $1 \leq p < \infty$. An usual value of p is 2. This method is also sometimes called Compromise Programming.

Step – 3: Now, solve the above single objective problem described in Step – 2 by GRG method to obtain the compromise solution.

Numerical example and discussion

Input data

A marketing company procures two types of items as rice and wheat from the three production sources and supply to two different destinations through the two different types of conveyances where availabilities, demands and conveyances capacity are interval in nature. i.e., we consider the following (3×2×2) MIIVSTP:

Supplies

$$O_1^1 = [10, 22], O_2^1 = [12, 24], O_3^1 = [20, 53], O_1^2 = [20, 31], O_2^2 = [12, 26], O_3^2 = [4, 17].$$

Demands

$$D_1^1 = [16, 25], D_2^1 = [19, 32], D_1^2 = [26, 36], D_2^2 = [17, 23].$$

Conveyances capacities

$$E_1 = [29, 44], E_2 = [19, 45].$$

Desired minimum total safety measure for the system (=B) = 5 (Tables 1, 2, 3 and 4)

Input for safety constraints with stochastic safety factors

Input for safety constraints with fuzzy-random safety factors

This problem is feasible since O∩D∩E = [78,173]∩[78,116]∩[48,89] = [78,89] $\neq \phi$ (is non-empty).

Model 1

This interval valued solid transportation problem without safety factors with the above data is reduced to

$$\text{Minimize ZZ} = \sum_{q=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{2}\left[C_{ijkL}^{q}, C_{ijkR}^{q}\right]x_{ijk}^{q}$$

Table 1 Interval transportation costs

Interval transportation costs									
		Item-1				Item-2			
j		1		2		1		2	
i	k	1	2	1	2	1	2	1	2
1		[42, 46]	[70, 82]	[72, 80]	[90, 99]	[75, 86]	[20, 28]	[92, 97]	[92, 98]
2		[84, 90]	[30, 32]	[60, 62]	[40, 48]	[81, 88]	[21, 24]	[71, 76]	[35, 45]
3		[10, 16]	[14, 20]	[40, 45]	[60, 70]	[14, 18]	[12, 16]	[32, 38]	[75, 82]
Crisp safety factors									
1		0.5	0.4	0.65	0.7	0.6	0.7	0.8	0.9
2		0.8	0.95	1	0.2	0.1	0.15	0.2	0.7
3		0.25	0.35	0.4	0.55	0.5	0.3	0.95	1
Fuzzy safety factors									
1		(.45,.5,.55)	(.35,.4,.45)	(.6,.65,.7)	(.65,.7,.75)	(.55,.6,.65)	(.65,.7,.75)	(.75,.8,.85)	(.85,.9,.95)
2		(.75,.8,.85)	(.9,.95,1)	(.9,.95,1)	(.15,.2,.25)	(.05,.1,.15)	(.1,.15,.2)	(.15,.2,.25)	(.65,.7,.75)
3		(.25,.3,.35)	(.3,.35,.4)	(.35,.4,.45)	(.5,.55,.6)	(.45,.5,.55)	(.25,.3,.35)	(.9,.95,1)	(.9,.95,1)
Interval safety factors									
1		[.45,.55]	[.35,.45]	[.6,.7]	[.65,.75]	[.55,.65]	[.65,.75]	[.75,.85]	[.85,.95]
2		[.75,.85]	[.9,. 1]	[.9,1]	[.15,.25]	[.05,.15]	[.1,.2]	[.15,.25]	[.65,.75]
3		[.25,.35]	[.3,.4]	[.35,.45]	[.5, .6]	[.45,.55]	[.25,.35]	[.9,1]	[.9,1]

Table 2 Input for safety constraints with stochastic safety factors

		Assumed values of $\bar{s}^{q}_{ijk}$								Assumed values of $\left(\sigma^2_{ijk}\right)^q$							
		Item-1				Item-2				Item-1				Item-2			
j		1		2		1		2		1		2		1		2	
i	k	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1		15	18	21	10	12	11	18	17	13	5	21	5	2	11	18	15
2		15	17	7	9	22	14	12	13	8	15	7	7	6	8	12	13
3		9	16	15	18	9	5	7	16	10	11	15	1	1	13	7	17

Subject to

$$\sum_{j=1}^{2}\sum_{k=1}^{2} x^1_{1jk} = [10,22], \sum_{j=1}^{2}\sum_{k=1}^{2} x^1_{2jk} = [12,24], \sum_{j=1}^{2}\sum_{k=1}^{2} x^1_{3jk} = [20,53],$$
$$\sum_{j=1}^{2}\sum_{k=1}^{2} x^2_{1jk} = [20,31], \sum_{j=1}^{2}\sum_{k=1}^{2} x^2_{2jk} = [12,26], \sum_{j=1}^{2}\sum_{k=1}^{2} x^2_{3jk} = [4,17]$$
$$\sum_{i=1}^{3}\sum_{k=1}^{2} x^1_{i1k} = [16,25], \sum_{i=1}^{3}\sum_{k=1}^{2} x^1_{i2k} = [19,32], \sum_{i=1}^{3}\sum_{k=1}^{2} x^2_{i1k} = [26,36]$$
$$\sum_{i=1}^{3}\sum_{k=1}^{2} x^2_{i1k} = [17,23], \sum_{q=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{2} x^q_{ij1} = [29,44], \sum_{q=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{2} x^q_{ij2} = [19,45]$$

$x^q_{ijk} \geq 0$ for all i, j, k, q.

In a Transportation problem, the feasibility constraints are always equality constraints. So if demand, supply and conveyances capacities are all interval numbers, an equality constraint with decision variables in the left side can be written as a deterministic set of constraints as follows:

Table 3 Input for safety constraints with fuzzy-random safety factors

		Item-1				Item-2			
j		1		2		1		2	
i	k	1	2	1	2	1	2	1	2
Assumed values of E $\left(\widehat{\tilde{s}}^{q}_{ijk}\right)$									
1		(14,15,16)	(17,18,19)	(20,21,22)	(9,10,11)	(11,12,13)	(10,11,12)	(17,18,19)	(16,17,18)
2		(14,15,16)	(16,17,18)	(6,7,8)	(8,9,10)	(21,22,23)	(13,14,15)	(11,12,13)	(12,13,14)
3		(8,9,10)	(15,16,17)	(14,15,16)	(17,18,19)	(8,9,10)	(4,5,6)	(6,7,8)	(15,16,17)
Assumed values ofVar $\left(\widehat{\tilde{s}}^{q}_{ijk}\right)$									
1		(12,13,14)	(4,5,6)	(20,21,22)	(4,5,6)	(1,2,3)	(10,11,12)	(17,18,19)	(14,15,16)
2		(7,8,9)	(14,15,16)	(6,7,8)	(6,7,8)	(5,6,7)	(7,8,9)	(11,12,13)	(12,13,14)
3		(9,10,11)	(10,11,12)	(14,15,16)	(1,2,3)	(1,2,3)	(12,13,14)	(6,7,8)	(16,17,18)

Table 4 Other assumed parametric values

Other assumed parametric values	
Model	Data
Model – 2a	$S \geq 5$
Model – 2b,2c,2d,2f, 2e	λ=1.5, m_B=(1, 2, 3), $\sigma_B^2 = (2,3,4)$
Model – 3a, 3b, 3c, 3e,3d	λ_1=2, m_B=1, $\sigma_B^2 = 3$

The composite Objective Function is in its final form,

$$\mathrm{Min\ ZZ} = \lambda\,\mathrm{Min(m(Z))} + (1-\lambda)\mathrm{Min(w(Z))}, 0 \leq \lambda \leq 1.$$

Now, with λ=0.5, we have

$$\begin{aligned}
&\mathrm{Min\ ZZ} = 0.5[\mathrm{Min(m(Z))} + \mathrm{Min(w(Z))}]\\
&= 23x_{111}^1 + 41x_{112}^1 + 40x_{121}^1 + 49.5x_{122}^1 + 45x_{211}^1 + 16x_{212}^1 + 31x_{221}^1 + 24x_{222}^1 + 8_{311}^1 + 10x_{312}^1\\
&+22.5x_{321}^1 + 35x_{322}^1 + 43x_{111}^2 + 14x_{112}^2 + 48.5x_{121}^2 + 49x_{122}^2 + 44x_{211}^2 + 12x_{212}^2 + 38x_{221}^2 + 22.5x_{222}^2\\
&+9x_{311}^2 + 8x_{312}^2 + 18.5x_{321}^2 + 41x_{322}^2.
\end{aligned}$$

Subject to

$$10 \leq x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 \leq 22 \tag{10a}$$

$$12 \leq x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 \leq 24 \tag{10b}$$

$$20 \leq x_{311}^1 + x_{312}^1 + x_{321}^1 + x_{322}^1 \leq 53 \tag{10c}$$

$$20 \leq x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 \leq 31 \tag{10d}$$

$$12 \leq x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 \leq 26 \tag{10e}$$

$$4 \leq x_{311}^2 + x_{312}^2 + x_{321}^2 + x_{322}^2 \leq 17 \tag{10f}$$

$$16 \leq x_{111}^1 + x_{112}^1 + x_{211}^1 + x_{212}^1 + x_{311}^1 + x_{312}^1 \leq 25 \tag{10g}$$

$$19 \leq x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 + x_{321}^1 + x_{322}^1 \leq 32 \tag{10h}$$

$$26 \leq x_{111}^2 + x_{112}^2 + x_{211}^2 + x_{212}^2 + x_{311}^2 + x_{312}^2 \leq 36 \tag{10i}$$

Table 5 Results of different models with different methods

Model	Method	Value of the decision variables	ZZ	m(Z)	w(Z)
1	Convex Combination	$x_{111}^1 = 10,\ x_{222}^1 = 12,\ x_{311}^1 = 13,\ x_{321}^1 = 7,\ x_{112}^2 = 20,\ x_{212}^2 = 5,\ x_{222}^2 = 7,$ $x_{312}^2 = 1,\ x_{321}^2 = 10.$	1470	2671	279
	Global Criteria	$x_{111}^1 = 10, x_{212}^1 = 12, x_{312}^1 = 1, x_{321}^1 = 19, x_{112}^2 = 14, x_{121}^2 = 4, x_{122}^2 = 2,$ $x_{212}^2 = 12, x_{321}^2 = 11,\ GC = 0.1640659.$	1695	3195.5	205.5
2a	Convex Combination	$x_{111}^1 = 9.84, x_{121}^1 = 0.16, x_{222}^1 = 12, x_{311}^1 = 13.16\ x_{321}^1 = 6.84, x_{112}^2 = 20,$ $x_{222}^2 = 12, x_{311}^2 = 7.23, x_{111}^1 = 5$	1487.51	2685.58	294.4
	Global Criteria	$x_{111}^1 = 10, x_{212}^1 = 12, x_{312}^1 = 1, x_{321}^1 = 19, x_{112}^2 = 14,$ $x_{121}^2 = 4,\ x_{122}^2 = 2, x_{212}^2 = 12, x_{321}^2 = 11, GC = 0.1617884$	1695	3195.5	205.5
2b	Convex Combination	$x_{111}^1 = 9.999, x_{121}^1 = 0.001, x_{221}^1 = 0.44, x_{222}^1 = 11.56$ $x_{311}^1 = 13, x_{321}^1 = 6.999, x_{112}^2 = 20, x_{122}^2 = 7.5, x_{212}^1 = 7.58$ $, x_{222}^2 = 4.4, x_{311}^2 = 0.21, x_{321}^2 = 12.58.$	1487.59	3095	274.7
	Global Criteria	$x_{111}^1 = 10, x_{212}^1 = 12, x_{321}^1 = 20, x_{112}^2 = 21, x_{212}^2 = 5,$ $x_{222}^2 = 7, x_{321}^2 = 14, GC = 0.2159576.$	1642.5	3048.5	250.5
2c	Convex Combination	$x_{122}^1 = 10, x_{212}^1 = 16, x_{321}^1 = 19.73, x_{322}^1 = 0.27,$ $x_{112}^2 = 14.36,\ x_{121}^2 = 12.63, x_{211}^2 = 11.63, x_{222}^2 = 0.36$ $x_{322}^2 = 4.$	2702.47	5146.38	257.2
	Global Criteria	$x_{111}^1 = 10, x_{212}^1 = 12, x_{312}^1 = 1, x_{321}^1 = 19, x_{112}^2 = 14,$ $x_{122}^2 = 4, x_{122}^2 = 2, x_{212}^2 = 12, x_{321}^2 = 11, GC = 0.1215162$	1695	3195.5	205.5
2d	Convex Combination	$x_{111}^1 = 9.87, x_{121}^1 = 0.13, x_{212}^1 = 11.56, x_{221}^1 = 0.54,$ $x_{322}^1 = 20, x_{111}^2 = 3.999, x_{112}^2 = 10.37, x_{121}^2 = 4.8,$ $x_{122}^2 = 0.8, x_{211}^2 = 11.2, x_{212}^2 = 0.197, x_{221}^2 = 0.62,$ $x_{311}^2 = 0.26, x_{321}^2 = 10.97.$	2447.69	4620.37	284.7
	Global Criteria	$x_{112}^1 = 10, x_{221}^1 = 14, x_{311}^1 = 15, x_{322}^1 = 5, x_{112}^2 = 16.5,$ $x_{121}^2 = 3.5, x_{222}^2 = 13.5, x_{311}^2 = 11.5, GC = 0.1695448.$	1947	3484.75	309.2
2e	Convex Combination	$x_{111}^1 = 10, x_{222}^1 = 12, x_{311}^1 = 13, x_{321}^1 = 7, x_{112}^2 = 20,$ $x_{212}^2 = 5.46, x_{222}^2 = 6.54, x_{312}^2 = 0.54, x_{321}^2 = 10.46.$	1470	2672.6	277.8

Table 5 (continued)

Model	Method	Value of the decision variables	ZZ	m(Z)	w(Z)
	Global Criteria	$x^1_{111} = 10, x^1_{212} = 12, x^1_{312} = 1, x^1_{321} = 19, x^2_{112} = 14,$ $x^2_{121} = 4, x^2_{122} = 2, x^2_{212} = 12, x^2_{321} = 11, GC = 0.1640659$	1695	3195.5	205.5
2f	Convex Combination	$x^1_{111} = 9.27, x^1_{121} = 1.09, x^1_{212} = 1.24, x^2_{221} = 1.32,$ $x^1_{222} = 9.44, x^1_{311} = 4.399, x^1_{322} = 15.6, x^2_{112} = 17.63,$ $x^2_{112} = 2.36.$	2286.9	4268.77	323.26
	Global Criteria	$x^1_{111} = 10, x^1_{211} = 6, x^1_{221} = 6, x^1_{322} = 20, x^2_{111} = 8.48,$ $x^2_{121} = 0.57, x^2_{122} = 10.95, x^2_{212} = 8.22, x^2_{221} = 3.78,$ $x^2_{311} = 8.77, x^2_{312} = 0.53, x^2_{322} = 1.7, GC = 0.4494937.$	2709.98	5147.87	268.63
3a	Convex Combination	$x^1_{111} = 6.53, x^1_{121} = 3.46, x^1_{122} = 0.014, x^1_{221} = 11.7,$ $x^1_{121} = 0.3, x^1_{311} = 13.7, x^1_{312} = 2.76, x^1_{321} = 3.53,$ $x^2_{112} = 21.4, x^2_{212} = 1.12, x^2_{222} = 16.49, x^2_{311} = 1.66,$ $x^2_{312} = 1.82, x^2_{321} = 0.51$	1598.69	2924.24	276.29
	Global Criteria	$x^1_{111} = 10, x^1_{212} = 0.75, x^1_{221} = 11.25, x^1_{311} = 11.89,$ $x^1_{312} = 0.36, x^1_{321} = 7.75, x^2_{112} = 17.1, x^2_{122} = 2.91,$ $x^2_{212} = 8.02, x^2_{222} = 10.98, x^2_{312} = 0.89, x^2_{321} = 3.1,$ $\mathrm{GC} = 0.03872063.$	1653.59	3067.02	243.23
3b	Convex Combination	$x^1_{111} = 10, x^1_{212} = 12, x^1_{311} = 1, x^1_{321} = 19, x^2_{112} = 20,$ $x^2_{212} = 3.1, x^2_{222} = 8.9, x^2_{311} = 2.9, x^2_{321} = 8.1$	1550.9	2868.15	241.75
	Global Criteria	$x^1_{111} = 10, x^1_{212} = 14.67, x^1_{321} = 19.998, x^2_{111} = 0.21,$ $x^2_{112} = 15.41, x^2_{121} = 1.6, x^2_{122} = 2.77, x^2_{211} = 0.43,$ $x^2_{212} = 9.96, x^2_{221} = 0.001, x^2_{222} = 1.6, x^2_{321} = 11,$ $\mathrm{GC} = 0.1921573.$	1730.75	2900.86	217.22
3c	Convex Combination	$x^1_{111} = 10, x^1_{212} = 0.42, x^1_{222} = 11.58, x^1_{311} = 12.58,$ $x^1_{321} = 7.4, x^2_{112} = 20, x^2_{222} = 12.47, x^2_{311} = 8.69,$ $x^2_{112} = 4.5$	1504.8	2715.92	296.21
	Global Criteria	$x^1_{111} = 10, x^1_{222} = 12, x^1_{312} = 13, x^1_{321} = 7, x^2_{111} = 20,$ $x^2_{222} = 16, x^2_{311} = 6, x^2_{321} = 1, GC = 0.6057821.$	1904.5	3867.5	329.5

Table 5 (continued)

Model	Method	Value of the decision variables	ZZ	m(Z)	w(Z)
3d	Convex Combination	$x_{111}^{1}=10, x_{222}^{1}=12, x_{311}^{1}=13, x_{321}^{1}=7, x_{112}^{2}=20,$ $x_{212}^{2}=5.46, x_{222}^{2}=6.54, x_{312}^{2}=0.54, x_{321}^{2}=10.46.$	1470	2672.6	277.85
	Global Criteria	$x_{111}^{1}=10, x_{212}^{1}=12, x_{312}^{1}=1, x_{321}^{1}=19, x_{112}^{2}=14,$ $x_{121}^{2}=4, x_{122}^{2}=2, x_{212}^{2}=12, x_{321}^{2}=11,$ GC $=0.1640659.$	1695	3195.5	205.5
3e	Convex Combination	$x_{112}^{1}=10, x_{221}^{1}=12, x_{312}^{1}=6, x_{322}^{1}=14, x_{111}^{2}=13.68,$ $x_{112}^{2}=4.04, x_{121}^{2}=2.28, x_{211}^{2}=1.04, x_{311}^{2}=0.28,$ $x_{321}^{2}=14.72.$	2539.52	3753.62	305.46
	Global Criteria	$x_{111}^{1}=2, x_{112}^{1}=8, x_{211}^{1}=6, x_{221}^{1}=6, x_{322}^{1}=20,$ $x_{111}^{2}=7, x_{122}^{2}=13, x_{211}^{2}=19, x_{321}^{2}=4, GC=0.9268636$	3378	6428	300

$$17 \leq x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 + x_{321}^2 + x_{322}^2 \leq 23 \tag{10j}$$

$$29 \leq x_{111}^1 + x_{121}^1 + x_{211}^1 + x_{221}^1 + x_{311}^1 + x_{321}^1 + x_{111}^2 + x_{121}^2 + x_{211}^2 + x_{221}^2 + x_{311}^2 + x_{321}^2 \leq 44 \tag{10k}$$

$$19 \leq x_{112}^1 + x_{122}^1 + x_{212}^1 + x_{222}^1 + x_{312}^1 + x_{322}^1 + x_{112}^2 + x_{122}^2 + x_{212}^2 + x_{222}^2 + x_{312}^2 + x_{322} \leq 45 \tag{10l}$$

$$x_{ijk}^q \geq 0 \text{ for all i, j, k, q.} \tag{10m}$$

Model 2a, 2b, 2c, 2d, 2e, 2f, 3a, 3b, 3c, 3d, 3e

Formulations of these models are same as (10a)–(10m) along with additional constraint(s) due to safety constraint which differs for different models.

For Model – 2a Crisp Safety constraint:

$$S = \sum_{q=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{2} s_{ijk}^q y_{ijk}^q$$

is reduced to

$$\begin{aligned}&\mathrm{S} = 0.5\mathrm{y}_{111}^1 + 0.4\mathrm{y}_{112}^1 + 0.65\mathrm{y}_{121}^1 + 0.7\mathrm{y}_{122}^1 + 0.8\mathrm{y}_{211}^1 + 0.95\mathrm{y}_{212}^1 + 1\mathrm{y}_{221}^1 + 0.2\mathrm{y}_{222}^1 + 0.25\mathrm{y}_{311}^1\\&+0.35y_{312}^1 + 0.4y_{321}^1 + 0.55y_{322}^1 + 0.6y_{111}^2 + 0.7y_{112}^2 + 0.8y_{121}^2 + 0.9y_{122}^2 + 0.1y_{211}^2 + 0.15y_{212}^2\\&+0.2y_{221}^2 + 0.7y_{222}^2 + 0.5y_{311}^2 + 0.3y_{312}^2 + 0.95y_{321}^2 + 1y_{322}^2 \geq 5.\end{aligned}$$

For Model – 2b

$$\sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} S_{ijk}^q y_{ijk}^q \geq \widehat{\widehat{B}}$$

$$\begin{aligned}&\Rightarrow \mathrm{prob}\left[\textstyle\sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \mathrm{S}_{ijk}^q \mathrm{y}_{ijk}^q \geq \widehat{\widehat{\mathrm{B}}}\right] \geq \varepsilon\\&\Rightarrow 0.5y_{111}^1 + 0.4y_{112}^1 + 0.65y_{121}^1 + 0.7y_{122}^1 + 0.8y_{211}^1 + 0.95y_{212}^1 + 1y_{221}^1 + 0.2y_{222}^1 + 0.25y_{311}^1\\&+0.35y_{312}^1 + 0.4y_{321}^1 + 0.55y_{322}^1 + 0.6y_{111}^2 + 0.7y_{112}^2 + 0.8y_{121}^2 + 0.9y_{122}^2 + 0.1y_{211}^2 + 0.15y_{212}^2\\&+0.2y_{221}^2 + 0.7y_{222}^2 + 0.5y_{311}^2 + 0.3y_{312}^2 + 0.95y_{321}^2 + 1y_{322}^2 \geq 21.\end{aligned}$$

For Model – 2c

$$\begin{aligned}&\textstyle\sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \widetilde{S}_{ijk}^q y_{ijk}^q \geq \widehat{\widehat{B}}.\\&\Rightarrow \mathrm{Prob}\left[\textstyle\sum_{q=1}^{Q} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \widetilde{\mathrm{S}}_{ijk}^q \mathrm{y}_{ijk}^q \geq \widehat{\widehat{\mathrm{B}}}\right] \geq \varepsilon\\&\Rightarrow 0.5y_{111}^1 + 0.4y_{112}^1 + 0.65y_{121}^1 + 0.7y_{122}^1 + 0.8y_{211}^1 + 0.95y_{212}^1 + 0.95y_{221}^1 + 0.2y_{222}^1 + 0.25y_{311}^1\\&+0.35y_{312}^1 + 0.4y_{321}^1 + 0.55y_{322}^1 + 0.6y_{111}^2 + 0.7y_{112}^2 + 0.8y_{121}^2 + 0.9y_{122}^2 + 0.1y_{211}^2 + 0.15y_{212}^2\\&+0.2y_{221}^2 + 0.7y_{222}^2 + 0.5y_{311}^2 + 0.3y_{312}^2 + 0.95y_{321}^2 + 0.95y_{322}^2 \geq 21\end{aligned}$$

For Model – 2d

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{S}_{ijk}^{q}y_{ijk}^{q} \geq \widehat{\widehat{B}}.$$

$$\Rightarrow \text{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{S}_{ijk}^{q}y_{ijk}^{q} \geq \widehat{\widehat{B}}\right] \geq \varepsilon.$$

$$\Rightarrow (15y_{111}^{1} + 18y_{112}^{1} + 15y_{211}^{1} + 17y_{212}^{1} + 9y_{311}^{1} + 16y_{312}^{1} + 12y_{111}^{2} + 11y_{112}^{2} + 22y_{211}^{2}$$
$$+ 14y_{212}^{2} + 9y_{311}^{2} + 5y_{312}^{2} + 21y_{121}^{1} + 10y_{122}^{1} + 7y_{221}^{1} + 9y_{222}^{1} + 15y_{321}^{1} + 18y_{322}^{1}$$
$$+ 18y_{121}^{2} + 17y_{122}^{2} + 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 16y_{322}^{2} - 9) \geq 1.5(13y_{111}^{1}+5y_{112}^{1}$$
$$+ 8y_{211}^{1} + 15y_{212}^{1} + 10y_{311}^{1} + 11y_{312}^{1} + 2y_{111}^{2} + 11y_{112}^{2} + 6y_{211}^{2} + 8y_{212}^{2} + 1y_{321}^{2}$$
$$+ 13y_{312}^{2} + 21y_{121}^{1} + 5y_{122}^{1} + 7y_{221}^{1} + 7y_{222}^{1} + 15y_{321}^{1} + 1y_{322}^{1} + 18y_{121}^{2} + 15y_{122}^{2}$$
$$+ 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 17y_{322}^{2} - 8$$

For Model – 2e

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left[S_{ijkL}^{q}, S_{ijkR}^{q}\right]y_{ijk}^{q} \geq \widehat{\widehat{B}}.$$

$$\Rightarrow 0.55y_{111}^{1} + 0.45y_{112}^{1} + 0.7y_{121}^{1} + 0.75y_{122}^{1} + 0.85y_{211}^{1} + 1y_{212}^{1} + 1y_{221}^{1} + 0.25y_{222}^{1} + 0.3y_{311}^{1}$$
$$+0.4y_{312}^{1} + 0.45y_{321}^{1} + 0.6y_{322}^{1} + 0.65y_{111}^{2} + 0.75y_{112}^{2} + 0.85y_{121}^{2} + 0.95y_{122}^{2} + 0.15y_{211}^{2} + 0.2y_{212}^{2}$$
$$+0.25y_{221}^{2} + 0.75y_{222}^{2} + 0.55y_{311}^{2} + 0.35y_{312}^{2} + 1y_{321}^{2} + 1y_{322}^{2} \geq 19$$

For Model – 2f

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{\widehat{S}}_{ijk}^{q}y_{ijk}^{q} \geq \widehat{\widehat{B}}.$$

$$\Rightarrow \text{prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{\widehat{S}}_{ijk}^{q}y_{ijk}^{q} \geq \widehat{\widehat{B}}\right] \geq \varepsilon.$$

$$\Rightarrow (15y_{111}^{1} + 18y_{112}^{1} + 15y_{211}^{1} + 17y_{212}^{1} + 9y_{311}^{1} + 16y_{312}^{1} + 12y_{111}^{2} + 11y_{112}^{2} + 22y_{211}^{2}$$
$$+ 14y_{212}^{2} + 9y_{311}^{2} + 5y_{312}^{2} + 21y_{121}^{1} + 10y_{122}^{1} + 7y_{221}^{1} + 9y_{222}^{1} + 15y_{321}^{1} + 18y_{322}^{1}$$
$$+ 18y_{121}^{2} + 17y_{122}^{2} + 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 16y_{322}^{2} - 9) \geq 1.5(13y_{111}^{1}+5y_{112}^{1}$$
$$+ 8y_{211}^{1} + 15y_{212}^{1} + 10y_{311}^{1} + 11y_{312}^{1} + 2y_{111}^{2} + 11y_{112}^{2} + 6y_{211}^{2} + 8y_{212}^{2} + 2y_{311}^{2}$$
$$+ 13y_{312}^{2} + 21y_{121}^{1} + 5y_{122}^{1} + 7y_{221}^{1} + 7y_{222}^{1} + 15y_{321}^{1} + 2y_{322}^{1} + 18y_{121}^{2} + 15y_{122}^{2}$$
$$+ 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 17y_{322}^{2} - 8)$$

For Model – 3a

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} S_{ijk}^{q} y_{ijk}^{q} \geq \widehat{B}.$$

$\Rightarrow \mathrm{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \mathrm{S}_{ijk}^{q}\mathrm{y}_{ijk}^{q} \geq \widehat{\mathrm{B}}\right] \geq \boldsymbol{\varepsilon}.$
$\Rightarrow 0.5y_{111}^{1} + 0.4y_{112}^{1} + 0.65y_{121}^{1} + 0.7y_{122}^{1} + 0.8y_{211}^{1} + 0.95y_{212}^{1} + 1y_{221}^{1} + 0.2y_{222}^{1} + 0.25y_{311}^{1}$
$+0.35y_{312}^{1} + 0.4y_{321}^{1} + 0.55y_{322}^{1} + 0.6y_{111}^{2} + 0.7y_{112}^{2} + 0.8y_{121}^{2} + 0.9y_{122}^{2} + 0.1y_{211}^{2} + 0.15y_{212}^{2}$
$+0.2y_{221}^{2} + 0.7y_{222}^{2} + 0.5y_{311}^{2} + 0.3y_{312}^{2} + 0.95y_{321}^{2} + 1y_{322}^{2} \geq 17.$

For Model – 3b

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widetilde{S}_{ijk}^{q} y_{ijk}^{q} \geq \widehat{B}.$$

$$\Rightarrow \mathrm{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widetilde{\mathrm{S}}_{ijk}^{q}\mathrm{y}_{ijk}^{q} \geq \widehat{\mathrm{B}}.\right] \geq \boldsymbol{\varepsilon}. \Rightarrow 0.5y_{111}^{1} + 0.4y_{112}^{1} + 0.65y_{121}^{1}$$
$$+ 0.7y_{122}^{1} + 0.8y_{211}^{1} + 0.95y_{212}^{1} + 0.95y_{221}^{1} + 0.2y_{222}^{1} + 0.25y_{311}^{1} + 0.35y_{312}^{1} + 0.4y_{321}^{1}$$
$$+ 0.55y_{322}^{1} + 0.6y_{111}^{2} + 0.7y_{112}^{2} + 0.8y_{121}^{2} + 0.9y_{122}^{2} + 0.1y_{211}^{2} + 0.15y_{212}^{2} + 0.2y_{221}^{2}$$
$$+ 0.7y_{222}^{2} + 0.5y_{311}^{2} + 0.3y_{312}^{2} + 0.95y_{321}^{2} + 0.95y_{322}^{2} \geq 17.$$

For Model – 3c

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widehat{S}_{ijk}^{q} y_{ijk}^{q} \geq \widehat{B}.$$

$$\Rightarrow \mathrm{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \widehat{\mathrm{S}}_{ijk}^{q}\mathrm{y}_{ijk}^{q} \geq \widehat{\mathrm{B}}.\right] \geq \boldsymbol{\varepsilon}$$

$$\Rightarrow 15y_{111}^{1} + 18y_{112}^{1} + 15y_{211}^{1} + 17y_{212}^{1} + 9y_{311}^{1} + 16y_{312}^{1} + 12y_{111}^{2} + 11y_{112}^{2} + 22y_{211}^{2}$$
$$+ 14y_{212}^{2} + 9y_{311}^{2} + 5y_{312}^{2} + 21y_{121}^{1} + 10y_{122}^{1} + 7y_{221}^{1} + 9y_{222}^{1} + 15y_{321}^{1} + 18y_{322}^{1}$$
$$+ 18y_{121}^{2} + 17y_{122}^{2} + 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 16y_{322}^{2} - 7) \geq 2(13y_{111}^{1} + 5y_{112}^{1}$$
$$+ 8y_{211}^{1} + 15y_{212}^{1} + 10y_{311}^{1} + 11y_{312}^{1} + 2y_{111}^{2} + 11y_{112}^{2} + 6y_{211}^{2} + 8y_{212}^{2} + 1y_{311}^{2}$$
$$+ 13y_{312}^{2} + 21y_{121}^{1} + 5y_{122}^{1} + 7y_{221}^{1} + 7y_{222}^{1} + 15y_{321}^{1} + 1y_{322}^{1} + 18y_{121}^{2} + 15y_{122}^{2}$$
$$+ 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 17y_{322}^{2} - 5)$$

For Model – 3d

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left[S_{ijkL}^{q}, S_{ijkR}^{q}\right]y_{ijk}^{q} \geq \widehat{B}.$$

$$\Rightarrow \mathrm{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left[\mathrm{S}_{\mathrm{ijkL}}^{\mathrm{q}}, \mathrm{S}_{\mathrm{ijkR}}^{\mathrm{q}}\right]\mathrm{y}_{\mathrm{ijk}}^{\mathrm{q}} \geq \widehat{\mathrm{B}}\right] \geq \varepsilon.$$
$$\Rightarrow 0.55y_{111}^{1} + 0.45y_{112}^{1} + 0.7y_{121}^{1} + 0.75y_{122}^{1} + 0.85y_{211}^{1} + 1y_{212}^{1} + 1y_{221}^{1} + 0.25y_{222}^{1} + 0.3y_{311}^{1}$$
$$+0.4y_{312}^{1} + 0.45y_{321}^{1} + 0.6y_{322}^{1} + 0.65y_{111}^{2} + 0.75y_{112}^{2} + 0.85y_{121}^{2} + 0.95y_{122}^{2} + 0.15y_{211}^{2} + 0.2y_{212}^{2}$$
$$+0.25y_{221}^{2} + 0.75y_{222}^{2} + 0.55y_{311}^{2} + 0.35y_{312}^{2} + 1y_{321}^{2} + 1y_{322}^{2} \geq 10.$$

For Model – 3e

$$\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{\widehat{S}}_{ijk}^{q}y_{ijk}^{q} \geq \widehat{B}.$$

$$\Rightarrow \mathrm{Prob}\left[\sum_{q=1}^{Q}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\widehat{\widehat{\mathrm{S}}}_{\mathrm{ijk}}^{\mathrm{q}}\mathrm{y}_{\mathrm{ijk}}^{\mathrm{q}} \geq \widehat{\mathrm{B}}\right] \geq \varepsilon.$$

$$\begin{aligned}\Rightarrow\ & 15y_{111}^{1} + 18y_{112}^{1} + 15y_{211}^{1} + 17y_{212}^{1} + 9y_{311}^{1} + 16y_{312}^{1} + 12y_{111}^{2} + 11y_{112}^{2} + 22y_{211}^{2}\\ & + 14y_{212}^{2} + 9y_{311}^{2} + 5y_{312}^{2} + 21y_{121}^{1} + 10y_{122}^{1} + 7y_{221}^{1} + 9y_{222}^{1} + 15y_{321}^{1} + 18y_{322}^{1}\\ & + 18y_{121}^{2} + 17y_{122}^{2} + 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 16y_{322}^{2} - 7) \geq 2\big(13y_{111}^{1}+5y_{112}^{1}\\ & + 8y_{211}^{1} + 15y_{212}^{1} + 10y_{311}^{1} + 11y_{312}^{1} + 2y_{111}^{2} + 11y_{112}^{2} + 6y_{211}^{2} + 8y_{212}^{2} + 2y_{311}^{2}\\ & + 13y_{312}^{2} + 21y_{121}^{1} + 5y_{122}^{1} + 7y_{221}^{1} + 7y_{222}^{1} + 15y_{321}^{1} + 2y_{322}^{1} + 18y_{121}^{2} + 15y_{122}^{2}\\ & + 12y_{221}^{2} + 13y_{222}^{2} + 7y_{321}^{2} + 17y_{322}^{2} - 5\big)\end{aligned}$$

Results

Solution by GRG The above constrained optimization problems are executed using LINGO 12.0 and the results of Models – 1, 2a, 2b, 2c, 2d, 2e, 2f,3a, 3b,3c, 3d, 3e are as follows: (Table 5)

Optimal result by global criteria method The above problem is solved by the LINGO 12.0 package for obtaining the optimal compromise solution of the problem. We get $GC = 0.4558780$ and optimal compromise solution as, $x_{111}^{1} = 10,\ x_{212}^{1} = 12,\ x_{311}^{1} = 1,\ x_{321}^{1} = 19,\ x_{112}^{2} = 17.6,\ x_{121}^{2} = 2.37,\ x_{212}^{2} = 8.37,\ x_{222}^{2} = 3.63,\ x_{321}^{2} = 11$ and rest all are zero.

The optimal value of each objective functions i.e., m(z) and w(z) are 2997.39 and 222.53 respectively.

Discussion

The result of the following models is to be expected. The optimal value of model-1 is minimum than any other model because we solve model-1 without adding the safety constraint but in the remaining models i.e., model 2a, 2b,2c, 2d, 2e, 2f, 3a, 3b, 3c, 3d, 3e we introduce the Safety constraints. In this Multi item interval valued solid transportation problem we consider desire safety measure as a fuzzy-stochastic variable and in this numerical illustration we consider desire safety measure is 5. In other models decision maker wants to have more total safety measure for the system i.e., 5. For this reason the transportation routes are rearranged so that total safety measure is greater or equal to 5. Thus, as a result, total mean cost is increased. These additional costs are incurred against the increased total safety measure.

Conclusion

The main goal is to represent the solution procedure of the Multi Item Interval Valued Solid Transportation Problem (MIIVSTP) with safety factor under Desire Safety Measure (DSM) fuzzy-stochastic and stochastic.To prepare this we proposed five models as Multi-itemSolid Transportation Problem. In this paper we solve all mathematical problems by using LINGO 12.0 Software. In our approach, we have presented two types of constraints one deterministic, another uncertain both fuzzy and stochastic senses. Since transportation problem play an important role in our daily life so our technique is highly fruitful. Practical numerical examples are provided to demonstrate the feasibility of all decision variables of the proposed methods.

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