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ORIGINAL RESEARCH

Relations on neutrosophic soft set and their application in decision making

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Abstract

Neutrosophic soft sets are a mathematical model put forward to overcome uncertainty with the contribution of a parameterization tool and neutrosophic logic by considering of information a falsity membership function, an indeterminacy membership function and a truth membership function. This set theory which is a very successful mathematical model, especially as it handles information in three different aspects, was first introduced to the literature by Maji (Ann Fuzzy Math Inf 5(1):157–168, 2013) and later modified by Deli and Broumi (J Intell Fuzzy Syst 28(5):2233–2241, 2015). In this way, they aimed to use neutrosophic soft sets more effectively for uncertainty problems encountered in most real life problems. Relations are a method preferred by researchers to explain the correspondences between objects. In this paper, neutrosophic soft relationships are discuss and define by referring to the theory of neutrosophic soft set proposed by Deli and Broumi (Ann Fuzzy Math Inf 9:169–182, 2015). Then, we present the concepts of composition, inverse of neutrosophic soft relations and functions along with some related properties and theorems. Moreover, the equivalence classes and equivalence relations of soft relations are given with support from real life examples and some of their properties are analyzed. Finally, we propose an algorithm to be used in expressing the correspondence between objects in solving uncertainty problems by using the soft relationship defined and an example is given to show how this algorithm can be applied for uncertainty problems.

Keywords Neutrosophic soft relation · Neutrosophic soft set · Decision making

Mathematics Subject Classification 03E72 · 08A72

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1 Introduction

It is very important to express the current state of uncertainty in a problems in the best way possible and then to be able to solve the problem in the most ideal way. In this regard, especially in recent years, two mathematical models have come to the fore in dealing with uncertain data. One of them is the theory of neutrosophic set (briefly NS) proposed by Smarandache [1] as a generalization of fuzzy set (briefly FS) [2] and intuitionistic fuzzy set [3]. This set theory is a logic set with three components, the falsity, indeterminacy and truth memberships, that are completely independent from each other to express uncertainty clearly. The first data on NSs are available in the book [4] published in 1998. One of the most important advantages of this set theory is its ability to process indeterminate data that are not considered by FS theory and intuitionistic fuzzy set theory.

One of the mathematical models frequently used by researchers to solve uncertainty problems is the soft set (briefly SS) theory brought to the literature by Molodsov [5] as a general mathematical tool used to handle imprecision, vagueness and uncertainties. The most important reason why this set theory is frequently used by researchers is that it eliminates the lack of a parameterization tool that is not available in other set models. Since the SS theory was introduced, many different mathematical models of SS have been developed such as vague soft sets [6], soft expert sets [7], interval-valued vague soft sets [8–10], fuzzy soft sets [11], neutrosophic soft set (briefly NSS) [12–15], neutrosophic parameterized neutrosophic soft set [16] and soft multi set theory [17]. Currently, SS continues to attract the attention of researchers, and accordingly, the field of application of the this theory continues to increase day by day [18–24].

Since these two set theories are quite successful in processing uncertain data, Maji [12] has combined these theories and introduced a novel mathematical model, which is called NSS, to the literature. After the NSSs were modified by Deli and Bromi [13] so that they could be used more practically in uncertainty problems, and their operations were also redefined and a comparison with Maji's [12] definition was given. This mathematical model has been found interesting by many researchers: For example; Mukherjee and Sarkar [25] solved a medical diagnosis decision-making problem using NSSs and discussed about the theory. Moreover, Şahin and Küçük [26] studied some algebraic properties by introducing a novel style of NSS theory. Apart from that, Hussain and Shabir [27] investigated on algebraic operations of theory. Sumathi and Arockiarani [28] also studied the NSSs. In addition to these, a lot of research has been done on the NSS [29–33].

The FS theory, which was put forward by Zadeh [2] to overcome uncertainty before all these set theories, has been successfully applied to uncertainty problems, especially in decision-making areas, making this theory a focal point for research. Therefore, the extensions of FS, intuitionistic FS [3], interval-valued intuitionistic FS [34], NS, single valued NS [35] and their hybrid models have been applied as mathematical models recommended to overcome uncertainty.

Because of the observation that objects can be related to each other in real life, fuzzy relationships were used at first. Using these relationships, a vagueness that expresses the relation degree of two objects can only be modeled, that is, they are insufficient in modeling the uncertainty. In order to overcome this situation, Bustince and Burillo [36] suggested the concept of intuitionistic fuzzy relations and then Dinda and Samanta [37] the concept of intuitionistic fuzzy soft relations. However, in these relationships can only express a certain degree of uncertainty, that is , it does not handle indeterminacy degree of membership. For this reason, neutrosophic soft relationships have been brought to the literature by Deli and Broumi [38], using the neutrosophic soft sets given by Maji [12]. In this way, a great progress has been made in order to express the relations between objects in the best way. The better we can express the relationships between objects, the more we can manage decision-making processes in uncertain environments. That's why, recently, many researchers have studied generalizations of fuzzy relationships [39–41], fuzzy soft relationships [42,43] and neutrorophic soft relationships [44–46].

NSSs given by Maji [12] were modified by Deli and Bromi [13] and it was aimed to use this set model more functionally in decision making problems. The aim of this paper is to discuss and introduce the concept of neutrosophic soft relationships (briefly NSR) based on NSSs modified by Deli and Bromi [38]. The organization of this paper is as follow: In Sect. 2, we give the basic definitions and results of NS theory, SS theory and NSS theory that are useful for subsequent discussions. In Sect. 3, we characterize the idea of NSR and present the composition and inverse operations of these relations with some basic properties. In Sect. 4, various types of NSRs are defined. In addition, the equivalence classes and partitions of NSSs are analyzed. In Sect. 5, the concept of neutrosophic soft function is defined and some special types of this concept are presented together with related theorems. In Sect. 6, we present an application of NSRs in a decision-making problem. Finally, we conclude the paper in Sect. 7.

2 Preliminaries

In this section, we recall some basic notions in NS, SS, and NSS.

Throughout this paper, let P be a set of parameters and K, L, M be non-empty subsets of P. Also, let U be an initial universe, and 2^U denotes the power set of U.

Definition 2.1 [1] A NS X on the universe of discourse U is defined as:

$$X = \{\langle u, T_X(u), I_X(u), F_X(u)\rangle : u \in U\} \tag{1}$$

where ${}^-0 \leq T_X(u) + I_X(u) + F_X(u) \leq 3^+$ and $T, I, F : U \rightarrow]^-0, 1^+[$.

Note that the set of all the NSs over U will be denoted by $2^{N(U)}$.

Definition 2.2 [5] A pair (Γ, K) is called a SS over U, where Γ is a mapping given by $\Gamma : K \rightarrow 2^U$. For $p \in K$, $\Gamma(p)$ may be considered as the set of p-approximate elements of the SS (Γ, K), i.e.,

$$(\Gamma, K) = \{(p, \Gamma(p)) : p \in K\}. \tag{2}$$

Firstly, NSS defined by Maji [12] and later this concept has been modified by Deli and Bromi [13] as given below:

Definition 2.3 [13] A NSS $(\widehat{\Gamma}, P)$ over U is a set defined by a set valued function $\widehat{\Gamma}$ representing a mapping $\widehat{\Gamma} : P \to 2^{N(U)}$ where $\widehat{\Gamma}$ is called approximate function of the NSS $(\widehat{\Gamma}, P)$, i.e.,

$$(\widehat{\Gamma}, P) = \left\{ \left(p, \langle u, T_{\widehat{\Gamma}(p)}(u), I_{\widehat{\Gamma}(p)}(u), F_{\widehat{\Gamma}(p)}(u) \rangle : u \in U \right) : p \in P \right\} \tag{3}$$

where $0 \le T_{\widehat{\Gamma}(p)}(u) + I_{\widehat{\Gamma}(p)}(u) + F_{\widehat{\Gamma}(p)}(u) \le 3$ and $T_{\widehat{\Gamma}(p)}(u), I_{\widehat{\Gamma}(p)}(u), F_{\widehat{\Gamma}(p)}(u) \in [0, 1]$ called the truth-membership, indeterminacy-membership, falsity-membership functions of $\widehat{\Gamma}(p)$.

Definition 2.4 [47]

(i) A NSS $(\widehat{\Gamma}, P)$ over U is said to be null NSS if $T_{\widehat{\Gamma}(p)}(u) = 0$, $I_{\widehat{\Gamma}(p)}(u) = 1$, $F_{\widehat{\Gamma}(p)}(u) = 1$; $\forall p \in P, \forall u \in U$. It is denoted by $\emptyset_U$.
(ii) A NSS $(\widehat{\Gamma}, P)$ over U is said to be absolute NSS if $T_{\widehat{\Gamma}(p)}(u) = 1$, $I_{\widehat{\Gamma}(p)}(u) = 0$, $F_{\widehat{\Gamma}(p)}(u) = 0$; $\forall p \in P, \forall u \in U$. It is denoted by 1_U.
Clearly, $\emptyset_U^c = 1_U$ and $1_U^c = \emptyset_U$.

Definition 2.5 [48] Let $(\widehat{\Gamma}, P)$ be NSS over U. The complement of $(\widehat{\Gamma}, P)$ is denoted by $(\widehat{\Gamma}, P)^c$ and is defined by:

$$(\widehat{\Gamma}, P)^c = \left\{ \left(p, \langle u, F_{\widehat{\Gamma}(p)}(u), 1 - I_{\widehat{\Gamma}(p)}(u), T_{\widehat{\Gamma}(p)}(u) \rangle : u \in U \right) : p \in P \right\} \tag{4}$$

Obvious that, $((\widehat{\Gamma}, P)^c)^c = (\widehat{\Gamma}, P)$.

Definition 2.6 [12] Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be two NSSs over U. $(\widehat{\Gamma}, P)$ is said to be neutrosophic soft subset of $(\widehat{\Lambda}, P)$ if $T_{\widehat{\Gamma}(p)}(u) \le T_{\widehat{\Lambda}(p)}(u)$, $I_{\widehat{\Gamma}(p)}(u) \le I_{\widehat{\Lambda}(p)}(u)$ and $F_{\widehat{\Gamma}(p)}(u) \ge F_{\widehat{\Lambda}(p)}(u)$ for all $p \in P$, $u \in U$. It is denoted by $(\widehat{\Gamma}, P) \widehat{\subseteq} (\widehat{\Lambda}, P)$.

$(\widehat{\Gamma}, P)$ is said to be neutrosophic soft equal to $(\widehat{\Lambda}, P)$ if $(\widehat{\Gamma}, P) \widehat{\subseteq} (\widehat{\Lambda}, P)$ and $(\widehat{\Lambda}, P) \widehat{\subseteq} (\widehat{\Gamma}, P)$. It is denoted by $(\widehat{\Gamma}, P) = (\widehat{\Lambda}, P)$

Definition 2.7 [49] Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be two NSSs over U. Then,

(i) their union is denoted by $(\widehat{\Gamma}, P) \widehat{\cup} (\widehat{\Lambda}, P) = (\widehat{\Omega}, P)$ and is defined by:

$$(\widehat{\Omega}, P) = \left\{ \left(p, \langle u, T_{\widehat{\Omega}(p)}(u), I_{\widehat{\Omega}(p)}(u), F_{\widehat{\Omega}(p)}(u) \rangle : u \in U \right) : p \in P \right\} \tag{5}$$

where

$$T_{\widehat{\Omega}(p)}(u) = max \left\{ T_{\widehat{\Gamma}(p)}(u), T_{\widehat{\Lambda}(p)}(u) \right\},$$
$$I_{\widehat{\Omega}(p)}(u) = max \left\{ I_{\widehat{\Gamma}(p)}(u), I_{\widehat{\Lambda}(p)}(u) \right\}$$

and

$$F_{\widehat{\Omega}(p)}(u) = min \left\{ F_{\widehat{\Gamma}(p)}(u), F_{\widehat{\Lambda}(p)}(u) \right\}.$$

(ii) their intersection is denoted by $(\widehat{\Gamma}, P)\widehat{\cap}(\widehat{\Lambda}, P) = (\widehat{\Omega}, P)$ and is defined by:

$$(\widehat{\Omega}, P) = \left\{\left(p, \langle u, T_{\widehat{\Omega}(p)}(u), I_{\widehat{\Omega}(p)}(u), F_{\widehat{\Omega}(p)}(u)\rangle : u \in U\right) : p \in P\right\} \tag{6}$$

where

$$T_{\widehat{\Omega}(p)}(u) = min\left\{T_{\widehat{\Gamma}(p)}(u), T_{\widehat{\Lambda}(p)}(u)\right\},$$
$$I_{\widehat{\Omega}(p)}(u) = min\left\{I_{\widehat{\Gamma}(p)}(u), I_{\widehat{\Lambda}(p)}(u)\right\}$$

and

$$F_{\widehat{\Omega}(p)}(u) = max\left\{F_{\widehat{\Gamma}(p)}(u), F_{\widehat{\Lambda}(p)}(u)\right\}.$$

3 Relations on NSSs

In this section, we first define the cartesian product (briefly CP) of two NSSs, then characterize the idea of NSR and present the composition and inverse operations of these relations with some basic properties.

Definition 3.1 Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be two NSSs over U, then $(\widehat{\Gamma}, P) \times (\widehat{\Lambda}, P) = (\widehat{\Omega}, P \times P)$ is the CP of $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$, where $(a, b) \in P \times P$, $\widehat{\Omega} : P \times P \to 2^{N(U)}$ and $\widehat{\Omega}(a, b) = \Gamma(a) \times \Lambda(b)$, i.e.,

$$(\widehat{\Omega}, P \times P) = \left\{\left((a, b), \langle u, T_{\widehat{\Omega}(a,b)}(u), I_{\widehat{\Omega}(a,b)}(u), F_{\widehat{\Omega}(a,b)}(u)\rangle : u \in U\right) : (a, b) \in P \times P\right\} \tag{7}$$

where $T_{\widehat{\Omega}(a,b)}(u)$, $I_{\widehat{\Omega}(a,b)}(u)$, $F_{\widehat{\Omega}(a,b)}(u)$ are the truth, indeterminacy and falsity membership functions of $(\widehat{\Omega}, P \times P)$ such that $T_{\widehat{\Omega}(a,b)}, I_{\widehat{\Omega}(a,b)}, F_{\widehat{\Omega}(a,b)}(u) : U \to [0, 1]$ and for all $u \in U$ and $(a, b) \in P \times P$ we have:

$$T_{\widehat{\Omega}(a,b)}(u) = min\left\{T_{\widehat{\Gamma}(a)}(u), T_{\widehat{\Lambda}(b)}(u)\right\},$$
$$I_{\widehat{\Omega}(a,b)}(u) = max\left\{I_{\widehat{\Gamma}(a)}(u), I_{\widehat{\Lambda}(b)}(u)\right\}$$

and

$$F_{\widehat{\Omega}(a,b)}(u) = max\left\{F_{\widehat{\Gamma}(a)}(u), F_{\widehat{\Lambda}(b)}(u)\right\}.$$

Example 3.2 Let $U = \{u_1, u_2, u_3\}$ be an universal set, $P = \{p_1, p_2\}$ be a set of parameters and

$$(\widehat{\Gamma}, P) = \left\{\begin{array}{l} (p_1, \langle u_1, 0.45, 0.6, 0.2\rangle, \langle u_2, 0.55, 0.43, 0.76\rangle, \langle u_3, 0.6, 0.2, 0.45\rangle), \\ (p_2, \langle u_1, 0.55, 0.3, 0.56\rangle, \langle u_2, 0.75, 0.6, 0.4\rangle, \langle u_3, 0.25, 0.6, 0.57\rangle) \end{array}\right\},$$

$$(\widehat{\Lambda}, P) = \left\{ \begin{array}{c} (p_1, \langle u_1, 0.34, 0.7, 0.35\rangle, \langle u_2, 0.76, 0.5, 0.6\rangle, \langle u_3, 0.65, 0.25, 0.7\rangle), \\ (p_2, \langle u_1, 0.45, 0.33, 0.6\rangle, \langle u_2, 0.8, 0.65, 0.7\rangle, \langle u_3, 0.2, 0.65, 0.5\rangle) \end{array} \right\}$$

be two NSSs over U. Then, the CP of $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ is

$$(\widehat{\Omega}, P \times P) = \left\{ \begin{array}{c} ((p_1, p_1), \langle u_1, 0.34, 0.7, 0.35\rangle, \langle u_2, 0.55, 0.5, 0.76\rangle, \langle u_3, 0.6, 0.25, 0.7\rangle), \\ ((p_1, p_2), \langle u_1, 0.45, 0.6, 0.6\rangle, \langle u_2, 0.55, 0.65, 0.76\rangle, \langle u_3, 0.2, 0.65, 0.45\rangle), \\ ((p_2, p_1), \langle u_1, 0.34, 0.7, 0.56\rangle, \langle u_2, 0.75, 0.6, 0.6\rangle, \langle u_3, 0.25, 0.6, 0.7\rangle), \\ ((p_2, p_2), \langle u_1, 0.45, 0.33, 0.6\rangle, \langle u_2, 0.75, 0.65, 0.7\rangle, \langle u_3, 0.2, 0.65, 0.57\rangle) \end{array} \right\}.$$

Definition 3.3 Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be NSSs over U, then a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ is a neutrosophic soft subset of $(\widehat{\Gamma}, P) \times (\widehat{\Lambda}, P)$, and is of the form $(\widehat{\mathfrak{R}}, K \times L)$, where $K \times L \subseteq P \times P$ and $\widehat{\mathfrak{R}}(a, b) \subseteq (\widehat{\Gamma}, P) \times (\widehat{\Lambda}, P)$, $\forall (a, b) \in K \times L$, i.e.,

$$(\widehat{\mathfrak{R}}, K \times L) = \left\{ \left((a, b), \langle u, T_{\widehat{\mathfrak{R}}(a,b)}(u), I_{\widehat{\mathfrak{R}}(a,b)}(u), F_{\widehat{\mathfrak{R}}(a,b)}(u)\rangle : u \in U \right) : (a, b) \in K \times L \subseteq P \times P \right\} \tag{8}$$

where for all $u \in U$ and $(a, b) \in K \times L \subseteq P \times P$,

$$T_{\widehat{\mathfrak{R}}(a,b)}(u) = min \left\{ T_{\widehat{\Gamma}(a)}(u), T_{\widehat{\Lambda}(b)}(u) \right\},$$
$$I_{\widehat{\mathfrak{R}}(a,b)}(u) = max \left\{ I_{\widehat{\Gamma}(a)}(u), I_{\widehat{\Lambda}(b)}(u) \right\}$$

and

$$F_{\widehat{\mathfrak{R}}(a,b)}(u) = max \left\{ F_{\widehat{\Gamma}(a)}(u), F_{\widehat{\Lambda}(b)}(u) \right\}.$$

If $(\widehat{\mathfrak{R}}, K \times L)$ is a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Gamma}, P)$, then is called a NSR on $(\widehat{\Gamma}, P)$.

Definition 3.4 Let $\widehat{\mathfrak{R}}$ be a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$. Then,

(i) the domain of $\widehat{\mathfrak{R}}$ $\left(DOM_{\widehat{\mathfrak{R}}}\right)$ is defined as the NSS $(\widehat{\mathfrak{D}}, K_1)$, where $K_1 = \{a \in K : \widehat{\mathfrak{R}}(a, b) \in \widehat{\mathfrak{R}}, \ for\ some\ b \in L\}$ and $\widehat{\mathfrak{D}}(a_1) = \Gamma(a_1), \forall a_1 \in K_1$.

(ii) the range of $\widehat{\mathfrak{R}}$ $\left(RAN_{\widehat{\mathfrak{R}}}\right)$ is defined as the NSS $(\widehat{RN}, L_1)$ where $L_1 = \{b \in L : \widehat{\mathfrak{R}}(a, b) \in \widehat{\mathfrak{R}}, \ for\ some\ a \in K\}$ and $\widehat{RN}(b_1) = \Lambda(b_1), \forall b_1 \in L_1$.

Example 3.5 Consider Example 3.2. Let $\widehat{\mathfrak{R}}$ be a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ as follows:

$$\widehat{\mathfrak{R}} = \left\{ \begin{array}{c} ((p_1, p_2), \langle u_1, 0.45, 0.6, 0.6\rangle, \langle u_2, 0.55, 0.65, 0.76\rangle, \langle u_3, 0.2, 0.65, 0.45\rangle), \\ ((p_2, p_2), \langle u_1, 0.45, 0.33, 0.6\rangle, \langle u_2, 0.75, 0.65, 0.7\rangle, \langle u_3, 0.2, 0.65, 0.57\rangle) \end{array} \right\}$$
$$\subseteq (\widehat{\Gamma}, P) \times (\widehat{\Lambda}, P).$$

Then $DOM_{\widehat{\mathfrak{R}}} = (\widehat{\mathfrak{D}}, K_1)$ where $K_1 = \{p_1, p_2\}$ and $\widehat{\mathfrak{D}}(a_1) = \Gamma(a_1), \forall a_1 \in K_1$, and $RAN_{\widehat{\mathfrak{R}}} = (\widehat{RN}, L_1)$ where $L_1 = \{p_2\}$ and $\widehat{RN}(b_1) = \Lambda(b_1), \forall b_1 \in L_1$.

Definition 3.6 The identity NSR $\widehat{I}_{(\widehat{\Gamma},P)}$ on a NSS $(\widehat{\Gamma}, P)$ is defined as $\widehat{\Gamma}(a)\widehat{I}_{(\widehat{\Gamma},P)}\widehat{\Gamma}(b)$ if and only if $a = b$.

Example 3.7 In Example 3.2, the relation

$$(\widehat{\mathfrak{R}}, K \times L) = \left\{ \begin{array}{l} ((p_1, p_1), \langle u_1, 0.34, 0.7, 0.35\rangle, \langle u_2, 0.55, 0.5, 0.76\rangle, \langle u_3, 0.6, 0.25, 0.7\rangle), \\ ((p_2, p_2), \langle u_1, 0.45, 0.33, 0.6\rangle, \langle u_2, 0.75, 0.65, 0.7\rangle, \langle u_3, 0.2, 0.65, 0.57\rangle) \end{array} \right\}$$

is an identity NSR.

Definition 3.8 Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be NSSs over U and $(\widehat{\mathfrak{R}}, K \times L)$ be a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$. Then the inverse of $(\widehat{\mathfrak{R}}, K \times L)$, $(\widehat{\mathfrak{R}}^{-1}, L \times K)$ is a NSR and is defined as:

$$\widehat{\mathfrak{R}}^{-1}(b, a) = \widehat{\mathfrak{R}}(a, b), \quad \forall (a, b) \in K \times L \subseteq P \times P.$$

Example 3.9 Consider Example 3.5. Then we have,

$$\widehat{\mathfrak{R}}^{-1} = \left\{ \begin{array}{l} ((p_2, p_1), \langle u_1, 0.34, 0.7, 0.56\rangle, \langle u_2, 0.75, 0.6, 0.6\rangle, \langle u_3, 0.25, 0.6, 0.7\rangle), \\ ((p_2, p_2), \langle u_1, 0.45, 0.33, 0.6\rangle, \langle u_2, 0.75, 0.65, 0.7\rangle, \langle u_3, 0.2, 0.65, 0.57\rangle) \end{array} \right\}.$$

Theorem 3.10 *Suppose* $(\widehat{\Gamma}, P)$ *and* $(\widehat{\Lambda}, P)$ *are NSSs over* U*, and* $(\widehat{\mathfrak{R}}_1, K \times L)$ *and* $(\widehat{\mathfrak{R}}_2, K \times L)$ *are NSRs from* $(\widehat{\Gamma}, P)$ *to* $(\widehat{\Lambda}, P)$*. Then,*

(i) $((\widehat{\mathfrak{R}}_1, K \times L)^{-1})^{-1} = (\widehat{\mathfrak{R}}_1, K \times L)$.
(ii) If $(\widehat{\mathfrak{R}}_1, K \times L)\widehat{\subseteq}(\widehat{\mathfrak{R}}_2, K \times L)$ *then* $(\widehat{\mathfrak{R}}_1, K \times L)^{-1}\widehat{\subseteq}(\widehat{\mathfrak{R}}_2, K \times L)^{-1}$.

Proof For all $(a, b) \in K \times L \subseteq P \times P$,

(i) $((\widehat{\mathfrak{R}}_1)^{-1})^{-1} = \widehat{\mathfrak{R}}_1(a, b)$, thus $((\widehat{\mathfrak{R}}_1, K \times L)^{-1})^{-1} = (\widehat{\mathfrak{R}}_1, K \times L)$.
(ii) If $\widehat{\mathfrak{R}}_1(a, b) \subseteq \widehat{\mathfrak{R}}_2(a, b)$, then $\widehat{\mathfrak{R}}_1^{-1}(b, a) \subseteq \widehat{\mathfrak{R}}_2^{-1}(b, a)$, and thus $(\widehat{\mathfrak{R}}_1, K \times L)^{-1}\widehat{\subseteq}(\widehat{\mathfrak{R}}_2, K \times L)^{-1}$. □

Definition 3.11 If $(\widehat{\Gamma}, P)$, $(\widehat{\Lambda}, P)$ and $(\widehat{\Omega}, P)$ are NSSs over U and $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ are NSRs from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ and from $(\widehat{\Lambda}, P)$ to $(\widehat{\Omega}, P)$ respectively, where $K \times L \subseteq P \times P$ and $L \times M \subseteq P \times P$, then the composition of the NSRs $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ denoted by $\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1$ from $(\widehat{\Gamma}, P)$ to $(\widehat{\Omega}, P)$ is defined as:

$$(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1, K \times M) = \left\{ \left((a, c), \langle u, \begin{array}{c} T_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}(u), \\ I_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}(u), \\ F_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}(u) \end{array} \rangle : u \in U \right) : (a, c) \in K \times M \subseteq P \times P \right\} \tag{9}$$

where for all $(a, b) \in K \times L \subseteq P \times P$ and $(b, c) \in L \times M \subseteq P \times P$,

$$T_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}(u) = max \left\{ T_{\widehat{\mathfrak{R}}_1(a,b)}(u), T_{\widehat{\mathfrak{R}}_2(b,c)}(u) \right\}$$

$$= max\left\{min\left\{T_{\widehat{\Gamma}(a)}(u), T_{\widehat{\Lambda}(b)}(u)\right\}, min\left\{T_{\widehat{\Lambda}(b)}(u), T_{\widehat{\Omega}(c)}(u)\right\}\right\},$$

$$I_{(\widehat{\mathfrak{R}}_2\circ\widehat{\mathfrak{R}}_1)(a,c)}(u) = min\left\{I_{\widehat{\mathfrak{R}}_1(a,b)}(u), I_{\widehat{\mathfrak{R}}_2(b,c)}(u)\right\}$$

$$= min\left\{max\left\{I_{\widehat{\Gamma}(a)}(u), I_{\widehat{\Lambda}(b)}(u)\right\}, max\left\{I_{\widehat{\Lambda}(b)}(u), I_{\widehat{\Omega}(c)}(u)\right\}\right\},$$

$$F_{(\widehat{\mathfrak{R}}_2\circ\widehat{\mathfrak{R}}_1)(a,c)}(u) = min\left\{F_{\widehat{\mathfrak{R}}_1(a,b)}(u), F_{\widehat{\mathfrak{R}}_2(b,c)}(u)\right\}$$

$$= min\left\{max\left\{F_{\widehat{\Gamma}(a)}(u), F_{\widehat{\Lambda}(b)}(u)\right\}, max\left\{F_{\widehat{\Lambda}(b)}(u), F_{\widehat{\Omega}(c)}(u)\right\}\right\}.$$

Example 3.12 Let $U = \{u_1, u_2, u_3\}$ be a set of staffs, $P = \{p_1, p_2, p_3, p_4\}$ be the set of parameters requested from staffs and $K, L, M \in P$. For $i = 1, 2, 3, 4$, the parameters p_i stand for "successful", "hardworking", "talented" and "experienced", respectively.

Suppose that the three experts' views on the staff with the help of the NSSs $(\widehat{\Gamma}, P)$, $(\widehat{\Lambda}, P)$ and $(\widehat{\Omega}, P)$ are defined as:

$$(\widehat{\Gamma}, P) = \left\{\begin{array}{c} (p_1, \langle u_1, 0.5, 0.65, 0.23\rangle, \langle u_2, 0.65, 0.443, 0.64\rangle, \langle u_3, 0.45, 0.25, 0.65\rangle), \\ (p_2, \langle u_1, 0.7, 0.43, 0.36\rangle, \langle u_2, 0.5, 0.6, 0.3\rangle, \langle u_3, 0.52, 0.48, 0.7\rangle), \\ (p_3, \langle u_1, 0.56, 0.46, 0.15\rangle, \langle u_2, 0.6, 0.74, 0.62\rangle, \langle u_3, 0.55, 0.24, 0.57\rangle), \\ (p_4, \langle u_1, 0.75, 0.32, 0.64\rangle, \langle u_2, 0.6, 0.67, 0.34\rangle, \langle u_3, 0.25, 0.6, 0.57\rangle) \end{array}\right\},$$

$$(\widehat{\Lambda}, P) = \left\{\begin{array}{c} (p_1, \langle u_1, 0.2, 0.35, 0.8\rangle, \langle u_2, 0.9, 0.5, 0.6\rangle, \langle u_3, 0.45, 0.25, 0.5\rangle), \\ (p_2, \langle u_1, 0.9, 0.5, 0.4\rangle, \langle u_2, 0.5, 0.65, 0.32\rangle, \langle u_3, 0.45, 0.5, 0.7\rangle), \\ (p_3, \langle u_1, 0.6, 0.7, 0.25\rangle, \langle u_2, 0.75, 0.34, 0.84\rangle, \langle u_3, 0.6, 0.2, 0.45\rangle), \\ (p_4, \langle u_1, 0.7, 0.34, 0.6\rangle, \langle u_2, 0.8, 0.65, 0.45\rangle, \langle u_3, 0.7, 0.64, 0.72\rangle) \end{array}\right\},$$

$$(\widehat{\Omega}, P) = \left\{\begin{array}{c} (p_1, \langle u_1, 0.67, 0.65, 0.24\rangle, \langle u_2, 0.7, 0.48, 0.64\rangle, \langle u_3, 0.56, 0.27, 0.58\rangle), \\ (p_2, \langle u_1, 0.74, 0.38, 0.76\rangle, \langle u_2, 0.65, 0.7, 0.46\rangle, \langle u_3, 0.28, 0.64, 0.72\rangle), \\ (p_3, \langle u_1, 0.65, 0.45, 0.18\rangle, \langle u_2, 0.67, 0.82, 0.92\rangle, \langle u_3, 0.34, 0.25, 0.65\rangle), \\ (p_4, \langle u_1, 0.75, 0.35, 0.64\rangle, \langle u_2, 0.85, 0.7, 0.45\rangle, \langle u_3, 0.54, 0.67, 0.83\rangle) \end{array}\right\}.$$

Accordingly, the relationships and effects between the parameters requested from staffs can be expressed. For example; if $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ are NSRs from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ and from $(\widehat{\Lambda}, P)$ to $(\widehat{\Omega}, P)$ respectively, where $K \times L \subseteq P \times P$ and $L \times M \subseteq P \times P$ for $K = \{p_2, p_3\}$, $L = \{p_1\}$, $M = \{p_4\}$, then these NSRs are defined as:

$$(\widehat{\mathfrak{R}}_1, K \times L) = \left\{\begin{array}{c} ((p_2, p_1), \langle u_1, 0.2, 0.43, 0.8\rangle, \langle u_2, 0.5, 0.6, 0.6\rangle, \langle u_3, 0.45, 0.25, 0.65\rangle), \\ ((p_3, p_1), \langle u_1, 0.2, 0.46, 0.8\rangle, \langle u_2, 0.6, 0.74, 0.62\rangle, \langle u_3, 0.45, 0.25, 0.57\rangle) \end{array}\right\},$$

$$(\widehat{\mathfrak{R}}_2, L \times M) = \left\{((p_1, p_4), \langle u_1, 0.2, 0.35, 0.8\rangle, \langle u_2, 0.85, 0.7, 0.6\rangle, \langle u_3, 0.45, 0.67, 0.83\rangle),\right\}.$$

Here, the NSR $(\widehat{\mathfrak{R}}_1, K \times L)$ describes the effect of parameters in K on being successful, where it measures the falsity, indeterminacy and true degrees for a hardworking or talented staff to be successful. Similarly, the NSR $(\widehat{\mathfrak{R}}_2, L \times M)$ expresses the effect of a successful staff on being experienced, where it measures the falsity, indeterminacy and true degrees for a successful staff to be experienced.

The combination of these two relationships can help us understand the impact that a hardworking or talented staff can have on being experienced. The composition between the NSRs $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ is:

$$(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1, K \times M) = \left\{ \begin{array}{l} ((p_2, p_4), \langle u_1, 0.2, 0.35, 0.8\rangle, \langle u_2, 0.85, 0.6, 0.6\rangle, \langle u_3, 0.45, 0.25, 0.65\rangle), \\ ((p_3, p_4), \langle u_1, 0.2, 0.35, 0.8\rangle, \langle u_2, 0.85, 0.7, 0.6\rangle, \langle u_3, 0.45, 0.25, 0.57\rangle) \end{array} \right\}$$

$T_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}$, $I_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}$ and $F_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}$ represent respectively, the degrees of true, indeterminacy and falsity for a hardworking or talented staff to be experienced. For example, the term $\langle u_2, 0.85, 0.6, 0.6\rangle$ for the parameter pair (p_2, p_4) means that the hardworking of the staff SS has a 0.85 truth degree of in being experienced, 0.6 indeterminacy degree of in being experienced, 0.6 falsity degree of in being experienced.

Theorem 3.13 *Let $(\widehat{\Gamma}, P)$, $(\widehat{\Lambda}, P)$ and $(\widehat{\Omega}, P)$ be NSSs over U and $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ be NSRs from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ and from $(\widehat{\Lambda}, P)$ to $(\widehat{\Omega}, P)$ respectively, where $K \times L \subseteq P \times P$ and $L \times M \subseteq P \times P$. Then $(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1, K \times M)^{-1} = (\widehat{\mathfrak{R}}_1^{-1} \circ \widehat{\mathfrak{R}}_2^{-1}, M \times K)$.*

Proof If $(\widehat{\mathfrak{R}}_1, K \times L)$ and $(\widehat{\mathfrak{R}}_2, L \times M)$ are NSRs from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ and $(\widehat{\Lambda}, P)$ to $(\widehat{\Omega}, P)$ respectively, then $(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1, K \times M)\widehat{\subseteq}(\widehat{\Gamma}, P) \times (\widehat{\Omega}, P)$. Now, for $(a, c) \in K \times M$, $u \in U$,

$$\begin{aligned}
T_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)^{-1}(c,a)}(u) &= T_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)(a,c)}(u) \\
&= max\left\{T_{\widehat{\mathfrak{R}}_1(a,b)}(u), T_{\widehat{\mathfrak{R}}_2(b,c)}(u)\right\} \\
&= max\left\{T_{\widehat{\mathfrak{R}}_2(b,c)}(u), T_{\widehat{\mathfrak{R}}_1(a,b)}(u)\right\} \\
&= max\left\{min\left\{T_{\widehat{\Lambda}(b)}(u), T_{\widehat{\Omega}(c)}(u)\right\}, min\left\{T_{\widehat{\Gamma}(a)}(u), T_{\widehat{\Lambda}(b)}(u)\right\}\right\} \\
&= max\left\{min\left\{T_{\widehat{\Omega}(c)}(u), T_{\widehat{\Lambda}(b)}(u)\right\}, min\left\{T_{\widehat{\Lambda}(b)}(u), T_{\widehat{\Gamma}(a)}(u)\right\}\right\} \\
&= max\left\{T_{\widehat{\mathfrak{R}}_2^{-1}(c,b)}(u), T_{\widehat{\mathfrak{R}}_1^{-1}(b,a)}(u)\right\} = T_{(\widehat{\mathfrak{R}}_1^{-1} \circ \widehat{\mathfrak{R}}_2^{-1})(c,a)}(u)
\end{aligned}$$

Similar results can be shown for components $I_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)^{-1}(c,a)}(u)$ and $F_{(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)^{-1}(c,a)}(u)$. This completes the proof. □

4 Partitions on NSSs

In this section, various types of NSRs given in the previous section are defined. In addition, the equivalence classes and partitions of NSSs are analyzed by giving them together with related theorems.

Definition 4.1 Let $\widehat{\mathfrak{R}}$ be a NSR on $(\widehat{\Gamma}, P)$, then

(i) $(\widehat{\mathfrak{R}}, K \times L)$ is reflexive NSR (briefly RNSR) if $\widehat{\mathfrak{R}}(a, a) \in (\widehat{\mathfrak{R}}, K \times L)$, $\forall a \in P$, $(a, a) \in K \times L \in P \times P$.
(ii) $(\widehat{\mathfrak{R}}, K \times L)$ is symmetric NSR (briefly SNSR) if $\widehat{\mathfrak{R}}(a, b) \in (\widehat{\mathfrak{R}}, K \times L) \Longrightarrow \widehat{\mathfrak{R}}(b, a) \in (\widehat{\mathfrak{R}}, K \times L)$, $\forall a, b \in P$ $(a, b) \in K \times L \subseteq P \times P$.

(iii) $(\widehat{\mathfrak{R}}, K \times L)$ is transitive NSR (briefly TNSR) if $\widehat{\mathfrak{R}}(a,b) \in (\widehat{\mathfrak{R}}, K \times L)$ and $\widehat{\mathfrak{R}}(b,c) \in (\widehat{\mathfrak{R}}, K \times L) \implies \widehat{\mathfrak{R}}(a,c) \in (\widehat{\mathfrak{R}}, K \times L)$, $\forall a,b,c \in P$ $(a,b),(a,c),(b,c) \in K \times L \subseteq P \times P$.
(iv) $(\widehat{\mathfrak{R}}, K \times L)$ is a neutrosophic soft equivalence relation (briefly NSER) if it is RNSR, SNSR and TNSR.

Example 4.2 Reconsider Example 3.2. Suppose a NSR $(\widehat{\mathfrak{R}}, K \times L)$ for the NSS $(\widehat{\Gamma}, P)$ given in this example is defined by:

$$(\widehat{\mathfrak{R}}, K \times L) = \left\{ \begin{array}{c} ((p_1,p_1), \langle u_1, 0.45, 0.6, 0.2\rangle, \langle u_2, 0.55, 0.43, 0.76\rangle, \langle u_3, 0.6, 0.2, 0.45\rangle), \\ ((p_1,p_2), \langle u_1, 0.45, 0.6, 0.56\rangle, \langle u_2, 0.55, 0.6, 0.76\rangle, \langle u_3, 0.25, 0.6, 0.57\rangle), \\ ((p_2,p_1), \langle u_1, 0.45, 0.6, 0.56\rangle, \langle u_2, 0.55, 0.6, 0.76\rangle, \langle u_3, 0.25, 0.6, 0.57\rangle), \\ ((p_2,p_2), \langle u_1, 0.55, 0.3, 0.56\rangle, \langle u_2, 0.75, 0.6, 0.4\rangle, \langle u_3, 0.25, 0.6, 0.57\rangle) \end{array} \right\}.$$

This NSR is a NSER.

Definition 4.3 Let $(\widehat{\Gamma}, P)$ be a NSS. Then the neutrosophic soft equivalence class of $\widehat{\Gamma}(a)$ is defined as

$$[\widehat{\Gamma}(a)] = \left\{ \widehat{\Gamma}(b) : \widehat{\Gamma}(b)\widehat{\mathfrak{R}}\widehat{\Gamma}(a), \forall a, b \in P \right\}. \tag{10}$$

Example 4.4 Consider Example 4.2. Then have $[\widehat{\Gamma}(p_1)] = \{\widehat{\Gamma}(p_1), \widehat{\Gamma}(p_2)\} = [\widehat{\Gamma}(p_2)]$.

Lemma 4.5 *Let $(\widehat{\mathfrak{R}}, K \times L)$ be an NSER on NSS $(\widehat{\Gamma}, P)$. For any $\widehat{\Gamma}(a), \widehat{\Gamma}(b) \in (\widehat{\Gamma}, P)$, $\widehat{\Gamma}(a)\widehat{\mathfrak{R}}\widehat{\Gamma}(b)$ if and only if $[\widehat{\Gamma}(a)] = [\widehat{\Gamma}(b)]$.*

Proof $(\Rightarrow)$ Let $\widehat{\Gamma}(a_1) \in [\widehat{\Gamma}(a)]$. Then $\widehat{\Gamma}(a_1)\widehat{\mathfrak{R}}\widehat{\Gamma}(a)$. Since $(\widehat{\mathfrak{R}}, K \times L)$ is an NSER (i.e., using the transitive property), then $\widehat{\Gamma}(a_1) \in [\widehat{\Gamma}(b)]$. Hence $[\widehat{\Gamma}(a)] \subseteq [\widehat{\Gamma}(b)]$. Similarly $[\widehat{\Gamma}(b)] \subseteq [\widehat{\Gamma}(a)]$. Thus, $[\widehat{\Gamma}(a)] = [\widehat{\Gamma}(b)]$.

$(\Leftarrow)$ Since $(\widehat{\mathfrak{R}}, K \times L)$ is an NSER (i.e., using the reflexive property), then $\widehat{\Gamma}(b)\widehat{\mathfrak{R}}\widehat{\Gamma}(b)$. Hence $\widehat{\Gamma}(b) \in [\widehat{\Gamma}(b)] = [\widehat{\Gamma}(a)]$ which gives $\widehat{\Gamma}(a)\widehat{\mathfrak{R}}\widehat{\Gamma}(b)$. □

Definition 4.6 A collection of nonempty neutrosophic soft subsets $\rho = \{(\widehat{\Gamma}_i, P_i) : i \in I\}$ of a NSS $(\widehat{\Gamma}, P)$ is called a NSS partition (briefly NSS-PART) of $(\widehat{\Gamma}, P)$ such that $(\widehat{\Gamma}, P) = \widehat{\cup}_i(\widehat{\Gamma}_i, P_i)$ and $P_i \cap P_j = \emptyset$, whenever $i \neq j$.

Example 4.7 Let $U = \{u_1, u_2\}$ be an universal set, $P = \{p_1, p_2, p_3\}$ be a set of parameters and

$$(\widehat{\Gamma}, P) = \left\{ \begin{array}{c} (p_1, \langle u_1, 0.77, 0.65, 0.4\rangle, \langle u_2, 0.6, 0.47, 0.8\rangle), \\ (p_2, \langle u_1, 0.8, 0.35, 0.6\rangle, \langle u_2, 0.7, 0.55, 0.62\rangle), \\ (p_3, \langle u_1, 0.67, 0.34, 0.76\rangle, \langle u_2, 0.14, 0.34, 0.45\rangle) \end{array} \right\}$$

be a NSS over U. Suppose $P_1 = \{p_2, p_3\}$, $P_2 = \{p_1\}$, where $(\widehat{\Gamma}_1, P_1) = \left\{\widehat{\Gamma}_1(p_2), \widehat{\Gamma}_1(p_3)\right\}$ and $(\widehat{\Gamma}_2, P_2) = \left\{\widehat{\Gamma}_2(p_1)\right\}$ are neutrosophic soft subsets of $(\widehat{\Gamma}, P)$ such that $\widehat{\Gamma}_i = \widehat{\Gamma}$ for $i = 1, 2$. Since $P_1 \cap P_2 = \emptyset$ and $(\widehat{\Gamma}_1, P_1)\widehat{\cup}(\widehat{\Gamma}_2, P_2) = (\widehat{\Gamma}, P)$, then $\left\{(\widehat{\Gamma}_1, P_1), (\widehat{\Gamma}_2, P_2)\right\}$ is a NSS-PART of $(\widehat{\Gamma}, P)$.

Definition 4.8 Elements of the NSS-PART are called a NSS-block of $(\widehat{\Gamma}, P)$.

Theorem 4.9 *Let* $\{(\widehat{\Gamma}_i, P_i) : i \in I\}$ *be a NSS-PART of the NSS* $(\widehat{\Gamma}, P)$. *The neutrosophic soft relation defined on* $(\widehat{\Gamma}, P)$ *as* $\widehat{\Gamma}(a)\widehat{\mathfrak{R}}\widehat{\Gamma}(b)$ *is an NSER if and only if* $\widehat{\Gamma}(a)$ *and* $\widehat{\Gamma}(b)$ *are elements of the same block.*

Proof (i) It is clear that any element of the NSS $(\widehat{\Gamma}, P)$ is in the same block itself. Hence $\widehat{\mathfrak{R}}(a, a) \in (\widehat{\mathfrak{R}}, K \times L)$ and $(\widehat{\mathfrak{R}}, K \times L)$ is RNSR.

(ii) If $\widehat{\mathfrak{R}}(a, b) \in (\widehat{\mathfrak{R}}, K \times L)$, then $\widehat{\Gamma}(a)$ and $\widehat{\Gamma}(b)$ are in the same block, i.e., $\widehat{\mathfrak{R}}(b, a) \in (\widehat{\mathfrak{R}}, K \times L)$ and so it is SNSR.

(iii) If $\widehat{\mathfrak{R}}(a, b) \in (\widehat{\mathfrak{R}}, K \times L)$ and $\widehat{\mathfrak{R}}(b, c) \in (\widehat{\mathfrak{R}}, K \times L)$, then $\widehat{\Gamma}(a)$, $\widehat{\Gamma}(b)$ and $\widehat{\Gamma}(c)$ are in the same block, i.e., $\widehat{\mathfrak{R}}(a, c) \in (\widehat{\mathfrak{R}}, K \times L)$ and so it is TNSR. □

Remark 4.10 The NSR given in Theorem 4.9 is called a NSER determined by the NSS-PART ρ. For example, the NSER determined by the partition $\rho = \{(\widehat{\Gamma}_1, P_1), (\widehat{\Gamma}_2, P_2)\}$ in Example 4.7 is as follows:

$$\widehat{\mathfrak{R}} = \{\widehat{\Gamma}(p_1) \times \widehat{\Gamma}(p_1), \widehat{\Gamma}(p_2) \times \widehat{\Gamma}(p_2), \widehat{\Gamma}(p_3) \times \widehat{\Gamma}(p_3), \widehat{\Gamma}(p_2) \times \widehat{\Gamma}(p_3), \widehat{\Gamma}(p_3) \times \widehat{\Gamma}(p_2)\}$$

Theorem 4.11 *Corresponding to every NSER defined on a NSS* $(\widehat{\Gamma}, P)$ *there exists a NSS-PART on* $(\widehat{\Gamma}, P)$ *and this NSS-PART precisely consists of the neutrosophic soft equivalence classes of* $(\widehat{\mathfrak{R}}, K \times L)$.

Proof Let $[\widehat{\Gamma}(a)]$ be neutrosophic soft equivalence class with respect to a NSR $(\widehat{\mathfrak{R}}, K \times L)$ on $(\widehat{\Gamma}, P)$. Here, we can denote $[\widehat{\Gamma}(a)]$ as $(\widehat{\Gamma}, P_a)$ for $P_a = \{b \in P : \widehat{\mathfrak{R}}(b, a) \in (\widehat{\mathfrak{R}}, K \times L)\}$. Thus, we have to show that the collection $\{(\widehat{\Gamma}, P_a) : a \in P\}$ is a NSS-PART ρ of $(\widehat{\Gamma}, P)$. For this we must show from Definition 4.6 that the following two conditions are satisfied:

(i) $(\widehat{\Gamma}, P) = \widehat{\bigcup}_{a \in P}(\widehat{\Gamma}, P_a)$,

(ii) $P_a \cap P_b = \emptyset$ for $a \neq b$.

Since $\widehat{\mathfrak{R}}$ is an NSER (i.e., using the reflexise property), $\widehat{\mathfrak{R}}(a, a) \in (\widehat{\mathfrak{R}}, K \times L)$, $\forall a \in P$, i.e., condition (i) is implemented.

Let $p \in P_a \cap P_b$ for the second part. Since $\widehat{\Gamma}(p) \in (\widehat{\Gamma}, P_a)$ and $\widehat{\Gamma}(p) \in (\widehat{\Gamma}, P_b)$, then $\widehat{\mathfrak{R}}(p, a) \in (\widehat{\mathfrak{R}}, K \times L)$ and $\widehat{\mathfrak{R}}(p, b) \in (\widehat{\mathfrak{R}}, K \times L)$. Then, using the transitive property of $\widehat{\mathfrak{R}}$, we have $\widehat{\mathfrak{R}}(a, b) \in (\widehat{\mathfrak{R}}, K \times L)$. Moreover, using Lemma 4.5 we have $[\widehat{\Gamma}(a)] = [\widehat{\Gamma}(b)]$. However, since $P_a = P_b$ is obtained from here, then this is a contradiction, i.e. $P_a \neq P_b$, hence $P_a \cap P_b = \emptyset$. □

Remark 4.12 The NSS-PART in Theorem 4.11 where we construct all the neutrosophic soft equivalence classes of $(\widehat{\mathfrak{R}}, K \times L)$ is called the quotient NSS of $(\widehat{\Gamma}, P)$ and is denoted by $(\widehat{\Gamma}, P) \setminus (\widehat{\mathfrak{R}}, K \times L)$.

5 Neutrosophic soft functions

In this section, the concept of neutrosophic soft function (briefly NSF) is defined and some special types of this concept are presented together with related theorems.

Definition 5.1 Let $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ be two nonempty NSS. Then a NSR $\mathfrak{f}$ from $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ is called a NSF if every element in the domain has a unique element in the range. We write $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$. If $\widehat{\Gamma}(a)\mathfrak{f}\widehat{\Lambda}(b)$ then $\mathfrak{f}(\widehat{\Gamma}(a)) = \widehat{\Lambda}(b)$, $\forall a, b \in P$.

Example 5.2 Consider Example 3.2. Then, the NSR $\mathfrak{f}$ forms a neutrosophic soft function from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ as follows:

$$\mathfrak{f} = \left\{ \begin{array}{c} ((p_1, p_1), \langle u_1, 0.34, 0.7, 0.35\rangle, \langle u_2, 0.55, 0.5, 0.76\rangle, \langle u_3, 0.6, 0.25, 0.7\rangle), \\ ((p_2, p_1), \langle u_1, 0.34, 0.7, 0.56\rangle, \langle u_2, 0.75, 0.6, 0.6\rangle, \langle u_3, 0.25, 0.6, 0.7\rangle) \end{array} \right\}.$$

Definition 5.3 Let $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ be a NSF. Then,

(i) $\mathfrak{f}$ is injective NSF (briefly INSF) if $\widehat{\Gamma}(a_1) \neq \widehat{\Gamma}(a_2)$ implying $\mathfrak{f}(\widehat{\Gamma}(a_1)) \neq \mathfrak{f}(\widehat{\Gamma}(a_2))$ for $\widehat{\Gamma}(a_1), \widehat{\Gamma}(a_2) \in (\widehat{\Gamma}, P)$.
(ii) $\mathfrak{f}$ is surjective NSF (briefly SNSF) if $\mathfrak{f}((\widehat{\Gamma}, P)) = (\widehat{\Lambda}, P)$.
(iii) $\mathfrak{f}$ is bijective NSF (briefly BNSF) if it is both INSF and SNSF.
(iv) $\mathfrak{f}$ is a constant NSF (briefly CNSF) if all elements in $DOM_{\mathfrak{f}}$ have the same image.
(v) $\mathfrak{f}$ is an identity NSF (briefly IDNSF) if the NSF $\widehat{I_{\mathfrak{f}}}$ on a NSS $(\widehat{\Gamma}, P)$ is defined by $\widehat{I_{\mathfrak{f}}} : (\widehat{\Gamma}, P) \to (\widehat{\Gamma}, P)$ as $\widehat{I_{\mathfrak{f}}}(\widehat{\Gamma}(a)) = \widehat{\Gamma}(a)$ for every $\widehat{\Gamma}(a)$ in $(\widehat{\Gamma}, P)$.

Definition 5.4 Let $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ and $\mathfrak{g} : (\widehat{\Lambda}, P) \to (\widehat{\Omega}, P)$ be two NSFs. Then $\mathfrak{g} \circ \mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ is also a NSF defined by $(\mathfrak{g} \circ \mathfrak{f})(\widehat{\Gamma}(a)) = \mathfrak{g}(\mathfrak{f}(\widehat{\Gamma}(a)))$.

Definition 5.5 Let $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ be a BNSF. Then the inverse NSR $\mathfrak{f}^{-1} : (\widehat{\Lambda}, P) \to (\widehat{\Gamma}, P)$ is called the inverse NSF.

Theorem 5.6 *If $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ is BNSF then $\mathfrak{f}^{-1} : (\widehat{\Lambda}, P) \to (\widehat{\Gamma}, P)$ is also a BNSF.*

Proof Let $\widehat{\Lambda}(b_1) \neq \widehat{\Lambda}(b_2)$ for $\widehat{\Lambda}(b_1), \widehat{\Lambda}(b_2) \in (\widehat{\Lambda}, P)$. In this case, this we should prove $\mathfrak{f}^{-1}(\widehat{\Lambda}(b_1)) \neq \mathfrak{f}^{-1}\widehat{\Lambda}(b_2)$. Let $\mathfrak{f}^{-1}(\widehat{\Lambda}(b_1)) = \widehat{\Gamma}(a_1)$ and $\mathfrak{f}^{-1}(\widehat{\Lambda}(b_2)) = \widehat{\Gamma}(a_2)$, i.e., $\widehat{\Lambda}(b_1) = \mathfrak{f}(\widehat{\Gamma}(a_1))$ and $\widehat{\Lambda}(b_2) = \mathfrak{f}(\widehat{\Gamma}(a_2))$. Since $\mathfrak{f}$ is BNSF, $\mathfrak{f}(\widehat{\Gamma}(a_1)) \neq \mathfrak{f}(\widehat{\Gamma}(a_2))$ implies $\widehat{\Gamma}(a_1) \neq \widehat{\Gamma}(a_2)$. Thus $\mathfrak{f}^{-1}$ is BNSF.

Let $\widehat{\Gamma}(a)$ is an element of $(\widehat{\Gamma}, P)$. Since $\mathfrak{f}$ is SNSF, there exists a unique element $\widehat{\Lambda}(b)$ in $(\widehat{\Lambda}, P)$ such that $\mathfrak{f}(\widehat{\Gamma}(a)) = \widehat{\Lambda}(b)$ implies $\widehat{\Gamma}(a) = \mathfrak{f}^{-1}(\widehat{\Lambda}(b))$ for $\widehat{\Gamma}(a)$ in $(\widehat{\Gamma}, P)$. Thus $\mathfrak{f}^{-1}$ is SNSF. Hence, $\mathfrak{f}^{-1}$ is BNSF. □

Theorem 5.7 *Let $\mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$, $\mathfrak{g} : (\widehat{\Lambda}, P) \to (\widehat{\Omega}, P)$ be two BNSFs. Then,*

(i) $\mathfrak{g} \circ \mathfrak{f} : (\widehat{\Gamma}, P) \to (\widehat{\Lambda}, P)$ *is a BNSF.*

(ii) $(\mathfrak{g} \circ \mathfrak{f})^{-1} = \mathfrak{f}^{-1} \circ \mathfrak{g}^{-1}$.

Proof (i) Let $\widehat{\Gamma}(a_1) \neq \widehat{\Gamma}(a_2)$, i.e., $\widehat{\Gamma}(a_1)$, $\widehat{\Gamma}(a_2)$ be two distinct elements of $(\widehat{\Gamma}, P)$. In this case, this we should prove $(\mathfrak{g} \circ \mathfrak{f})(\widehat{\Gamma}(a_1)) \neq (\mathfrak{g} \circ \mathfrak{f})(\widehat{\Gamma}(a_2))$. Since $\mathfrak{f}$ and $\mathfrak{g}$ are BNSFs, then we have $\mathfrak{f}(\widehat{\Gamma}(a_1)) \neq \mathfrak{f}(\widehat{\Gamma}(a_2))$ and $\mathfrak{g}\left(\mathfrak{f}(\widehat{\Gamma}(a_1))\right) \neq \mathfrak{g}\left(\mathfrak{f}(\widehat{\Gamma}(a_2))\right)$. Hence $(\mathfrak{g} \circ \mathfrak{f}$ is BNSF.

Let $\widehat{\Omega}(c)$ is an element of $(\widehat{\Omega}, P)$. Since $\mathfrak{g}$ is SNSF, then there exists $\widehat{\Lambda}(b)$ in $(\widehat{\Lambda}, P)$ such that $\mathfrak{g}(\widehat{\Lambda}(b)) = \widehat{\Omega}(c)$. Moreover; since $\mathfrak{f}$ is SNSF, then there exists $\widehat{\Gamma}(a)$ in $(\widehat{\Gamma}, P)$ such that $\mathfrak{f}(\widehat{\Gamma}(a)) = \widehat{\Lambda}(b)$. Finally; since $(\mathfrak{g} \circ \mathfrak{f})(\widehat{\Gamma}(a)) = \widehat{\Omega}(c)$ for every $\widehat{\Omega}(c)$ in $(\widehat{\Omega}, P)$, then $\mathfrak{g} \circ \mathfrak{f}$ is SNSF. Hence, $\mathfrak{g} \circ \mathfrak{f}$ is BNSF.

(ii) We have from Theorem 3.13 that $(\widehat{\mathfrak{R}}_2 \circ \widehat{\mathfrak{R}}_1)^{-1} = \widehat{\mathfrak{R}}_1^{-1} \circ \widehat{\mathfrak{R}}_2^{-1}$. Thus, since $\mathfrak{f}$, $\mathfrak{g}$ and $\mathfrak{g} \circ \mathfrak{f}$ are BNSFs, then $(\mathfrak{g} \circ \mathfrak{f})^{-1} = \mathfrak{f}^{-1} \circ \mathfrak{g}^{-1}$. □

6 Decision making

In this section, we present an application of NSRs in a decision-making problem. Some of it is quoted from in [12,50–52].

The uncertainty problem we are dealing with is as bellow:

Suppose some individuals want to choose the house that suits them best. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of existing houses, $P = \{p_1, p_2, p_3\}$ be the set containing all the parameters that individuals want from the house they will live in according to their own preferences. For $i = 1, 2, 3$, the parameters p_i stand for "comfortable", "cheap" and "surrounded by green", respectively.

Suppose two real estate agents evaluate existing houses based on selected parameters with the help of NSSs $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ are defined as:

$$(\widehat{\Gamma}, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{c} \langle u_1, 0.5, 0.45, 0.42\rangle, \langle u_2, 0.6, 0.4, 0.62\rangle, \langle u_3, 0.5, 0.32, 0.5\rangle, \\ \langle u_4, 0.7, 0.3, 0.5\rangle, \langle u_5, 0.8, 0.35, 0.6\rangle, \langle u_6, 0.6, 0.15, 0.1\rangle \end{array} \right), \\ \left(p_2, \begin{array}{c} \langle u_1, 0.56, 0.3, 0.4\rangle, \langle u_2, 0.9, 0.5, 0.6\rangle, \langle u_3, 0.64, 0.25, 0.53\rangle, \\ \langle u_4, 0.58, 0.64, 0.24\rangle, \langle u_5, 0.82, 0.35, 0.64\rangle, \langle u_6, 0.68, 0.25, 0.52\rangle \end{array} \right), \\ \left(p_3, \begin{array}{c} \langle u_1, 0.74, 0.63, 0.25\rangle, \langle u_2, 0.56, 0.35, 0.64\rangle, \langle u_3, 0.69, 0.27, 0.54\rangle, \\ \langle u_4, 0.57, 0.65, 0.28\rangle, \langle u_5, 0.58, 0.39, 0.62\rangle, \langle u_6, 0.87, 0.24, 0.52\rangle \end{array} \right) \end{array} \right\},$$

$$(\widehat{\Lambda}, P) = \left\{ \begin{array}{l} \left(p_1, \begin{array}{c} \langle u_1, 0.8, 0.65, 0.23\rangle, \langle u_2, 0.95, 0.3, 0.63\rangle, \langle u_3, 0.64, 0.25, 0.56\rangle, \\ \langle u_4, 0.57, 0.65, 0.2\rangle, \langle u_5, 0.56, 0.39, 0.63\rangle, \langle u_6, 0.96, 0.27, 0.65\rangle \end{array} \right), \\ \left(p_2, \begin{array}{c} \langle u_1, 0.65, 0.67, 0.28\rangle, \langle u_2, 0.59, 0.48, 0.46\rangle, \langle u_3, 0.67, 0.25, 0.75\rangle, \\ \langle u_4, 0.57, 0.68, 0.25\rangle, \langle u_5, 0.15, 0.83, 0.26\rangle, \langle u_6, 0.92, 0.25, 0.63\rangle \end{array} \right), \\ \left(p_3, \begin{array}{c} \langle u_1, 0.88, 0.65, 0.22\rangle, \langle u_2, 0.57, 0.34, 0.66\rangle, \langle u_3, 0.65, 0.28, 0.52\rangle, \\ \langle u_4, 0.59, 0.64, 0.25\rangle, \langle u_2, 0.15, 0.63, 0.64\rangle, \langle u_3, 0.63, 0.25, 0.12\rangle \end{array} \right) \end{array} \right\},$$

By applying the algorithm below, we can choose the house where the relationship between the parameters given in the problem is the best. (In other words, the harmony between parameters is desired to be the best.)

Step 1 Input the NSSs $(\widehat{\Gamma}, P)$ and $(\widehat{\Lambda}, P)$ over U.
Step 2 Construct a NSR $(\widehat{\mathfrak{R}}, K \times L)$ as requested for $K \times L \subseteq P \times P$.

Table 1 Comparison table

U	(p_1, p_2)	(p_1, p_3)	(p_2, p_3)
u_1	**0.75**	**0.73**	0.81
u_2	0.45	0.31	0.41
u_3	0.07	0.3	0.4
u_4	**0.75**	**0.73**	**0.97**
u_5	0.38	0.14	0.14
u_6	0.26	**0.73**	0.36

Table 2 Comparison table

(p_1, p_2)	(p_1, p_3)	(p_2, p_3)
u_1, u_4	u_1, u_4, u_6	u_4
0.75	0.73	0.97

Step 3 Calculate the score of the relationship between parameters according to $\widehat{\mathfrak{R}}$ for each object using the formula:

$$T_{\widehat{\mathfrak{R}}(a,b)}(u) + I_{\widehat{\mathfrak{R}}(a,b)}(u) - F_{\widehat{\mathfrak{R}}(a,b)}(u), \quad \forall u \in U, \quad (a, b) \in K \times L \subseteq P \times P.$$

Step 4 Create a comparison table for the calculations in Step 3.
Step 5 Determine the highest values for each column in the created table.
Step 6 Create the score table showing the objects with the highest values corresponding to each pair of parameters (Tables 1 and 2).
Step 7 Calculate the score Ξ_u for each object by taking the sum of these values.
Step 8 Find u_k, for which $\Xi_{u_k} = max_{u \in U} \{\Xi_u\}$.

Remark 6.1 If there is more than one value u_k, any of these objects can be selected.

Now let's express a NSR from $(\widehat{\Gamma}, P)$ to $(\widehat{\Lambda}, P)$ by using the existing NSSs in the uncertainty problem as follows:

$$\widehat{\mathfrak{R}} = \left\{ \begin{array}{l} \left((p_1, p_2), \begin{array}{c} \langle u_1, 0.5, 0.67, 0.42\rangle, \langle u_2, 0.59, 0.48, 0.62\rangle, \langle u_3, 0.5, 0.32, 0.75\rangle, \\ \langle u_4, 0.57, 0.68, 0.5\rangle, \langle u_5, 0.15, 0.83, 0.6\rangle, \langle u_6, 0.64, 0.25, 0.63\rangle \end{array} \right), \\ \left((p_1, p_3), \begin{array}{c} \langle u_1, 0.5, 0.65, 0.42\rangle, \langle u_2, 0.57, 0.4, 0.66\rangle, \langle u_3, 0.5, 0.32, 0.52\rangle, \\ \langle u_4, 0.59, 0.64, 0.5\rangle, \langle u_5, 0.15, 0.63, 0.64\rangle, \langle u_6, 0.6, 0.25, 0.12\rangle \end{array} \right), \\ \left((p_2, p_3), \begin{array}{c} \langle u_1, 0.56, 0.65, 0.4\rangle, \langle u_2, 0.57, 0.5, 0.66\rangle, \langle u_3, 0.65, 0.28, 0.53\rangle, \\ \langle u_4, 0.58, 0.64, 0.25\rangle, \langle u_5, 0.15, 0.63, 0.64\rangle, \langle u_6, 0.63, 0.25, 0.52\rangle \end{array} \right) \end{array} \right\}.$$

By calculating as expressed in the algorithm, the comparison table and score table were constructed as follows:

The score Ξ_u of each object by sum of these values are:

$$\Xi_{u_1} = 1.48, \quad \Xi_{u_2} = 0, \quad \Xi_{u_3} = 0, \quad \Xi_{u_4} = 2.45, \quad \Xi_{u_5} = 0, \quad \Xi_{u_6} = 0.73.$$

Finally, the highest value can be chosen by $\Xi_{u_4} = max_{u \in U} \{\Xi_u\}$. Thus, we determined that u_4 is the most suitable object selected depending on the relationships in the parameter pairs considered in the NSR $\widehat{\mathfrak{R}}$. In other words, the object u_4 can provide the relationship between the parameters in the selected relationship better than the other objects.

Noted that the parameter pairs given in the NSR $\widehat{\mathfrak{R}}$ and the relationship values between them may mean that the selected object may change.

7 Conclusion

In this paper, the concept of neutrosophic soft relation, which can be discussed as a generalization of intuitionistic fuzzy soft relation, fuzzy soft relation, is discussed and defined for neutrosophic soft sets modified by Deli and Bromi [13]. Since these relationships are given for neutrosophic soft sets that can express as a falsity membership function, an indeterminacy membership function and a truth membership function of the information, these develops relationships based on hybrid sets that have been established based on fuzzy sets, intuitionistic fuzzy sets and soft sets. In addition, the contribution of a parameterization tool to successfully express the uncertainty problems encountered in most real life is another important feature of the given relationships. The concepts of composition and inverse of neutrosophic soft relations and functions were also presented along with some related properties and theorems. Moreover, the ideas of equivalence, symmetric, reflexive and transitive neutrosophic soft relations were given and analyzed in detail along with associated properties. Finally, an algorithm based on soft relationships was proposed for uncertainty problems and it was shown how it can be applied over a problem.

Thanks to the neutrosophic soft sets, which offer a very successful approach in expressing uncertainty situations, the relationships defined in this paper can be reconsidered by generalizing the neutrosophic soft sets over many mathematical models such as soft expert set [7], refined neutrosophic set [53], intuitionistic neutrosophic soft set [54], interval-valued neutrosophic soft set [14], bipolar neutrosophic soft set [55], (plithogenic) hypersoft set [56,57], single(multi)-valued neutrosophic hypersoft set [58]. The structure of the neutrosophic soft relation enables it to describe the relations between inconsistent and indeterminate data. For this reason, neutrosophic soft relation can be applied to express inconsistent and indeterminate data that can be encountered in many fields such as engineering, physics, medical diagnosis more easily.

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