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Are standard asset pricing factors long-range dependent?

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Abstract Factor portfolios derived from phenomena identified in the cross-section of stock returns have become vital parts of modern investment products and financial models. Even though much has been learned about the properties of these portfolios in recent years, one issue still remains unaddressed. Are factor returns long-range dependent (LRD)? We seek to answer this important research question because if factor returns were LRD, optimal portfolio decisions and traditional asset pricing methods/tests based on these factors would be severely biased and the validity of a large strand of prior research would be compromised. Specifically, using Hurst exponent approaches within rescaled range and detrended fluctuation frameworks, we analyse the presence of LRD in the returns of factor portfolios formed based on size, book-to-market, momentum and beta characteristics. For the periods from 1931 to 2014 (US market) and 1990 to 2014 (20 international markets) and supported by several robustness checks, we find no systematic evidence of persistence or anti-persistence in the factor returns. This implies that the factor use can be considered unproblematic in both asset management and asset pricing.

Keywords Hurst exponent · Rescaled range analysis · Detrended fluctuation analysis · Size effect · Book-to-market effect · Momentum effect · Beta effect

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1 Introduction

In recent decades, the identification and deeper analysis of exploitable cross-sectional stock market effects has received considerable attention in both practice and academia (see Chordia et al. 2014). Among the vast number of revealed phenomena, the size, book-to-market, momentum and beta effects can be considered the most important because related arbitrage portfolios have become important components of modern investment products and/or benchmark variables in standard asset pricing models. According to Fama and French (1992), the size (book-to-market) effect implies that returns are negatively (positively) related to firm size (the book-to-market ratio). Jegadeesh and Titman (1993) show that stocks exhibit momentum behaviour such that buying past winners and selling past losers can lead to substantially high returns. Finally, Frazzini and Pedersen (2014) document that high-beta stocks earn significantly lower returns than low-beta stocks.

The properties of the arbitrage portfolio returns exploiting these effects (hereforth, factor returns) have been the subject of numerous empirical studies.[1] For example, they have been shown to be predictors of economic growth (see Liew and Vassalou 2000), to proxy for variables that describe investment opportunities (see Petkova 2006) and to have strong co-movement across asset classes (see Asness et al. 2013b). However, one important question has not yet been answered: Do factor returns show signs of long-range dependence (LRD)? This issue is especially salient because the presence of LRD in factor returns would have significant impact on many applications in modern financial economics. First, as size, book-to-market, momentum and beta portfolios are typical components of financial products (see, for example, AQR Capital Management, www.aqr.com), optimal consumption/savings and portfolio decisions involving these products would become extremely sensitive to the investment horizon if the factor returns were LRD (see Lo 1991). Second, problems would arise in the pricing of derivative securities (where the arbitrage portfolios are the underlyings) with martingale methods since the class of continuous time stochastic processes most commonly employed is inconsistent with long-term memory (see Maheswaran and Sims 1993; Ohanissian et al. 2004). Finally, traditional tests of capital asset pricing models and the arbitrage pricing theory, in which factor return factors have become standard explanatory variables, are no longer valid since the usual forms of statistical inference do not apply to time-series exhibiting such persistence (see Lo 1991).[2]

In economics and finance, LRD has a long history (see Mandelbrot 1997). It is a specific departure from random walk behaviour because LRD time-series exhibit an unusually high degree of persistence so that observations in the remote past are

[1] We use the term ‘factor returns’ because we do not wish to take sides in the debate about whether they are anomalous returns or represent premia compensating for certain types of risk.

[2] Also, note that the conclusions of some tests of the efficient market hypothesis or stock market rationality also hang precariously on the presence or absence of long-term memory (see Merton 1987).

nontrivially correlated with observations in the distant future, even as the time span between the two observations increases. Thus, the defining characteristic of LRD has been taken by many to be a slow (hyperbolic) decay of the autocorrelation function (see Grau-Carles 2000).[3] To detect LRD, various estimators have been proposed in the literature (see Baillie 1996; Kantelhardt 2009; Fernandez 2011). In this article, we focus on the 'fractal class' of estimators. Specifically, we use rescaled range analysis (RRA; also often called Hurst R/S analysis) and the detrended fluctuation analysis (DFA) to gain insights into the dynamics of factor returns. These two methods (and their modifications) are the most popular ones in the field and have been extensively applied in recent studies of the return properties of equities (see Cajueiro and Tabak 2004a,b, 2005a,b; Kristoufek and Vosvrda 2013; Hull and McGroarty 2014; Sensoy and Tabak 2015), exchange rates (see Ausloos 2000; Ivanova and Ausloos 2002; Norouzzadeh and Rahmani 2006), commodities (see Tabak and Cajueiro 2007; Alvarez-Ramirez et al. 2008; Wang and Liu 2010; Batten et al. 2013) investment funds (see Crato and Ray 2003; Souza et al. 2004; Wang et al. 2005) and futures (see Crato and Ray 2000; Souza et al. 2008; Wang et al. 2011).[4] They both provide estimates of the Hurst exponent, a simple metric to judge the degree of LRD.

The contributions of our study can be summarised as follows. First, to analyse LRD in factor returns, we construct arbitrage portfolios that seek to exploit the size, book-to-market, momentum and beta effects in the US stock market (1931 - 2014) and the stock markets of 20 other developed countries (1990 - 2014).[5] This allows an illustrative look at whether there are differences in the magnitude of the factor returns across countries. Second, in addition to producing this by-product, we focus on the research question of whether we can find evidence of LRD within these returns. In the US, our rich sample allows investigation of the dynamics of LRD by using a local Hurst exponent approach for RRA and DFA considering the time-dependence of the Hurst exponent which has been identified in several recent studies (see Carbone et al. 2004; Batten et al. 2008). Third, we apply a more general filter technique for the RRA than used in the previous literature. It is well-known that short-range dependence which is substantial in stock returns (see Lo and MacKinlay 1988) and may be associated with lingering liquidity effects in financial markets can distort the RRA (see Lo 1991). Thus, recent studies apply ARMA filters to eliminate short-range dependences from the data (see Szilagyi and Batten 2007; Batten and Hamada 2009; Batten et al. 2013). However, as the RRA is also sensitive to heteroscedasticity (see Lo 1991) which is another typical feature of stock returns (see Schwert and Seguin 1990), we extend this approach to an ARMA-GARCH framework. Finally,

[3]A typical example of LRD is given by Granger-type fractionally differenced (FD) time-series models (see Campbell et al. 1997). Consider an AR(1) series with slope $\phi = 0.5$ and a FD series with differencing parameter $\kappa = 1/3$. Although both series have first-order autocorrelation of 0.500, at lag 5 (10, 25) the AR(1) correlation is 0.031 (0.001, 0.000) whereas the FD series has correlation 0.295 (0.235, 0.173), declining to only 0.109 at lag 100.

[4]Recent applications show that they are useful as standards that can distinguish emerging capital markets from mature capital markets (see Eom et al. 2008; Auer, 2016b).

[5]There are some online sources offering international factor returns (e.g., the data library of Kenneth French). However, they often do not provide the beta factor and also do not cover as many countries as we do.

we support our results by a variety of robustness checks. Specifically, we examine their sensitivity with respect to alternative parameterisations and specifications of our methodology, additional approaches to estimate the Hurst exponent and the use of factor returns collected from publicly available databases of well-known researchers.

The remainder of the article is organised as follows. Section 2 provides a brief description of our dataset, the portfolio construction methods and our way of estimating the Hurst exponent based on the ARMA-GARCH-adjusted RRA and the DFA. Section 3 presents a compact descriptive analysis of size, book-to-market, momentum and beta portfolio returns. Section 4 contains our results on LRD in factor returns and the outcomes of several robustness checks. Section 5 summarises and concludes.

2 Data and methodology

2.1 Data sources and portfolio construction

In our study, we utilise a dataset of four kinds of factor returns; the returns of size, book-to-market, momentum and beta portfolios which are constructed based on literature standards. Pricing and accounting data for portfolio formation are from the union of the CRSP and the Compustat/XpressFeed Global databases. US data include all available common stocks in the merged CRSP/XpressFeed data. International data include all common stocks on the Compustat/XpressFeed Global database for 20 developed markets.[6] All returns are in US dollars and do not include any currency hedging (see Fama and French 1998; Griffin 2002). Excess returns are above the US Treasury bill rate (see Fama and French 2012).

The portfolio construction is in line with Fama and French (1992,1993,1996), Asness et al. (2013a) and Frazzini and Pedersen (2014). The variables needed for the portfolio sorts are defined as follows. (i) Size is the total market value of equity. (ii) The book-to-market ratio is the ratio of book equity to the total market value of equity.[7] To ensure that variables are known before they are used, the standard convention is to align accounting variables at the end of a firm's fiscal year ending anywhere in calendar year $t-1$ to June of calendar year t (see Asness et al. 2013a). Furthermore, to compute book-to-market ratios, book equity is scaled by the total value of equity at fiscal year end.[8] (iii) Momentum returns are the returns over

[6]Individual issues are assigned to markets based on the location of the primary exchange. For companies traded in multiple markets, the primary trading vehicle identified by Compustat/XpressFeed is used.

[7]Book equity is obtained as shareholders' equity minus the preferred stock value (PSTKRV, PSTKL or PSKT depending on availability). Shareholders' equity is measured by stockholders' equity (SEQ) or, if not available, the sum of common equity (CEQ) and preferred stocks (PSTK). If both SEQ and CEQ are unavailable, shareholders' equity is proxied by total assets (TA) minus the sum of total liability (LT) and minority interest (MIB).

[8]For firms with fiscal year ending in December this approach of Asness et al. (2013a) delivers the same measure as in Fama and French (1992). For firms with fiscal year not ending in December, prices at the fiscal year end date are used while Fama and French (1992) use December prices for all firms.

the prior 12 months, skipping the most recent month (see Fama and French 1996).[9] (iv) Betas are obtained from rolling regressions of daily excess returns on daily market excess returns. Specifically, the time-series (TS) estimate of beta for stock i is given by $\hat{\beta}_i^{TS} = \hat{\rho}_{im} \cdot \hat{\sigma}_i / \hat{\sigma}_m$, where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are the estimated volatilities for the stock and the market, $\hat{\rho}_{im}$ is their correlation and the market return is equal to the value-weighted returns of all stocks in a given country. A one-year (five-year) horizon of daily (overlapping three-day) log-returns is used to estimate volatilities (correlations) under the condition that at least 120 (750) trading days of non-missing data are available. This accounts for the fact that correlations move more slowly than volatilities (see De Santis and Gérard 1997) and controls for non-synchronous trading (see Frazzini and Pedersen 2014). Finally, the time-series estimate of beta is shrinked towards the cross-sectional (CS) mean. In other words, the final beta estimate is $\hat{\beta}_i = w\hat{\beta}_i^{TS} + (1-w)\hat{\beta}^{CS}$ with $w = 0.6$ and $\hat{\beta}^{CS} = 1$. This takes into account the tendency of betas to revert towards one (see Tofallis 2008) and serves as an outlier reduction mechanism (see Frazzini and Pedersen 2014).

The size and book-to-market portfolios are constructed using six value-weighted portfolios formed on size and book-to-market ratios. At the end of each calendar month, stocks are assigned to two size-sorted portfolios (small and big) based on their market capitalisation.[10] Then, each portfolio is divided into sub-portfolios of stocks with high, medium and low book-to-market ratios (value, neutral and growth stocks).[11] Based on these six sub-portfolios, the size portfolio (SMB, small minus big) is the average return on the three small portfolios minus the average return on the three big portfolios:

$$\begin{aligned} r_{t+1}^{SMB} &= \frac{1}{3}(r_{t+1}^{small,value} + r_{t+1}^{small,neutral} + r_{t+1}^{small,growth}) \\ &- \frac{1}{3}(r_{t+1}^{big,value} + r_{t+1}^{big,neutral} + r_{t+1}^{big,growth}). \end{aligned} \quad (1)$$

In contrast, the book-to-market portfolio (HML, high minus low) is the average return on the two value portfolios minus the average return on the two growth portfolios:

$$r_{t+1}^{HML} = \frac{1}{2}(r_{t+1}^{small,value} + r_{t+1}^{big,value}) - \frac{1}{2}(r_{t+1}^{small,growth} + r_{t+1}^{big,growth}). \quad (2)$$

The SMB and HML portfolios are both value-weighted. Size and book-to-market breakpoints are refreshed in June of each calendar year, and the portfolios are rebalanced every calendar month to maintain value weights.

To construct the momentum portfolio, we create six portfolios, sorted first by size and afterwards by the returns of the previous year (excluding the most recent

[9] This definition is in line with recent evidence that short-term prior returns contribute little to momentum profits (see Novy-Marx 2012). Its slightly different from the seminal momentum studies of Jegadeesh and Titman (1993,2001) using returns over the past 3 to 12 months.

[10] For US securities, the size breakpoint is the median NYSE market equity. For international securities, it is the 80th percentile by country.

[11] The book-to-market breakpoints are the 70th and 30th percentiles. Also note that Fama and French (1992,1993,1996,2012) use independent sorts. However, we prefer conditional sorts, as proposed by Asness et al. (2013a), because they ensure a balanced number of stocks in each portfolio.

month).[12] This yields sub-portfolios with stocks characterised by previous upward, intermediate and downward development. The momentum factor (UMD, up minus down; also called WML, winner minus loser) is then the average return on the two high return portfolios minus the average return on the two low return portfolios:

$$r_{t+1}^{UMD} = \frac{1}{2}\left(r_{t+1}^{small,up} + r_{t+1}^{big,up}\right) - \frac{1}{2}\left(r_{t+1}^{small,down} + r_{t+1}^{big,down}\right). \tag{3}$$

Size and momentum breakpoints are refreshed every calendar month and the portfolios are rebalanced every calendar month to maintain value weights.

Following Frazzini and Pedersen (2014), portfolios exploiting the beta effect should be long in low-beta stocks, short in high-beta stocks and apply a weighting scheme different from the portfolios described above. To construct such portfolios, stocks are ranked in ascending order on the basis of their estimated betas and the ranked stocks are assigned to one of two sub-portfolios: low-beta and high-beta. In each sub-portfolio, stocks are weighted based on their ranked betas (lower-beta stocks have larger weights in the low-beta sub-portfolio and high-beta stocks have larger weights in the high-beta sub-portfolio). Both portfolios are rescaled to have a beta of one at portfolio formation which makes the total position market neutral. As with the previously discussed portfolios, the beta portfolio is designed to be self-financing.

More formally, the returns of this portfolio are obtained as follows. Let z be the $n \times 1$ vector of beta ranks $z_i = rank(\hat{\beta}_{it})$ at portfolio formation, and let $\bar{z} = 1_n' z/n$ be the average rank, where n is the number of stocks and 1_n is a $n \times 1$ vector of ones. The portfolio weights of the sub-portfolios are given by $w^H = k(z - \bar{z})^+$ and $w^L = k(z - \bar{z})^-$, where $k = 2/1_n'|z - \bar{z}|$ is a normalising constant and x^+ and x^- indicate positive and negative elements of a vector x. By construction, we have $1_n' w^H = 1$ and $1_n' w^L = 1$. Thus, the beta portfolio earns:

$$r_{t+1}^{BAB} = \frac{1}{\hat{\beta}_t^L}\left(r_{t+1}^L - r_f\right) - \frac{1}{\hat{\beta}_t^H}\left(r_{t+1}^H - r_f\right), \tag{4}$$

where $r_{t+1}^L = r_{t+1}' w^L$ and $r_{t+1}^H = r_{t+1}' w^H$ are the returns of the low-beta and high-beta portfolios, $\hat{\beta}_t^L = \hat{\beta}_t' w^L$ and $\hat{\beta}_t^H = \hat{\beta}_t' w^H$ are the corresponding portfolio betas and r_f is the risk-free rate.

After a subtraction of the riskfree rate, our portfolio construction yields monthly factor excess returns from January 1931 to December 2014 for the US market and from July 1990 to December 2014 for the 20 international markets Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Hong Kong, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Singapore and Sweden.[13] Because our fractal methods require the use of continuously compounded returns (see Peters 1992), we convert these simple returns to log returns.

[12] The momentum breakpoints are the 70th and 30th percentiles.

[13] This choice of countries is motivated by a focus on developed markets listed in the MSCI market classification (see https://www.msci.com/market-classification). Some developed markets (Israel and Portugal), emerging markets and frontier markets in the MSCI classification cannot be considered because of insufficient sample sizes.

2.2 Hurst exponent estimators

Rescaled range analysis. Among the first to have analysed statistical dependence in asset returns was Mandelbrot (1971), who proposed the RRA to detect LRD in economic time-series. This approach was originally developed by the hydrologist Hurst (1951) in his studies of river discharges and allows the calculation of the self-similarity parameter H, called the Hurst exponent, which measures the intensity of LRD. Several studies have demonstrated the superiority of RRA to more conventional methods of determining LRD, such as analysing autocorrelations, variance ratios, and spectral decompositions (see Campbell et al. 1997). For example, Mandelbrot and Wallis (1969) and Kristoufek (2012) show that it can detect LRD even in highly non-Gaussian time-series with large skewness and/or kurtosis. Furthermore, Mandelbrot (1972, 1975) reports almost-sure convergence of the RRA for stochastic processes with infinite variances. This provides an important advantage over autocorrelations and variance ratios because they need not be well-defined in this case. Finally, Mandelbrot (1972) argues that unlike spectral analysis which detects periodic cycles, RRA can detect non-periodic cycles, i.e., cycles with periods equal to or longer than the sample period.

By now there is no question that LRD can indeed be detected by RRA (see Baillie 1996). However, perhaps the most important shortcoming of the method is that it also detects short memory without differentiating it from long memory (see Wallis and Matalas 1970). To overcome this limitation, Lo (1991) develops a modification to account for the effects of short memory. Based on this new method, Lo (1991) and Jacobsen (1996) show that, what the earlier literature had assumed was evidence of LRD in stock returns may well be the result of quickly decaying short memory instead. However, problems with the conservatism (see Willinger et al. 1999; Teverovsky et al. 1999; Giraitis et al. 2003) and with choosing its truncation lag (see Di Matteo 2007) made researchers look for alternatives to account for the effects of short-range dependence. One strand of this literature considers subdividing a given time-series into blocks of 5, 10 or 20 observations and shuffling each block randomly to destroy the structure of autocorrelation within these blocks (see Tabak and Cajueiro 2007; Souza et al. 2008). Another strand advocates the application of ARMA filters (see Szilagyi and Batten 2007; Batten and Hamada 2009; Batten et al. 2013).

Because the RRA is also sensitive to the presence of heteroscedasticity (see Lo 1991; Teverovsky et al. 1999), we extend the latter approach to an ARMA-GARCH filter. This means that the classic RRA is not applied to the original returns but to the standardised residuals of a fitted ARMA-GARCH model. Given a series of log excess returns r_t for $t = 1, ..., T$, we estimate the ARMA(v, w)-GARCH(p, q) model:

$$r_t = \alpha + \sum_{i=1}^{v} \beta_i r_{t-i} + \sum_{j=1}^{w} \gamma_j \epsilon_{t-j} + \epsilon_t \quad \text{with} \quad \epsilon_t | \psi_{t-1} \sim N(0, h_t) \tag{5}$$

$$h_t = \delta + \sum_{k=1}^{p} \zeta_k \epsilon_{t-k}^2 + \sum_{l=1}^{q} \eta_l h_{t-l}. \tag{6}$$

Here, the excess return r_t on a portfolio at day t is considered to be linearly related to its lagged values r_{t-i}, $i = 1, ..., v$, the error term ϵ_t and lagged values ϵ_{t-j}, $j = 1, ..., w$ of the error term. ϵ_t depends on past information ψ_{t-1} and is assumed to follow a conditional normal distribution. Thus, our model allows a conditionally heteroscedastic error distribution and, as a direct consequence, even captures fat-tail behaviour (see Tsay 2005).[14] The conditional variance h_t depends upon the squared residuals ϵ^2_{t-k}, $k = 1, ..., p$, of the process and lagged values h_{t-l}, $l = 1, ..., q$, of the conditional variance.

Based on the filtered series $s_t = \epsilon_t / h_t^{0.5}$, RRA is performed as follows (see Sánchez Granero et al. 2008; Souza et al. 2008): We begin by dividing the filtered time-series into d sub-series of length n. Next, for each sub-series $m = 1, ..., d$, we (i) find the mean μ_m and the standard deviation σ_m by means of maximum likelihood, (ii) normalize the data by subtracting the sample mean, i.e., $X_{i,m} = s_{i,m} - \mu_m$ for $i = 1, ..., n$, (iii) create a cumulative time-series $Y_{i,m} = \Sigma_{j=1}^{i} X_{j,m}$ for $i = 1, ..., n$, (iv) find the range $R_m = max(Y_{1,m}, ..., Y_{n,m}) - min(Y_{1,m}, ..., Y_{n,m})$, and (v) rescale the range by R_m/σ_m. This yields the classic formula for the rescaled range of sub-series m:

$$Q_m = \frac{1}{\sigma_m} \cdot \left[\max_{1 \le i \le n} \sum_{j=1}^{i} (s_{j,m} - \mu_m) - \min_{1 \le i \le n} \sum_{j=1}^{i} (s_{j,m} - \mu_m) \right]. \tag{7}$$

It is used in a last step (vi) to obtain the mean value $\bar{Q}_n = d^{-1} \Sigma_{m=1}^{d} Q_m$ of the rescaled range over all sub-series of length n. Since the sum of all n deviations $X_{i,m}$ is zero, the maximum (minimum) in Eq. 7 is always non-negative (non-positive). Thus, R_m is always non-negative and hence $Q_m \ge 0 \rightarrow \bar{Q}_n \ge 0$.

It can be shown that the $\bar{Q}_n$ statistic asymptotically follows the relation $\bar{Q}_n \sim cn^H$, where c is a constant and H is the Hurst exponent (see Mandelbrot 1975). Thus, H can be obtained by running a linear regression over a sample of increasing time horizons:

$$\log(\bar{Q}_n) = \log(c) + H \log(n) + e, \tag{8}$$

where e is the residual of the regression. Equivalently, we could plot $\bar{Q}_n$ against n on a double-logarithmic paper. If the process is white noise, then the plot will roughly be a straight line with slope 0.5. If the process is persistent (large values followed by large values and small values followed by small values) then the slope will be greater than 0.5. If it is anti-persistent (ergodic or mean reverting process) then the slope will be less than 0.5 (see Ellis and Hudson 2007).[15] Thus, both $H > 0.5$ and $H < 0.5$ indicate LRD.

[14] Note that the tail behaviour of this kind of GARCH specifications often remains too short (see Bollerslev and Wooldridge 1992). However, this is no disadvantage for our analysis because RRA is robust to heavy tails.

[15] While this is the most frequently used procedure, there are also versions that differ in the sub-sample (distinct vs. overlapping) and scatter-plot construction (averages vs. all points) (see Mielniczuk and Wojdyłło 2007).

For small n, there is a significant deviation from the 0.5 slope (see Anis and Lloyd 1976). Therefore, the theoretical (i.e. for white noise) values of $\bar{Q}_n$ are usually approximated by:

$$E(\bar{Q}_n) = \begin{cases} \frac{n-0.5}{n} \cdot \frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{n}{2}\right)} \cdot \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}} & \text{for } n \leq 340 \\ \frac{n-0.5}{n} \cdot \frac{1}{\sqrt{n\frac{\pi}{2}}} \cdot \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}} & \text{for } n > 340, \end{cases} \tag{9}$$

where Γ is the Euler gamma function (see Peters 1994). Using this expression, Weron (2002) proposes to estimate H as 0.5 plus the slope of a regression (8) with $\bar{Q}_n - E(\bar{Q}_n)$ as dependent variable. We have adopted this approach.[16]

Detrended fluctuation analysis. Our second method to measure LRD is the DFA proposed by Moreira et al. (1994) and Peng et al. (1994). The advantage of this method over RRA is that it avoids spurious detection of apparent long-range correlation that is an artefact of non-stationarity (see Kantelhardt 2009). However, in contrast to the RRA, research on the theoretical properties of the DFA is rather limited (see Mielniczuk and Wojdyłło 2007) and its behaviour in situations, for which the RRA has proven to be robust, is not completely known. Thus, in our application, the DFA is merely intended to back up our RRA results which may be considered dependent on the choice of filter process.

The DFA can be summarized as follows (see Grau-Carles 2000; Grech and Mazur 2004). First, we divide the time-series of log excess returns into d sub-series of length n. Next, for each sub-series $m = 1, ..., d$, we (i) create a cumulative time-series $Y_{i,m} = \Sigma_{j=1}^{i} X_{j,m}$ for $i = 1, ..., n$, where, in contrast to the RRA, excess return deviations and not residual deviations are used, (ii) fit a least squares line $Y_{i,m} = a_m + b_m i + e$ to $\{Y_{1,m}, ..., Y_{n,m}\}$, and (iii) calculate the root mean square fluctuation (i.e. standard deviation) of the integrated and detrended time-series:

$$F_m = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_{i,m} - a_m - b_m i)^2}. \tag{10}$$

Finally, we (iv) calculate the mean value $\bar{F}_n$ of the root mean square fluctuation for all sub-series of length n. Similar to the RRA, a power-law behavior $\bar{F}_n \sim cn^H$ is expected (see Peng et al. 1994; Taqqu et al. 1995) from which H can be extracted from log-log linear fit.

2.3 Significance testing

Testing the null hypothesis of no or weak dependence against the alternative of strong dependence based on H estimates from RRA and DFA is not as clear and straightforward as for other estimators because no asymptotic distribution theory has been derived for these statistics so far. Fortunately, however, a number of

[16]For more details and potential drawbacks of Eq. 9, see Sánchez Granero et al. (2008). Also note that our results do not change significantly when the correction concerning (9) is omitted.

researchers have suggested simulation-based approaches to evaluate the significance of estimated Hurst exponents (see Barunik and Kristoufek 2010). In our article, we follow the approach of Weron (2002).[17] Using a quantile-based simulation procedure, they derive approximate confidence intervals for estimates from RRA and DFA. Specifically, for a confidence level of 99%, they propose using following intervals:

	Lower bound	Upper bound
RRA	$0.5 - \exp[-7.19 \log(\log \psi) + 4.34]$	$0.5 + \exp[-7.51 \log(\log \psi) + 4.48]$
DFA	$0.5 - \exp[-2.67 \log \psi + 4.06]$	$0.5 + \exp[-3.19 \log \psi + 5.28]$,

where $\psi = log_2 T$.[18] As argued by Weron (2002), decisions based on these confidence intervals are more accurate than heuristic values proposed by some authors in previous research. Also note that the confidence interval for the RRA is wider than for the DFA.

3 Descriptive statistics

To get a first impression of the properties of our factor excess returns, Table 1 provides some descriptive statistics for the richer US sample. Besides reporting basic statistics (minimum, maximum, mean, standard deviation, skewness and kurtosis), we calculate the Sharpe ratios (the ratios of mean excess returns to the standard deviations of excess returns) to measure their risk-adjusted performance (see Sharpe 1966). To detect differences in portfolio performance, we also conduct the Ledoit and Wolf (2008) bootstrap test and report the p-values for testing the null hypothesis of equal Sharpe ratios in parentheses.

The lowest (highest) mean excess returns are realised by the size (beta) portfolios. The lowest (highest) volatility can be observed for size (momentum) portfolios. Size and book-to-market (momentum and beta) portfolios are positively (negatively) skewed indicating that in comparison to a symmetric distribution, there is a higher (lower) probability of large losses. The most extreme (thinnest) tails can be found for the momentum (beta) portfolio.

As far as the Sharpe ratios are concerned, we find a negative one for the size portfolio and the highest positive one for the beta portfolio. The performance of the latter portfolio is significantly higher than the ones of the size and book-to-market portfolios and is more than twice (albeit not significant) that of the momentum portfolio. Interestingly, we find a rising Sharpe ratio when we move along our table from the left to the right. As we have ordered the portfolios by the time periods the corresponding cross-sectional effects have been extensively analysed (from earliest to latest), this tendency is consistent with the recent evidence of Chordia et al. (2014) and McLean

[17]Interestingly, in our application, this simple procedure yields results similar to the bootstrap test proposed by Grau-Carles (2005) and recently used by Cajueiro and Tabak (2008, 2010) and Souza et al. (2008).

[18]These intervals refer to minimum sub-sample sizes of $n_{min} > 50$. However, they are also good approximations for smaller n_{min}. Detailed values for other sizes and confidence levels are tabulated in Weron (2002).

Table 1 Factor excess returns in the US stock market

	Size	Book-to-market	Momentum	Beta
Min	−10.312	−11.728	−65.636	−22.048
Max	34.538	31.217	15.381	15.705
Mean	−0.043	0.013	0.226	0.349
StdDev	3.047	3.320	5.123	3.152
Skewness	2.096	1.955	−5.181	−1.094
Kurtosis	21.981	18.518	58.075	10.089
Sharpe ratio	−0.014	0.004	0.044	0.111
(Size)	–	(0.696)	(0.324)	(0.021)
(Book-to-market)	–	–	(0.542)	(0.041)
(Momentum)	–	–	–	(0.222)

For the period from January 1931 to December 2014, this table shows the key properties of portfolios formed based on size, book-to-market, momentum and beta characteristics (as outlined in Section 2.1) in the US stock market. Besides the minimum, maximum, mean, standard deviation, skewness and kurtosis of monthly percentage excess returns (over the risk-free rate), we also report the Sharpe ratios and test for differences in Sharpe ratios using the bootstrap test of Ledoit and Wolf (2008) in its standard specification with a block size of 24 months. The p-values for testing the null hypothesis of equal Sharpe ratios are given in parentheses.

and Pontiff (2016). They show that cross-sectional effects tend to attenuate after they begin to receive increasing interest in scientific research. It is also worth noting that the negative Sharpe ratio for the size portfolio is not without precedent in the literature. A number of recent studies find that the classic size effect is either waning (see van Dijk 2011) or no longer existent (see Artmann et al. 2012).[19]

The performance results for the 20 international markets (and the US market for comparison) are summarised in Fig. 1 showing the monthly Sharpe ratios of the factor portfolios. Starting with a look at the size portfolios, we again find evidence of unfavourable size portfolio performance. In all countries, this kind of portfolio realises a negative Sharpe ratio. For the book-to-market portfolios, we find mixed results. While 13 of 20 international markets show positive Sharpe ratios, the remaining seven realise negative Sharpe ratios. As far as the momentum and beta portfolios are concerned, we observe impressive positive performance in almost all countries. The only exceptions are Singapore for the momentum portfolio and Japan for the momentum and beta portfolios. However, the latter result is not very surprising because research from other fields (for example, consumption-based asset pricing; see Hamori 1992) shows that Japan appears to be special with respect to many market dynamics.

[19]This is why a new strand of the literature seeks to construct new types of size factors that may revive the size effect. One prominent example in this field is the size factor of Asness et al. (2015). Its main idea is to control for 'junk', i.e., stocks of companies that are small, have low average returns and are typically distressed or illiquid.

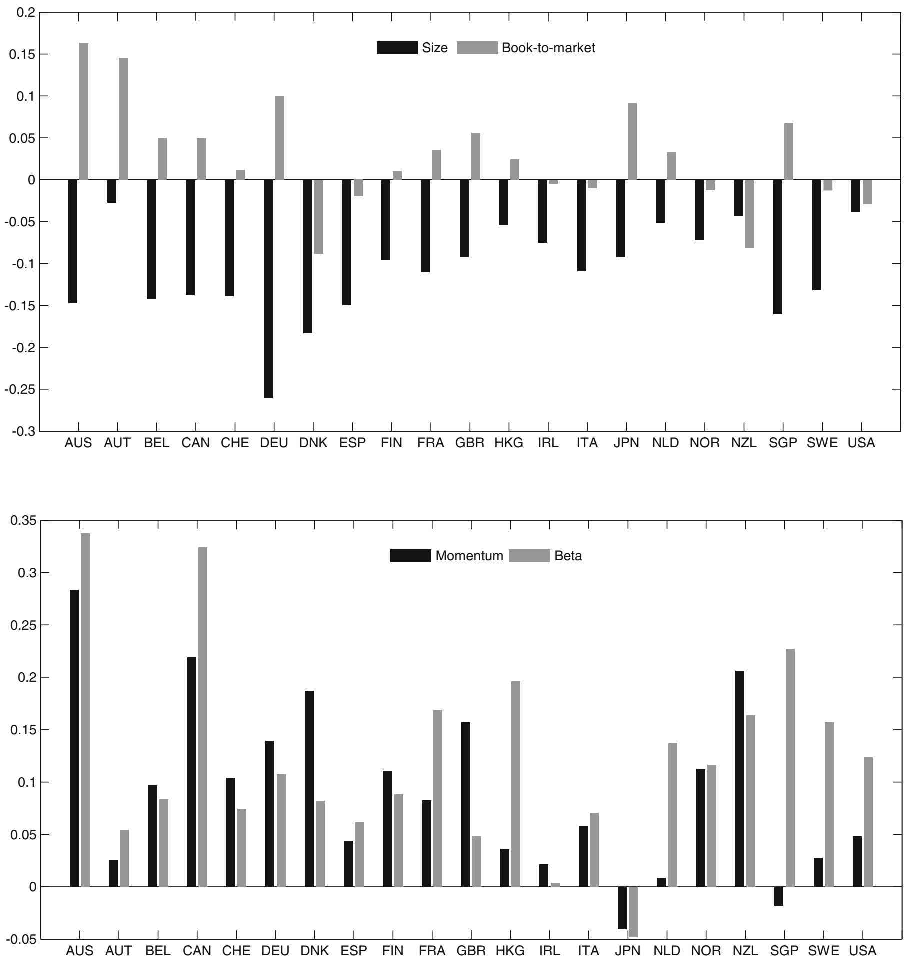

Fig. 1 Factor excess returns in international stock markets. For the period from July 1990 to December 2014, this figure shows the monthly Sharpe ratios of portfolios formed based on size, book-to-market, momentum and beta characteristics (as outlined in Section 2.1) in 20 international stock markets. For comparison, the Sharpe ratios for the US stock market are also included. Abbreviations are used as follows: AUS = Australia, AUT = Austria, BEL = Belgium, CAN = Canada, CHE = Switzerland, DEU = Germany, DNK = Denmark, ESP = Spain, FIN = Finland, FRA = France, GBR = United Kingdom, HKG = Hong Kong, IRL = Ireland, ITA = Italy, JPN = Japan, NLD = Netherlands, NOR = Norway, NZL = New Zealand, SGP = Singapore, SWE = Sweden, USA = United States

4 Empirical analysis

4.1 US results

We begin our discussion of potential LRD in factor excess returns with the RRA analysis of the US market. In line with typical autocorrelation characteristics of stock returns (see Campbell et al. 1997), systematic analysis reveals that an AR(1)-GARCH(1,1) model provides the best fit (in terms of the Akaike information criterion) for our four time-series of factor excess returns.[20] Based on the resulting monthly filtered data, we perform a local Hurst analysis which allows an investigation of time-varying LRD and is designed as follows (see Grech and Mazur 2004; Grech and Pamula 2008). For a given month $t = \tau$, the corresponding H_τ value will be calculated according to the process outlined in Section 2.2 for the time window $[\tau - N + 1, \tau]$ of N months. Moving the time-window from month to month, we are able to produce the history of Hurst exponents over time. Because a too small window size N can introduce huge statistical uncertainty, we follow Chamoli et al. (2007) and use a window of $N = 600$ months to avoid problems associated with short time-series. For the estimation of the Hurst exponent within a specific time-window, we use a minimum sub-sample size of $n_{min} = 48$ months which is in line with the suggestion of Weron (2002).

Figure 2 shows the results of our calculations supplemented by 99 % confidence intervals. For the size and beta portfolios, we find some indications of persistence because the Hurst exponents are above 0.5 in all time-windows. However, the estimated values cannot be regarded as statistically significant because they never breach the upper bound of the confidence interval.[21] For book-to-market and momentum portfolios, we can observe a fluctuation of the Hurst exponent around 0.5. The magnitude of the estimates does not allow a rejection of the null hypothesis of no or weak LRD and thus there is neither evidence of strong persistence or anti-persistence for the excess returns of these portfolios. Also note that the estimated Hurst exponents are not only statistically insignificant; they also have dimensions that are typically considered economically insignificant (see Willinger et al. 1999).

For the DFA, we use the same window and sub-sample sizes as for the RRA. However, instead of filtered returns, the DFA is based on unfiltered returns because it basically contains its own filter. Because this filter represents a simple ordinary least squares trend regression, it is sensitive to outliers in the underlying data (see Lucey 2004). This is why we perform a 99 % winsorization (as used, for example, in Bali et al. 2011). That is, we set all data below (above) the 1 % (99 %) percentile to the 1 % (99 %) percentile.

The results of the DFA are also reported in Fig. 2. In contrast to the RRA, the DFA detects long phases of significant persistence for the size portfolio returns. Thus, the DFA suggests that they are realisations of trend-reinforcing processes (see Mulligan 2004) such that the return should either increase or decrease depending on whether the previous change has been positive or negative. Such a result is not in conflict with the empirical evidence on the fading presence of the size effect (see van Dijk 2011) because the Hurst exponent does not make any statement regarding what kind of performance is likely to persist (see De Souza and Gokcan 2004). While, in the past, the positive performance of the size portfolios persisted, the persistence at the end of our sample relates to the potential continuation of negative performance. As far as the

[20] For the sake of brevity, we do not report the filter results. However, they are available upon request.

[21] Switching to a 95 % confidence interval causes a few breaches. However, a picture of insignificant LRD in most time-windows remains.

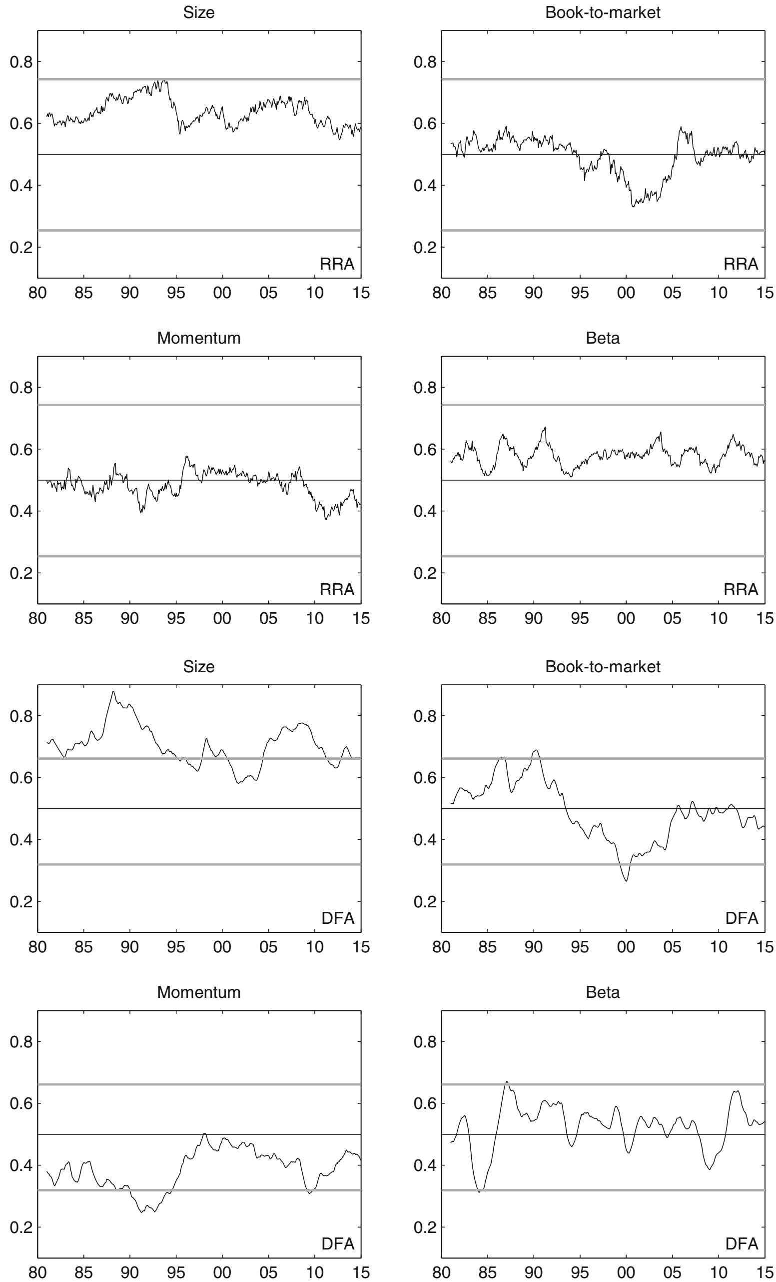

Fig. 2 Time-varying Hurst exponents for the US market. For the period from January 1981 to December 2014, this figure shows the time-varying Hurst exponents of our size, book-tomarket, momentum and beta portfolios built for the US market. The coefficients are estimated using rescaled range analysis (RRA) and detrended fluctuation analysis (DFA) for rolling time-windows of size N = 600 months on filtered (RRA) and unfiltered (DFA) excess returns. Weron (2002) 99 % confidence intervals for the evaluation of significant differences from 0.5 are given as bold grey lines

other portfolio types are concerned, we can detect some incidents of significant anti-persistence for the book-to-market and momentum portfolios which indicate a mean reverting-mechanism in returns such that a movement towards the equilibrium tends to follow a movement way from equilibrium and vice versa. However, this cannot be regarded as a systematic effect. Finally, with a few exceptions where the bounds of the confidence interval are touched, the beta portfolio does not significantly depart from a Hurst exponent of 0.5. Thus, appart from the results for the size portfolio, the DFA supports the results of the RRA by also not detecting systematic evidence for LRD.

4.2 International results

Repeating our analysis for the 20 international stock markets provides the results of Table 2, where we have used RRA and DFA specifications similar to Section 4.1. However, for reasons of sample size, we only obtain one H estimate per country, portfolio and estimation method instead of a time-series of estimates. Furthermore, we use $n_{min} = 12$ because this increases the number of observations for the Hurst exponent regressions.

We find that, even though size, book-to-market and beta (momentum) appear to have a tendency towards persistence (anti-persistence) in the majority of cases, there is no systematic evidence of significant LRD. For the RRA, no Hurst exponent is statistically significant. For the DFA, only the size portfolio for Switzerland and the beta portfolios of Australia, Denmark and the US show Hurst exponents significantly different from 0.5. Thus, LRD appears to be no crucial problem for traditional inference from classic multi-factor asset pricing models implemented in international stock market analysis.

4.3 Robustness checks

As our main results are subject to several issues of arbitrary choice, this section describes a variety of supplementary calculations that verify the robustness of our results.

Publicly available factors. In a first sensitivity check, we analyse alternative asset pricing factors for the US market. We collect the classic factors SMB, HML and UMD proposed by Fama and French (1993) and Carhart (1997) from the data library of Kenneth French because this data is used by most empirical asset pricing studies. Furthermore, we extend our analysis to cover other types of freely available factors. Also from Kenneth French's database, we extract the factors RMW (robust minus weak), CMA (conservative minus aggressive), STREV (short-term reversal) and LTREV (long-term reversal) used, for example, in Fama and French (2015). From the website of Robert Novy-Marx, we obtain the factors PMU (profitable minus unprofitable), IIPMU (intra industry PMU) and IAPMU (industry adjusted PMU) proposed in Novy-Marx (2013). Finally, the factors QMJ (quality minus junk) used in Asness et al. (2013a) and LIQ (liquidity) used in Pastor and Stambaugh (2003), which we download from the websites of Lasse Petersen and Lubos Pastor, respectively, complete our selection of factors.

Table 2 Hurst exponents for 20 international stock markets

	RRA				DFA			
	SMB	HML	UMD	BAB	SMB	HML	UMD	BAB
AUS	0.501	0.542	0.561	0.637	0.558	0.575	0.661	0.791*
AUT	0.436	0.533	0.436	0.532	0.339	0.544	0.540	0.515
BEL	0.524	0.545	0.458	0.583	0.514	0.605	0.462	0.564
CAN	0.575	0.547	0.489	0.549	0.656	0.590	0.629	0.655
CHE	0.640	0.477	0.469	0.444	0.723*	0.431	0.445	0.476
DEU	0.640	0.542	0.513	0.562	0.678	0.521	0.562	0.576
DNK	0.642	0.595	0.483	0.739	0.711	0.573	0.458	0.851*
ESP	0.498	0.533	0.490	0.571	0.481	0.571	0.549	0.581
FIN	0.517	0.637	0.472	0.570	0.584	0.691	0.419	0.692
FRA	0.530	0.505	0.452	0.522	0.465	0.549	0.459	0.560
GBR	0.530	0.538	0.382	0.556	0.514	0.601	0.468	0.643
HKG	0.512	0.602	0.420	0.539	0.544	0.650	0.596	0.607
IRL	0.540	0.538	0.555	0.492	0.597	0.530	0.498	0.524
ITA	0.458	0.512	0.390	0.570	0.497	0.510	0.363	0.654
JPN	0.574	0.593	0.451	0.447	0.629	0.590	0.441	0.428
NLD	0.650	0.519	0.439	0.488	0.647	0.578	0.426	0.554
NOR	0.499	0.550	0.411	0.550	0.454	0.608	0.392	0.589
NZL	0.565	0.520	0.416	0.401	0.634	0.482	0.498	0.417
SGP	0.506	0.577	0.447	0.572	0.595	0.655	0.582	0.653
SWE	0.402	0.551	0.476	0.608	0.395	0.578	0.532	0.640
USA	0.592	0.530	0.406	0.676	0.569	0.543	0.451	0.784*
H > 0.5	16	20	3	16	15	19	8	18
H < 0.5	5	1	18	5	6	2	13	3

For the period from July 1990 to December 2014, this table shows the Hurst exponents for size (SMB), book-to-market (HML), momentum (UMD) and beta (BAB) portfolios formed in 20 international stock markets (and for comparison also for the US market). Estimation is performed using rescaled range analysis (RRA) and detrended fluctuation analysis (DFA). The Weron (2002) 99 % confidence intervals for evaluating significant differences from 0.5 take values of 0.135 - 0.865 for RRA and 0.265 - 0.715 for DFA. Statistical significance is indicated by an asterix. Country abbreviations are used as in Fig. 1. For descriptive purposes, the table also contains the numbers of Hurst exponents larger and smaller than 0.5.

Table 3 presents the RRA and DFA Hurst exponents and the corresponding confidence intervals for the excess returns of these factors. Interestingly, the results for the alternative factors are in line with our main finding. While the size and short-term reversal factor show significant LRD (for DFA), there is no such evidence for all other factors (for RRA and DFA).

Alternative estimator. Because Taqqu et al. (1995) highlights that, in certain applications, the choice of Hurst exponent estimator can crucially influence decision making, we supplement our analysis by implementing one more estimator.

Table 3 Hurst exponents for alternative US factor excess returns

	Sample	RRA			DFA		
		H	CI (LB)	CI (UB)	H	CI (LB)	CI (UB)
French database							
SMB	07-1926 - 12-2014	0.660	0.312	0.683	0.662*	0.350	0.631
HML	07-1926 - 12-2014	0.481	0.312	0.683	0.484	0.350	0.631
RMW	07-1963 - 12-2014	0.535	0.256	0.740	0.435	0.320	0.660
CMA	07-1963 - 12-2014	0.432	0.256	0.740	0.514	0.320	0.660
UMD	01-1927 - 12-2014	0.547	0.312	0.683	0.508	0.350	0.631
STREV	02-1926 - 12-2014	0.645	0.312	0.683	0.695*	0.350	0.631
LTREV	01-1931 - 12-2014	0.616	0.308	0.687	0.618	0.348	0.633
Novy-Marx database							
PMU	07-1963 - 12-2012	0.623	0.251	0.745	0.603	0.318	0.663
IIPMU	07-1963 - 12-2012	0.513	0.251	0.745	0.482	0.318	0.663
IAPMU	07-1963 - 12-2012	0.596	0.251	0.745	0.640	0.318	0.663
Pedersen database							
QMJ	07-1956 - 12-2012	0.559	0.267	0.728	0.563	0.326	0.654
Pastor database							
LIQ	01-1968 - 12-2014	0.602	0.245	0.751	0.544	0.315	0.666

For a selection of publicly available US asset pricing factor excess returns, this table reports the Hurst exponents estimated by rescaled range analysis (RRA) and detrended fluctuation analysis (DFA) and the lower bounds (LB) and upper bounds (UB) of corresponding Weron (2002) 99 % confidence intervals (CI). Statistical significance is indicated by an asterix. Abbreviations are used as follows: SMB = small minus big, HML = high minus low, UMD = up minus down, RMW = robust minus weak, CMA = conservative minus aggressive, STREV = short-term reversal, LTREV = long-term reversal, PMU = profitable minus unprofitable, IIPMU = intra industry PMU, IAPMU = industry adjusted PMU, QMJ = quality minus junk, LIQ = liquidity. Source of the underlying series are the data libraries of Kenneth French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html), Robert Novy-Marx (http://rnm.simon.rochester.edu/data_lib/OSoV/index.html), Lasse Pedersen (http://www.lhpedersen.com/data) and Lubos Pastor (http://faculty.chicagobooth.edu/lubos.pastor/research).

Specifically, we apply the periodogram regression method (PRM) of Geweke and Porter-Hudak (1983) whose basic idea is to estimate the differencing parameter κ of a general fractionally integrated model. As the spectral density function of such a model is identical to that of a fractional Gaussian noise with $H = \kappa + 0.5$, the PRM can be used to estimate H. In contrast to RRA and DFA, the asymptotic properties of the PRM are known and inference can be based on asymptotic normality.[22] Because the PRM is not generally consistent in the non-stationary case, it also requires the application of filter techniques (see Andrews and Guggenberger 2003).

Using the PRM specification of Grau-Carles (2000) for our filtered US market data, we can obtain the local Hurst exponent estimates and confidence intervals

[22]Note that Weron (2002) also constructs his simulated confidence intervals for the PRM and finds that the resulting values are close to the classic normality-based interval of the method.

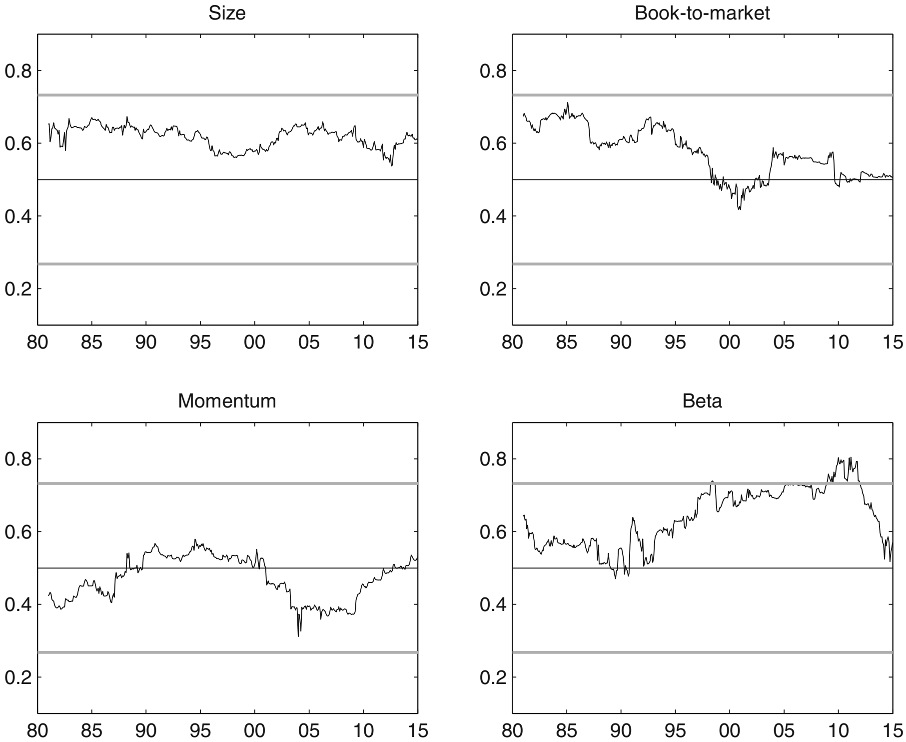

Fig. 3 PRM results in the US market. For the period from January 1981 to December 2014, this figure shows the time-varying Hurst exponents of our size, book-to-market, momentum and beta portfolios built for the US market. The coefficients are estimated using the periodogram regression method (PRM) for rolling time-windows of size $N = 600$ months on filtered excess returns. The corresponding normality-based 99 % confidence intervals for the evaluation of significant differences from 0.5 are given as bold grey lines

shown in Fig. 3. Even though the estimates are partially quite different from the ones derived from RRA and DFA in Fig. 2, our conclusions regarding their non-significance are quite similar. This also holds in an application of the PRM to our international data (not reported here).

Alternative parameterisations. In a final robustness check, we vary some of the settings in our estimation procedures. (i) We use filters of the AR(2)-GARCH(1,1) and AR(3)-GARCH(1,1) forms and find that, in line with Batten et al. (2013), filters of higher orders have little effect on the size of the Hurst exponents from RRA.[23] We also drop the GARCH part from our filter and even repeat our calculations without any filter. In this case, we can detect that the magnitude of the estimated Hurst coefficients rises with each eliminated filter dimension. This is consistent with the fact that

[23]Similar arguments hold when we follow Kang et al. (2009) and Mohammadi and Su (2010) by using alternative GARCH types, namely the TGARCH, EGARCH, CGARCH, IGARCH and FIGARCH models.

the RRA estimator also captures short-term dependence (see Lo 1991) and highlights that our filter procedure does exactly what it was intended for.[24] (ii) Besides linear detrending, we also use quadratic and cubic trend functions in the DFA. However, consistent with Vandewalle et al. (1997), the additional variables do not add any valuable information. (iii) Finally, we find that smaller time-windows $N = 120, 240, 480$ in the analysis of the US market and a minimum block size of $n_{min} = 12$ (in the US sample) and $n_{min} = 48$ (international sample) for obtaining the Hurst sub-samples (as used by Weron 2002) do not qualitatively influence our conclusions.

5 Conclusion

Portfolios seeking to exploit the returns associated with the cross-sectional stock market effects of size, book-to-market, momentum and beta have become standard asset pricing factors in scientific research and/or the basis of a variety of financial products in practice. Even though long-range dependence in the returns of such portfolios can negatively influence portfolio decisions and asset pricing tests, no scientific study has analysed whether their use is actually unproblematic. In this article, we fill this gap in the literature.

For the US market and 20 other developed markets, we analyse the issue of long-range dependence in factor portfolio excess returns based on rescaled range and detrended fluctuation analysis, where the former is augmented by a new filter technique. Supported by a variety of robustness checks, we find only weak indications for the presence of long memory. This is in line with previous research studying long memory in stock markets (see Jacobsen 1996; Willinger et al. 1999) and has several implications for academic research and investment practice.

First, we can argue that, at least for our selection of countries and factors, there appears to be no need to modify decision models and asset pricing tests in order to take into account (anti-)persistence in factor returns since we do not find strong evidence of statistically and economically significant (anti-)persistence. This result also means that we do not have to expect crucial distortions in traditional asset management applications. Thus, the concerns on the consequences of long-range dependence raised in earlier literature (see Lo 1991; Barkoulas and Baum 1996) can be considered as purely theoretical.

Second, it is instructive to have a closer look at the presented results from an investors point of view. Finding no strong evidence of long memory implies that we are potentially unable to predict future factor returns based on past factor returns. However, this does not imply that the factor portfolios are unsuccessful. They can still show impressive performance, as we have seen in the case of the momentum and beta factors, because momentum and beta differences between individual stocks

[24]To back up this result, we have extended the study of Kristoufek (2012), which compares the performance of various Hurst exponent approaches in a variety of different memory and distribution settings, by our filtered procedure. Our results show that (i) non-normal GARCH residuals do not bias the H estimator and that (ii) the application of the filter leads to more precise estimates (in terms of a lower mean absolute error) of the population H than the estimator of Lo (1991).

predict future returns and thus determine factor portfolio performance (see Jacobs 2015; Harvey et al. 2016).[25]

Finally, while recent research has shown that Hurst exponents can provide valuable investment information when they are used in neural stock return networks (see Qian and Rasheed 2004, 2006), carry trade strategies (see Auer and Hoffmann 2016), hedge fund selection (see Auer 2016c) and precious metal trading (see Auer 2016a), our mostly insignificant Hurst exponents suggest that Hurst-based trading strategies are unlikely to be fruitful when applied to factor portfolio returns. Nonetheless, future research might have an explicit look at such trading rules because, for example in the gold market, simple rule-of-thumb strategies, which do not test the statistical significance of Hurst exponents but simply use heuristically set Hurst exponent boundaries, work reasonably well (see Batten et al. 2013).

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[25] If one interprets cross-sectional effects as market anomalies, absence of long memory in factor returns does not imply efficient markets because the performance of the portfolios still originates from abnormal predictability.

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