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ORIGINAL RESEARCH

Competition graphs under complex Pythagorean fuzzy information

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Abstract
A complex Pythagorean fuzzy set, an extension of Pythagorean fuzzy set, is useful model to deal the vagueness with the degrees whose ranges are extended from real to complex subset with unit circle. This set deals with vagueness and periodicity more precisely as compared to complex fuzzy set and complex intuitionistic fuzzy set. In this paper, we propose a new graph, complex Pythagorean fuzzy competition graph by combining the complex Pythagorean fuzzy information with competition graph. We also investigate the two extensions of complex Pythagorean fuzzy competition graphs, namely, complex Pythagorean fuzzy k-competition and complex Pythagorean fuzzy p-competition graphs. Moreover, we present complex Pythagorean fuzzy neighborhood graphs and m-step complex Pythagorean fuzzy competition graphs. In addition, we illustrate an application of complex Pythagorean fuzzy competition graphs with algorithm to highlight the importance of these graphs in real life.

Keywords Complex Pythagorean fuzzy sets · k-competition · m-step competition · p-competition

Mathematics Subject Classification 03E72 · 68R10 · 68R05

1 Introduction

Arrangement of node connections has a vast area of applications in distinct fields of life. They may represent physical networks, such as organic molecules, roadways and electric circuits. They are also used in representing fewer interactions as might arise in databases, ecosystems, in the flow of control, in a sociological relationship, or in

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the computer program. Any mathematical object concerning points and connections between them is called a graph which is first introduced by Euler [15] in 1739. There are numerous utilizations of graph theory in distinct areas of life, including data mining, image capturing, clustering and computer science. The idea of competition graphs was first given by Cohen [13] in 1968 for the determination of the problem in ecology. Competition graphs have many other applications as in channel assignment, coding, modeling of economic systems, energy systems, modeling of complex economic and in channel assignment, besides ecosystem. After the initial motivation of application of competition between species in ecosystem several variations of competition graphs are found in literature, namely, common enemy graph of digraph [21], competition common enemy graph of digraph [34], competition hypergraphs [35], p-competition graphs of digraph [19,20] and tolerance competition graphs [12]. In 2000, another generalization of competition graphs, named, the m-step competition graph of a directed graph was introduced by Cho et al. [14]. All these above graphs are crisp graphs through which information about the real world competitions cannot be modeled.

In 1965, Zadeh [40] originally proposed the concept of fuzzy set (FS) as a novel approach to represent uncertainty occurring in distinct fields by introducing the membership function whose range lies between 0 and 1. In 1983, Atanassov [11] extended the fuzzy set and introduced the new set called intuitionistic fuzzy set (IFS) by adding a new component which determine the degree of non-membership with the restriction that the sum of membership and non-membership grade should not exceed 1. The idea of Pythagorean fuzzy set (PFS) was originally proposed by Yager [38], as an extension of IFS, which is an efficient tool for conducting the uncertainty more properly as compared to FS and IFS. The membership grade χ and non-membership grade λ of IFS are required to satisfy the condition $\chi + \lambda \leq 1$ but PFS relax the constraint with $\chi^2 + \lambda^2 \leq 1$. Akram and Habib [1] considered q-rung picture fuzzy graphs.

FS, IFS and PFS are useful models in dealing with uncertainties but in some realistic scenarios these theories are not enough to handle incomplete, inconsistent and imprecise information of periodic or two-dimensional nature. They are employed in many fields but there is one major deficiency of these sets which is the lack of capability to handle the periodicity. In order to overcome this issue, Ramot et al. [25] put forward the notion of complex fuzzy set (CFS) by extending the range of FS from unit interval [0,1] to the closed unit circle in the complex plane. Ramot et al. achieved his concept by adding a new term in definition of FS proposed by Zadeh, called phase term which is the distinguishing factor between CFS theory and all other theories such as FS, IFS, PFS. The phase term of CFS ensures the existence of some cases where the periodic dimension (second dimension) of membership function may require. After that, Zhang et al. [43] investigated many operations on CFS. Further, Alkouri and Salleh [7] extended this concept for complex IFSs by adding a new component called non-membership function. After that Alkouri and Salleh discussed some operations on complex intuitionistic fuzzy set [8] and also introduced the concept of complex Atanassov intuitionistic fuzzy relation [9] in 2013. In 2019, Ullah et al. [37] proposed the complex Pythagorean fuzzy set (CPFS) which is the generalization of all the existing theories. The idea of fuzzy graph was introduced by Rosenfeld [27] in 1975. After that Thirunavukarasu [36] gave the concept of complex fuzzy graph. In 2019, Yaqoob et al. [39] combine the concept of complex intuitionistic fuzzy set with graph theory and introduced the notion of complex intuitionistic fuzzy graph. Later on, Luqman et al. [23] worked on complex neutrosophic hypergraphs and complex intuitonistic fuzzy hypergraphs [22]. A lot of work has been done hypergraphs [16,17]. In 2019, Akram et al. [6] introduced the conception of complex Pythagorean fuzzy graphs in which the pairs of membership grades depict the two dimensional information. Fuzzy competition graphs (FCGs) were firstly introduced by Samanta and Pal [30] to express the partialness of preys and species concerning their extent of competition. After that, Raychaudhuri and Roberts [26] generalized the concept of competition graphs. Later on, Samanta et al. investigated fuzzy k-competition and fuzzy p-competition graphs in 2013 [30] and introduced some more results on fuzzy k-competition graphs in 2014 [31]. Further, m-step fuzzy competition graphs were introduced by Samanta et al. [29] in 2015. Recently, Sahoo and Pal [28] introduced the idea of intuitionistic fuzzy competition graphs to extend the capacity to model human knowledge. Further, some operations on intuitionistic fuzzy competition graphs were discussed by Nasir et al. [24]. Alshehri and Akram [10] introduced the concept of bipoler fuzzy competition graphs. Moreover, Akram et al. [32] discussed bipolar fuzzy competition graphs in 2017. Akram and Sarwar [5] discussed m-polar fuzzy competition graphs. Sarwar et al. [33] introduced fuzzy competition hypergraphs. Further, Habib et al. [18] introduced q-rung orthopair fuzzy competition graphs in 2019. Akram et al. proposed a decision making framework based on fuzzy competition hypergraphs [42]. Several other decision making techniques were investigated in [2–4,10,41,42,44,45]. The existing competition graphs are used to represent the real world competitions but there are some competitions which cannot be represented by these graphs because of their periodic or two dimensional nature. For example, some species of ecology may be strong or weak for some specific time interval. Similarly, the preys may be strong, digestive and harmful under some specific time interval. The term weak, strong, tasty, etc. are fuzzy in nature. For this periodic information or two-dimensional information about species and prey, the existing competition graphs are not enough. This encourage us for the development of complex fuzzy competition graphs.

In this paper, we propose the innovative concept of complex Pythagorean fuzzy competition graphs by the combination of competition graphs with CPFSs. Further, we introduce the two extensions of complex Pythagorean fuzzy competition graphs, namely, complex Pythagorean fuzzy k-competition and complex Pythagorean fuzzy p-competition graphs. Further, we present complex Pythagorean fuzzy neighborhood graphs, m-step complex Pythagorean fuzzy competition graphs and m-step complex Pythagorean fuzzy neighborhood graphs. We also describe an application of complex Pythagorean fuzzy competition graphs to highlight the importance of these graphs in real life.

2 Preliminaries

This section presents some basic definitions which are helpful in further developments.

Definition 2.1 [31] Let $\overrightarrow{G} = (\widetilde{A}, \overrightarrow{\widetilde{B}})$ be a fuzzy digraph (FDG). Then the fuzzy competition graph (FCG) $\mathfrak{C}(\overrightarrow{G})$ of $\overrightarrow{G}$ is an undirected graph $G = (\widetilde{A}, \widetilde{B})$ having same fuzzy vertex set as in $\overrightarrow{G}$ and an edge exists between two distinct nodes $s, w \in Y$ in $C(\overrightarrow{G})$ if and only if $N^+(s) \cap N^+(w) \neq \emptyset$ in $\overrightarrow{G}$ and the membership grade of the edge (s, w) in $C(\overrightarrow{G})$ is defined as:

$$\mu_{\widetilde{B}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w))H(N^+(s) \cap N^+(w)),$$

Definition 2.2 [37] A complex Pythagorean fuzzy set (CPFS) on a universal set Z, is an object of the form

$$\widetilde{\mathbb{M}} = \{(t, \mu_{\widetilde{\mathbb{M}}}(t)e^{i\theta_{\widetilde{\mathbb{M}}}(t)}, \nu_{\widetilde{\mathbb{M}}}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}}(t)}) : t \in Z\},$$

where $\mu_{\widetilde{\mathbb{M}}}(t), \nu_{\widetilde{\mathbb{M}}}(t) \in [0, 1], \theta_{\widetilde{\mathbb{M}}}(t), \vartheta_{\widetilde{\mathbb{M}}}(t) \in [0, 2\pi], i = \sqrt{-1}$ and for every $t \in Z$, $0 \leq \mu^2_{\widetilde{\mathbb{M}}}(t) + \nu^2_{\widetilde{\mathbb{M}}}(t) \leq 1$.

For every $t \in Z$, $\mu_{\widetilde{\mathbb{M}}}(t)$ and $\theta_{\widetilde{\mathbb{M}}}(t)$ are amplitude and phase terms for the membership function of t, and $\nu_{\widetilde{\mathbb{M}}}(t)$ and $\vartheta_{\widetilde{\mathbb{M}}}(t)$ are amplitude and phase terms for the non-membership function of t.

Definition 2.3 [6] A complex Pythagorean fuzzy relation (CPFR) $R(Y, Z)$ is defined as the subset of $Y \times Z$, is characterized by the membership and non-membership grades and is of the form:

$$R(Y, Z) = \{\langle (s, w), \mu_R(s, w)e^{i\theta_R(s,w)}, \nu_R(s, w)e^{i\vartheta_R(s,w)}\rangle \mid (s, w) \in Y \times Z\},$$

where $i = \sqrt{-1}$, $\mu_R(s, w), \nu_R(s, w) \in [0, 1]$, $\theta_R(s, w), \vartheta_R(s, w) \in [0, 2\pi]$ and $0 \leq \mu^2_R(s, w) + \nu^2_R(s, w) \leq 1$.

For every $(s, w) \in Y \times Z$, $\mu_R(s, w)$ and $\theta_R(s, w)$ are amplitude and phase terms for the membership function of (s,w), and $\nu_R(s, w)$ and $\vartheta_R(s, w)$ are amplitude and phase terms for the non-membership function of (s,w).

Now, we define complex Pythagorean fuzzy graph (CPFG) which is an extension of existing theories such as fuzzy graph, Pythagorean fuzzy graph and complex fuzzy graph. CPFGs are more suitable in the process of decision making and have the capability to consider the two-dimensional information about vertices and edges in a single set.

Definition 2.4 [6] A complex Pythagorean fuzzy graph (CPFG) on Y, is a triplet $\xi = (Y, \widetilde{A}, \widetilde{B})$, where $\widetilde{A}$ and $\widetilde{B}$ are CPFS and CPFR on Y, respectively such that:

$$\begin{aligned}
\mu_{\widetilde{B}}(s, w) &\leq \mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w), \\
\nu_{\widetilde{B}}(s, w) &\leq \nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w), \quad \text{(for amplitude terms)} \\
\theta_{\widetilde{B}}(s, w) &\leq \theta_{\widetilde{A}}(s) \wedge \theta_{\widetilde{A}}(w), \\
\vartheta_{\widetilde{B}}(s, w) &\leq \vartheta_{\widetilde{A}}(s) \vee \vartheta_{\widetilde{A}}(w), \quad \text{(for phase terms)}
\end{aligned}$$

and

$$0 \leq \mu^2_{\widetilde{B}}(s, w) + \nu^2_{\widetilde{B}}(s, w) \leq 1, \ \theta_{\widetilde{B}}(s, w), \vartheta_{\widetilde{B}}(s, w) \in [0, 2\pi], \quad \text{for all } s, w \in Y.$$

Definition 2.5 [6] A complex Pythagorean fuzzy digraph (CPFDG) on Y, is a triplet $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$, where $\widetilde{A}$ and $\widetilde{B}$ are CPFS and CPFR on Y, respectively such that:

$$\begin{aligned} &\mu_{\widetilde{B}}\overrightarrow{(s,w)} \leq \mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w), \\ &\nu_{\widetilde{B}}\overrightarrow{(s,w)} \leq \nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w), \quad \text{(for amplitude terms)} \end{aligned}$$

$$\begin{aligned} &\theta_{\widetilde{B}}\overrightarrow{(s,w)} \leq \theta_{\widetilde{A}}(s) \wedge \theta_{\widetilde{A}}(w), \\ &\vartheta_{\widetilde{B}}\overrightarrow{(s,w)} \leq \vartheta_{\widetilde{A}}(s) \vee \vartheta_{\widetilde{A}}(w), \quad \text{(for phase terms)} \end{aligned}$$

and

$$0 \leq \mu^2_{\widetilde{B}}\overrightarrow{(s,w)} + \nu^2_{\widetilde{B}}\overrightarrow{(s,w)} \leq 1,\ \theta_{\widetilde{B}}\overrightarrow{(s,w)}, \vartheta_{\widetilde{B}}\overrightarrow{(s,w)} \in [0, 2\pi], \quad \text{for all } s, w \in Y.$$

3 Complex Pythagorean fuzzy competition graphs

In this section, we discuss our main objective of this paper, complex Pythagorean fuzzy competition graphs (CPFCG). Before discussing CPFCGs we first discuss complex Pythagorean fuzzy out-neighborhood (CPF-out-neighborhood) of the vertex, complex Pythagorean fuzzy in-neighborhood (CPF-in-neighborhood) of the vertex, intersection of two CPFSs, height and cardinality of complex Pythagorean fuzzy set which will be used for the further devolvements.

Definition 3.1 Complex Pythagorean fuzzy out-neighborhood (CPF-out-neighborhood) of a vertex s of a CPFDG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ is a CPFS defined by:

$$\widetilde{\mathbb{N}}^p(s) = (S^p_s, t^p_s e^{i\phi^p_s}, f^p_s e^{i\psi^p_s}),$$

where

$$S^p_s = \{w \mid \mu_{\widetilde{B}}(\overrightarrow{s,w}) > 0 \; or \; \nu_{\widetilde{B}}(\overrightarrow{s,w}) > 0\},$$

such that $t^p_s : S^p_s \to [0,1]$ defined by $t^p_s(w) = \mu_{\widetilde{B}}(\overrightarrow{s,w})$, $\phi^p_s : S^p_s \to [0, 2\pi]$ defined by $\phi^p_s(w) = \theta_{\widetilde{B}}(\overrightarrow{s,w})$, $f^p_s : S^p_s \to [0,1]$ defined by $f^p_s(w) = \nu_{\widetilde{B}}(\overrightarrow{s,w})$ and $\psi^p_s : S^p_s \to [0, 2\pi]$ defined by $\psi^p_s(w) = \vartheta_{\widetilde{B}}(\overrightarrow{s,w})$.

Definition 3.2 Complex Pythagorean fuzzy in-neighborhood (CPF-in-neighborhood) of a vertex s of a CPFDG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ is a CPFS defined by:

$$\widetilde{\mathbb{N}}^n(s) = (S^n_s, t^n_s e^{i\phi^n_s}, f^n_s e^{i\psi^n_s}),$$

where

$$S^n_s = \{w \mid \mu_{\widetilde{B}}(\overrightarrow{w,s}) > 0 \; or \; \nu_{\widetilde{B}}(\overrightarrow{w,s}) > 0\},$$

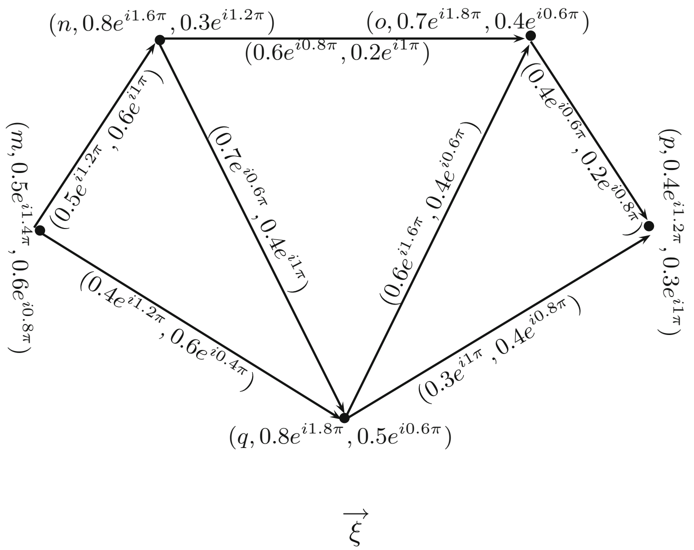


Fig. 1 Complex Pythagorean fuzzy digraph

such that $t_s^n : S_s^n \to [0, 1]$ defined by $t_s^n(w) = \mu_{\widetilde{B}}(\overrightarrow{w, s})$, $\phi_s^n : S_s^n \to [0, 2\pi]$ defined by $\phi_s^n(w) = \theta_{\widetilde{B}}(\overrightarrow{w, s})$, $f_s^n : S_s^n \to [0, 1]$ defined by $f_s^n(w) = \nu_{\widetilde{B}}(\overrightarrow{w, s})$ and $\psi_s^n : S_s^n \to [0, 2\pi]$ defined by $\psi_s^n(w) = \vartheta_{\widetilde{B}}(\overrightarrow{w, s})$.

Example 3.1 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG as displayed in Fig. 1, defined by:

$$\widetilde{A} = \Big\langle \Big(\frac{m}{0.5e^{i1.4\pi}}, \frac{n}{0.8e^{i1.6\pi}}, \frac{o}{0.7e^{i1.8\pi}}, \frac{p}{0.4e^{i1.2\pi}}, \frac{q}{0.8e^{i1.8\pi}}\Big), \Big(\frac{m}{0.6e^{i0.8\pi}}, \frac{n}{0.3e^{i1.2\pi}}, \frac{o}{0.4e^{i0.6\pi}}, \frac{p}{0.3e^{i1\pi}}, \frac{q}{0.5e^{i0.6\pi}}\Big) \Big\rangle,$$

$$\overrightarrow{\widetilde{B}} = \Bigg\langle \Bigg(\frac{\overrightarrow{(m, n)}}{0.5e^{i1.2\pi}}, \frac{\overrightarrow{(n, o)}}{0.6e^{i0.8\pi}}, \frac{\overrightarrow{(o, p)}}{0.4e^{i0.6\pi}}, \frac{\overrightarrow{(m, q)}}{0.4e^{i1.2\pi}}, \frac{\overrightarrow{(n, q)}}{0.7e^{i0.6\pi}}, \frac{\overrightarrow{(q, o)}}{0.6e^{i1.6\pi}}, \frac{\overrightarrow{(q, p)}}{0.3e^{i1\pi}} \Bigg), \Bigg(\frac{\overrightarrow{(m, n)}}{0.6e^{i1\pi}}, \frac{\overrightarrow{(n, o)}}{0.2e^{i1\pi}}, \frac{\overrightarrow{(o, p)}}{0.2e^{i0.8\pi}}, \frac{\overrightarrow{(m, q)}}{0.6e^{i0.4\pi}}, \frac{\overrightarrow{(n, q)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(q, o)}}{0.4e^{i0.6\pi}}, \frac{\overrightarrow{(q, p)}}{0.4e^{i0.8\pi}} \Bigg) \Bigg\rangle.$$

CPF-out-neighborhood and CPF-in-neighborhood of the vertices are shown in Table 1.

Definition 3.3 Let $\widetilde{\mathbb{M}}_1$ and $\widetilde{\mathbb{M}}_2$ be a two CPFSs on Z, where

$$\widetilde{\mathbb{M}}_1 = \{\langle (t, \mu_{\widetilde{\mathbb{M}}_1}(t)e^{i\theta_{\widetilde{\mathbb{M}}_1}(t)}, \nu_{\widetilde{\mathbb{M}}_1}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}_1}(t)}) \rangle : t \in Z\},$$
$$\widetilde{\mathbb{M}}_2 = \{\langle (t, \mu_{\widetilde{\mathbb{M}}_2}(t)e^{i\theta_{\widetilde{\mathbb{M}}_2}(t)}, \nu_{\widetilde{\mathbb{M}}_2}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}_2}(t)}) \rangle : t \in Z\}.$$

Table 1 CPP-out neighborhood and CPF-in neighborhood of the vertices

x	$\widetilde{\mathbb{N}}^p(x)$	$\widetilde{\mathbb{N}}^n(x)$
m	$\{(n, 0.5e^{i1.2\pi}, 0.6e^{i1\pi}),$ $(q, 0.4e^{i1.2\pi}, 0.6e^{i0.4\pi})\}$	$\emptyset$
n	$\{(o, 0.6e^{i0.8\pi}, 0.2e^{i1\pi}),$ $(q, 0.7e^{i0.6\pi}, 0.4e^{i1\pi})\}$	$\{(m, 0.5e^{i1.2\pi}, 0.6e^{i1\pi})\}$
o	$\{(p, 0.4e^{i0.6\pi}, 0.2e^{i0.8\pi})\}$	$\{(q, 0.6e^{i1.6\pi}, 0.4e^{i0.6\pi}),$ $(n, 0.6e^{i0.8\pi}, 0.2e^{i1\pi})\}$
p	$\emptyset$	$\{(o, 0.4e^{i0.6\pi}, 0.2e^{i0.8\pi}),$ $(q, 0.3e^{i1\pi}, 0.4e^{i0.8\pi})\}$
q	$\{(o, 0.6e^{i1.6\pi}, 0.4e^{i0.6\pi}),$ $(p, 0.3e^{i1\pi}, 0.4e^{i0.8\pi})\}$	$\{(m, 0.4e^{i1.2\pi}, 0.6e^{i0.4\pi}),$ $(q, 0.7e^{i0.6\pi}, 0.4e^{i1\pi})\}$

The CPF intersection of $\widetilde{\mathbb{M}}_1$ and $\widetilde{\mathbb{M}}_2$, denoted by $\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2$, is specified by the function;

$$\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2 = \{\langle(t, \mu_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)e^{i\theta_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)}, \nu_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)})\}\rangle : t \in Z\},$$

where

$$\mu_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)e^{i\theta_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)} = \min(\mu_{\widetilde{\mathbb{M}}_1}(t), \mu_{\widetilde{\mathbb{M}}_2}(t))e^{i\min(\theta_{\widetilde{\mathbb{M}}_1}(t), \theta_{\widetilde{\mathbb{M}}_2}(t))},$$
$$\nu_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}_1 \cap \widetilde{\mathbb{M}}_2}(t)} = \max(\nu_{\widetilde{\mathbb{M}}_1}(t), \nu_{\widetilde{\mathbb{M}}_2}(t))e^{i\max(\vartheta_{\widetilde{\mathbb{M}}_1}(t), \vartheta_{\widetilde{\mathbb{M}}_2}(t))}.$$

Definition 3.4 Let $\widetilde{\mathbb{M}} = \{(t, \mu_{\widetilde{\mathbb{M}}}(t)e^{i\theta_{\widetilde{\mathbb{M}}}(t)}, \nu_{\widetilde{\mathbb{M}}}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}}(t)} \mid t \in Z)\}$ be a CPFS. Then the Cardinality of CPFS $\widetilde{\mathbb{M}}$, denoted by $|\widetilde{\mathbb{M}}| = (|\widetilde{\mathbb{M}}|_\mu e^{i|\widetilde{\mathbb{M}}|_\theta}, |\widetilde{\mathbb{M}}|_\nu e^{i|\widetilde{\mathbb{M}}|_\vartheta})$, is the sum of membership and non-membership grades of elements of Z, and is defined as:

$$|\widetilde{\mathbb{M}}| = \left(\sum_{t_i \in Z} \mu_{\widetilde{\mathbb{M}}}(t_i)e^{i\sum_{t_i \in Z}\theta_{\widetilde{\mathbb{M}}}(t_i)}, \sum_{t_i \in Z} \nu_{\widetilde{\mathbb{M}}}(t_i)e^{i\sum_{t_i \in Z}\vartheta_{\widetilde{\mathbb{M}}}(t_i)}\right),$$
$$= (|\widetilde{\mathbb{M}}|_\mu e^{i|\widetilde{\mathbb{M}}|_\theta}, |\widetilde{\mathbb{M}}|_\nu e^{i|\widetilde{\mathbb{M}}|_\vartheta}),$$

for all $t_i \in Z$.

Definition 3.5 Let $\widetilde{\mathbb{M}} = \{(t, \mu_{\widetilde{\mathbb{M}}}(t)e^{i\theta_{\widetilde{\mathbb{M}}}(t)}, \nu_{\widetilde{\mathbb{M}}}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}}(t)} \mid t \in Z)\}$ be CPFS. Then the height of CPFS $\widetilde{\mathbb{M}}$, denoted by $\mathbb{h}(\widetilde{\mathbb{M}}) = (\mathbb{h}_\mu(\widetilde{\mathbb{M}})e^{i\mathbb{h}_\theta(\widetilde{\mathbb{M}})}, \mathbb{h}_\nu(\widetilde{\mathbb{M}})e^{i\mathbb{h}_\vartheta(\widetilde{\mathbb{M}})})$, is defined as:

$$\mathbb{h}(\widetilde{\mathbb{M}}) = (\max(\mu_{\widetilde{\mathbb{M}}}(t)e^{i\max\theta_{\widetilde{\mathbb{M}}}(t)}, \min(\nu_{\widetilde{\mathbb{M}}}(t)e^{i\min\vartheta_{\widetilde{\mathbb{M}}}(t)}),$$
$$= (\mathbb{h}_\mu(\widetilde{\mathbb{M}})e^{i\mathbb{h}_\theta(\widetilde{\mathbb{M}})}, \mathbb{h}_\nu(\widetilde{\mathbb{M}})e^{i\mathbb{h}_\vartheta(\widetilde{\mathbb{M}})}).$$

A complex Pythagorean fuzzy competition graph (CPFCG) is defined below.

Definition 3.6 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. Then complex Pythagorean fuzzy competition graph (CPFCG) $\mathfrak{C}(\overrightarrow{\xi})$ of a CPFDG $\overrightarrow{\xi}$ is an undirected CPFG $\xi =$

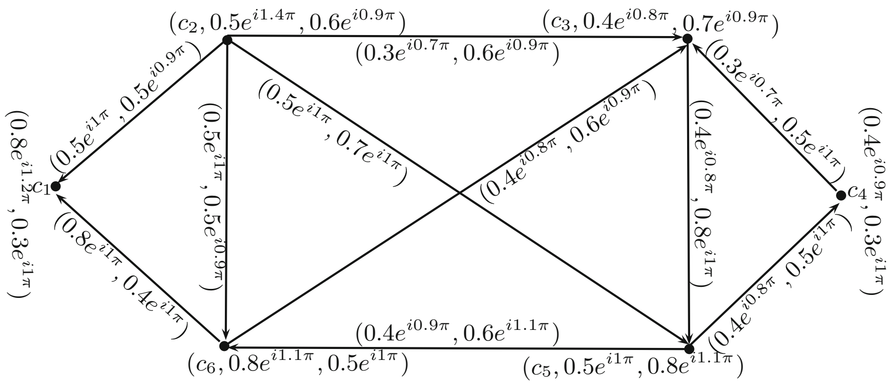


Fig. 2 Complex Pythagorean fuzzy digraph

$(Y, \widetilde{A}, \widetilde{B})$, where the vertex set of $\mathfrak{C}(\overrightarrow{\xi})$ is same as in $\overrightarrow{\xi}$ and a CPF edge exists between two distinct nodes $s, w \in Y$ in $\mathfrak{C}(\overrightarrow{\xi})$ if and only if the CPFS $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w) \neq \emptyset$ in $\overrightarrow{\xi}$. The membership and non-membership grades of the edge (s, w) in $\mathfrak{C}(\overrightarrow{\xi})$ are defined as:

$$\mu_{\widetilde{B}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$
$$\nu_{\widetilde{B}}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$
$$\theta_{\widetilde{B}}(s, w) = 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi} \right],$$
$$\vartheta_{\widetilde{B}}(s, w) = 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi} \right].$$

Example 3.2 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG as given in Fig. 2, defined by;

$$\widetilde{A} = \Big\langle \Big(\frac{c_1}{0.8e^{i1.2\pi}}, \frac{c_2}{0.5e^{i1.4\pi}}, \frac{c_3}{0.4e^{i0.8\pi}}, \frac{c_4}{0.4e^{i0.9\pi}}, \frac{c_5}{0.5e^{i1\pi}}, \frac{c_6}{0.8e^{i1.1\pi}} \Big),$$
$$\Big(\frac{c_1}{0.3e^{i1\pi}}, \frac{c_2}{0.6e^{i0.9\pi}}, \frac{c_3}{0.7e^{i0.9\pi}}, \frac{c_4}{0.3e^{i1\pi}}, \frac{c_5}{0.8e^{i1.1\pi}}, \frac{c_6}{0.5e^{i1\pi}} \Big) \Big\rangle$$
$$\overrightarrow{\widetilde{B}} = \Big\langle \Big(\frac{\overrightarrow{(c_2, c_1)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_2, c_3)}}{0.3e^{i0.7\pi}}, \frac{\overrightarrow{(c_2, c_5)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_2, c_6)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_3, c_5)}}{0.4e^{i0.8\pi}}, \frac{\overrightarrow{(c_4, c_3)}}{0.3e^{i0.7\pi}}, \frac{\overrightarrow{(c_5, c_4)}}{0.4e^{0.8i\pi}}, \frac{\overrightarrow{(c_5, c_6)}}{0.4e^{i0.9\pi}}, \frac{\overrightarrow{(c_6, c_1)}}{0.8e^{i1\pi}}, \frac{\overrightarrow{(c_6, c_3)}}{0.4e^{i0.8\pi}} \Big),$$
$$\Big(\frac{\overrightarrow{(c_2, c_1)}}{0.5e^{i0.9\pi}}, \frac{\overrightarrow{(c_2, c_3)}}{0.6e^{i0.9\pi}}, \frac{\overrightarrow{(c_2, c_5)}}{0.7e^{i1\pi}}, \frac{\overrightarrow{(c_2, c_6)}}{0.5e^{i0.9\pi}}, \frac{\overrightarrow{(c_3, c_5)}}{0.8e^{i1\pi}}, \frac{\overrightarrow{(c_4, c_3)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_5, c_4)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_5, c_6)}}{0.6e^{i1.1\pi}}, \frac{\overrightarrow{(c_6, c_1)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(c_6, c_3)}}{0.6e^{i0.9\pi}} \Big) \Big\rangle.$$

The CPF-out-neighborhood of the vertices are given in Table 2.

The CPFS $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$ are calculated in Table 3.

Thus, it can be seen form Table 3 that there is an edge between c_2 and c_3; c_2 and c_4; c_2 and c_5; c_2 and c_6; and c_4 and c_6. The corresponding CPFCG is displayed in Fig. 3.

Table 2 CPF out-neighborhood of the vertices

x	$\widetilde{\mathbb{N}}^p(x)$
c_1	$\emptyset$
c_2	$\{(c_1, 0.5e^{i1\pi}, 0.5e^{i0.9\pi}), (c_3, 0.3e^{i0.7\pi}, 0.6e^{i0.9\pi}), (c_5, 0.5e^{i1\pi}, 0.7e^{i1\pi}),$ $(c_6, 0.5e^{i1\pi}, 0.5e^{i0.9\pi})\}$
c_3	$\{(c_5, 0.4e^{i0.8\pi}, 0.8e^{i1\pi})\}$
c_4	$\{(c_3, 0.3e^{i0.7\pi}, 0.5e^{i1\pi})\}$
c_5	$\{(c_4, 0.4e^{i0.8\pi}, 0.5e^{i1\pi}), (c_6, 0.4e^{i0.9\pi}, 0.6e^{i1.1\pi})\}$
c_6	$\{(c_1, 0.8e^{i1\pi}, 0.4e^{i1\pi}), (c_3, 0.4e^{i0.8\pi}, 0.6e^{i0.9\pi})\}$

Table 3 CPFSs $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$

s	w	$\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$	$\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$
c_1	c_2	$\emptyset$	$\emptyset$
c_1	c_3	$\emptyset$	$\emptyset$
c_1	c_4	$\emptyset$	$\emptyset$
c_1	c_5	$\emptyset$	$\emptyset$
c_1	c_6	$\emptyset$	$\emptyset$
c_2	c_3	$\{(c_5, 0.4e^{i0.8\pi}, 0.8e^{i1\pi})\}$	$\{(0.4e^{i0.8\pi}, 0.8e^{i1\pi})\}$
c_2	c_4	$\{(c_3, 0.3e^{i0.7\pi}, 0.6e^{i1\pi})\}$	$\{(0.3e^{i0.7\pi}, 0.6e^{i1\pi})\}$
c_2	c_5	$\{(c_6, 0.4e^{i0.9\pi}, 0.6e^{i1.1\pi})\}$	$\{(0.4e^{i0.9\pi}, 0.6e^{i1.1\pi})\}$
c_2	c_6	$\{(c_3, 0.3e^{i0.7\pi}, 0.6e^{i0.9\pi})\}$	$\{(0.3e^{i0.7\pi}, 0.6e^{i0.9\pi})\}$
c_3 9 c_4	$\emptyset$	$\emptyset$	
c_3	c_5	$\emptyset$	$\emptyset$
c_3	c_6	$\emptyset$	$\emptyset$
c_4	c_5	$\emptyset$	$\emptyset$
c_4	c_6	$\{(c_3, 0.3e^{i0.7\pi}, 0.6e^{i1\pi})\}$	$\{(0.3e^{i0.7\pi}, 0.6e^{i1\pi})\}$
c_5	c_6	$\emptyset$	$\emptyset$

Definition 3.7 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG. An edge (s, w) in a CPFG is said to be strong if

$$\begin{aligned}
\mu_{\widetilde{B}}(s, w) &> \frac{1}{2}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)),\\
\nu_{\widetilde{B}}(s, w) &< \frac{1}{2}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)),\\
\theta_{\widetilde{B}}(s, w) &> \frac{1}{2}\left[2\pi\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right)\right],\\
\vartheta_{\widetilde{B}}(s, w) &< \frac{1}{2}\left[2\pi\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right)\right],
\end{aligned}$$

for all $s, w \in Y$.

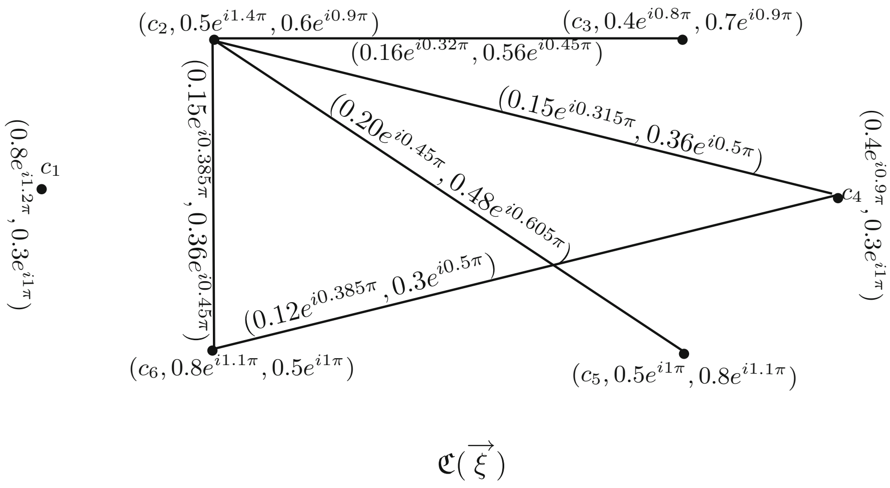


Fig. 3 Complex Pythagorean fuzzy competition graph

Theorem 3.1 *Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. If there exist only one element in CPFS $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$, then the edge (s, w) of $C(\overrightarrow{\xi})$ is strong if and only $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\mu > \frac{1}{2}$, $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\nu < \frac{1}{2}$, $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\theta > 1\pi$ and $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\vartheta < 1\pi$.*

Proof Here $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. Let $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w) = (c, qe^{i\alpha}, re^{i\beta})$, where $qe^{i\alpha}$ and $re^{i\beta}$ are the membership and non-membership grades of either the edge (s, c) or (w, c). So, $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = (qe^{i\alpha}, re^{i\beta}) = |\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|$. Then,

$$
\begin{aligned}
|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\mu &= q = \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),\\
|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\nu &= r = \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),\\
|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\theta &= \alpha = \mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),\\
|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)|_\vartheta &= \beta = \mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),
\end{aligned}
$$

So, according to the definition 3.5 the membership and non-membership grades of the edge (s, w) in the corresponding CPFCG is defined as

$$
\begin{aligned}
\mu_{\widetilde{B}}(s, w) &= (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times q,\\
\nu_{\widetilde{B}}(s, w) &= (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times r,\\
\theta_{\widetilde{B}}(s, w) &= 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi} \right]\\
&= 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\alpha}{2\pi} \right],
\end{aligned}
$$

$$\vartheta_{\widetilde{B}}(s,w) = 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi}\right]$$
$$= 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\beta}{2\pi}\right].$$

Hence, edge (s, w) is strong if and only if $q > \frac{1}{2}$, $r < \frac{1}{2}$, $\alpha > 1\pi$ and $\beta < 1\pi$. □

4 Complex Pythagorean fuzzy *k*-competition graphs

This section presents an extension of complex Pythagorean fuzzy competition graph called complex Pythagorean fuzzy k-competition graph (CPF k-competition graph) and some related theorems about these graphs below.

Definition 4.1 Let k be a complex number given by $k = xe^{i\gamma}$ and $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)| = (x^{'}e^{i\gamma^{'}}, y^{'}e^{i\eta^{'}})$. The CPF k-competition graph $\mathfrak{C}_k(\overrightarrow{\xi})$ of CPFDG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ is an undirected CPFG $\xi = (Y, \widetilde{A}, \widetilde{B})$, where the CPF vertex set of $\mathfrak{C}_k(\overrightarrow{\xi})$ is same as in $\overrightarrow{\xi}$ and CPF edge exists between two distinct nodes s and w in $\mathfrak{C}_k(\overrightarrow{\xi})$ if and only if $x^{'} > x$, $\gamma^{'} > \gamma$ for membership grade and $y^{'} > x$, $\eta^{'} > \gamma$ for non-membership grade. The membership and non-membership grades of the edge (s, w) in $\mathfrak{C}_k(\overrightarrow{\xi})$ are defined as:

$$\mu_{\widetilde{B}}(s,w) = \frac{x^{'}-x}{x^{'}}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$
$$\nu_{\widetilde{B}}(s,w) = \frac{y^{'}-x}{y^{'}}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_{\nu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$

$$\theta_{\widetilde{B}}(s,w) = 2\pi\left[\frac{\gamma^{'}-\gamma}{\gamma^{'}}\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi}\right],$$
$$\vartheta_{\widetilde{B}}(s,w) = 2\pi\left[\frac{\eta^{'}-\gamma}{\eta^{'}}\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi}\right].$$

Example 4.1 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be CPFDG as given in Fig. 4, defined by;

$$\widetilde{A} = \left\langle \left(\frac{s_1}{0.4e^{i1.6\pi}}, \frac{s_2}{0.3e^{i1.4\pi}}, \frac{s_3}{0.5e^{i1.2\pi}}, \frac{s_4}{0.9e^{i1.4\pi}}, \frac{s_5}{0.8e^{i1.4\pi}}, \frac{s_6}{0.6e^{i1.6\pi}}\right), \left(\frac{s_1}{0.7e^{i1.2\pi}}, \frac{s_2}{0.8e^{i1\pi}}, \frac{s_3}{0.6e^{i0.6\pi}}, \frac{s_4}{0.2e^{i0.8\pi}}, \frac{s_5}{0.3e^{i1\pi}}, \frac{s_6}{0.5e^{i1.2\pi}}\right)\right\rangle,$$

$$\overrightarrow{\widetilde{B}} = \left\langle \left(\frac{\overrightarrow{(s_1,s_2)}}{0.2e^{i1.3\pi}}, \frac{\overrightarrow{(s_1,s_3)}}{0.3e^{i1.2\pi}}, \frac{\overrightarrow{(s_2,s_4)}}{0.3e^{i1\pi}}, \frac{\overrightarrow{(s_4,s_3)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(s_5,s_1)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(s_5,s_2)}}{0.3e^{i1\pi}}, \frac{\overrightarrow{(s_5,s_6)}}{0.5e^{i1.2\pi}}, \frac{\overrightarrow{(s_6,s_3)}}{0.4e^{i1.1\pi}}, \frac{\overrightarrow{(s_6,s_4)}}{0.3e^{i1.2\pi}}\right), \left(\frac{\overrightarrow{(s_1,s_2)}}{0.8e^{i1.2\pi}}, \frac{\overrightarrow{(s_1,s_3)}}{0.6e^{i1\pi}}, \frac{\overrightarrow{(s_2,s_4)}}{0.5e^{i0.9\pi}}, \frac{\overrightarrow{(s_4,s_3)}}{0.5e^{i0.7\pi}}, \frac{\overrightarrow{(s_5,s_1)}}{0.5e^{i1.1\pi}}, \frac{\overrightarrow{(s_5,s_2)}}{0.7e^{i1\pi}}, \frac{\overrightarrow{(s_5,s_6)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(s_6,s_3)}}{0.6e^{i1\pi}}, \frac{\overrightarrow{(s_6,s_4)}}{0.4e^{i1\pi}}\right)\right\rangle.$$

The CPF-out-neighborhood of the vertices are shown in Table 4.

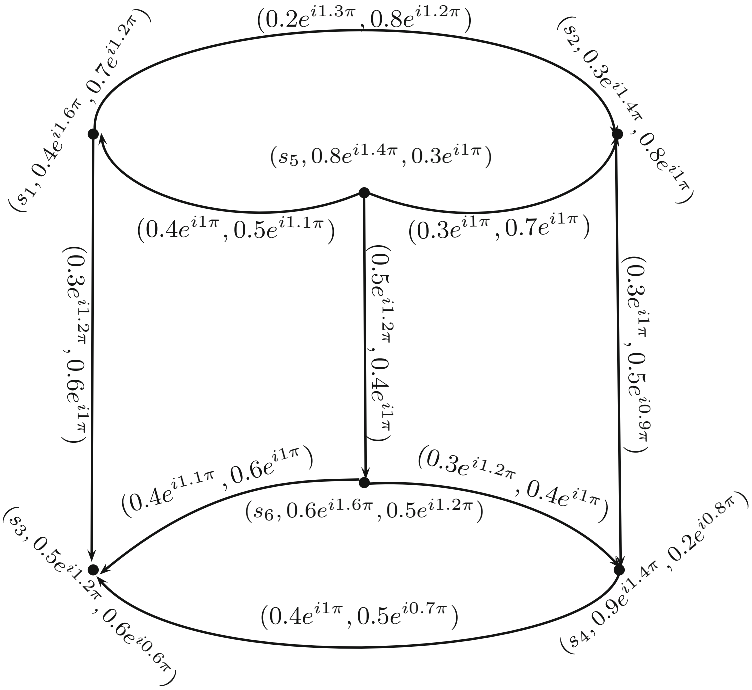


Fig. 4 Complex Pythagorean fuzzy digraph

Table 4 CPF-out-neighborhoods of the vertices

x	$\widetilde{\mathbb{N}}^p(x)$
s_1	$\{(s_2, 0.2e^{i1.3\pi}, 0.8e^{i1.2\pi}), (s_3, 0.3e^{i1.2\pi}, 0.6e^{i1\pi})\}$.
s_2	$\{(s_4, 0.3e^{i1\pi}, 0.5e^{i0.9\pi})\}$.
s_3	$\emptyset$
s_4	$\{(s_3, 0.4e^{i1\pi}, 0.5e^{i0.7\pi})\}$.
s_5	$\{(s_1, 0.4e^{i1\pi}, 0.5e^{i1.1\pi}), (s_2, 0.3e^{i1\pi}, 0.7e^{i1\pi}), (s_6, 0.5e^{i1.2\pi}, 0.4e^{i1\pi})\}$.
s_6	$\{(s_3, 0.4e^{i1.1\pi}, 0.6e^{i1\pi}), (s_4, 0.3e^{i1.2\pi}, 0.4e^{i1\pi})\}$.

The CPFSs $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$ are given in Table 5.

Now, let $k = 0.2e^{i0.5\pi}$, then corresponding CPF $0.2e^{i0.5\pi}$-competition graph has only four edges since

$$
\begin{aligned}
&|\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_4)|_\mu = 0.3 > 0.2, \quad |\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_4)|_\nu = 0.6 > 0.2,\\
&|\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_4)|_\theta = 1 > 0.5, \quad |\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_4)|_\vartheta = 1 > 0.5,\\
&|\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_6)|_\mu = 0.3 > 0.2, \quad |\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_6)|_\nu = 0.6 > 0.2,\\
&|\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_6)|_\theta = 1 > 0.5, \quad |\widetilde{\mathbb{N}}^p(s_1) \cap \widetilde{\mathbb{N}}^p(s_6)|_\vartheta = 1 > 0.5,\\
&|\widetilde{\mathbb{N}}^p(s_4) \cap \widetilde{\mathbb{N}}^p(s_6)|_\mu = 0.4 > 0.2, \quad |\widetilde{\mathbb{N}}^p(s_4) \cap \widetilde{\mathbb{N}}^p(s_6)|_\nu = 0.6 > 0.2,\\
&|\widetilde{\mathbb{N}}^p(s_4) \cap \widetilde{\mathbb{N}}^p(s_6)|_\theta = 1 > 0.5, \quad |\widetilde{\mathbb{N}}^p(s_4) \cap \widetilde{\mathbb{N}}^p(s_6)|_\vartheta = 1 > 0.5,
\end{aligned}
$$

Table 5 Height and cardinality of CPFS $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$

s	w	$\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$	$\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$	$\lvert\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)\rvert$
s_1	s_2	$\emptyset$	$\emptyset$	$\emptyset$
s_1	s_3	$\emptyset$	$\emptyset$	$\emptyset$
s_1	s_4	$\{(s_3, 0.3e^{i1\pi}, 0.6e^{i1\pi})\}$	$\{(0.3e^{i1\pi}, 0.6e^{i1\pi})\}$	$(0.3e^{i1\pi}, 0.6e^{i1\pi})$
s_1	s_5	$\{(s_2, 0.2e^{i1\pi}, 0.8e^{i1.2\pi})\}$	$\{(0.2e^{i1\pi}, 0.8e^{i1.2\pi})\}$	$(0.2e^{i1\pi}, 0.8e^{i1.2\pi})$
s_1	s_6	$\{(s_3, 0.3e^{i1.1\pi}, 0.6e^{i1\pi})\}$	$\{(0.3e^{i1.1\pi}, 0.6e^{i1\pi})\}$	$(0.3e^{i1.1\pi}, 0.6e^{i1\pi})$
s_2	s_3	$\emptyset$	$\emptyset$	$\emptyset$
s_2	s_4	$\emptyset$	$\emptyset$	$\emptyset$
s_2	s_5	$\emptyset$	$\emptyset$	$\emptyset$
s_2	s_6	$\{(s_4, 0.3e^{i1\pi}, 0.5e^{i1\pi})\}$	$\{(0.3e^{i1\pi}, 0.5e^{i1\pi})\}$	$(0.3e^{i1\pi}, 0.5e^{i1\pi})$
s_3	s_4	$\emptyset$	$\emptyset$	$\emptyset$
s_3	s_5	$\emptyset$	$\emptyset$	$\emptyset$
s_3	s_6	$\emptyset$	$\emptyset$	$\emptyset$
s_4	s_5	$\emptyset$	$\emptyset$	$\emptyset$
s_4	s_6	$\{(s_3, 0.4e^{i1\pi}, 0.6e^{i1\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1\pi})\}$	$(0.4e^{i1\pi}, 0.6e^{i1\pi})$
s_5	s_6	$\emptyset$	$\emptyset$	$\emptyset$

$$
\begin{aligned}
&|\widetilde{\mathbb{N}}^p(s_2) \cap \widetilde{\mathbb{N}}^p(s_6)|_\mu = 0.3 > 0.2, \quad |\widetilde{\mathbb{N}}^p(s_2) \cap \widetilde{\mathbb{N}}^p(s_6)|_\nu = 0.5 > 0.2,\\
&|\widetilde{\mathbb{N}}^p(s_2) \cap \widetilde{\mathbb{N}}^p(s_6)|_\theta = 1 > 0.5, \quad |\widetilde{\mathbb{N}}^p(s_2) \cap \widetilde{\mathbb{N}}^p(s_6)|_\vartheta = 1 > 0.5,
\end{aligned}
$$

The CPF $0.2e^{i0.5\pi}$-competition graph is show in Fig. 5.

Theorem 4.1 *Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. If $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = (1e^{i2\pi}, 1e^{i2\pi})$ and $x^{'} > 2x$, $\gamma^{'} > 2\gamma$, $y^{'} < 2x$, and $\eta^{'} < 2\gamma$ then the edge (s, w) is strong in $\mathfrak{C}_k(\overrightarrow{\xi})$. Here, $|\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)| = (x^{'}e^{i\gamma^{'}}, y^{'}e^{i\eta^{'}})$.*

Proof Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. Let $\mathfrak{C}_k(\overrightarrow{\xi}) = (Y, \widetilde{A}, \widetilde{B})$ be the corresponding CPF k-competition graph. According to the given statement if $\mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = 1$ for amplitude term and $\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = 2\pi$ for the phase term of the membership grade then according to definition 3.5 the membership grade of the edge (s, w) in the corresponding CPF k-competition graph $\mathfrak{C}_k(\overrightarrow{\xi})$ is

$$
\begin{aligned}
\mu_{\widetilde{B}}(s, w) &= \frac{x^{'} - x}{x^{'}}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),\\
&= \frac{x^{'} - x}{x^{'}}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)),\\
\theta_{\widetilde{B}}(s, w) &= 2\pi\left[\frac{\gamma^{'} - \gamma}{\gamma^{'}}\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi}\right],\\
&= 2\pi\left[\frac{\gamma^{'} - \gamma}{\gamma^{'}}\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{2\pi}{2\pi}\right].
\end{aligned}
$$

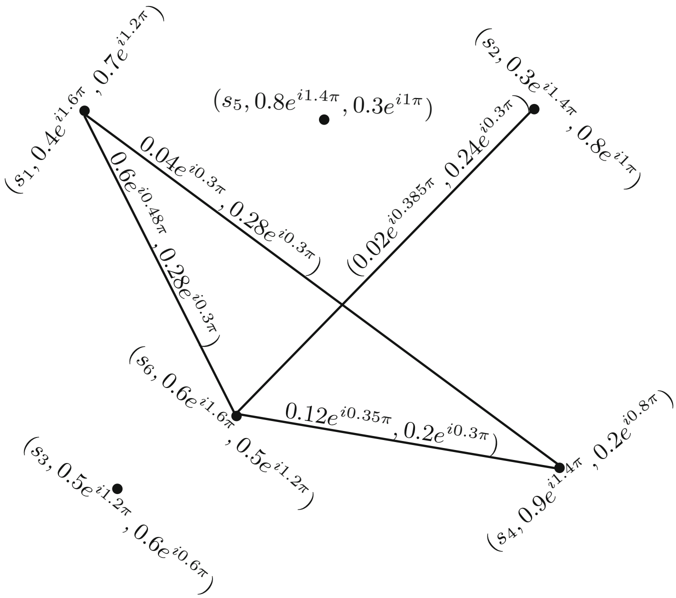


Fig. 5 Complex Pythagorean fuzzy $0.2e^{i0.5\pi}$-competition graph

Clearly this gives that $\mu_{\widetilde{B}}(s, w) > \frac{1}{2}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w))$ *as* $\frac{x^{'} - x}{x^{'}} > \frac{1}{2}$ and $\theta_{\widetilde{B}}(s, w) > \frac{1}{2}[2\pi(\frac{\theta_{\widetilde{B}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{B}}(w)}{2\pi})]$ *as* $\frac{\gamma^{'} - \gamma}{\gamma^{'}} > \frac{1}{2}$. Similarly, if $\mathbb{h}_{\nu}(\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w)) = 1$ for the amplitude term and $\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w)) = 2\pi$ for phase term of the non-membership function, then the non-membership value for the edge (s, w) in $\mathfrak{C}_{k}(\overrightarrow{\xi})$ is

$$\begin{aligned}
\nu_{\widetilde{B}}(s, w) &= \frac{y^{'} - x}{y^{'}}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_{\nu}(\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w)),\\
&= \frac{y^{'} - x}{y^{'}}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)),\\
\vartheta_{\widetilde{B}}(s, w) &= 2\pi\left[\frac{\eta^{'} - \gamma}{\eta^{'}}\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w))}{2\pi}\right],\\
&= 2\pi\left[\frac{\eta^{'} - \gamma}{\eta^{'}}\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{2\pi}{2\pi}\right].
\end{aligned}$$

This gives that $\nu_{\widetilde{B}}(s, w) < \frac{1}{2}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w))$ as $\frac{y^{'} - x}{y^{'}} < \frac{1}{2}$ and $\vartheta_{\widetilde{B}}(s, w) < \frac{1}{2}(\vartheta_{\widetilde{A}}(s) \vee \vartheta_{\widetilde{A}}(w))$ *as* $\frac{\eta^{'} - \gamma}{\eta^{'}} < \frac{1}{2}$. Hence, the edge (s, w) is strong. □

5 *p*-competition complex Pythagorean fuzzy graphs

In this section, we define another extension of CPFCG called p-competition complex Pythagorean fuzzy graph (p-competition CPFG). Before defining p-competition CPFG, we first define support of CPFS below.

Definition 5.1 Let $\widetilde{\mathbb{M}} = \{(t, \mu_{\widetilde{\mathbb{M}}}(t)e^{i\theta_{\widetilde{\mathbb{M}}}(t)}, \nu_{\widetilde{\mathbb{M}}}(t)e^{i\vartheta_{\widetilde{\mathbb{M}}}(t)} | t \in Z)\}$ be a CPFS. Then the support of CPFS $\widetilde{\mathbb{M}}$ is subset $\widetilde{\mathbb{M}}_o$ of Z, and is defined as:

$$\widetilde{\mathbb{M}}_o = \{t \in Z : \mu_{\widetilde{\mathbb{M}}}(t) \neq 0, \nu_{\widetilde{\mathbb{M}}}(t) \neq 0\}.$$

Definition 5.2 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be CPFDG. The p-competition CPFG $\mathfrak{C}^p(\overrightarrow{\xi})$ of the CPFDG $\overrightarrow{\xi}$ is an undirected CPFG $\xi = (Y, \widetilde{A}, \widetilde{B})$, where the CPF vertex set of $\mathfrak{C}^p(\overrightarrow{\xi})$ is same as $\overrightarrow{\xi}$ and a CPF edge exists between two distinct nodes $s, w \in Y$ in $\mathfrak{C}^p(\xi)$ if and only if $|supp(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))| \geq p$. The membership and non-membership grades of the edge (s, w) is be defined as:

$$\mu_{\widetilde{B}}(s, w) = \frac{(a-p)+1}{a}[\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)] \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$
$$\nu_{\widetilde{B}}(s, w) = \frac{(a-p)+1}{a}[\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)] \times \mathbb{h}_{\nu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$

$$\theta_{\widetilde{B}}(s, w) = 2\pi \left[\frac{(a-p)+1}{a} \left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi} \right],$$
$$\vartheta_{\widetilde{B}}(s, w) = 2\pi \left[\frac{(a-p)+1}{a} \left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))}{2\pi} \right],$$

where $a = |supp(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))|$.

Theorem 5.1 *Let* $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ *be a CPFDG. If* $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = (1e^{i2\pi}, 0e^{i0\pi})$ *in* $\mathfrak{C}^{\frac{[a]}{2}}(\overrightarrow{\xi})$, *then the edge* (s, w) *is strong, where* $a = |supp(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))|$. *(Note that for any real number* a, $[a]$ *is the greatest integer not exceeding* a*).*

Proof Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG and $\xi = (Y, \widetilde{A}, \widetilde{B})$ be the corresponding $\frac{[a]}{2}$-competition CPFG. According to the statement if $\mathbb{h}_{\mu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = 1$ and $\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)) = 2\pi$ in $\mathfrak{C}^{\frac{[a]}{2}}(\overrightarrow{\xi})$. Then membership grade of the edge (s, w) in $\mathfrak{C}^{\frac{[a]}{2}}(\overrightarrow{\xi})$ is defined as

$$\mu_{\widetilde{B}}(s, w) = \frac{(a-p)+1}{a}[\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)] \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)),$$
$$= \frac{(a-p)+1}{a}[\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)] \times 1,$$

$$\theta_{\widetilde{B}}(s,w) = 2\pi\left[\frac{(a-p)+1}{a}\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi}\wedge\frac{\theta_{\widetilde{A}}(w)}{2\pi}\right)\times\frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w))}{2\pi}\right],$$
$$= 2\pi\left[\frac{(a-p)+1}{a}\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi}\wedge\frac{\theta_{\widetilde{A}}(w)}{2\pi}\right)\times\frac{2\pi}{2\pi}\right],$$

Thus, clearly $\mu_{\widetilde{B}}(s,w) > \frac{1}{2}(\mu_{\widetilde{A}}(s)\wedge\mu_{\widetilde{A}}(w))$ and $\theta_{\widetilde{B}}(s,w) > \frac{1}{2}[2\pi(\frac{\theta_{\widetilde{A}}(s)}{2\pi}\wedge\frac{\theta_{\widetilde{A}}(w)}{2\pi})]$ as $\frac{(a-p)+1}{a} > \frac{1}{2}$. Similarly, if $\mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w)) = 0$ and $\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w)) = 0$ in $\mathfrak{C}^{\frac{[a]}{2}}(\overrightarrow{\xi})$, then the non-membership grade (s,w) of the edge in $\mathfrak{C}^{\frac{[a]}{2}}(\overrightarrow{\xi})$ is

$$\nu_{\widetilde{B}}(s,w) = \frac{(a-p)+1}{a}[\nu_{\widetilde{A}}(s)\vee\nu_{\widetilde{A}}(w)]\times\mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w)),$$
$$= \frac{(a-p)+1}{a}[\nu_{\widetilde{A}}(s)\vee\nu_{\widetilde{A}}(w)]\times 0,$$
$$\nu_{\widetilde{B}}(s,w) = [\nu_{\widetilde{A}}(s)\vee\nu_{\widetilde{A}}(w)]\times 0$$
$$\vartheta_{\widetilde{B}}(s,w) = 2\pi\left[\frac{(a-p)+1}{a}\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi}\vee\frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right)\times\frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w))}{2\pi}\right],$$
$$= 2\pi\left[\frac{(a-p)+1}{a}\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi}\vee\frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right)\times\frac{0}{2\pi}\right],$$
$$= 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi}\vee\frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right)\times 0\right].$$

This gives that $\nu_{\widetilde{B}}(s,w) < \frac{1}{2}(\nu_{\widetilde{A}}(s)\vee\nu_{A(w)})$ and $\vartheta_{\widetilde{B}}(s,w) < \frac{1}{2}\left[2\pi(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi}\vee\frac{\vartheta_{\widetilde{A}}(w)}{2\pi})\right]$ as $0 < \frac{1}{2}$. □

Example 5.1 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be CPFDG as shown in Fig. 6, defined by;

$$\widetilde{A} = \Big\langle\Big(\frac{a_1}{0.8e^{i1.6\pi}},\frac{a_2}{0.4e^{i1.4\pi}},\frac{a_3}{0.7e^{i1.8\pi}},\frac{a_4}{0.6e^{i1.6\pi}},\frac{a_5}{0.7e^{i1.4\pi}},\frac{a_6}{0.5e^{i1\pi}}\Big),$$
$$\Big(\frac{a_1}{0.3e^{i1.2\pi}},\frac{a_2}{0.8e^{i1.1\pi}},\frac{a_3}{0.6e^{i0.6\pi}},\frac{a_4}{0.8e^{i0.8\pi}},\frac{a_5}{0.7e^{i0.9\pi}},\frac{a_6}{0.8e^{i1\pi}}\Big)\Big\rangle,$$
$$\overrightarrow{\widetilde{B}} = \Big\langle\Big(\frac{(a_1,a_4)}{0.6e^{i1.6\pi}},\frac{(a_1,a_5)}{0.7e^{i1.4\pi}},\frac{(a_1,a_6)}{0.4e^{i0.9\pi}},\frac{(a_2,a_4)}{0.4e^{i1.2\pi}},\frac{(a_2,a_5)}{0.3e^{i1\pi}},\frac{(a_2,a_6)}{0.4e^{i1\pi}},\frac{(a_3,a_4)}{0.5e^{i1.2\pi}},\frac{(a_3,a_5)}{0.5e^{i1.2\pi}},\Big),$$
$$\Big(\frac{(a_1,a_4)}{0.5e^{i1\pi}},\frac{(a_1,a_5)}{0.5e^{i1\pi}},\frac{(a_1,a_6)}{0.6e^{i1.1\pi}},\frac{(a_2,a_4)}{0.6e^{i1.1\pi}},\frac{(a_2,a_5)}{0.7e^{i1\pi}},\frac{(a_2,a_6)}{0.8e^{i0.9\pi}},\frac{(a_3,a_4)}{0.7e^{i0.9\pi}},\frac{(a_3,a_5)}{0.7e^{i0.9\pi}},\Big)\Big\rangle$$

The CPF-out neighborhood of the vertices are given in Table 6.

The height, support and cardinality of support, of CPFS $\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w)$, for all $s, w \in Y$ are shown in Table 7.

For p = 2 the corresponding $\mathfrak{C}^2(\xi)$ is shown in Fig. 7.

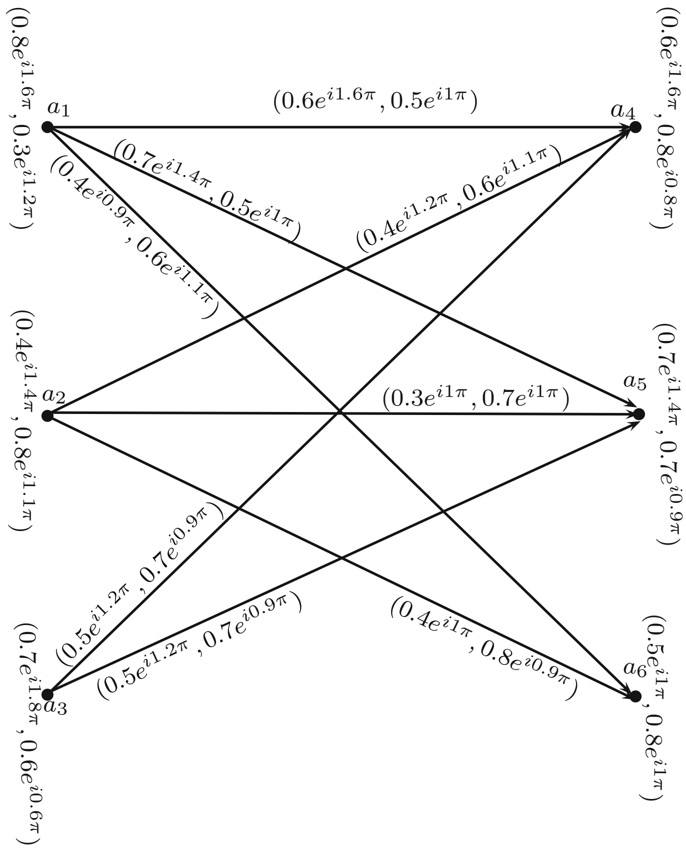


Fig. 6 Complex Pythagorean fuzzy digraph

Table 6 CPF-out neighborhood of the vertices

x	$\widetilde{\mathbb{N}}^p(x)$
a_1	$\{(a_4, 0.6e^{i1.6\pi}, 0.5e^{i1\pi})(a_5, 0.7e^{i1.4\pi}, 0.5e^{i1\pi})(a_6, 0.4e^{i0.9\pi}, 0.6e^{i1.1\pi})\}$
a_2	$\{(a_4, 0.4e^{i1.2\pi}, 0.6e^{i1.1\pi})(a_5, 0.3e^{i1\pi}, 0.7e^{i1\pi})(a_6, 0.4e^{i1\pi}, 0.8e^{i1\pi})\}$
a_3	$\{(a_4, 0.5e^{i1.2\pi}, 0.7e^{i0.9\pi})(a_5, 0.5e^{i1.2\pi}, 0.7e^{i0.9\pi})\}$
a_4	$\emptyset$
a_5	$\emptyset$
a_6	$\emptyset$

6 Complex Pythagorean fuzzy neighborhood graphs

In this section, we define complex Pythagorean fuzzy open and close neighborhood of a vertex, complex Pythagorean fuzzy open neighborhood graph and complex Pythagorean fuzzy closed neighborhood graph below.

Table 7 Height, support and cordinality of support of CPFS $\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w)$

s	w	$\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w)$	$N = \mathbb{h}(\widetilde{\mathbb{N}}^{p}(s) \cap \widetilde{\mathbb{N}}^{p}(w))$	$a = supp(N)$	$\lvert a \rvert$
a_1	a_2	$\{(a_4, 0.4e^{i1.2\pi}, 0.6e^{i1.1\pi})(a_5, 0.3e^{i1\pi}, 0.7e^{i1\pi})$ $(a_6, 0.4e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$	$(0.4e^{i1.2\pi}, 0.6e^{i1\pi})$	$\{(a_4, a_5, a_6)\}$	3
a_1	a_3	$\{(a_4, 0.6e^{i1.2\pi}, 0.7e^{i1\pi})(a_5, 0.5e^{i1.2\pi}, 0.7e^{i0.9\pi})\}$	$(0.6e^{i1.2\pi}, 0.7e^{i0.9\pi})$	$\{(a_4, a_5, a_6)\}$	2
a_1	a_4	$\emptyset$	$\emptyset$	$\emptyset$	0
a_1	a_5	$\emptyset$	$\emptyset$	$\emptyset$	0
a_1	a_6	$\emptyset$	$\emptyset$	$\emptyset$	0
a_2	a_3	$\{(a_4, 0.4e^{i1.2\pi}, 0.7e^{i1.1\pi})(a_5, 0.3e^{i1\pi}, 0.7e^{i1\pi})\}$	$(0.4e^{i1.2\pi}, 0.7e^{i1\pi})$	$\{(a_4, a_5)\}$	2
a_2	a_4	$\emptyset$	$\emptyset$	$\emptyset$	0
a_2	a_5	$\emptyset$	$\emptyset$	$\emptyset$	0
a_2	a_6	$\emptyset$	$\emptyset$	$\emptyset$	0
a_3	a_4	$\emptyset$	$\emptyset$	$\emptyset$	0
a_3	a_5	$\emptyset$	$\emptyset$	$\emptyset$	0
a_3	a_6	$\emptyset$	$\emptyset$	$\emptyset$	0

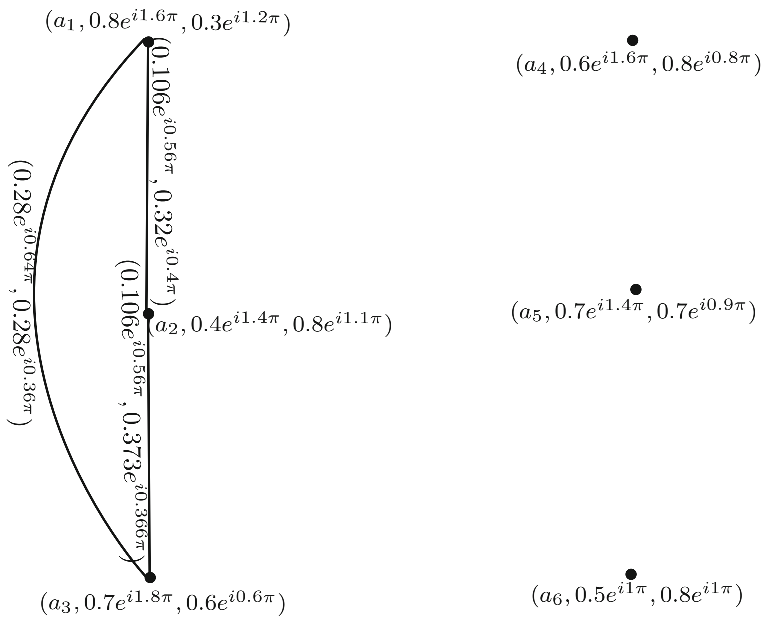


Fig. 7 2-competition complex Pythagorean fuzzy graph

Definition 6.1 Complex Pythagorean fuzzy open neighborhood of a vertex s of a CPFG $\xi = (Y, \widetilde{A}, \widetilde{B})$ is a CPFS defined by:

$$\widetilde{\mathbb{N}}(s) = (S_s, t_s e^{i\phi_s}, f_s e^{i\psi_s}),$$

where

$$S_s = \{w \mid \mu_{\widetilde{B}}(s, w) > 0 \ or \ \nu_{\widetilde{B}}(s, w) > 0\},$$

such that $t_s : S_s \to [0, 1]$ defined by $t_s(w) = \mu_{\widetilde{B}}(s, w)$, $\phi_s : S_s \to [0, 2\pi]$ defined by $\phi_s(w) = \theta_{\widetilde{B}}(s, w)$, $f_s : S_s \to [0, 1]$ defined by $f_s(w) = \nu_{\widetilde{B}}(s, w)$ and $\psi_s : S_s \to [0, 2\pi]$ defined by $\psi_s(w) = \vartheta_{\widetilde{B}}(s, w)$. For every vertex $s \in Y$, complex Pythagorean fuzzy singleton set can be defined as:

$$\widetilde{A}_s = (\{s\}, \mu'_{\widetilde{A}} e^{i\theta'_{\widetilde{A}}}, \nu'_{\widetilde{A}} e^{i\vartheta'_{\widetilde{A}}}),$$

where $\mu'_{\widetilde{A}} : \{s\} \to [0, 1]$ defined by $\mu'_{\widetilde{A}}(s) = \mu_{\widetilde{A}}(s)$, $\theta'_{\widetilde{A}} : \{s\} \to [0, 2\pi]$ defined by $\theta'_{\widetilde{A}}(s) = \theta_{\widetilde{A}}(s)$, $\nu'_{\widetilde{A}} : \{s\} \to [0, 1]$ defined by $\nu'_{\widetilde{A}}(s) = \nu_{\widetilde{A}}(s)$ and $\vartheta'_{\widetilde{A}} : \{s\} \to [0, 2\pi]$ defined by $\vartheta'_{\widetilde{A}}(s) = \vartheta_{\widetilde{A}}(s)$. Complex fuzzy closed neighborhood of a vertex s is defined as:

$$\widetilde{\mathbb{N}}[s] = \widetilde{\mathbb{N}}(s) \cup \widetilde{A}_s.$$

Definition 6.2 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG. The complex Pythagorean fuzzy open-neighborhood graph (CPFONG) of ξ is a CPFG $\mathfrak{N}(\xi) = (Y, \widetilde{A}, \widetilde{C})$, where the CPF vertex set of $\mathfrak{N}(\xi)$ remains same as in ξ and a CPF edge exists between two distinct nodes $s, w \in Y$ in $\mathfrak{N}(\xi)$ if and only if the CPFS $\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w) \neq \emptyset$ in ξ. The membership and non-membership grades of the edge (s, w) in $\mathfrak{N}(\xi)$ are defined as:

$$\mu_{\widetilde{C}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w)),$$
$$\nu_{\widetilde{C}}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_{\nu}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w)),$$
$$\theta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w))}{2\pi} \right],$$
$$\vartheta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w))}{2\pi} \right].$$

Definition 6.3 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG. The complex Pythagorean fuzzy closed-neighborhood graph (CPFCNG) of ξ is a CPFG $\mathfrak{N}[\xi] = (Y, \widetilde{A}, \widetilde{C})$, where the CPF vertex set of $\mathfrak{N}(\xi)$ remains same as in ξ and a CPF edge exists between two distinct nodes $s, w \in Y$ in $\mathfrak{N}[\xi]$ if and only if the CPFS $\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w] \neq \emptyset$ in ξ. The membership and non-membership grades of the edge (s, w) in $\mathfrak{N}[\xi]$ are defined as:

$$\mu_{\widetilde{C}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]),$$
$$\nu_{\widetilde{C}}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_{\nu}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]),$$

$$\theta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w])}{2\pi} \right],$$
$$\vartheta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\vartheta}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w])}{2\pi} \right].$$

Theorem 6.1 *For every edge of CPFG ξ there is one edge in $\mathfrak{N}[\xi]$.*

Proof Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG and $\mathfrak{N}[\xi] = (Y, \widetilde{A}, \widetilde{B}')$ be the corresponding CPFCNG. Let (s, w) be an edge of CPFG ξ. Then, $s, w \in \widetilde{\mathbb{N}}[s]$ and $s, w \in \widetilde{\mathbb{N}}[w]$. So $s, w \in \widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]$. Thus, $\mathbb{h}_{\mu}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0$ and $\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0$. So, according to the definition 6.3 the membership grade of the edge (s, w) in $\mathfrak{N}[\xi]$ is defined as

$$\mu_{\widetilde{B}'}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_{\mu}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0,$$
$$\theta_{\widetilde{B}'}(s, w) = 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_{\theta}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w])}{2\pi} \right] \neq 0.$$

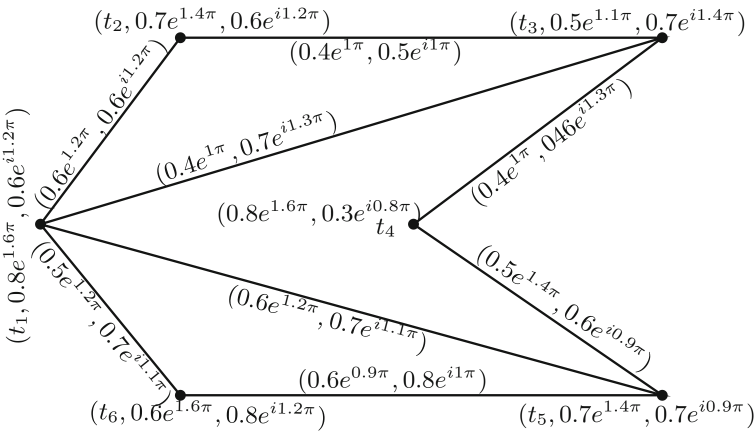


Fig. 8 Complex Pythagorean fuzzy digraph

Similarly, $\mathbb{h}_\nu(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0$ and $\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0$ for the height of non-membership function. So,

$$\nu_{\widetilde{B}'}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]) \neq 0,$$

$$\vartheta_{\widetilde{B}'}(s, w) = 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w])}{2\pi} \right] \neq 0.$$

Hence, for every edge (s, w) in ξ there exist an edge (s, w) in $\mathfrak{N}[\xi]$. □

Example 6.1 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG as shown in Fig. 8, defined by;

$$\widetilde{A} = \left\langle \left(\frac{t_1}{0.8e^{i1.6\pi}}, \frac{t_2}{0.7e^{i1.4\pi}}, \frac{t_3}{0.5e^{i1.1\pi}}, \frac{t_4}{0.8e^{i1.6\pi}}, \frac{t_5}{0.7e^{i1.4\pi}}, \frac{t_6}{0.6e^{i1.6\pi}} \right), \left(\frac{t_1}{0.6e^{i1.2\pi}}, \frac{t_2}{0.6e^{i1.2\pi}}, \frac{t_3}{0.7e^{i1.4\pi}}, \frac{t_4}{0.3e^{i0.8\pi}}, \frac{t_5}{0.7e^{i0.9\pi}}, \frac{t_6}{0.8e^{i1.2\pi}} \right) \right\rangle,$$

$$B = \left\langle \left(\frac{(t_1, t_2)}{0.6e^{i1.2\pi}}, \frac{(t_1, t_3)}{0.4e^{i1\pi}}, \frac{(t_1, t_5)}{0.6e^{i1.2\pi}}, \frac{(t_1, t_6)}{0.5e^{i1.2\pi}}, \frac{(t_2, t_3)}{0.4e^{i1\pi}}, \frac{(t_3, t_4)}{0.4e^{i1\pi}}, \frac{(t_4, t_5)}{0.5e^{i1.4\pi}}, \frac{(t_5, t_6)}{0.6e^{i0.9\pi}} \right), \left(\frac{(t_1, t_2)}{0.6e^{i1.2\pi}}, \frac{(t_1, t_3)}{0.7e^{i1.3\pi}}, \frac{(t_1, t_5)}{0.7e^{i1.1\pi}}, \frac{(t_1, t_6)}{0.7e^{i1.1\pi}}, \frac{(t_2, t_3)}{0.5e^{i1\pi}}, \frac{(t_3, t_4)}{0.4e^{i1.3\pi}}, \frac{(t_4, t_5)}{0.6e^{i0.9\pi}}, \frac{(t_5, t_6)}{0.8e^{i1\pi}} \right) \right\rangle.$$

The CPF open and closed neighborhoods of the vertices are shown in Table 8.

For CPFONG the CPFS $\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w))$, for all $s, w \in Y$ is given in Table 9.

The corresponding CPFONG is shown in Fig. 9.

Similarly, for CPFCNG the CPFS $\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]$ and $\mathbb{h}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w))$, for all $s, w \in Y$ are given in Table 10.

The corresponding CPFCNG is shown in Fig. 10.

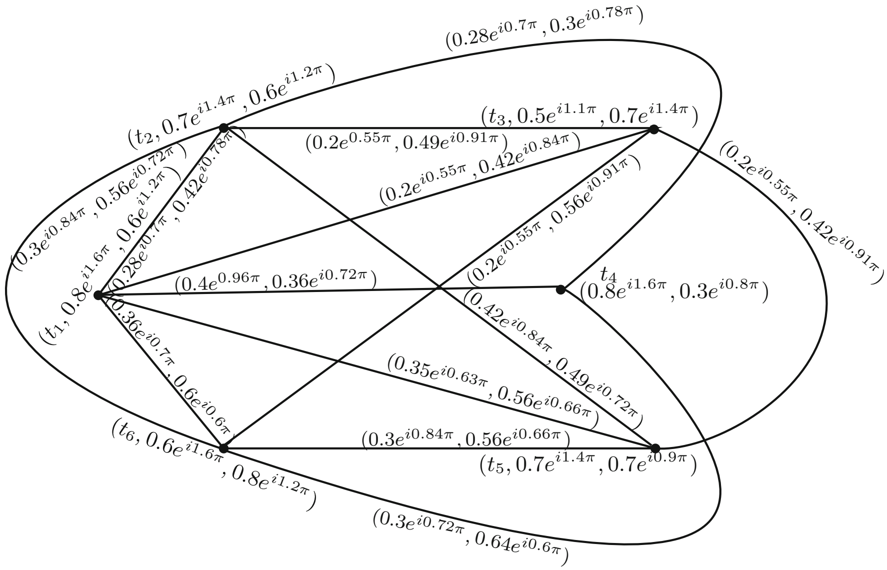
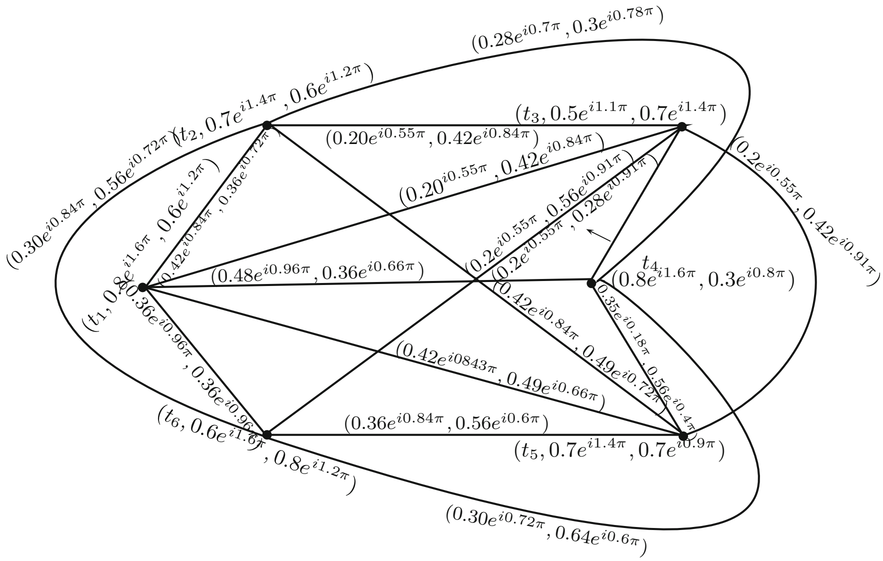


Fig. 9 Complex Pythagorean fuzzy open neighborhood graph



$\mathfrak{N}[\xi]$

Fig. 10 Complex Pythagorean fuzzy closed neighborhood graph

Table 8 CPF open and closed neighborhoods

s	$\widetilde{\mathbb{N}}(s)$	$\widetilde{\mathbb{N}}[s]$
t_1	$\{(t_2, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi}), ((t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}))$ $(t_5, 0.6e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_6, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$	$\{(t_2, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi}), ((t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}))$ $\{(t_5, 0.6e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_6, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$ $\cup\{t_1, 0.8e^{i1.6\pi}, 0.6e^{i1.2\pi}\}$
t_2	$\{(t_1, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi}), (t_3, 0.4e^{i1\pi}, 0.5e^{i1\pi})\}$	$\{(t_1, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi}), (t_3, 0.4e^{i1\pi}, 0.5e^{i1\pi})\}$ $\cup\{(t_2, 0.7e^{i1.4\pi}, 0.6e^{i1.2\pi})\}$
t_3	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_2, 0.4e^{i1\pi}, 0.5e^{i1\pi})$ $(t_4, 0.4e^{i1\pi}, 0.4e^{i1.3\pi})\}$	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_2, 0.4e^{i1\pi}, 0.5e^{i1\pi})$ $(t_4, 0.4e^{i1\pi}, 0.4e^{i1.3\pi})\} \cup \{(t_3, 0.5e^{i1.1\pi}, 0.7e^{i1.4\pi})\}$
t_4	$\{(t_3, 0.4e^{i1\pi}, 0.4e^{i1.3\pi}), (t_5, 0.5e^{i1.4\pi}, 0.6e^{i0.9\pi})\}$	$\{(t_3, 0.4e^{i1\pi}, 0.4e^{i1.3\pi}), (t_5, 0.5e^{i1.4\pi}, 0.6e^{i0.9\pi})\}$ $\cup\{(t_4, 0.8e^{i1.6\pi}, 0.3e^{i0.8\pi})\}$
t_5	$\{(t_1, 0.6e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_4, 0.5e^{i1.4\pi}, 0.6e^{i0.9\pi})$ $(t_6, 0.6e^{i0.9\pi}, 0.8e^{i01\pi})\}$	$\{(t_1, 0.6e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_4, 0.5e^{i1.4\pi}, 0.6e^{i0.9\pi})$ $(t_6, 0.6e^{i0.9\pi}, 0.8e^{i1\pi}) \cup \{(t_5, 0.7e^{i1.4\pi}, 0.7e^{i0.9\pi})\}$
t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_5, 0.6e^{i0.9\pi}, 0.8e^{i1\pi})\}$	$(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi}), (t_5, 0.6e^{i0.9\pi}, 0.8e^{i1\pi})\}$ $\cup\{(t_6, 0.6e^{i1.6\pi}, 0.8e^{i1.2\pi})\}$

Table 9 CPFSs $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$

s	w	$\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w)$	$\mathbb{h}(\widetilde{\mathbb{N}}(s) \cap \widetilde{\mathbb{N}}(w))$
t_1	t_2	$\{(t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$
t_1	t_3	$\{(t_2, 0.4e^{i1\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1.2\pi})\}$
t_1	t_4	$\{(t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_5, 0.5e^{i1.2\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.5e^{i1.2\pi}, 0.6e^{i1.2\pi})\}$
t_1	t_5	$\{(t_6, 0.5e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$	$\{(0.5e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$
t_1	t_6	$\{(t_5, 0.6e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$	$\{(0.6e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$
t_2	t_3	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$
t_2	t_4	$\{(t_3, 0.4e^{i1\pi}, 0.5e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.5e^{i1.3\pi})\}$
t_2	t_5	$\{(t_1, 0.6e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$
t_2	t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.5e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$
t_3	t_4	$\emptyset$	$\emptyset$
t_3	t_5	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_4, 0.4e^{i1\pi}, 0.6e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1.3\pi})\}$
t_3	t_6	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$
t_4	t_5	$\emptyset$	$\emptyset$
t_4	t_6	$\{(t_5, 0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$	$\{(0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$
t_5	t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$	$\{(0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$

7 *m*-step complex Pythagorean fuzzy competition graphs

If a prey d is assaulted by the predator c then the connection between them can be represented by an arc $\overrightarrow{(c, d)}$ in a CPFDG. But, if the predator want an assistance of several arbitrators $(c_1, c_2, \ldots c_{m-1})$ then the connection among them is represented by CPFDP

Table 10 CPFSs $\widetilde{\mathbb{N}}^p[s] \cap \widetilde{\mathbb{N}}^p[w]$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p[s] \cap \widetilde{\mathbb{N}}^p[w])$

s	w	$\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w]$	$\mathbb{h}(\widetilde{\mathbb{N}}[s] \cap \widetilde{\mathbb{N}}[w])$
t_1	t_2	$\{(t_1, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi}), (t_2, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi})$ $(t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.6e^{i1.2\pi})\}$
t_1	t_3	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_2, 0.4e^{i1\pi}, 0.6e^{i1.2\pi})$ $(t_3, 0.4e^{i1\pi}, 0.7e^{i1.4\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1.2\pi})\}$
t_1	t_4	$\{(t_3, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_5, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$ $(t_2, 0.6e^{i1.2\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.6e^{i1.1\pi})\}$
t_1	t_5	$\{(t_1, 0.6e^{i1.2\pi}, 0.7e^{i1.2\pi}), (t_5, 0.6e^{i1.2\pi}, 0.7e^{i1.1\pi})$ $(t_6, 0.5e^{i0.9\pi}, 0.8e^{i1.1\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$
t_1	t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.2\pi}), (t_5, 0.6e^{i0.9\pi}, 0.8e^{i1.1\pi})$ $(t_6, 0.5e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$
t_2	t_3	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_2, 0.4e^{i1\pi}, 0.6e^{i1.2\pi})$ $(t_3, 0.4e^{i1\pi}, 0.7e^{i1.4\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1.2\pi})\}$
t_2	t_4	$\{(t_3, 0.4e^{i1\pi}, 0.5e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.5e^{i1.3\pi})\}$
t_2	t_5	$\{(t_1, 0.6e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$
t_2	t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.5e^{i1.2\pi}, 0.7e^{i1.2\pi})\}$
t_3	t_4	$\{(t_3, 0.4e^{i1\pi}, 0.7e^{i1.4\pi}), (t_4, 0.4e^{i1\pi}, 0.4e^{i1.3\pi})$	$\{(0.4e^{i1\pi}, 0.4e^{i1.3\pi})\}$
t_3	t_5	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi}), (t_4, 0.4e^{i1\pi}, 0.6e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.6e^{i1.3\pi})\}$
t_3	t_6	$\{(t_1, 0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$	$\{(0.4e^{i1\pi}, 0.7e^{i1.3\pi})\}$
t_4	t_5	$\{(t_5, 0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$	$\{(0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$
t_4	t_6	$\{(t_5, 0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$	$\{(0.5e^{i0.9\pi}, 0.8e^{i1\pi})\}$
t_5	t_6	$\{(t_1, 0.5e^{i1.2\pi}, 0.7e^{i1.1\pi})\}$ $(t_5, 0.6e^{i0.9\pi}, 0.8e^{i1\pi}), (t_6, 0.6e^{i0.9\pi}, 0.8e^{i1.2\pi})\}$	$\{(0.6e^{i1.2\pi}, 0.7e^{i1\pi})\}$

$\overrightarrow{P}^m_{(c,d)}$ in a CPFDG. In this section, we first define m-step complex Pythagorean fuzzy digraph (CPFDG). Then we define CPF m-step out-neighborhood, CPF m-step in-neighborhood of the vertex and then m-step complex Pythagorean fuzzy competition graph.

Definition 7.1 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ be CPFDG. The m-step complex Pythagorean fuzzy digraph (m-step CPFDG) of $\overrightarrow{\xi}$ is denoted by $\overrightarrow{\xi}_m = (\overrightarrow{\xi}, \widetilde{A}, \overrightarrow{\overrightarrow{C}})$, which has same CPF vertex set as $\overrightarrow{\xi}$ and has a complex Pythagorean fuzzy edge between two distinct nodes s and w in $\overrightarrow{\xi}_m$ if there exists a complex Pythagorean fuzzy directed path (CPFDP) of length m from s to w, i.e., $\overrightarrow{P}^m_{(s,w)}$ in $\overrightarrow{\xi}$.

Definition 7.2 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ be CPFDG. Then complex Pythagorean fuzzy m-step out-neighborhood of a vertex s of a CPFDG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ is CPFS

$$\widetilde{\mathbb{N}}^p_m(s) = (S^p_s, t^p_s e^{i\phi^p_s}, f^p_s e^{i\psi^p}),$$

where

$$S_s^p = \{w | there\ exists\ a\ CPFDP\ of\ length\ m\ from\ s\ to\ w,\ \overrightarrow{P}^m_{(s,w)}\}$$

such that $t_s^p : S_s^p \to [0, 1]$ defined by $t_s^p(w) = \{\min \mu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(s,w)}\}$, $\phi_s^p : S_s^p \to [0, 2\pi]$, defined by $\phi_s^p(w) = \{\min \theta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(s,w)}\}$, $f_s^p : S_s^p \to [0, 1]$, defined by $f_s^p(w) = \{\max \nu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(s,w)}\}$, and $\psi_s^p : S_s^p \to [0, 2\pi]$ defined by $\psi_s^p(w) = \{\max \vartheta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(s,w)}\}$.

Definition 7.3 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be CPFDG. Then complex Pythagorean fuzzy m-step in-neighborhood of a vertex s of a CPFDG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ is CPFS

$$\widetilde{\mathbb{N}}^n_m(s) = (S_s^n, t_s^n e^{i\phi_s^n}, f_s^n e^{i\psi^n}),$$

where

$$S_s^n = \{w |\ there\ exists\ a\ CPFDP\ of\ length\ m\ from\ w\ to\ s,\ \overrightarrow{P}^m_{(w,s)}\}$$

such that $t_s^n : S_s^n \to [0, 1]$ defined by $t_s^n(w) = \{\min \mu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(w,s)}\}$, $\phi_s^n : S_s^n \to [0, 2\pi]$, defined by $\phi_s^n(w) = \{\min \theta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(w,s)}\}$, $f_s^n : S_s^n \to [0, 1]$, defined by $f_s^n(w) = \{\max \nu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(w,s)}\}$, and $\psi_s^n : S_s^n \to [0, 2\pi]$ defined by $\psi_s^n(w) = \{\max \vartheta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $\overrightarrow{P}^m_{(w,s)}\}$.

Example 7.1 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG as depicted in Fig. 11, defined by:

$$\begin{aligned}\widetilde{A} &= \left\langle \left(\frac{d_1}{0.8e^{i1.2\pi}}, \frac{d_2}{0.5e^{i1.4\pi}}, \frac{d_3}{0.4e^{i1.5\pi}}, \frac{d_4}{0.6e^{i1.4\pi}} \right), \right. \\ &\quad \left. \left(\frac{d_1}{0.5e^{i1\pi}}, \frac{d_2}{0.6e^{i0.9\pi}}, \frac{d_3}{0.7e^{i0.9\pi}}, \frac{d_4}{0.3e^{i1\pi}} \right) \right\rangle, \\ \overrightarrow{\widetilde{B}} &= \left\langle \left(\frac{\overrightarrow{(d_1, d_2)}}{0.45e^{i1.1\pi}}, \frac{\overrightarrow{(d_1, d_3)}}{0.4e^{i0.3\pi}}, \frac{\overrightarrow{(d_1, d_4)}}{0.55e^{i1.3\pi}} \frac{\overrightarrow{(d_3, d_4)}}{0.35e^{i1\pi}}, \frac{\overrightarrow{(d_4, d_2)}}{0.5e^{i0.7\pi}} \right), \right. \\ &\quad \left. \left(\frac{\overrightarrow{(d_1, d_2)}}{0.55e^{i0.8\pi}}, \frac{\overrightarrow{(d_1, d_3)}}{0.6e^{i0.9\pi}}, \frac{\overrightarrow{(d_1, d_4)}}{0.45e^{i1\pi}} \frac{\overrightarrow{(d_3, d_4)}}{0.66e^{i0.9\pi}}, \frac{\overrightarrow{(d_4, d_2)}}{0.5e^{i0.9\pi}} \right) \right\rangle.\end{aligned}$$

The CPF 2-step out and in-neighborhoods of the vertices are given in Table 11.

Definition 7.4 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFG. The m-step CPFCG of ξ is denoted by $\mathfrak{C}_m(\overrightarrow{\xi}) = (Y, \widetilde{A}, \widetilde{C})$, where the CPF vertex set of $\mathfrak{C}_m(\overrightarrow{\xi})$ is same as in $\overrightarrow{\xi}$ and a

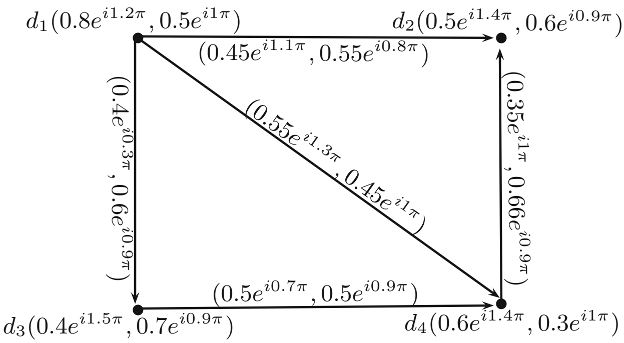


Fig. 11 Complex Pythagorean fuzzy digraph

Table 11 CPF-2-step neighborhoods

s	$\widetilde{\mathbb{N}}^p_2(s)$	$\widetilde{\mathbb{N}}^n_2(s)$
d_1	$(d_4, 0.4e^{i0.3\pi}, 0.6e^{i0.9\pi})$, $(d_2, 0.35e^{i1\pi}, 0.66e^{i1\pi})$	$\emptyset$
d_2	$\emptyset$	$(d_3, 0.35e^{i0.7\pi}, 0.66e^{i0.9\pi})$.
d_3	$(d_2, 0.35e^{i0.7\pi}, 0.66e^{i0.9\pi})$	$\emptyset$
d_4	$\emptyset$	$(d_1, 0.4e^{i0.3\pi}, 0.6e^{i0.9\pi})$

complex Pythagorean fuzzy edge exists between two distinct nodes s and w in $\mathfrak{C}_m(\overrightarrow{\xi})$ if and only if $\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w) \neq \emptyset$ in CPFDG. The membership and non-membership grades of the edge (s, w) are defined as:

$$\mu_{\widetilde{C}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),$$
$$\nu_{\widetilde{C}}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),$$
$$\theta_{\widetilde{C}}(s, w) = 2\pi\left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi}\right],$$
$$\vartheta_{\widetilde{C}}(s, w) = 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi}\right].$$

Example 7.2 Let $\overrightarrow{G} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG as shown in Fig. 12, defined by;

$$\widetilde{A} = \left\langle \left(\frac{d_1}{0.8e^{i1.2\pi}}, \frac{d_2}{0.7e^{i1\pi}}, \frac{d_3}{0.5e^{i1.4\pi}}, \frac{d_4}{0.4e^{i1.4\pi}}, \frac{d_5}{0.5e^{i1.8\pi}}, \frac{d_6}{0.8e^{i1.3\pi}}, \frac{d_7}{0.8e^{i1.2\pi}}\right), \left(\frac{d_1}{0.6e^{i1.1\pi}}, \frac{d_2}{0.7e^{i1.3\pi}}, \frac{d_3}{0.6e^{i1.2\pi}}, \frac{d_4}{0.7e^{i0.9\pi}}, \frac{d_5}{0.7e^{i0.5\pi}}, \frac{d_6}{0.4e^{i1\pi}}, \frac{d_7}{0.6e^{i1.1\pi}}\right)\right\rangle,$$

$$\overrightarrow{\widetilde{B}} = \left\langle \left(\frac{\overrightarrow{(d_1, d_2)}}{0.7e^{i1\pi}}, \frac{\overrightarrow{(d_1, d_3)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(d_2, d_3)}}{0.5e^{i0.9\pi}}, \frac{\overrightarrow{(d_4, d_2)}}{0.3e^{i0.9\pi}}, \frac{\overrightarrow{(d_4, d_3)}}{0.4e^{i1.2\pi}}, \frac{\overrightarrow{(d_5, d_4)}}{0.3e^{i1.2\pi}}, \frac{\overrightarrow{(d_5, d_6)}}{0.4e^{i1.2\pi}}, \frac{\overrightarrow{(d_6, d_4)}}{0.3e^{i1.2\pi}}, \frac{\overrightarrow{(d_7, d_4)}}{0.3e^{i1\pi}}\right), \left(\frac{\overrightarrow{(d_1, d_2)}}{0.7e^{i1.2\pi}}, \frac{\overrightarrow{(d_1, d_3)}}{0.6e^{i1.2\pi}}, \frac{\overrightarrow{(d_2, d_3)}}{0.7e^{i1.1\pi}}, \frac{\overrightarrow{(d_4, d_2)}}{0.7e^{i1.2\pi}}, \frac{\overrightarrow{(d_4, d_3)}}{0.6e^{i0.9\pi}}, \frac{\overrightarrow{(d_5, d_4)}}{0.7e^{i0.9\pi}}, \frac{\overrightarrow{(d_5, d_6)}}{0.4e^{i1\pi}}, \frac{\overrightarrow{(d_6, d_4)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(d_7, d_4)}}{0.6e^{i1\pi}}\right)\right\rangle.$$

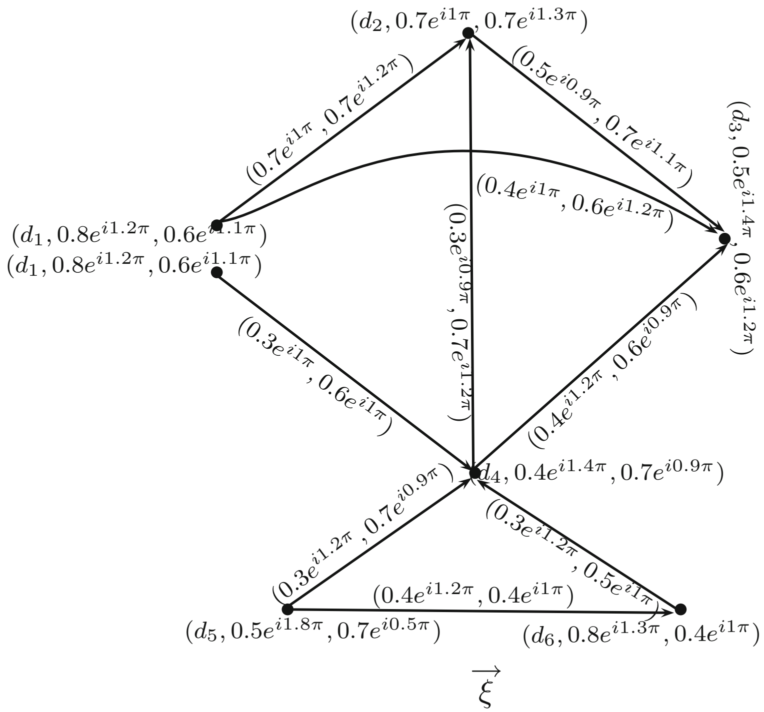


Fig. 12 Complex Pythagorean fuzzy digraph

Table 12 CPF 2-step out-neighborhoods of the vertices

s	$\widetilde{\mathbb{N}}_2^p(s)$
d_1	$\{(d_3, 0.5e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$
d_2	$\emptyset$
d_3	$\emptyset$
d_4	$\{(d_3, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$
d_5	$\{(d_4, 0.3e^{i1.2\pi}, 0.5e^{i1\pi}), (d_2, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$
d_6	$\{(d_2, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$
d_7	$\{(d_3, 0.3e^{i1\pi}, 0.6e^{i1\pi})\}$

The CPF 2-step out-neighborhood of the vertices are shown in Table 12.

Therefore $\widetilde{\mathbb{N}}_2^p(d_1) \cap \widetilde{\mathbb{N}}_2^p(d_7) = \{(d_3, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$, $\widetilde{\mathbb{N}}_2^p(d_1) \cap \widetilde{\mathbb{N}}_2^p(d_4) = \{(d_3, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$, $\widetilde{\mathbb{N}}_2^p(d_4) \cap \widetilde{\mathbb{N}}_2^p(d_7) = \{(d_3, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$ and $\widetilde{\mathbb{N}}_2^p(d_5) \cap \widetilde{\mathbb{N}}_2^p(d_6) = \{(d_2, 0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$. Hence $\mathbb{h}(\widetilde{\mathbb{N}}_2^p(d_1) \cap \widetilde{\mathbb{N}}_2^p(d_4)) = \{(0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$, $\mathbb{h}(\widetilde{\mathbb{N}}_2^p(d_4) \cap \widetilde{\mathbb{N}}_2^p(d_7)) = \{(0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$, $\mathbb{h}(\widetilde{\mathbb{N}}_2^p(d_1) \cap \widetilde{\mathbb{N}}_2^p(d_7)) = \{(0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$ and $\widetilde{\mathbb{N}}_2^p(d_5) \cap \widetilde{\mathbb{N}}_2^p(d_2) = \{(0.3e^{i0.9\pi}, 0.7e^{i1.2\pi})\}$. The corresponding 2-step CPFCG is shown in Fig. 13.

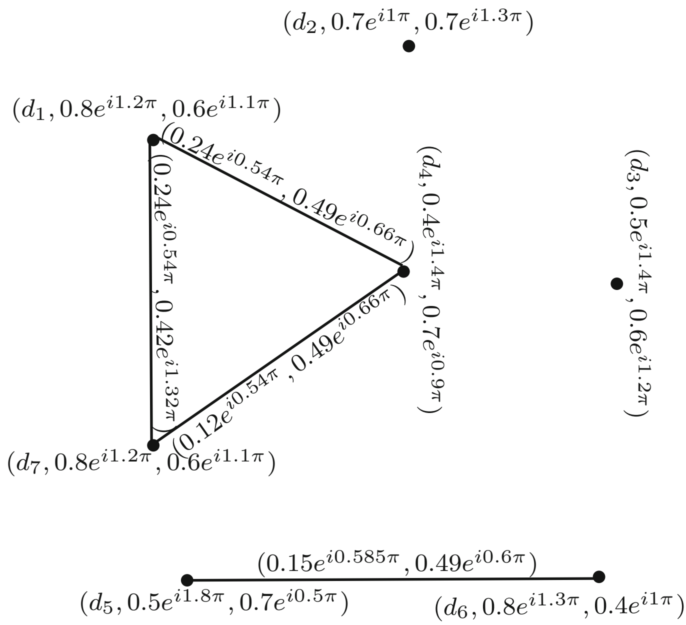


Fig. 13 2-Step complex Pythagorean fuzzy competition graph

Theorem 7.1 *Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ be CPFDG. If $m > |Y|$ then the edge set of $C_m(\overrightarrow{\xi})$ is empty.*

Proof Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ be CPFDG and $C_m(\overrightarrow{\xi}) = (Y, \widetilde{A}, \widetilde{C})$ be the corresponding m-step CPFCG. So according to the definition 7.4 the values for the membership and non-membership grades of the edge (s, w) in $C_m(\overrightarrow{\xi})$ are

$$\mu_{\widetilde{C}}(s, w) = (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),$$
$$\nu_{\widetilde{C}}(s, w) = (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),$$

$$\theta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi} \right],$$
$$\vartheta_{\widetilde{C}}(s, w) = 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi} \right],$$

for all $s, w \in Y$. If $|m| > Y$, there does not exist any CPFDP of length m in $\overrightarrow{\xi}$. So, the CPFS $(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) = \emptyset$. Then $\mu_{\widetilde{C}}(s, w) = 0$, $\theta_{\widetilde{C}}(s, w) = 0$ for the membership grade and $\nu_{\widetilde{C}}(s, w) = 0$, $\vartheta_{\widetilde{C}}(s, w) = 0$ for the non-membership grade. Hence $\mathfrak{C}_m(\overrightarrow{\xi})$ has no edges. □

Definition 7.5 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be CPFG. Then complex Pythagorean fuzzy m-step neighborhood of a vertex s of a CPFG $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ is CPFS

$$\widetilde{\mathbb{N}}_m(s) = (S_s, te_s^{i\phi_s}, f_s e^{i\psi_s}),$$

where

$$S_s = \{w| \, there \; exist \; a \; CPFDP \; of \; length \; m \; between \; w \; and \; s, \; P^m_{(w,s)}\}$$

such that $t_s : S_s \to [0, 1]$ defined by $t_s(w) = \{\min \mu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $P^m_{(w,s)}\}$, $\phi_s : S_s \to [0, 2\pi]$ defined by $\phi_s(w) = \{\min \theta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $P^m_{(w,s)}\}$, $f_s : S_s \to [0, 1]$ defined by $f_s(w) = \{\max \nu_{\widetilde{B}}(d, g), (d, g)$ is an edge of $P^m_{(w,s)}\}$, and $\psi_s : S_s \to [0, 2\pi]$ defined by $\psi_s(w) = \{\max \vartheta_{\widetilde{B}}(d, g), (d, g)$ is an edge of $P^m_{(w,s)}\}$.

Definition 7.6 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be a CPFG. The m-step CPFNG of ξ is denoted by $\mathfrak{N}_m(\xi) = (Y, \widetilde{A}, \widetilde{C})$, where the CPF vertex set of $\mathfrak{N}_m(\xi)$ is same as in ξ and a complex Pythagorean fuzzy edge exists between two distinct nodes s and w in $\mathfrak{N}_m(\xi)$ if and only if $\widetilde{\mathbb{N}}_m(s) \cap \widetilde{\mathbb{N}}_m(w) \neq \emptyset$ in CPFG. The membership and non-membership grades of the edge (s, w) are defined as:

$$\begin{aligned}
\mu_{\widetilde{C}}(s, w) &= (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}_m(s) \cap \widetilde{\mathbb{N}}_m(w)), \\
\nu_{\widetilde{C}}(s, w) &= (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}_m(s) \cap \widetilde{\mathbb{N}}_m(w)), \\
\theta_{\widetilde{C}}(s, w) &= 2\pi \left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}_m(s) \cap \widetilde{\mathbb{N}}_m(w))}{2\pi} \right], \\
\vartheta_{\widetilde{C}}(s, w) &= 2\pi \left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi} \right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}_m(s) \cap \widetilde{\mathbb{N}}_m(w))}{2\pi} \right].
\end{aligned}$$

Example 7.3 Let $\xi = (Y, \widetilde{A}, \widetilde{B})$ be as CPFG as shown in Fig. 14 defined by;

$$\begin{aligned}
\widetilde{A} = \Bigg\langle & \left(\frac{n_1}{0.55e^{i1.2\pi}}, \frac{n_2}{0.66e^{i1\pi}}, \frac{n_3}{0.7e^{i1.3\pi}}, \frac{n_4}{0.9e^{i1.5\pi}}, \frac{n_5}{0.5e^{i1\pi}}, \frac{n_6}{0.7e^{i0.9\pi}} \right), \\
& \left(\frac{n_1}{0.8e^{i1.3\pi}}, \frac{n_2}{0.5e^{i0.9\pi}}, \frac{n_3}{0.7e^{i0.9\pi}}, \frac{n_4}{0.3e^{i1\pi}}, \frac{n_5}{0.6e^{i1\pi}}, \frac{n_6}{0.7e^{i1.3\pi}} \right) \Bigg\rangle, \\
B = \Bigg\langle & \left(\frac{(n_1, n_2)}{0.5e^{i1\pi}}, \frac{(n_2, n_3)}{0.65e^{i0.8\pi}}, \frac{(n_3, n_4)}{0.65e^{i1.1\pi}}, \frac{(n_4, n_5)}{0.45e^{i1\pi}}, \frac{(n_4, n_6)}{0.6e^{i0.8\pi}}, \frac{(n_6, n_1)}{0.4e^{i0.9\pi}} \right), \\
& \left(\frac{(n_1, n_2)}{0.6e^{i1.2\pi}}, \frac{(n_2, n_3)}{0.7e^{i0.9\pi}}, \frac{(n_3, n_4)}{0.5e^{i1\pi}}, \frac{(n_4, n_5)}{0.45e^{i0.8\pi}}, \frac{(n_4, n_6)}{0.6e^{i1.2\pi}}, \frac{(n_6, n_1)}{0.8e^{i1.3\pi}} \right) \Bigg\rangle.
\end{aligned}$$

The CPF 2-step neighborhood of the vertices are shown in Table 13.
The CPFS $\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w))$ are shown in Table 14.
The corresponding 2-step CPFNG is shown in Fig. 15.

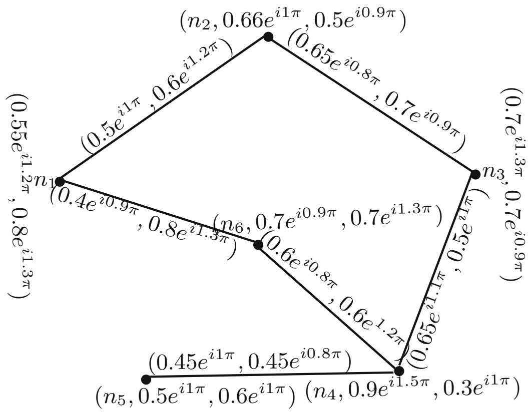


Fig. 14 Complex Pythagorean fuzzy graph

Table 13 CPF 2-step neighborhood of the vertices

s	$\widetilde{\mathbb{N}}_2(s)$
n_1	$\{(n_3, 0.5e^{i0.8\pi}, 0.7e^{i1.2\pi}), (n_4, 0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$
n_2	$\{(n_4, 0.65e^{i0.8\pi}, 0.7e^{i1\pi}), (n_6, 0.4e^{i0.9\pi}, 0.8e^{i1.3\pi})\}$
n_3	$\{(n_1, 0.5e^{i0.8\pi}, 0.7e^{i1.2\pi}), (n_5, 0.45e^{i1\pi}, 0.5e^{i1\pi}), (n_6, 0.6e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$
n_4	$\{(n_1, 0.5e^{i0.8\pi}, 0.6e^{i1.2\pi}), (n_2, 0.65e^{i0.8\pi}, 0.7e^{i1\pi})\}$
n_5	$\{(n_3, 0.45e^{i1\pi}, 0.5e^{i1\pi}), (n_6, 0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$
n_6	$\{(n_2, 0.4e^{i0.9\pi}, 0.8e^{i1.3\pi}), (n_3, 0.6e^{i0.8\pi}, 0.6e^{i1.2\pi}), (n_5, 0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$

Table 14 CPFSs $\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w))$

s	w	$\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w)$	$\mathbb{h}(\widetilde{\mathbb{N}}_2(s) \cap \widetilde{\mathbb{N}}_2(w))$
n_1	n_2	$\{(n_4, 0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$	$\{(0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$
n_1	n_3	$\emptyset$	$\emptyset$
n_1	n_4	$\emptyset$	$\emptyset$
n_1	n_5	$\{(n_3, 0.45e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.45e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$
n_1	n_6	$\{(n_3, 0.5e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.5e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$
n_2	n_3	$\{(n_6, 0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$	$\{(0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$
n_2	n_4	$\emptyset$	$\emptyset$
n_2	n_5	$\{(n_6, 0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$	$\{(0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$
n_2	n_6	$\emptyset$	$\emptyset$
n_3	n_4	$\{(n_1, 0.5e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$	$\{(0.5e^{i0.8\pi}, 0.7e^{i1.2\pi})\}$
n_3	n_5	$\{(n_6, 0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$
n_3	n_6	$\{(n_5, 0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$
n_4	n_5	$\emptyset$	$\emptyset$
n_4	n_6	$\{(n_2, 0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$	$\{(0.4e^{i0.8\pi}, 0.8e^{i1.3\pi})\}$
n_5	n_6	$\{(n_3, 0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$	$\{(0.45e^{i0.8\pi}, 0.6e^{i1.2\pi})\}$

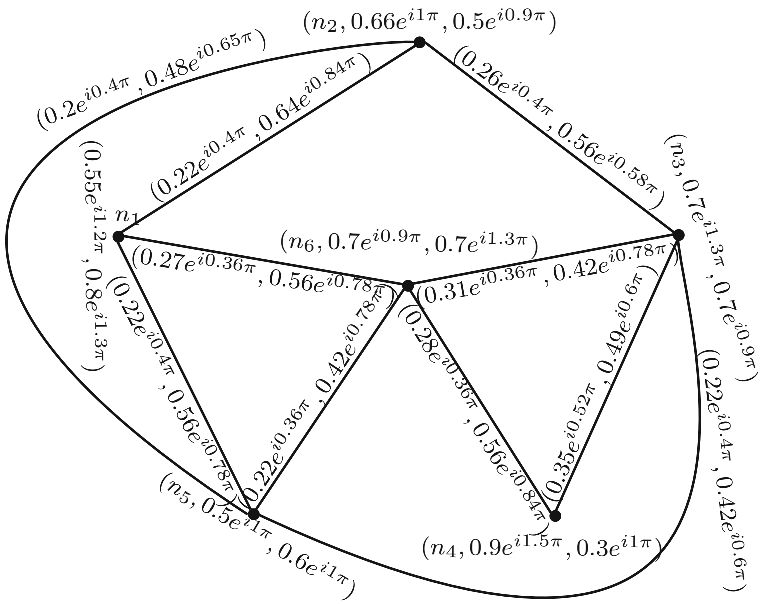


Fig. 15 2-Step complex Pythagorean fuzzy neighborhood graph

Definition 7.7 Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG. Let the common vertex of m-step out-neighborhoods of vertices $b_1, b_2, \ldots b_n$ is z. Also, let $\mu_{\widetilde{B}}\overrightarrow{(q_1, w_1)}e^{i\theta_{\widetilde{B}}\overrightarrow{(q_1,w_1)}}$, $\mu_{\widetilde{B}}\overrightarrow{(q_2, w_2)}e^{i\theta_{\widetilde{B}}\overrightarrow{(q_2,w_2)}}, \ldots \mu_{\widetilde{B}}\overrightarrow{(q_n, w_n)}e^{i\theta_{\widetilde{B}}\overrightarrow{(q_n,w_n)}}$ represent the minimum membership grades of the edges of paths $\overrightarrow{P}^{m}_{(b_1,z)}, \overrightarrow{P}^{m}_{(b_2,z)}, \ldots \overrightarrow{P}^{m}_{(b_n,z)}$, and $\nu_{\widetilde{B}}\overrightarrow{(q_1, w_1)}$ $e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_1,w_1)}}$, $\nu_{\widetilde{B}}\overrightarrow{(q_2, w_2)}e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_2,w_2)}}, \cdots \nu_{\widetilde{B}}\overrightarrow{(q_n, w_n)}e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_n,w_n)}}$ represent the maximum membership grades of the edges of paths $\overrightarrow{P}^{m}_{(b_1,z)}, \overrightarrow{P}^{m}_{(b_2,z)}, \ldots \overrightarrow{P}^{m}_{(b_n,z)}$, respectively. The m-step vertex $z \in Y$ is strong if $\mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)} > \dfrac{1}{2}$ and $\theta_{\widetilde{B}}\overrightarrow{(q_k, w_k)} > 1\pi$ for membership grades and $\nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)} < \dfrac{1}{2}$ and $\vartheta_{\widetilde{B}}\overrightarrow{(q_k, w_k)} < 1\pi$ for non-membership grades, for all $k = 1, 2, 3, \ldots, n$.

The strength of prey z denoted by $(s_T(z), s_F(z))$, where $s_T{:}Y \to \{x|x \in \mathbf{C}{:}|x| \leq 1\}$ and $s_F{:}Y \to \{x^{'}|x^{'} \in \mathbf{C} : |x^{'}| \leq 1\}$, is defined as:

$$s_T(z) = \frac{\sum_{k=1}^{n} \mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} e^{i\frac{\sum_{j=1}^{n}\theta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}{n}},$$

$$s_F(z) = \frac{\sum_{k=1}^{n} \nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} e^{i\frac{\sum_{j=1}^{n}\vartheta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}{n}}.$$

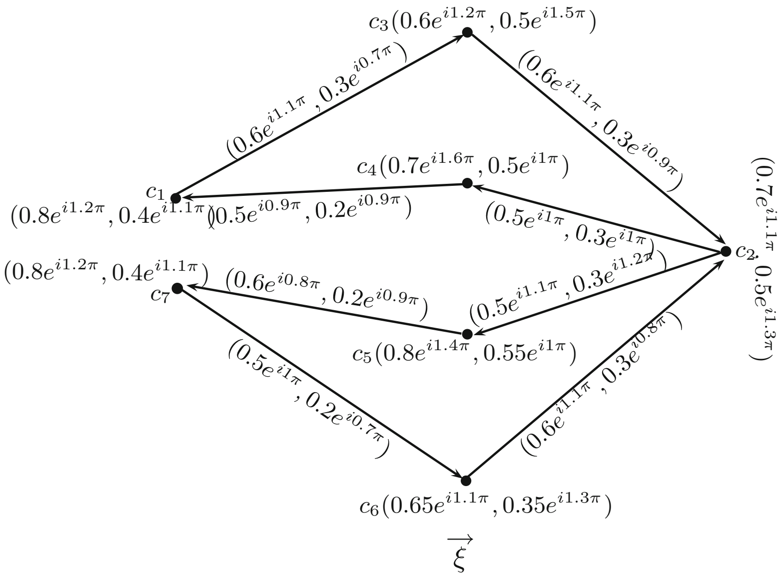


Fig. 16 Complex Pythagorean fuzzy digraph

Example 7.4 Let $\xi = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG as shown in Fig. 16, defined by:

$$\widetilde{A} = \left\langle \left(\frac{c_1}{0.8e^{i1.2\pi}}, \frac{c_2}{0.7e^{i1.1\pi}}, \frac{c_3}{0.6e^{i1.2\pi}}, \frac{c_4}{0.7e^{i1.6\pi}}, \frac{c_5}{0.8e^{i1.4\pi}}, \frac{c_6}{0.65e^{i1.1\pi}}, \frac{c_7}{0.8e^{i1.2\pi}} \right), \right.$$
$$\left. \left(\frac{c_1}{0.4e^{i1.1\pi}}, \frac{c_2}{0.5e^{i1.3\pi}}, \frac{c_3}{0.5e^{i1.5\pi}}, \frac{c_4}{0.5e^{i1\pi}}, \frac{c_5}{0.55e^{i1\pi}}, \frac{c_6}{0.35e^{i1.3\pi}}, \frac{c_7}{0.4e^{i1.1\pi}} \right) \right\rangle,$$
$$\overrightarrow{\widetilde{B}} = \left\langle \left(\frac{\overrightarrow{(c_1, c_3)}}{0.6e^{i1.1\pi}}, \frac{\overrightarrow{(c_2, c_4)}}{0.5e^{i1\pi}}, \frac{\overrightarrow{(c_2, c_5)}}{0.5e^{i1.1\pi}}, \frac{\overrightarrow{(c_3, c_2)}}{0.6e^{i1.1\pi}}, \frac{\overrightarrow{(c_4, c_1)}}{0.5e^{i0.9\pi}}, \frac{\overrightarrow{(c_5, c_7)}}{0.6e^{i0.8\pi}}, \frac{\overrightarrow{(c_6, c_2)}}{0.6e^{i1.1\pi}}, \frac{\overrightarrow{(c_7, c_6)}}{0.5e^{i1\pi}} \right), \right.$$
$$\left. \left(\frac{\overrightarrow{(c_1, c_3)}}{0.3e^{i0.7\pi}}, \frac{\overrightarrow{(c_2, c_4)}}{0.3e^{i1\pi}}, \frac{\overrightarrow{(c_2, c_5)}}{0.3e^{i1.2\pi}}, \frac{\overrightarrow{(c_3, c_2)}}{0.3e^{i0.9\pi}}, \frac{\overrightarrow{(c_4, c_1)}}{0.2e^{i0.9\pi}}, \frac{\overrightarrow{(c_5, c_7)}}{0.2e^{i0.9\pi}}, \frac{\overrightarrow{(c_6, c_2)}}{0.3e^{i0.8\pi}}, \frac{\overrightarrow{(c_7, c_6)}}{0.2e^{i0.7\pi}}, \right) \right\rangle.$$

In Fig. 16, the strength of the vertex c_2 is

$$s_T(c_2) = \frac{0.6 + 0.5}{2} e^{i \frac{1.1\pi + 1\pi}{2}} = 0.55e^{i1.05\pi}$$

and

$$s_F(c_2) = \frac{0.3 + 0.3}{2} e^{i \frac{0.9\pi + 0.8\pi}{2}} = 0.3e^{i0.85\pi}.$$

Hence the vertex c_2 is strong 2-step prey as $0.55 > \frac{1}{2}$, $1.05\pi > 1\pi$ and $0.3 < \frac{1}{2}$, $0.85\pi < 1\pi$.

Theorem 7.2 *If all the vertices in CPFDG $\overrightarrow{\xi}$ are strong, then in $\mathfrak{C}_m(\overrightarrow{\xi})$,*

1. $\mu_{\widetilde{B}}(s,w) > \frac{1}{2}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w))$ *and* $\theta_{\widetilde{B}}(s,w) > \frac{1}{2}(\theta_{\widetilde{A}}(s) \wedge \theta_{\widetilde{A}}(w))$,
2. $\nu_{\widetilde{B}}(s,w) < \frac{1}{2}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w))$ *and* $\vartheta_{\widetilde{B}}(s,w) < \frac{1}{2}(\vartheta_{\widetilde{A}}(s) \vee \vartheta_{\widetilde{A}}(w))$,

for all $s, w \in Y$.

Proof Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\widetilde{B}})$ be a CPFDG and let all the vertices of $\overrightarrow{\xi}$ are strong. Let $\mathfrak{C}_m(\overrightarrow{\xi}) = (Y, \widetilde{A}, \widetilde{C})$ be the corresponding m-step CPFCG. Here we consider two cases. *Case 1* Let $(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) = \emptyset$. Then there does not exist edge between s and w in $\mathfrak{C}_m(\overrightarrow{\xi})$. Hence, there is nothing to proof. *Case 2* Let $(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))$ be a non-empty set. Then, clearly $\mathbb{h}_\mu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) > \frac{1}{2}$ and $\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) > \frac{1}{2}$ in $\overrightarrow{\xi}$ as all the vertices are strong. Then the membership grade of the edge (s, w) in $\mathfrak{C}_m(\overrightarrow{\xi})$ is

$$
\begin{aligned}
\mu_{\widetilde{C}}(s,w) &= (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),\\
\mu_{\widetilde{C}}(s,w) &> (\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w)) \times \frac{1}{2}\\
\theta_{\widetilde{C}}(s,w) &= 2\pi\left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi}\right],\\
\theta_{\widetilde{C}}(s,w) &> 2\pi\left[\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{1\pi}{2\pi}\right],
\end{aligned}
$$

Then, $\mu_{\widetilde{C}}(s,w) > \frac{1}{2}(\mu_{\widetilde{A}}(s) \wedge \mu_{\widetilde{A}}(w))$ and $\theta_{\widetilde{C}}(s,w) > \frac{1}{2}\left[2\pi\left(\frac{\theta_{\widetilde{A}}(s)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(w)}{2\pi}\right)\right]$ as $\mathbb{h}_\mu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) > \frac{1}{2}$ and $\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) > \frac{1}{2}$.
Similarly, $\mathbb{h}_\nu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) < \frac{1}{2}$ and $\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) < \frac{1}{2}$. Then non-membership grade of the edge (s, w) is

$$
\begin{aligned}
\nu_{\widetilde{C}}(s,w) &= (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)),\\
\nu_{\widetilde{C}}(s,w) &< (\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w)) \times \frac{1}{2}\\
\vartheta_{\widetilde{C}}(s,w) &= 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w))}{2\pi}\right],\\
\vartheta_{\widetilde{C}}(s,w) &< \left[2\pi\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right) \times \frac{1\pi}{2\pi}\right],
\end{aligned}
$$

Then, $\nu_{\widetilde{C}}(s,w) < \frac{1}{2}(\nu_{\widetilde{A}}(s) \vee \nu_{\widetilde{A}}(w))$ and $\vartheta_{\widetilde{C}}(s,w) < \frac{1}{2}\left[2\pi\left(\frac{\vartheta_{\widetilde{A}}(s)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(w)}{2\pi}\right)\right]$ as $\mathbb{h}_\nu(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) < \frac{1}{2}$ and $\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p_m(s) \cap \widetilde{\mathbb{N}}^p_m(w)) < \frac{1}{2}$.
This proves the result. □

Theorem 7.3 *If a vertex z of $\overrightarrow{\xi}$ is strong, then in strength of z,*

1. $\dfrac{\sum_{k=1}^{n} \mu_{\widetilde{B}}(q_k, w_k)}{n} > \dfrac{1}{2}$ *and* $\dfrac{\sum_{j=1}^{n} \theta_{\widetilde{B}}(q_k, w_k)}{n} > 1\pi$ *for* $s_T(z)$,

2. $\dfrac{\sum_{j=1}^{n} \nu_{\widetilde{B}}(q_k, w_k)}{n} < \dfrac{1}{2}$ *and* $\dfrac{\sum_{j=1}^{n} \vartheta_{\widetilde{B}}(q_k, w_k)}{n} < 1\pi$ *for* $s_F(z)$.

Proof Let $\overrightarrow{\xi} = (Y, \widetilde{A}, \overrightarrow{\overrightarrow{B}})$ be a CPFDG. Let z be the common vertex of m-step out-neighborhood of vertices $b_1, b_2, \dots, b_n$, i.e., there exist CPFDPs $\overrightarrow{P}^m_{(b_1,z)}, \overrightarrow{P}^m_{(b_2,z)}, \cdots \overrightarrow{P}^m_{(b_n,z)}$, in $\overrightarrow{\xi}$. Also, let $\mu_{\widetilde{B}}\overrightarrow{(q_1, w_1)}e^{\iota\theta_{\widetilde{B}}\overrightarrow{(q_1,w_1)}}$, $\mu_{\widetilde{B}}\overrightarrow{(q_2, w_2)}e^{\iota\theta_{\widetilde{B}}\overrightarrow{(q_2,w_2)}}, \cdots \mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)} e^{\iota\theta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}$ be the minimum membership grades of the edges of paths $\overrightarrow{P}^m_{(b_1,z)}, \overrightarrow{P}^m_{(b_2,z)}, \cdots \overrightarrow{P}^m_{(b_n,z)}$, respectively and $\nu_{\widetilde{B}}\overrightarrow{(q_1, w_1)}e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_1,w_1)}}$, $\nu_{\widetilde{B}}\overrightarrow{(q_2, w_2)}e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_2,w_2)}}, \cdots \nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}e^{i\vartheta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}$ be the maximum membership grades of the edges of paths $\overrightarrow{P}^m_{(b_1,z)}, \overrightarrow{P}^m_{(b_2,z)}, \cdots \overrightarrow{P}^m_{(b_n,z)}$, respectively. If z is strong, each arc $\overrightarrow{(q_k, w_k)}$, $k = 1, 2, \dots, n$ is strong. So $\mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)} > \frac{1}{2}$, $\theta_{\widetilde{B}}\overrightarrow{(q_k, w_k)} > 1\pi$ and $\nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)} < \frac{1}{2}$, $\vartheta_{\widetilde{B}}\overrightarrow{(q_k, w_k)} < 1\pi$. Now,

$$s_T(z) = \frac{\sum_{k=1}^{n} \mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} e^{i\frac{\theta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}{n}},$$

where

$$\frac{\sum_{k=1}^{n} \mu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} > \frac{\frac{1}{2} + \frac{1}{2} + \cdots (n - times) + \frac{1}{2}}{n} > \frac{1}{2},$$
$$\frac{\sum_{k=1}^{n} \theta_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} > \frac{1\pi + 1\pi + \cdots (n - times) + 1\pi}{n} > 1\pi.$$

Similarly,

$$s_F(z) = \frac{\sum_{k=1}^{n} \nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} e^{i\frac{\vartheta_{\widetilde{B}}\overrightarrow{(q_k,w_k)}}{n}},$$

where

$$\frac{\sum_{k=1}^{n} \nu_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} < \frac{\frac{1}{2} + \frac{1}{2} + \cdots (n - times) + \frac{1}{2}}{n} < \frac{1}{2},$$
$$\frac{\sum_{k=1}^{n} \vartheta_{\widetilde{B}}\overrightarrow{(q_k, w_k)}}{n} < \frac{1\pi + 1\pi + \cdots (n - times) + 1\pi}{n} < 1\pi.$$

This proves the result. □

8 Application

Fuzzy competition graphs are becoming significant as they are applicable to many areas where there is a competition between distinct real world entities. However, there exist

Table 15 Membership and non-membership grades of vertices

Name of candidate	CPF values	Name of designation	CPF values
$Oliva$	$(0.8e^{i1.2\pi}, 0.6e^{i1.1\pi})$	CEO	$(0.9e^{i1.2\pi}, 0.3e^{i1\pi})$
$Jacob$	$(0.5e^{i0.8\pi}, 0.7e^{i1.3\pi})$	COO	$(0.5e^{i1.2\pi}, 0.6e^{i0.9\pi})$
$Oscar$	$(0.4e^{i1.2\pi}, 0.8e^{i0.9\pi})$	CCO	$(0.7e^{i1.3\pi}, 0.3e^{i0.8\pi})$
$Edward$	$(0.5e^{i1.1\pi}, 0.7e^{i1\pi})$	$CHRO$	$(0.7e^{i1\pi}, 0.5e^{i0.9\pi})$
$William$	$(0.3e^{i1.2\pi}, 0.8e^{i1\pi})$	CLO	$(0.8e^{i1.8\pi}, 0.4e^{i0.2\pi})$
$Robart$	$(0.4e^{i1.3\pi}, 0.6e^{i0.9\pi})$		
$Joseph$	$(0.7e^{i1.5\pi}, 0.4e^{i0.7\pi})$		
$Thomas$	$(0.9e^{i1.8\pi}, 0.4e^{i1.3\pi})$		

some competitions of real world which cannot be represented through these graphs. To represent all the competitions we propose CPFCGs which have the larger ability to show all the competitions of real world. To fully understand the concept of CPFCGs we construct an application of competition graphs under complex Pythagorean fuzzy environment with an algorithm below.

Let us consider an example of eight persons competing for designations in private limited company. Let us consider a set candidates $\{Oliva, Jacob, Oscar, Edward, William, Robart, Joseph, Thomas\}$ and {Chief executive officer (CEO), Chief operating officer (COO), Chief costumer officer (CCO), Chief human resources officer (CHRO), Chief legal officer (CLO)} be the set of particular designations for the candidates in company. The amplitude terms of the membership and non-membership grade of each candidate represent the degree of loyalty and disloyalty of candidate towards his designation respectively. The phase term of membership grade represents the percentage of effectiveness of candidate to fulfill the goals towards his designation while the phase term of non-membership grade represents the ineffectiveness to fulfill the goals, respectively. Similarly, the membership and non-membership grades of each designation represent the percentage that how much the designation is suitable and not suitable for candidate, respectively while the phase terms of the membership and non membership grade represent the availability and non-availability, respectively. Similarly, the membership and non-membership values for other candidates and designations are shown in Table 15.

The amplitude term of the membership function of each directed edge between candidate and designation represent the eligibility and non-eligibility of candidate towards the particular designation. The phase term of the membership grade of each directed edge represents past experience of candidate about the designation while the phase term of the non-membership function represents that the candidate has no experience. The corresponding CPFDG is shown in Fig. 17 in which candidates and designation are taken as vertices while the relation between them make a graph which shows the competition between applicants for the designations.

CPF-out-neighborhoods of the vertices are shown in Table 16.

The CPFSs $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$ are shown in Table 17.

The corresponding CPFCG is shown in Fig. 18.

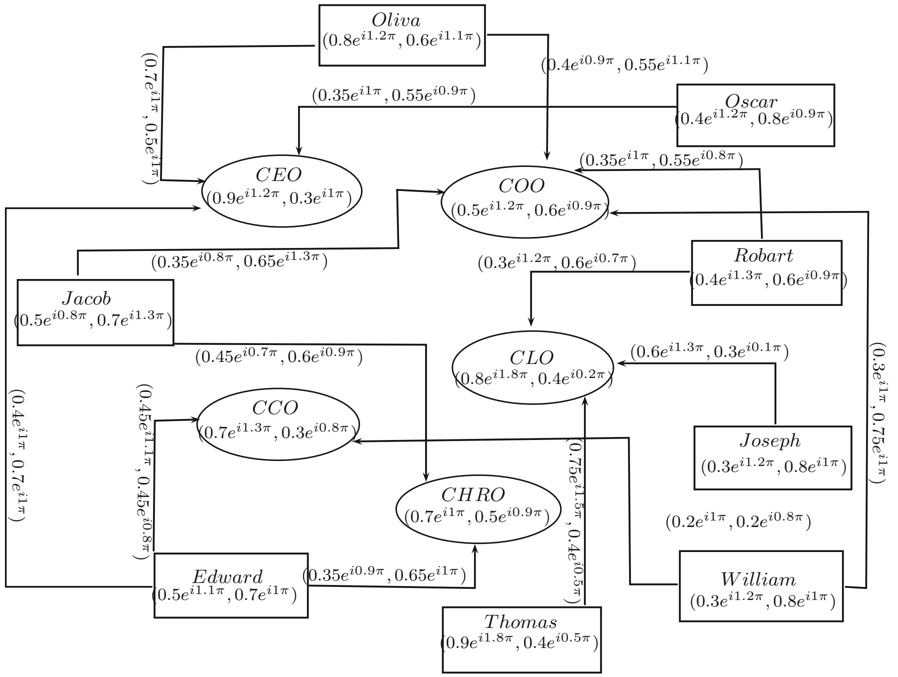


Fig. 17 Complex Pythagorean fuzzy digraph

Table 16 CPF-out-neighborhoods of the vertices

Candidates	$\widetilde{\mathbb{N}}^p(Candidates)$
Olive	$\{(CEO, 0.7e^{i1\pi}, 0.5e^{i1\pi}), (COO, 0.4e^{i0.9\pi}, 0.55e^{i1.1\pi})\}$
Jacob	$\{(COO, 0.35e^{i0.8\pi}, 0.65ee^{i1.3\pi}), (CHRO, 0.45e^{i0.7\pi}, 0.6e^{i0.9\pi})\}$
Oscar	$\{(CEO, 0.35e^{i1\pi}, 0.55e^{i0.9\pi})\}$
Edward	$\{(CEO, 0.4e^{i1\pi}, 0.7e^{i1\pi}), (CCO, 0.45e^{i1.1\pi}, 0.45e^{i0.8\pi}),$ $(CHRO, 0.35e^{i0.9\pi}, 0.65e^{i1\pi})\}$
Thomas	$\{(CLO, 0.75e^{i1.5\pi}, 0.4e^{i0.5\pi})\}$
Robart	$\{(COO, 0.35e^{i1\pi}, 0.55e^{i0.8\pi}), (CLO, 0.3e^{i1.2\pi}, 0.6e^{i0.7\pi})\}$
Joseph	$\{(CCO, 0.2e^{i1\pi}, 0.2e^{i0.8\pi})\}$
William	$\{(COO, 0.3e^{i1\pi}, 0.75e^{i1\pi}), (CCO, 0.2e^{i1\pi}, 0.2e^{i0.8\pi})\}$

The dotted lines show the candidates competing for the particular seats while the solid lines indicate the competition among the candidates. Thus, clearly it can be seen from Fig. 18 that there are five candidates in competition with $Oliva$, namely, $Jacob$, $Oscar$, $Edward$, $Robart$, and $William$; four candidates in competition with $Jacob$, namely, $Edward$, $Robart$, $William$, and $Oliva$; and similarly there is a competition between $Oscar$ and $Edward$; $Edward$ and $Joseph$; $Edward$ and $William$;

Table 17 CPFSs $\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$ and $\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$

s	w	$\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w)$	$\mathbb{h}(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$
$Oliva$	$Jacob$	$\{(COO, 0.35e^{i0.8\pi}, 0.65e^{i1.3\pi})\}$	$\{(COO, 0.35e^{i0.8\pi}, 0.65e^{i1.3\pi})\}$
$Oliva$	$Oscar$	$\{(CEO, 0.35e^{i1\pi}, 0.55e^{i1\pi})\}$	$\{(CEO, 0.35e^{i1\pi}, 0.55e^{i1\pi})\}$
$Oliva$	$Edward$	$\{(CEO, 0.4e^{i1\pi}, 0.7e^{i1\pi})\}$	$\{(CEO, 0.4e^{i1\pi}, 0.7e^{i1\pi})\}$
$Oliva$	$Thomas$	$\emptyset$	$\emptyset$
$Oliva$	$Robart$	$\{(COO, 0.35e^{i0.9\pi}, 0.55e^{i1.1\pi})\}$	$\{(COO, 0.35e^{i0.9\pi}, 0.55e^{i1.1\pi})\}$
$Oliva$	$Joseph$	$\emptyset$	$\emptyset$
$Oliva$	$William$	$\{(COO, 0.3e^{i0.9\pi}, 0.75e^{i1.1\pi})\}$	$\{(COO, 0.3e^{i0.9\pi}, 0.75e^{i1.1\pi})\}$
$Jacob$	$Oscar$	$\emptyset$	$\emptyset$
$Jacob$	$Edward$	$\{(CHRO, 0.35e^{i0.7\pi}, 0.65e^{i1.1\pi})\}$	$\{(CHRO, 0.35e^{i0.7\pi}, 0.65e^{i1.1\pi})\}$
$Jacob$	$Thomas$	$\emptyset$	$\emptyset$
$Jacob$	$Robart$	$\{(COO, 0.35e^{i0.8\pi}, 0.65e^{i1.3\pi})\}$	$\{(COO, 0.35e^{i0.8\pi}, 0.65e^{i1.3\pi})\}$
$Jacob$	$Joseph$	$\emptyset$	$\emptyset$
$Jacob$	$William$	$\{(COO, 0.3e^{i0.8\pi}, 0.75e^{i1.3\pi})\}$	$\{(COO, 0.3e^{i0.8\pi}, 0.75e^{i1.3\pi})\}$
$Oscar$	$Edward$	$\{(CEO, 0.35e^{i1\pi}, 0.7e^{i1\pi})\}$	$\{(CEO, 0.35e^{i1\pi}, 0.7e^{i1\pi})\}$
$Oscar$	$Thomas$	$\emptyset$	$\emptyset$
$Oscar$	$Robart$	$\emptyset$	$\emptyset$
$Oscar$	$Joseph$	$\emptyset$	$\emptyset$
$Oscar$	$William$	$\emptyset$	$\emptyset$
$Edward$	$Thomas$	$\emptyset$	$\emptyset$
$Edward$	$Robart$	$\emptyset$	$\emptyset$
$Edward$	$Joseph$	$\{(CCO, 0.2e^{i1\pi}, 0.45e^{i0.8\pi})\}$	$\{(CCO, 0.2e^{i1\pi}, 0.45e^{i0.8\pi})\}$
$Edward$	$William$	$\{(CCO, 0.2e^{i1\pi}, 0.45e^{i0.8\pi})\}$	$\{(CCO, 0.2e^{i1\pi}, 0.45e^{i0.8\pi})\}$
$Thomas$	$Robart$	$\{(CLO, 0.3e^{i1.2\pi}, 0.6e^{i0.7\pi})\}$	$\{(CLO, 0.3e^{i1.2\pi}, 0.6e^{i0.7\pi})\}$
$Thomas$	$Joseph$	$\emptyset$	$\emptyset$
$Thomas$	$William$	$\emptyset$	$\emptyset$
$Robart$	$Joseph$	$\emptyset$	$\emptyset$
$Robart$	$William$	$\{(COO, 0.3e^{i1\pi}, 0.75e^{i1\pi})\}$	$\{(COO, 0.3e^{i1\pi}, 0.75e^{i1\pi})\}$
$Joseph$	$William$	$\{(COO, 0.3e^{i1\pi}, 0.75e^{i1\pi})\}$	$\{(COO, 0.3e^{i1\pi}, 0.75e^{i1\pi})\}$

$Thomas$ and $Robart$; $Thomas$ and $William$; $Robart$ and $William$; and $Joseph$ and $William$. The method which is used in our application is shown in Table 18.

9 Comparative analysis

Competition graphs are becoming significant as they are applicable in different areas where there arise competition between distinct real world entities. But, there are some competitions in which the entities possess the two-dimensional or periodic information. In 2019, q-rung orthopair fuzzy competition graphs were proposed by Habib et al. [18] which have the greater ability in dealing with the incomplete and vague informa-

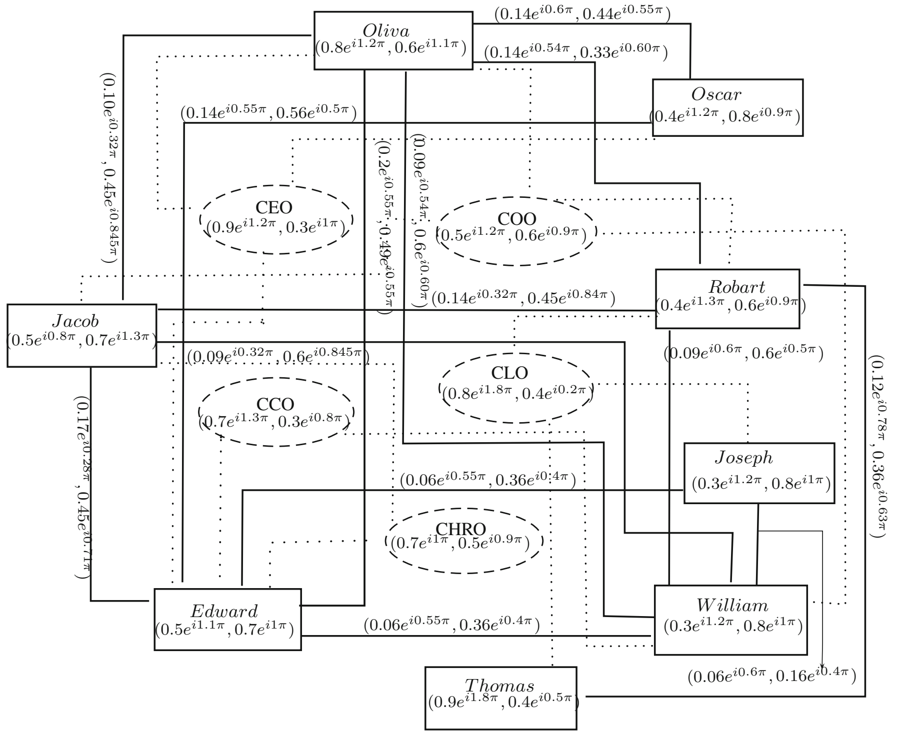


Fig. 18 Corresponding complex Pythagorean fuzzy competition graph

tion about the vertices and edges. q-rung orthopair fuzzy competition graphs increase with its various types of applications in various fields of life. Here, we discuss one of the application of q-rung orthopair fuzzy competition graphs for q=2, by considering a set of candidates competing for the particular designations (seats) in private limited company. Let us consider a set of five persons

$$\{(p_1, 0.8, 0.3), (p_2, 0.8, 0.4), (p_3, 0.5, 0.6), (p_4, 0.7, 0.4), (p_5, 0.5, 0.5)\}$$

competing for particular seats $\{d_1, 0.5, 0.6, d_2, 0.5, 0.6, d_3, 0.9, 0.3\}$. Let the membership and non-membership grades of persons represent the degree of loyalty and disloyalty of persons for the particular seats in the company while the membership and non-membership grades of seats (designations) depict the availability and non-availability of number of seats. Let $\{(\overrightarrow{p_1d_1}, 0.5, 0.6), (\overrightarrow{p_3d_2}, 0.3, 0.4), (\overrightarrow{p_5d_3}, 0.5, 0.5), (\overrightarrow{p_4d_3}, 0.6, 0.4)\}$ be the set of edges where the directed edge shows that the person is competing for the particular seat. The membership and non-membership grades of the edges show the eligibility and non-eligibility of persons for particular seats. The given information about persons and seats is incomplete because sometimes the criteria may change to select the candidates as some companies like to hire the person who has some past experience about the work in relevant field or effective in fulfilling the goals of the company. Similarly, the information about the number of seats is not enough

Table 18 Algorithm 1

Algorithm	
Step 1.	Assign the membership and non-membership grades for the set of candidates and designations.
Step 2.	If for any two vertices a_i and a_j, $\mu_{\widetilde{B}}(\overrightarrow{a_i, a_j}) > 0$, $\nu_{\widetilde{B}}(\overrightarrow{a_i, a_j}) > 0$, then $(a_j, \mu_{\widetilde{\mathbb{B}}}(\overrightarrow{a_i, a_j})e^{i\theta_{\widetilde{\mathbb{B}}}(\overrightarrow{a_i, a_j})}, \nu_{\widetilde{\mathbb{B}}}(\overrightarrow{a_i, a_j})e^{i\vartheta_{\widetilde{\mathbb{B}}}(\overrightarrow{a_i, a_j})}) \in \widetilde{\mathbb{N}}^p(a_i)$.
Step 3.	Find out the CPF-out-neighborhoods $\widetilde{\mathbb{N}}^p(a_i)$ for all vertices a_i, a_j by following the step 2.
Step 4.	Compute the CPFS $\widetilde{\mathbb{N}}^p(a_i) \cap \widetilde{\mathbb{N}}^p(a_j)$.
Step 5.	Compute the CPFS $\mathbb{h}(\widetilde{\mathbb{N}}^p(a_i) \cap \widetilde{\mathbb{N}}^p(a_j))$.
Step 6.	If $\widetilde{\mathbb{N}}^p(a_i) \cap \widetilde{\mathbb{N}}^p(a_j) \neq \emptyset$ then draw an edge (a_i, a_j).
Step 7.	Repeat the above step for all the different vertices of the CPFDG.
Step 8.	Assign the membership and non-membership grades by using the following conditions: $\mu_{\widetilde{B}}(a_i, a_j) = (\mu_{\widetilde{A}}(a_i) \wedge \mu_{\widetilde{A}}(a_j) \times \mathbb{h}_\mu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$, $\nu_{\widetilde{B}}(a_i, a_j) = (\nu_{\widetilde{A}}(a_i) \vee \nu_{\widetilde{A}}(a_j) \times \mathbb{h}_\nu(\widetilde{\mathbb{N}}^p(s) \cap \widetilde{\mathbb{N}}^p(w))$, and $\theta_{\widetilde{B}}(a_i, a_j) = 2\pi\left[\left(\frac{\theta_{\widetilde{A}}(a_i)}{2\pi} \wedge \frac{\theta_{\widetilde{A}}(a_j)}{2\pi}\right) \times \frac{\mathbb{h}_\theta(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w))}{2\pi}\right]$, $\vartheta_{\widetilde{B}}(a_i, a_j) = 2\pi\left[\left(\frac{\vartheta_{\widetilde{A}}(a_i)}{2\pi} \vee \frac{\vartheta_{\widetilde{A}}(a_j)}{2\pi}\right) \times \frac{\mathbb{h}_\vartheta(\widetilde{\mathbb{N}}^p(s)\cap\widetilde{\mathbb{N}}^p(w))}{2\pi}\right]$.

for the candidate because the candidate may want to know that how much the job is beneficial (good salary, transport facility, etc.). This lack information about seats and persons motivate us to use more generalized model, CPFCG, which has the larger potential in dealing with two dimensional information. An application is designed in our proposed model about the competition among candidates for the particular designations in a private limited company. The amplitude term of membership and non-membership grades represent the degree of loyalty and disloyalty while the phase term give the information about effectiveness and ineffectiveness of candidates to fulfill the goals of designation in company. Similarly, the amplitude terms of membership and non-membership values of the directed edges from candidate to designation show the eligibility and non eligibility while the phase term depicts the past experience of candidate about the designation. Therefore, CPFCGs are more useful as these graphs handle the two dimensional phenomena. The potential of these graphs for representing the two dimensional phenomena make it superior to handle the intuitive and ambiguous information.

10 Conclusion

The conception of graph theory is widely growing and it is playing significant role in communication network, computer science, operational research, sociology, mathematics and science. Complex Pythagorean fuzzy model, an extended structure of complex fuzzy and complex intuitionistic fuzzy models give more compatibility and flexibility in dealing with vagueness associated to both the membership and non-membership functions as compared to CF and CIF models, as it broads the space of vague information. In this research paper, we have introduced the concept of competition graphs under complex Pythagorean fuzzy environment. We have defined two generalizations of complex Pythagorean fuzzy competition graphs as complex Pythagorean fuzzy k-competition graphs and complex Pythagorean fuzzy p-competition graphs. We have also discussed complex Pythagorean fuzzy neighborhood graphs and m-step complex Pythagorean fuzzy neighborhood graphs. Some related theorems about these new graphs have also been proved. We have also designed an application of complex Pythagorean fuzzy competition graphs. Our next aim is to extend our work to complex q-rung orthopair fuzzy competition graphs.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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