J Econ Finan (2008) 32:25–34
DOI 10.1007/s12197-007-9005-3

Duration dependence in US business cycles: An analysis using the modulated power law process

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Published online: 5 October 2007

Abstract The modulated power law process is used to analyze the duration dependence in US business cycles. The model makes less restricting assumptions than traditional models do and measures both the local and global performance of business cycles. The results indicate evidence of positive duration dependence in the U.S. business cycles. Structural change after WWII in both expansion and contraction phases of business cycles is also documented. Hypothesis tests confirm that the model fits US business cycles.

Keywords Weibull process · Renewal process · Maximum likelihood estimation · Modulated power law process

JEL Classifications E3 · C1

1 Introduction

Previous studies have applied nonparametric and parametric methods to analyze the duration dependence in business cycles. Examples of early works that applied nonparametric χ^2-type goodness-of-fit tests include McCulloch (1975), Savin (1977), So (1994), Leeuw (1987), and Diebold and Rudebusch (1987).

Later parametric studies mainly used the Weibull distribution because of its parametric nature and of its flexibility in modeling different types of duration dependence. Using the National Bureau of Economic Research (NBER) monthly reference cycle chronology from 1854 to 1990, Sichel (1991) applied the Weibull

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hazard function to examine the duration dependence of business cycles and found positive duration dependence in prewar expansions and postwar contractions. Cochran and Defina (1995) also used the Weibull hazard function to investigate duration dependence in US stock market cycles over the January 1885 to July 1992 period. They found evidence of duration dependence in prewar expansions and in postwar contractions but did not find evidence of duration dependence in prewar contractions and postwar expansions. Using a generalized Weibull model to analyze the NBER monthly reference cycles chronology from December 1854 to March 2001, Zuehlke (2003) documented evidence of duration dependence for all samples in the study. Harman and Zuehlke (in press) applied the generalized Weibull model to analyze the US stock market data and document positive duration dependence for both prewar and postwar samples of stock market expansions and contractions.

One potential drawback of using a renewal process is that it assumes the sample observations are from an independent and identically distributed (i.i.d.) stochastic process, i.e., the durations between consecutive events are positive, independent, and identically distributed. This assumption is sound if a physical renewal process involves successive replacements of failed mechanical components. However, with business cycles of the past 150 years, it may not be reasonable to assume the interarrival times between economic troughs, or peaks, follow the same distribution. For example, Basu and Taylor (1999) have observed a significant decline in the volatility of measured US output.

Instead, this study uses a generalized model, the modulated power law process (MPLP), which is a compromise between a renewal process and a non-homogeneous Poisson process, to study the duration dependence in the US business cycles. Using the modulated power law process, the results indicate the presence of positive duration dependence in our samples of US business cycles. The results are robust to different sampling processes. Hypothesis tests of the parameters confirm that the MPLP model is appropriate in modeling US business cycles.

This study contributes to the literature in that it introduces a new statistical model to analyze the duration dependence of business cycles. The model relies on less restrictive assumptions about the underlying process of business cycles, but offers a more complete description of the statistical process. While traditional models measure only the local performance, the MPLP model measures both the global and local performance of the underlying statistical process. The model can also be applied to analyze other economic or financial cycles.

The remainder of the paper is organized as follows. Section 2 describes the modulated power law process and hypotheses regarding parameters in the MPLP model. The data used in this study and the sampling process are explained in Section 3. Section 4 presents the estimation of the MPLP parameters and the results of the hypothesis tests on the parameters, while Section 5 offers brief concluding remarks.

2 The modulated power law process

As a special case of the non-homogeneous gamma process introduced by Berman (1981), the MPLP model was introduced by Lakey and Rigdon (1992) and studied further by Black and Rigdon (1996). Let X_1, X_2,…,X_n be n i.i.d. gamma random variables with shape parameter β and unit scale parameter; that is, the PDF of X is given by:

$$f(x;\beta) = \frac{x^{\beta-1}}{\Gamma(\beta)}\exp(-x), \qquad x > 0. \tag{1}$$

This distribution has mean $\mu=\beta$ and standard deviation $\sigma = \sqrt{\beta}$. Now define $Y_i = \sum_{j=1}^{i} X_j$, i=1, 2,…, n. The Y_i's thus represent the event times of a renewal process in which the intervals between events follow an i.i.d. gamma process. If $\beta>1$, the gamma distribution has an increasing hazard function. If a system follows such a process, a repaired system is in better condition than it was just before the failure. When β increases, the coefficient of variation

$$\frac{\sigma}{\mu} = \frac{\sqrt{\beta}}{\beta} = \frac{1}{\sqrt{\beta}} \tag{2}$$

decreases. Consequently, the event times of the gamma renewal process will be nearly evenly spaced. If the Y_i's were the actual failure times, a failed and repaired unit would be in better condition than it was just before the failure. Now define

$$T_i = \theta Y_i^{1/\kappa}, \quad i = 1, 2, \ldots, n. \tag{3}$$

The process $T_1<T_2<\ldots T_n$ is a modulated power law process with parameters θ, κ and β. Depending on the size of κ, the transformed system may be improving ($\kappa<1$), or deteriorating ($\kappa>1$) over time. When $\beta>1$, the Y_i's tend to be evenly spaced, and therefore the actual failure times $T_1<T_2<\ldots T_n$ will also tend to be evenly spaced.

The MPLP model is preferred because it provides both a local performance measure, duration dependence, and a global performance measure of business cycles, i.e., whether the underlying statistical process is improving or deteriorating in the long run. In the model, κ *globally* measures the system's improvement or deterioration over time, while β measures duration dependence of the system. If $\kappa>1$, the system is deteriorating. As time goes along, the observed failures of the system tend to get closer and closer. If $\kappa<1$, the system is improving, and as time passes the observed failures of the system tend to get further and further apart. If $\kappa=1$, there is no change in the performance of the system over the long run. In the model, β locally measures duration dependence of business cycles. When $\beta>1$, there exists positive duration dependence in the process, which means that the probability that a cycle will end shortly increases as the cycle increases in length. Negative duration dependence exists if $\beta<1$. If $\beta=1$, the system reduces to the power law process and no duration dependence exists in the system.

The maximum likelihood estimates parameters of a modulated power law process can be approximated using the Newton-Raphson iterative method. Different hypotheses regarding the parameters can be conducted using likelihood ratio tests.

3 Data

This study uses the NBER business cycle data, measured in months, from December 1854 to July 2004, including thirty-two complete observations of expansion and contraction, and one time-truncated expansion. The last observation is time-truncated because the NBER announced on July 17, 2004 that the date of most recent trough is November 2001.Therefore, by July 2004, we have one time-truncated observation of expansion, which began in November 2001 and was truncated in July 2004 when NBER made the announcement, a total of 33 months. Although the actual peak may have already passed, however, without a formal announcement from NBER, the authors treat the last observation as a time-truncated case. Table 1 reports the implied durations of contractions and expansions from the dates of peaks and troughs, while Table 2 presents their descriptive statistics.

Table 1 US business cycle expansions and contractions

Contractions and expansions of US business cycles			
Peak	Trough	Contraction	Expansion
Pre-WWII			
Jun-1857	Dec-1858	18	30
Oct-1860	Jun-1861	8	22
Apr-1865	Dec-1867	**32**	**46**
Jun-1869	Dec-1870	18	18
Oct-1873	Mar-1879	65	34
Mar-1882	May-1885	38	36
Mar-1887	Apr-1888	13	22
Jul-1890	May-1891	10	27
Jan-1893	Jun-1894	17	20
Dec-1895	Jun-1897	18	18
Jun-1899	Dec-1900	18	24
Sep-1902	Aug-1904	23	21
May-1907	Jun-1908	13	33
Jan-1910	Jan-1912	24	19
Jan-1913	Dec-1914	23	12
Aug-1918	Mar-1919	**7**	**44**
Jan-1920	Jul-1921	18	10
May-1923	Jul-1924	14	22
Oct-1926	Nov-1927	13	27
Aug-1929	Mar-1933	43	21
May-1937	Jun-1938	13	50
Feb-1945	Oct-1945	**8**	**80**
Post-WWII			
Nov-1948	Oct-1949	11	37
Jul-1953	May-1954	**10**	**45**
Aug-1957	Apr-1958	8	39
Apr-1960	Feb-1961	10	24
Dec-1969	Nov-1970	**11**	**106**
Nov-1973	Mar-1975	16	36
Jan-1980	Jul-1980	6	58
Jul-1981	Nov-1982	16	12
Jul-1990	Mar-1991	8	92
Mar-2001	Nov-2001	8	120
Aug-2004	?	?	33

Report of the NBER US business cycles data from December 1854 to July 2004. The data are available at http://www.nber.org/cycles.html/. The last expansion observation is time-truncated at July 2004. Boldface indicates wartime business cycles.

Table 2 US business cycle expansions and contractions—report on the descriptive statistics of the business cycles

	Summary statistics					
	Complete sample		Pre-WWII sample		Post-WWII sample	
	Contraction	Expansion	Contraction	Expansion	Contraction	Expansion
Mean	17.44	37.15	20.64	28.91	10.4	53.64
Median	13.5	27	18	23	10	39
Std. dev.	12.2946	26.6354	13.5666	15.4732	3.34	36.3188
Skewness	2.2522	1.7537	1.8598	1.7597	0.7069	0.7565
Kurtosis	8.5328	5.4095	6.3999	6.4586	2.4221	2.1633

Table 1 shows that business cycles are not uniform in length. Contractions during the period covered in Table 1 have been as short as 6 months and as long as 65 months, while expansions have varied between 10 and 120 months. Table 1 also suggests that US business cycles have changed over time—in particular, durations of expansion have become longer since 1945, while those of contractions have become shorter. Table 2 shows that the average duration of expansion increases from 29 months in the prewar period to 54 months in the postwar period, while the average duration of contraction decreases from 29 months to 11 months, respectively. Figure 1 contrasts the average duration of expansions and contractions of the business cycles over three distinct periods: 1854–1919, 1919–1945, and 1945–2001.

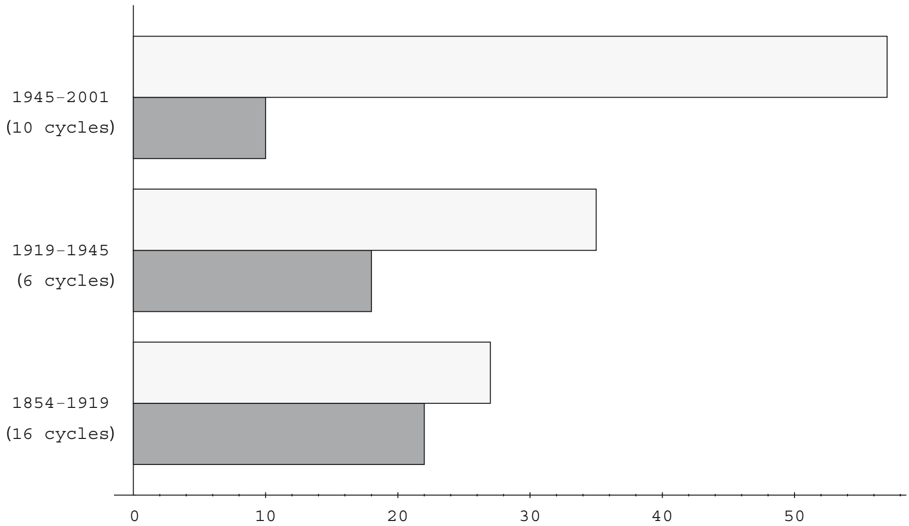

Fig. 1 The average durations of expansions and contractions in US business cycles over three different periods. The average duration of US expansions (*white*) gets longer, while that of US contractions (*gray*) gets shorter over time. This only chart includes the complete business cycles, and therefore, does not include the right-truncated expansion that started in November 2001

In order to test possible mean-duration changes after World War II, the sample data was divided into two subsamples: prewar and postwar. Furthermore, in order to eliminate any potential war impact on the business cycles, we also examine the peacetime subsample, which excludes all the wartime observations.

4 Empirical results

4.1 Estimation of MPLP parameters

Three parameters, κ, β, and θ, of the MPLP model are estimated for four different samples: complete, prewar, postwar, and peacetime samples. Table 3 reports the MLEs for the expansion data, and reports the results for the contraction data.

For the expansion data, the estimated shape parameter, $\widehat{\kappa}$, of the complete sample is 0.8196, implying that the expansions are improving over the long run. After eliminating the effects of wars, the estimated $\widehat{\kappa}$ is 0.8354, which suggests that wars do not change the improving trend of business cycles. Pre-WWII has a similar trend like that of the complete and the peacetime sample. The estimated κ is 0.9796, which is significantly (at 5%) less than 1.0 based on the hypothesis test. The same trend is evident over the post-WWII period, since $\widehat{\kappa}$ of the postwar business cycles is 0.9077.

In summary, the estimated κ suggests that, in the long run, the duration of expansions are getting longer and longer. The result is robust to different samples of US business cycles. This finding is consistent with the analysis from Fig. 1.

While κ measures the global performance of the expansions, β measures the local performance of the expansions after a failure. $\beta>1$ indicates the existence of positive duration dependence in the process, which means that the probability that an

Table 3 Point estimation and likelihood ratio test of the MPLP parameters

	Complete	Peacetime	Prewar	Postwar
Results of expansion data				
$\widehat{\kappa}$	0.8196**	0.8354**	0.9796**	0.9077
p-value: H_0: κ=1	(0.0142)	(0.0106)	(0.0325)	(0.3558)
$\widehat{\beta}$	3.1635***	3.4695***	4.6659***	2.5819**
p-value: H_0: β=1	(<0.0001)	(<0.0001)	(<0.0001)	(0.0217)
p-value: H_0: β=κ=1	(<0.0001)	(0.0002)	(<0.0001)	(0.1238)
$\widehat{\theta}$	4.2228	3.7815	5.6254	14.784
Results of contraction data				
$\widehat{\kappa}$	1.2665***	1.2108**	1.1097	1.0247
p-value: H_0: κ=1	(0.0053)	(0.0452)	(0.3127)	(0.2865)
$\widehat{\beta}$	3.4882***	3.4536***	3.313***	11.436***
p-value: H_0: β=1	(<0.0001)	(<0.0001)	(0.0002)	(<0.0001)
p-value: H_0: β=κ=1	(<0.0001)	(0.0001)	(0.0013)	(0.0002)
$\widehat{\theta}$	13.4813	11.5719	9.5179	1.0195

The table reports maximum likelihood estimates of β, κ and θ, results of hypothesis tests and p values for β and κ of the complete, peacetime, prewar and postwar samples. For each sample, the results are shown in columns. The estimates of the parameters are first reported. Significant parameters at the 5% level are highlighted in boldface. The p values of hypothesis tests of β=1 and κ=1 are reported in parenthesis. NBER business cycle data from December 1854 to July 2004 is used.

expansion will end shortly increases as the duration of expansion increases. Negative duration dependence exists when $\beta<1$. The estimated values of β are significantly greater than one, implying strong positive duration dependence in expansions. The result is robust for the impact of WWII, because all the estimated βs are significantly greater than 1. A significant decrease in duration dependence after the WWII is also documented. For the prewar sample, the estimated β is 4.6659, while the estimated β is 2.5819 for the postwar sample. Third, wars have little effect on the expansions: the estimated β is 3.1635 for the complete sample, and is 3.4695 for the peace sample. In all three cases, the β parameters are significantly different from one.

The results are not surprising because the coefficient of variation increases from 0.54 (=15.4732/28.91) to 0.68 (=36.3188/53.64) after the war, which implies a more dispersed distribution of the duration for expansions. In Fig. 2a the pre- and post-war duration and expansion times are plotted. It is clear that the prewar expansions are more evenly spaced, although they have a smaller mean duration of 29 months. This finding supports the implications from the above estimated βs.

The contraction data indicate quite a different picture. Contrary to those for the expansions, all of the estimated κs are greater than 1, implying shorter contractions. The economic implication is that in the long-run, the duration of contractions is getting shorter and shorter, which is consistent with the finding in Fig. 1. Also, there is little variation for the estimated κ from different samples.

For the complete sample, the estimated β is 3.4882, and is statistically significant. The large value of estimated β implies strong duration dependence in the

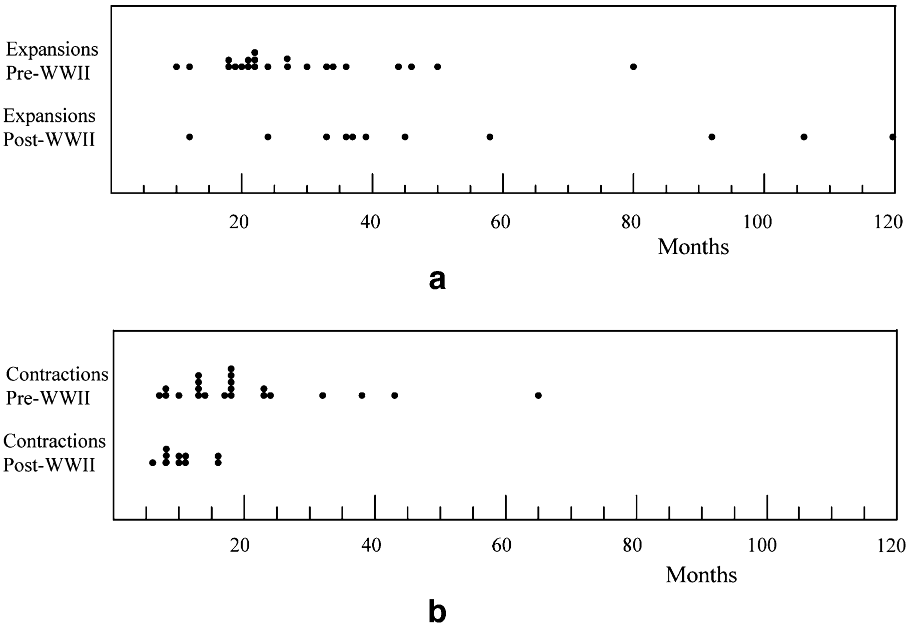

Fig. 2 **a** Plots of prewar and postwar expansion (**b** contraction) durations. The high and low horizontal lines plot the postwar and prewar expansion (contraction) durations, respectively. Each *dot* represents an occurrence of an expansion (contraction). The length of line from the vertical axis to the asterisk represents the duration of an expansion. Multiple durations are indicated by stacking the dots

contractions. This trend does not change after the possible effects of wars are removed: the estimated β of the peacetime sample is 3.4536. However, a change is observed in the trend after WWII. Before the war, the estimated β is 3.3130, while it is 11.4360 after the war. The economic implication is that the duration dependence in contractions after the war is much stronger than before the war (11.436 vs. 3.313), and the durations of contraction are more evenly distributed (all $\widehat{\kappa}$'s are significantly larger than 1.0).

This implication conforms to the coefficients of variation before and after the war. The coefficients of variation are 0.68 (=13.5666/20.64) and 0.32 (=3.34/10.4), respectively. On the two horizontal lines in Fig. 2b, the duration of the prewar and postwar contractions are plotted, where the length from the vertical axis to the dot represents the duration of contractions. It is clear that the postwar contractions are more clustered around the mean, implying strong duration dependence for the postwar period.

These results differ from those of Sichel (1991). He found evidence of positive duration dependence in prewar expansions but no evidence of duration dependence in post expansions, and he found evidence of positive duration dependence in postwar contractions but no evidence in prewar contractions. One possible explanation of the different findings is the different assumptions used in the models. The Weibull renewal process requires the underlying statistical process to be i.i.d., while the modulated power law process does not. The findings of this study are similar to those in Zuehlke (2003). He finds duration dependence in all of his samples. While his model provides measurement of the local performance of business cycle durations, this study provides measurement for both the global and local performance of the business cycles. Besides, the MPLP model captures the structural change on the duration dependence of both expansions and contractions after the WWII.

Some researchers have tried to explain the relative economic stability after the WWII. Romer (1999) attributes the prolonged postwar expansion and less frequent occurrence of severe depression to counter-cyclical monetary policy and automatic stabilizers, while Taylor (1998) states that monetary policy deserves much of the credit for the unprecedented degree of economic stability in 1990s. However, a detailed discussion of the causes is beyond the scope of this study.

4.2 Hypothesis tests of the MPLP parameters

This study tests three hypotheses about the estimated MPLP parameters in US business cycles,

1. H_{0a}: $\beta=1$ (the model reduces to the power law process), versus H_{1a}: $\beta\neq1$
2. H_{0b}: $\kappa=1$ (the model reduces to a gamma renewal process), versus H_{1b}: $\kappa\neq1$
3. H_{0c}: $\beta=1$ and $\kappa=1$ [the model reduces to the homogeneous Poisson process (HPP)], versus H_{1c}: $\beta\neq1$ or $\kappa\neq1$

Different scenarios are illustrated in Fig. 3. Failing to reject any of the hypotheses would suggest that the MPLP may not be proper for modeling the duration dependence of US business cycles.

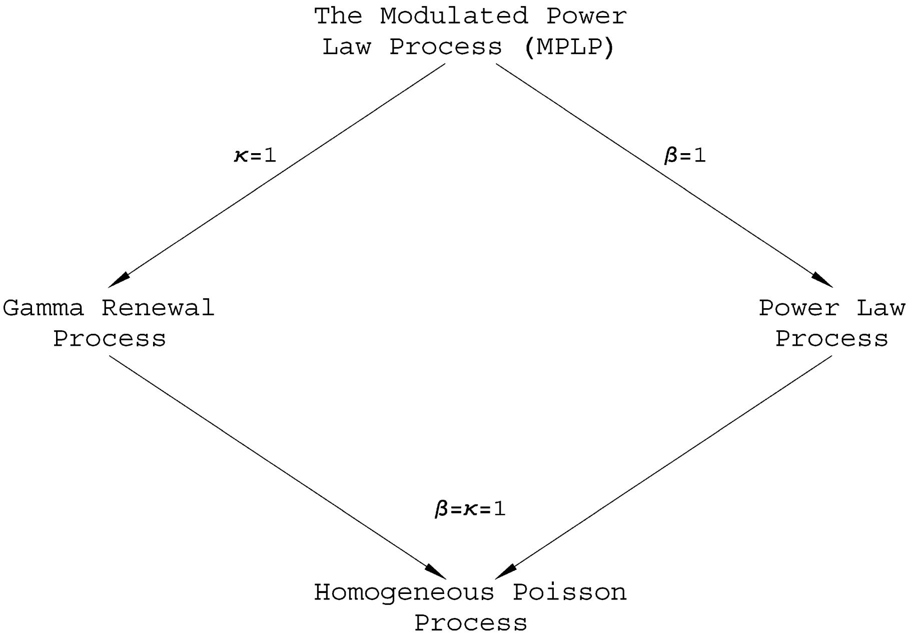

Fig. 3 Some special cases of the modulated power law process. When κ=1, the MPLP reduces to the power law process. When β=1, the times between failure are i.i.d. gamma random variables, so the process becomes a gamma renewal process. Finally, when both β=1 and κ=1, the process reduces to the homogeneous Poisson process. The MPLP is thus a generalization of the power law process and the gamma renewal process

The results are presented in Table 3—the test results for expansion samples and the results for contraction samples. For the hypothesis H_{0a}: β=1, the null hypothesis is rejected at the 0.05 level. The result confirms that the non-homogeneous Poisson process should not be used to model US business cycle.

For the hypothesis H_{0b}: κ=1. The test statistics lead to the rejection of the null hypothesis that κ=1 at 0.05 level for the complete sample, peacetime sample and prewar sample. They fail to reject the null hypothesis that κ=1 for the postwar sample. The results suggest that the observed business cycles are not derived from a constant Gamma process.

Similar results are obtained for the third hypothesis H_{0c}: β=κ=1. They reject the null hypothesis at the 1% significance level for the complete, peacetime and prewar samples, but fail to reject the hypothesis in the postwar sample (The p value is 0.12). The results imply that the system does show improvement in the complete, peacetime and prewar expansion samples, and that the homogenous Poison process is not an adequate model for business expansions.

The results of the hypothesis tests for the contraction data, which are presented in Table 3, are similar to those of the expansions. They reject the hypothesis that κ=1 for the complete sample at the 1% significance level, and for peacetime sample at the 5% significance level, while the hypothesis for the prewar and postwar expansions cannot be rejected. The results suggest that a renewal process should not be used to model the complete and peacetime samples. For the complete sample and the three sub-samples, the hypothesis that $\beta=1$ can be rejected and is significant at the 1% level. Similarly, for the complete sample and three sub-samples, the null hypothesis that $\beta=\kappa=1$ can be rejected at the 1% level.

5 Conclusions and implications

Applying the modulated power law process, this study finds various degrees of positive duration dependence in all samples for both the expansions and the contractions.

Evidence of structural change after WWII is also found: duration dependence of expansions decreases while for contractions it increases after WWII. The results conform to the descriptive statistics of the samples. For example, although both the prewar and postwar expansions exhibit positive duration dependence, the larger magnitude of duration dependence for the prewar period is consistent with its small coefficient of variation in the more evenly distributed expansions before the war. The results are similar for peacetime business cycles.

Acknowledgement We thank James E. Payne (the Editor) and an anonymous referee for providing helpful comments that have significantly improved the paper. We are grateful to James R. Webb, Jacky Yuk-chow So, Mostafa Mashayekhi, Ma. Zenia N. Agustin, James R. Schmidt, Edward Lawrence and participants at the finance seminar at the University of Nebraska-Lincoln and at the 2004 Southern Economic Association annual meeting for helpful discussions and comments.

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