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ORIGINAL RESEARCH

New oscillation criterion for Emden–Fowler type nonlinear neutral delay differential equations

Hui Li[1] · Yige Zhao[1] · Zhenlai Han[1]

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Abstract

In this paper, we consider the following Emden–Fowler type nonlinear neutral delay differential equations

$$\left(r(t)(z'(t))^{\alpha}\right)' + q(t)y^{\beta}(\sigma(t)) = 0,$$

where $z(t) = y(t) + p(t)y(\tau(t))$. Some new oscillatory and asymptotic properties are obtained by means of the inequality technique and the Riccati transformation. It is worth pointing out that the oscillatory and asymptotic behaviors for our studied equation are ensured by only one condition and $\alpha, \beta \in \mathbb{R}$ are arbitrary quotients of two odd positive integers, which are completely new compared with previous references. Thus, this paper improves and generalizes some known results. Two illustrative examples are presented at last.

Keywords Oscillation · Emden–Fowler · Neutral · Delay · Differential equation

Mathematics Subject Classification 34C10 · 34A34 · 34K40

1 Introduction

The Emden–Fowler differential equation is derived from the study of the electrodynamic potential inside the atomic nucleus. It is widely applied in many important

✉ Zhenlai Han
hanzhenlai@163.com

Hui Li
lihui4727@163.com

Yige Zhao
zhaoeager@126.com

[1] School of Mathematical Sciences, University of Jinan, Jinan 250022, Shandong, People's Republic of China

fields, such as nuclear physics, astrophysics, fluid mechanics and gas dynamics. In 1973, Wong [1] firstly established the oscillation criteria for the classical Emden–Fowler differential equation as follows:

$$x''(t) + a(t)|x(t)|^{\gamma}\operatorname{sgn}x(t) = 0,$$

in the super-linear case, i.e., $\gamma > 1$. These results were generalized by Philos [2] for general super-linear case. Since then, a number of researches on this class of differential equations have been carried out and attracted enormous attentions.

The neutral functional differential equation arises in the design of high-speed computer lossless transmission lines. It also finds wide applications in certain high-tech fields, such as control, communication, mechanical engineering, biomedicine, physics, mechanics, economics and so on. What has to be mentioned here is, the first work devoted of the oscillatory properties of the neutral equations [3].

From what have been discussed above, we can see that the investigation of oscillatory and asymptotic behaviors for the Emden–Fowler type neutral delay differential equations is of great significance in both theory and application. They have attracted increasing interest of numerous scholars successfully because of their general applications in both engineering and natural science. During the last three decades, lots of papers and monographs about the oscillation of delay differential equations have been published. Among the many notable results, it is worthwhile to stress the pioneers' work, we refer the reader to [4–9] and the references cited therein. However, during the past decade, there are few papers considering the oscillatory and asymptotic behaviors of the Emden–Fowler type differential equations, see [10–12], the Emden–Fowler type neutral delay differential equations, see [13–18] and the references cited therein.

In 2007, Han et al. [10] studied the second-order Emden– Fowler delay dynamic equations on time scales

$$x^{\Delta\Delta}(t) + p(t)x^{\gamma}(\tau(t)) = 0,$$

where γ is a quotient of two odd positive integers. They established some new oscillation criteria by means of the Riccati transformation and the inequality technique. Results in this paper unify the oscillation of the second-order Emden–Fowler delay differential and difference equations.

At the same year, Xu and Liu [13] presented some Philos-type oscillation criteria for the Emden–Fowler neutral delay differential equations

$$[|x'(t)|^{\gamma-1}x'(t)]' + q_1(t)|y(t-\sigma)|^{\alpha-1}y(t-\sigma) + q_2(t)|y(t-\sigma)|^{\beta-1}y(t-\sigma) = 0,$$

where $x(t) = y(t) + p(t)y(t-\tau)$. The results obtained improved some known results in the literature.

In 2016, Agarwal et al. [14] considered a second-order nonlinear neutral differential equation of the following form

$$\left(r(t)\left((x(t) + p(t)x(\tau(t)))'\right)^{\alpha}\right)' + q(t)x^{\alpha}(\sigma(t)) = 0,$$

where α is a quotient of two odd positive integers. They established a new criterion which amended some known results. They proved that if there exist two functions $\rho,\ \delta \in C^1([t_0,\infty),(0,\infty))$ such that

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[\rho(s)Q(s)-\frac{((\rho'(s))_+)r(\sigma(s))}{(\alpha+1)^{\alpha+1}\rho^{\alpha}(s)(\sigma'(s))^{\alpha}}\right]ds=\infty$$

and

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[\psi(s)-\frac{\delta(s)r(s)((\varphi(s))_+)^{\alpha+1}}{(\alpha+1)^{\alpha+1}}\right]ds=\infty,$$

then the equation above is oscillatory.

Recently, Džurina and Jadlovská [15] studied a second-order half-linear delay differential equation of the following form

$$(r(t)(y'(t))^{\alpha})'+q(t)y^{\alpha}(\tau(t))=0,$$

where α is a quotient of two odd positive integers. The oscillation of the studied equation was attained via only one condition. A particular example of Euler type equation is provided in order to illustrate the significance of their main results.

We think that the mathematical conclusions should be concise and easy to be verified. Therefore, in this paper, we are committed to obtain a criterion which depends on only one condition. The conclusions in this paper are simpler and the method is different from others. We mainly consider the following Emden–Fowler type nonlinear neutral delay differential equations

$$\left(r(t)(z'(t))^{\alpha}\right)'+q(t)y^{\beta}(\sigma(t))=0, \tag{1.1}$$

where $z(t)=y(t)+p(t)y(\tau(t))$, α and β are quotients of two odd positive integers, $r(t)>0$, $p(t)\geq 0$, $\tau(t),\sigma(t)\leq t$, $\sigma'(t)\geq 0$, $\tau(t),\sigma(t)\to\infty$ as $t\to\infty$, $q(t)\geq 0$ and $q(t)\not\equiv 0$.

From the description above, we can see that, firstly, equation in this paper is neutral form in comparison to [10] and [15]. Secondly, the delays in our equation are variable compared with [13]. Moreover, the indices α and β are mutual independent with each other in contrast to [14] and [15]. In addition, since the oscillation conditions in this paper are simpler than the references above, it is easy to verify. In summary, the research object in this paper is more general while the conclusions are easier to verify.

Set $t_a=\tau(t_b)$ for some $t_b\geq t_0$ and $t_c=\sigma(t_d)$ for some $t_d\geq t_0$. By a solution of Eq. (1.1), we mean a function y which is continuous and satisfies Eq. (1.1) on $[t_1,\infty)$ with $t_1=\min\{t_a,t_c\}$ (By the derivative at $t=t_1$, we mean the right-hand side derivative). We only discuss these solutions of Eq. (1.1) which exist on some half-line $[t_1,\infty)$ and satisfy $\sup\{|x(t)|: t_e\leq t<\infty\}>0$ for any $t_e\geq t_1$. As usual, such a solution y of Eq. (1.1) is said to be oscillatory, if it is neither eventually positive nor eventually negative. Otherwise, it is called non-oscillatory. Equation (1.1) is called oscillatory, if all its solutions are oscillatory. Otherwise, it is called non-oscillatory.

The main work of this paper can be presented as follows: Firstly, the new oscillation criterion obtained is simple and concise since we do not need any other auxiliary functions. Secondly, we are concerned with a more general model. More concretely, we add a neutral term in the studied equation and the constants α and β are independent with each other. We investigate the oscillation criteria for a nonlinear differential equation rather than a half-linear one. Thirdly, by a new inequality technique, we obtain our oscillation criterion.

This paper is organized as follows. In Sect. 2, we present some necessary knowledge. Section 3 is dedicated in addressing our main results. At last, we give some illustrative examples.

2 Preliminaries

In this section, we will present some necessary knowledge.

Definition 2.1 Equation (1.1) is called in the non-canonical form if $\pi(t_0) < \infty$, where

$$\pi(t) = \int_t^\infty r^{-1/\alpha}(s)ds < \infty. \tag{2.1}$$

Throughout this paper, we will investigate the oscillatory and asymptotic properties of Eq. (1.1) in the non-canonical form, i.e., Eq. (1.1) which satisfies condition (2.1). And we assume that the neutral coefficient $p(t)$ satisfies that $p(t)\frac{\pi(\tau(t))}{\pi(t)} < c < 1$ for large t, where c is a positive constant and $\pi(t)$ is defined as above.

3 Main results

We are now in a position to state and prove our main results in this paper.

Theorem 3.1 *Suppose that*

$$\int^\infty \left(\frac{1}{r(t)}\int^t q(s)ds\right)^{1/\alpha} dt = \infty. \tag{3.1}$$

Then every solution $y(t)$ of Eq. (1.1) is oscillatory or satisfies $\lim_{t\to\infty} y(t) = 0$.

Proof By contradiction, suppose that Eq. (1.1) is nonoscillatory and $y(t)$ is a nonoscillatory solution for Eq. (1.1). Without loss of generality, we will assume that $y(t)$ is eventually positive. Then there exists some $t_2 > t_0$ such that for any $t > t_2$, we have $y(t) > 0$, $y(\tau(t)) > 0$ and $y(\sigma(t)) > 0$. Then from Eq. (1.1) we know that

$$\left(r(t)(z'(t))^\alpha\right)' = -q(t)y^\beta(\sigma(t)) < 0. \tag{3.2}$$

Thus, $r(t)(z'(t))^\alpha$ is decreasing for all $t \geq t_2$, which implies that $r(t)(z'(t))^\alpha$ does not change sign eventually, neither does $z'(t)$. That is, there exists a $t_3 \geq t_2$ such that either $z'(t) < 0$ or $z'(t) > 0$ for any $t \geq t_3$. In what follows, we will discuss the two cases above, respectively.

Case (i) $z'(t) > 0$ for all $t \geq t_3$. On one hand, since $z(t) = y(t) + p(t)y(\tau(t))$, we can obtain

$$y(t) = z(t) - p(t)y(\tau(t)) \geq z(t) - p(t)z(\tau(t)).$$

From (3.2), we know that $r(t)(z'(t))^{\alpha}$ is decreasing. Then for any $s \geq t \geq t_2$, it follows that

$$r(t)(z'(t))^{\alpha} \geq r(s)(z'(s))^{\alpha},$$

i.e.,

$$z'(s) \leq \left(\frac{r(t)}{r(s)}\right)^{1/\alpha} z'(t).$$

Integrating this inequality from t to v with respect to s, then we have

$$z(v) - z(t) \leq r^{1/\alpha}(t)z'(t) \int_t^v r^{-1/\alpha}(s)ds.$$

Letting $v \to \infty$, we have $z(t) \geq -r^{1/\alpha}(t)z'(t)\pi(t)$. Then

$$\left(\frac{z(t)}{\pi(t)}\right)' = \frac{z'(t)\pi(t) - z(t)\pi'(t)}{\pi^2(t)} = \frac{z'(t)\pi(t) + z(t)r^{-1/\alpha}(t)}{\pi^2(t)} \geq 0,$$

which means that $\frac{z(t)}{\pi(t)}$ is nondecreasing. Therefore, $\frac{z(t)}{\pi(t)} \geq \frac{z(\tau(t))}{\pi(\tau(t))}$, i.e.,

$$z(\tau(t)) \leq \frac{\pi(\tau(t))}{\pi(t)} z(t). \tag{3.3}$$

On the other hand, by means of (3.3), we get

$$\begin{aligned} y(t) &\geq z(t) - p(t)z(\tau(t)) \\ &\geq z(t) - p(t)\tfrac{\pi(\tau(t))}{\pi(t)} z(t) = \left(1 - p(t)\tfrac{\pi(\tau(t))}{\pi(t)}\right) z(t). \end{aligned}$$

From Eq. (1.1) and the inequality above, we have

$$\left(r(t)(z'(t))^{\alpha}\right)' = -q(t)y^{\beta}(\sigma(t)) \leq -q(t)\left(1 - p(\sigma(t))\frac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}\right)^{\beta} z^{\beta}(\sigma(t)). \tag{3.4}$$

Letting $w(t) = \frac{r(t)(z'(t))^{\alpha}}{z^{\beta}(\sigma(t))}$, we have

$$\begin{aligned} w'(t) &= \tfrac{(r(t)(z'(t))^{\alpha})' z^{\beta}(\sigma(t)) - \beta r(t)(z'(t))^{\alpha} z^{\beta-1}(\sigma(t)) z'(\sigma(t))\sigma'(t)}{z^{2\beta}(\sigma(t))} \\ &\leq -q(t)\left(1 - p(\sigma(t))\tfrac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}\right)^{\beta} - \tfrac{\beta r(t)(z'(t))^{\alpha} z'(\sigma(t))\sigma'(t)}{z^{\beta+1}(\sigma(t))} \\ &\leq -q(t)\left(1 - p(\sigma(t))\tfrac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}\right)^{\beta}. \end{aligned}$$

Integrating the inequality above from t_3 to t, one has

$$w(t) - w(t_3) \leq -\int_{t_3}^{t} q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^{\beta} ds.$$

It follows from (2.1) and (3.1) that

$$\int_{t_3}^{\infty} q(s)ds = \infty,$$

and so

$$\int_{t_3}^{\infty} q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^{\beta} ds = \infty.$$

In fact, since $p(t)\frac{\pi(\tau(t))}{\pi(t)} < c < 1$, we know that $0 < 1 - c < 1 - p(\sigma(t))\frac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}$, which implies that

$$\int_{t_3}^{\infty} q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^{\beta} ds = \infty.$$

Thus

$$w(t) \leq w(t_3) - \int_{t_3}^{t} q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^{\beta} ds.$$

Letting $t \to \infty$. we obtain a contradiction.

Case (ii) $z'(t) < 0$ for all $t \geq t_3$. We claim that $\lim_{t\to\infty} z(t) = 0$. In fact, if we assume that $\lim_{t\to\infty} z(t) = L \neq 0$, then $L > 0$. So for any M with $0 < M < L$, we have $z(t) > M$. Similarly to the calculation of (3.3) and (3.4), one has

$$\begin{aligned} \left(r(t)(z'(t))^{\alpha}\right)' &\leq -q(t)\left(1 - p(\sigma(t))\tfrac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}\right)^{\beta} z^{\beta}(\sigma(t)) \\ &< -q(t)M^{\beta}\left(1 - p(\sigma(t))\tfrac{\pi(\tau(\sigma(t)))}{\pi(\sigma(t))}\right)^{\beta}. \end{aligned}$$

Integrating the inequality above from t_3 to t, we can obtain

$$r(t)(z'(t))^{\alpha} - r(t_3)(z'(t_3))^{\alpha} < -M^{\beta}\int_{t_3}^{t} q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^{\beta} ds.$$

Since $z'(t) < 0$, we get

$$r(t)(z'(t))^\alpha < -M^\beta \int_{t_3}^t q(s)\left(1 - p(\sigma(s))\frac{\pi(\tau(\sigma(s)))}{\pi(\sigma(s))}\right)^\beta ds,$$

i.e.,

$$z'(t) < -M^{\beta/\alpha}\left(\frac{1}{r(t)}\int_{t_3}^t q(s)ds\right)^{1/\alpha}.$$

Integrating the above inequality from t_3 to t, we obtain

$$z(t) < z(t_3) - M^{\beta/\alpha}\int_{t_3}^t \left(\frac{1}{r(s)}\int_{t_3}^s q(v)dv\right)^{1/\alpha} ds,$$

which is a contradiction when $t \to \infty$. Therefore, $\lim_{t\to\infty} z(t) = 0$. By the property of the limit, we know that $\lim_{t\to\infty} y(t) = 0$. The proof is completed.

In what follows, we will present a corollary for the following equation

$$(r(t)(y'(t))^\alpha)' + q(t)y^\alpha(\sigma(t)) = 0, \quad t \geq t_0, \tag{3.5}$$

where $\alpha > 0$ is a quotient of odd positive integers, $r \in C^1([t_0, \infty), \ [0, \infty))$, $q \in C([t_0, \infty), \ (0, \infty))$ and $q(t) \not\equiv 0$, $\sigma(t) \leq t$, $\sigma'(t) \geq 0$ and $\lim_{t\to\infty} \sigma(t) = \infty$ on the half-line of the form $[t_*, \ \infty)$, $t_* \geq t_0$.

Corollary 3.1 *Suppose that* (3.1) *holds, then every solution* $y(t)$ *of Eq.* (3.5) *is oscillatory or satisfies* $\lim_{t\to\infty} y(t) = 0$.

This is a result of [15] without any additional conditions, since $p(t)\frac{\pi(\tau(t))}{\pi(t)} < c < 1$ is naturally satisfied when $p(t) \equiv 0$, which implies that our paper is a direct generalization of [15, Theorem 1].

4 Examples

Two examples will be presented in this section to illustrate our main results.

Example 4.1 Consider the following second-order nonlinear neutral delay differential equation

$$\left(t\left(\left(y(t) + \frac{1}{8t}y\left(\frac{t}{2}\right)\right)'\right)^{\frac{1}{3}}\right)' + (t-1)y\left(\frac{t}{3}\right) = 0, \ t \geq 1. \tag{4.1}$$

Compared with (1.1), we can see that $r(t) = t$, $\tau(t) = \frac{t}{2}$, $p(t) = \frac{1}{8t}$, $\sigma(t) = \frac{t}{3}$, $\alpha = \frac{1}{3}$, $\beta = 1$ and $q(t) = t - 1$. We will verify the conditions of Theorem 3.1, respectively. Firstly, it is obvious that α and β are quotients of two positive odd integers, $p(t) \geq 0$,

$\tau(t),\ \sigma(t) \le t$, $\sigma'(t) = \frac{1}{3} > 0$, $q(t) \ge 0$ and $q(t) \not\equiv 0$ for all $t \ge 1$. Moreover, $\lim_{t\to\infty} \tau(t) = \lim_{t\to\infty} \sigma(t) = \infty$. Secondly, we will show that (4.1) is in the non-canonical form, i.e., (2.1) holds. In fact,

$$\int_t^\infty \frac{1}{r^{1/\alpha}(s)} ds = \int_t^\infty s^{-3} ds = \frac{1}{2t^2} < \infty \text{ for any } t \ge 1.$$

Thirdly,

$$p(t)\frac{\pi(\tau(t))}{\pi(t)} = \frac{1}{8t}\frac{t^2}{\tau^2(t)} = \frac{1}{2t} \le \frac{1}{2} < 1.$$

Next, we verify the condition (3.1), i.e.,

$$\int^\infty \left(\frac{1}{r(t)}\int^t q(s)ds\right)^{1/\alpha} dt = \int_1^\infty \left(\frac{1}{t}\int_1^t (s-1)ds\right)^3 dt = \infty.$$

It implies that every solution $y(t)$ of Eq. (4.1) is oscillatory or satisfies $\lim_{t\to\infty} y(t) = 0$.

Example 4.2 Consider the second-order nonlinear delay differential equation

$$\left(t^{\frac{3}{2}} y'(t)\right)' + y(t) = 0,\ t \ge 1. \tag{4.2}$$

Compared with (1.1), it is obvious that $r(t) = t^{\frac{3}{2}}$, $\sigma(t) = t$, $\alpha = 1$, $\beta = 1$ and $q(t) = 1$. we will verify the conditions of Corollary 3.1, respectively. We can see that α and β are two quotients of two positive odd integers, $\sigma(t) \le t$, $\sigma'(t) = 1 > 0$, $q(t) \not\equiv 0$ for all $t \ge 1$. What is more, $\lim_{t\to\infty} \sigma(t) = \infty$. Next, we will show that (4.2) is in the non-canonical form, i.e., (2.1) holds. In fact,

$$\int_t^\infty \frac{1}{r^{1/\alpha}(s)} ds = \int_t^\infty s^{-\frac{3}{2}} ds = 2t^{-\frac{1}{2}} < \infty \text{ for any } t \ge 1.$$

Since $p(t)\frac{\pi(\tau(t))}{\pi(t)} < c < 1$ is naturally satisfied when $p(t) \equiv 0$, we only need to verify the condition (3.1). Then

$$\int^\infty \left(\frac{1}{r(t)}\int^t q(s)ds\right)^{1/\alpha} dt = \int_1^\infty \frac{1}{t^{\frac{3}{2}}}\int_1^t ds dt = \int_1^\infty \left(t^{-\frac{1}{2}} - t^{-\frac{3}{2}}\right) dt = \infty.$$

It implies that every solution $y(t)$ of Eq. (4.2) is oscillatory or satisfies $\lim_{t\to\infty} y(t) = 0$ (the image of the solution $y(t)$ and $y'(t)$ is shown in Fig. 1).

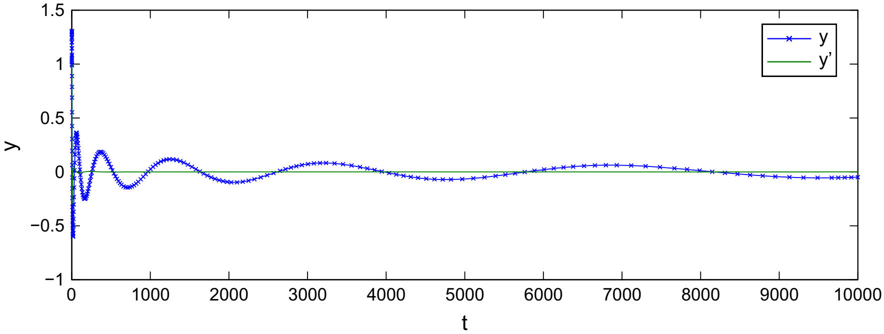

Fig. 1 The trajectory of the solution of Eq. (4.2) and its derivative

5 Conclusion

A new oscillation criterion is presented in this paper by means of the Riccati transformation and some new inequality techniques. The oscillation criterion established here depends only on one condition, which is simpler compared with [14]. Moreover, we investigate a nonlinear neutral delay equation where α and β are independent, which is more general than equations in [14,15]. Thus, this is an improvement and generalization of [14,15] and some other related references.

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