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ORIGINAL RESEARCH

Newton iterative algorithm based modeling and proportional derivative controller design for second-order systems

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Abstract This paper proposes an identification method for estimating the parameters of a stable second-order system based on the impulse response experiment. From the impulse response experiment, the measured data are collected for implementing parameter estimation. By defining and minimizing a cost function, a Newton iterative algorithm is derived for estimating the parameters of the system. The multi-point identification method is used to show the effectiveness of the proposed Newton iterative algorithm. The results show that the estimated model by the proposed Newton iterative estimation method has higher accuracy. Based on the estimated model, a design method of the proportional derivative controller is presented according to the system dynamical performance. The simulation test shows that the proposed controller design method can meet the desired dynamic specifications.

Keywords Newton iterative method · Mathematical modeling · Parameter estimation · Controller design · Impulse response · Multi-point identification

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1 Introduction

System identification problems for dynamical systems have been attracting much attentions [1–4]. System identification is building the mathematical models of systems by minimizing a cost function from the measured data [5–10]. Various experiments were used to generate the measured data for system identification [11]. The impulse response test, the step response test [12–16] and the relay feedback test [17] are used widely for system identification. For examples, Hidayat et al proposed a Laguerre domain identification method for continuous linear time-delay systems from the impulse response data [18]. Deb et al studied the transfer function identification from impulse response via orthogonal hybrid functions [19]. Liu et al proposed a frequency domain step response identification method for continuous-time processes with time delay [20]. Mei et al presented a decentralized identification method for multivariable integrating processes with time delays from closed-loop step tests [21]. Panda et al presented a parameter estimation algorithm for integrating and time delay processes using single relay feedback test [22]. Among these experiments for system identification, the impulse response test is easy to realize. So, the impulse response test is used to generate the measured data in this paper.

System identification plays an important role in designing controller parameters. After estimating the system mathematical model, some methods can be used to design the controller [23]. Malek et al studied the identification and controller design methods for time delay systems [24]. Baran et al presented an approach to the model identification and the proportional integral controller tuning based on a model-based experimental design [25]. Mohideen et al studied the modeling and control of level processes [26]. System identification is the prerequisite for the system control.

The iterative algorithm is widely used in system identification and for solving the solutions of nonlinear equations or matrix equations. [27–29]. Ding et al presented gradient based and least squares-based iterative algorithms for Hammerstein systems [30]. Ding et al proposed a two-stage least squares based iterative estimation algorithm for CARARMA systems [31]. Yun utilized iterative methods for obtaining all the roots of a nonlinear equation. [32]. Chidume et al provided an iterative method for approximating solutions of Hammerstein nonlinear integral equations. [33]. Noor et al proposed iterative methods for solving nonlinear equations by using the homotopy perturbation approach [34]. Wu et al presented an iterative algorithm for solving complex conjugate and transpose matrix equations. [35]. Sharma et al proposed a weighted-Newton methods for solving systems of nonlinear equations. [36]. Li presented a parameter estimation algorithm based on the Newton iteration for Hammerstein CARARMA systems [37]. Among the iterative algorithms, the Newton iterative method is the well known iterative algorithm for solving nonlinear equation and estimating parameters. By applying the iterative identification methods, the systems parameters can be estimated. The iterative identification algorithm can be derived by means of defining and minimizing an output error criterion function. On the basis of the Newton iterative principle, this paper derives the Newton iterative identification algorithm to estimate the parameters of a stable second-order system.

In many research areas, the plants to be controlled can be described as second-order system models. Bruschetta et al builded a second-order model for linear mechanical systems [38]. Delis et al constructed second-order macroscopic traffic flow models [39]. For the control systems, the stability and good dynamic performance are the basic requirements [40,41]. In order to meet these requirements, designing an efficient controller is very important. The proportional integral derivative controller is the most popular controller in process control [42–45]. Many design methods have been presented for designing the proportional integral derivative controller based on mathematical models of systems [46–48]. Ramasamy et al proposed a proportional integral derivative tuning method for single-input single-output systems using the impulse response [49]. Chen et al designed a fractional-order proportional integral derivative controller for the hydraulic turbine regulating system using the genetic algorithm [50]. Yuan et al proposed the proportional spatial derivative control method to the synchronization of the coupled distributed parameter system with time delay [51]. In this paper, a proportional derivative controller is designed according to some stability specifications.

The rest of this paper is organized as follows. Sect. 2 derives the Newton iterative algorithm to estimate the parameters of the second-order system with different poles and one zero. Sect. 3 gives the design procedure of a proportional derivative controller in terms of the estimated second-order system. Sect. 4 provides a simulation example and compares with multi-point identification method to show the effectiveness of the proposed algorithm. Finally, Sect. 5 offers some concluding remarks.

2 The Newton iterative estimate algorithm

Consider a stable second-order system with the following transfer function

$$G(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}, \tag{1}$$

where K is the gain, $-z_1$ is the zero, and $-p_1$ and $-p_2$ are the poles. When these parameters are unknown, some identification methods can be used to estimate these parameters from the measured data. The gain K can be estimated according to the steady output value [16]. But it is difficult to obtain the accurate steady-state value of the system because disturbances exist in practical processes. In order to avoid using the steady-state value, we change the transfer function of the second-order system in (1) into the sum of partial fractions,

$$G(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)} = \frac{a_3}{s+a_1} + \frac{a_4}{s+a_2}.$$

For the sake of finding the relations of the parameters of both sides of the above equation, we have the following analysis,

$$\frac{a_3}{s+a_1} + \frac{a_4}{s+a_2} = \frac{a_3(s+a_2)+a_4(s+a_1)}{(s+a_1)(s+a_2)}$$

$$= \frac{(a_3 + a_4)s + a_2 a_3 + a_1 a_4}{(s + a_1)(s + a_2)}$$
$$= \frac{(a_3 + a_4)(s + \frac{a_2 a_3 + a_1 a_4}{a_3 + a_4})}{(s + a_1)(s + a_2)}$$
$$= \frac{K(s + z_1)}{(s + p_1)(s + p_2)}.$$

Then, the relations among the parameters are given by

$$a_1 = p_1,\ a_2 = p_2,\ a_3 = \frac{Kp_1 - Kz_1}{p_1 - p_2},\ a_4 = \frac{Kz_1 - Kp_2}{p_1 - p_2}. \tag{2}$$

Assume that the input $r(t)$ is a unit impulse signal $r(t) = \delta(t)$.

$$r(t) = \begin{cases} \infty,\ t = 0, \\ 0,\ t \neq 0. \end{cases}$$

The Laplace transform of $r(t)$ is given by

$$R(s) = \mathscr{L}[r(t)] = \int_0^{\infty} \delta(t) \mathrm{e}^{-st} \mathrm{d}t = 1,$$

where $R(s)$ denotes the Laplace transform of the input $r(t)$.

The Laplace transform of the system output is

$$Y(s) = G(s)R(s) = \frac{a_3}{s + a_1} + \frac{a_4}{s + a_2}.$$

The system output response can be obtained by the inverse Laplace transform

$$y(t) = \mathscr{L}^{-1}[Y(s)] = \mathscr{L}^{-1}\left[\frac{a_3}{s + a_1} + \frac{a_4}{s + a_2}\right] = a_3 \mathrm{e}^{-a_1 t} + a_4 \mathrm{e}^{-a_2 t},$$

where $a_1,\ a_2,\ a_3$ and a_4 are the parameters to be estimated.

Define the parameter vector $\boldsymbol{\theta}$ as

$$\boldsymbol{\theta} = [a_1, a_2, a_3, a_4]^{\mathrm{T}} \in \mathbb{R}^4.$$

Define the residual

$$\varepsilon_i = y(t_i) - a_3 \mathrm{e}^{-a_1 t_i} - a_4 \mathrm{e}^{-a_2 t_i},$$

where $(t_i, y(t_i)\ i = 1, 2, ..., L)$ is the measured data, L is the data length. The parameters of the second-order system can be estimated by defining and minimizing a cost function. Define the cost function

$$J(\boldsymbol{\theta}) = \frac{1}{2}\sum_{i=1}^{L}\varepsilon_i^2.$$

Minimizing $J(\boldsymbol{\theta})$ and using the Newton iterative method give the Newton iterative estimate algorithm [52],

$$\hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} - \boldsymbol{H}^{-1}(\hat{\boldsymbol{\theta}}_{k-1})\boldsymbol{F}(\hat{\boldsymbol{\theta}}_{k-1}), \tag{3}$$

$$\boldsymbol{F}(\hat{\boldsymbol{\theta}}_k) = [f_1(k),\ f_2(k),\ f_3(k),\ f_4(k)]^{\mathrm{T}}, \tag{4}$$

$$\boldsymbol{H}(\hat{\boldsymbol{\theta}}_k) = \begin{bmatrix} h_{11}(k) & h_{12}(k) & h_{13}(k) & h_{14}(k) \\ h_{21}(k) & h_{22}(k) & h_{23}(k) & h_{24}(k) \\ h_{31}(k) & h_{32}(k) & h_{33}(k) & h_{12}(k) \\ h_{41}(k) & h_{42}(k) & h_{43}(k) & h_{44}(k) \end{bmatrix}, \tag{5}$$

$$f_1(k) = \sum_{i=1}^{L}\varepsilon_{ik}\hat{a}_{3k}t_i\mathrm{e}^{-\hat{a}_{1k}t_i}, \tag{6}$$

$$f_2(k) = \sum_{i=1}^{L}\varepsilon_{ik}\hat{a}_{4k}t_i\mathrm{e}^{-\hat{a}_{2k}t_i}, \tag{7}$$

$$f_3(k) = -\sum_{i=1}^{L}\varepsilon_{ik}\mathrm{e}^{-\hat{a}_{1k}t_i}, \tag{8}$$

$$f_4(k) = -\sum_{i=1}^{L}\varepsilon_{ik}\mathrm{e}^{-\hat{a}_{2k}t_i}, \tag{9}$$

$$h_{11}(k) = \sum_{i=1}^{L}\hat{a}_{3k}t_i^2(\hat{a}_{3k}\mathrm{e}^{-\hat{a}_{1k}t_i} - \varepsilon_{ik})\mathrm{e}^{-\hat{a}_{1k}t_i}, \tag{10}$$

$$h_{12}(k) = 2\sum_{i=1}^{L}\hat{a}_{3k}\hat{a}_{4k}t_i^2\mathrm{e}^{-(\hat{a}_{1k}+\hat{a}_{2k})t_i}, \tag{11}$$

$$h_{13}(k) = \sum_{i=1}^{L}t_i(\varepsilon_{ik} - \hat{a}_{3k}\mathrm{e}^{-\hat{a}_{1k}t_i})\mathrm{e}^{-\hat{a}_{1k}t_i}, \tag{12}$$

$$h_{14}(k) = -\sum_{i=1}^{L}\hat{a}_{3k}t_i\mathrm{e}^{-(\hat{a}_{1k}+\hat{a}_{2k})t_i}, \tag{13}$$

$$h_{22}(k) = \sum_{i=1}^{L}\hat{a}_{4k}t_i^2(\hat{a}_{4k}\mathrm{e}^{-\hat{a}_{2k}t_i} - \varepsilon_{ik})\mathrm{e}^{-\hat{a}_{2k}t_i}, \tag{14}$$

$$h_{23}(k) = -\sum_{i=1}^{L}\hat{a}_{4k}t_i\mathrm{e}^{-(\hat{a}_{1k}+\hat{a}_{2k})t_i}, \tag{15}$$

$$h_{24}(k) = \sum_{i=1}^{L} t_i \mathrm{e}^{-\hat{a}_{2k}t_i}(\varepsilon_{ik} - \hat{a}_{4k}\mathrm{e}^{-\hat{a}_{2k}t_i}), \tag{16}$$

$$h_{33}(k) = \sum_{i=1}^{L} \mathrm{e}^{-2\hat{a}_{1k}t_i}, \tag{17}$$

$$h_{34}(k) = \sum_{i=1}^{L} \mathrm{e}^{-(\hat{a}_{1k}+\hat{a}_{2k})t_i}, \tag{18}$$

$$h_{44}(k) = \sum_{i=1}^{L} \mathrm{e}^{-2\hat{a}_{2k}t_i}, \tag{19}$$

$$h_{21}(k) = h_{12}(k), \quad h_{31}(k) = h_{13}(k), \quad h_{41}(k) = h_{14}(k), \tag{20}$$

$$h_{32}(k) = h_{23}(k), \quad h_{42}(k) = h_{24}(k), \quad h_{43}(k) = h_{34}(k), \tag{21}$$

$$\varepsilon_{ik} = y(t_i) - \hat{a}_{3k}\mathrm{e}^{-\hat{a}_{1k}t_i} - \hat{a}_{4k}\mathrm{e}^{-\hat{a}_{2k}t_i}, \tag{22}$$

$$\hat{K}_k = \hat{a}_{3k} + \hat{a}_{4k}, \quad \hat{z}_{1k} = (\hat{a}_{2k}\hat{a}_{3k} + \hat{a}_{1k}\hat{a}_{4k})/(\hat{a}_{3k} + \hat{a}_{4k}),$$
$$\hat{p}_{1k} = \hat{a}_{1k}, \quad \hat{p}_{2k} = \hat{a}_{2k}. \tag{23}$$

The steps of implementing the Newton iterative estimate algorithm are listed in the following.

1. Collect the measured data $\{(t_i, y(t_i))\colon i = 1, 2, 3, \ldots, L\}$, where L is the data length.
2. To initialize: let $k = 1$, $\hat{\boldsymbol{\theta}}_0 = [a_{10}, a_{20}, a_{30}, a_{40}]^{\mathrm{T}}$ be a random vector, and give a small number $\varepsilon > 0$.
3. Compute ε_{ik} by (22).
4. Compute $f_l(k),\ l = 1, 2, 3, 4$ by (6)–(9), and form $\boldsymbol{F}(\hat{\boldsymbol{\theta}}_k)$ by (4).
5. Compute $h_{mn}(k),\ m = 1, 2, 3, 4,\ n = 1, 2, 3, 4$ by (10)–(21), and form $\boldsymbol{H}(\hat{\boldsymbol{\theta}}_k)$ by (5).
6. Update the parameter estimate $\hat{\boldsymbol{\theta}}_k$ by (3).
7. If $\|\hat{\boldsymbol{\theta}}_k - \hat{\boldsymbol{\theta}}_{k-1}\| > \varepsilon$, increase k by 1 and go to step 3; otherwise terminate the procedure and obtain the parameter estimate $\hat{\boldsymbol{\theta}}_k$.
8. Compute $\hat{K}_k = \hat{a}_{3k} + \hat{a}_{4k}$, $\hat{z}_{1k} = (\hat{a}_{2k}\hat{a}_{3k} + \hat{a}_{1k}\hat{a}_{4k})/(\hat{a}_{3k} + \hat{a}_{4k})$, $\hat{p}_{1k} = \hat{a}_{1k}$, $\hat{p}_{2k} = \hat{a}_{2k}$.

3 The proportional derivative controller design

In this section, a proportional derivative controller is designed to control the second-order system. In general, the transfer function of the proportional derivation controller in the following form

$$C(s) = K_p(1 + K_d s),$$

where K_p and K_d are the parameters of the controller to be designed.

In order to improve the robustness of the system, a low-pass filter is introduced. Then, the transfer function of the proportional derivative controller with a low-filter is given by

$$C(s) = K_p(1 + \frac{K_d s}{\tau s + 1}), \tag{24}$$

where τ is a small positive real number. The parameters K_p and K_d of the proportional derivative controller can be obtained according to some specifications. For the sake of ensuring enough stability and good dynamic performance, we use the gain crossover frequency ω_c and the phase margin ϕ_m as the specifications for designing the proportional derivation controller. The gain crossover frequency ω_c specification is

$$|C(j\omega_c)G(j\omega_c)| = 1.$$

The phase margin ϕ_m specification is

$$\mathrm{argC(j\omega_c)G(j\omega_c)} = -\pi + \phi_\mathrm{m}.$$

The system open-loop transfer function is given by

$$G_k(s) = C(s)G(s). \tag{25}$$

Letting $s := j\omega$ and substituting it into (25) give

$$G_k(j\omega) = C(j\omega)G(j\omega).$$

The above equation is the system frequency characteristics. Substituting (1) and (24) into (25) and letting $s := j\omega$ give

$$\begin{aligned} G_k(j\omega) &= \frac{KK_p(j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)} \times \left(\frac{1 + jK_d\omega + j\tau\omega}{j\tau\omega + 1}\right) \\ &= \frac{KK_p(j\omega + z_1)(jK_d\omega + j\tau\omega + 1)}{(j\omega + p_1)(j\omega + p_2)(j\tau\omega + 1)}. \end{aligned}$$

According to the gain crossover frequency specification ω_c, we get the following equations

$$\begin{aligned} |G_k(j\omega_c)| &= \left|\frac{KK_p(j\omega_c + z_1)(jK_d\omega_c + j\tau\omega_c) + 1)}{(j\omega_c + p_1)(j\omega_c + p_2)}\right| \\ &= \frac{KK_p\sqrt{(\omega_c^2 + z_1^2)(K_d^2\omega_c^2 + \tau^2\omega_c^2 + 1)}}{\sqrt{(\omega_c^2 + p_1^2)(\omega_c^2 + p_2^2)(\tau^2\omega_c^2 + 1)}} = 1. \end{aligned} \tag{26}$$

According to the phase margin ϕ_m specification, we get

$$\begin{aligned}\arg \mathrm{G_k}(\mathrm{j}\omega_\mathrm{c}) &= \arctan\frac{\omega_c}{z_1} + \arctan(K_d+\tau)\omega_c - \arctan\frac{\omega_c}{p_1} - \arctan\frac{\omega_c}{p_2} - \arctan\tau\omega_c\\ &= \phi_m - \pi.\end{aligned}$$

Let $\beta := \arctan\frac{\omega_c}{z_1} - \arctan\frac{\omega_c}{p_1} - \arctan\frac{\omega_c}{p_2} - \arctan\tau\omega_c$. Then we have

$$\arg \mathrm{G_k}(\mathrm{j}\omega_\mathrm{c}) = \beta + \arctan \mathrm{K_d}\omega_\mathrm{c} = \phi_\mathrm{m} - \pi. \tag{27}$$

K_d is an unknown parameter in (27). Solving (27) gives

$$K_d = \frac{\tan(\phi_\mathrm{m} - \pi - \beta)}{\omega_c}. \tag{28}$$

Substituting (28) into (26) gives

$$\frac{KK_p\sqrt{(\omega_c^2+z_1^2)[\tan^2(\phi_m-\pi-\beta)+1]}}{\sqrt{(\omega_c^2+p_1^2)(\omega_c^2+p_2^2)(\tau^2\omega_c^2+1)}} = 1. \tag{29}$$

From (29), we get

$$K_p = \frac{\sqrt{(\omega_c^2+p_1^2)(\omega_c^2+p_2^2)(\tau^2\omega_c^2+1)}}{K\sqrt{(\omega_c^2+z_1^2)[\tan^2(\phi_m-\pi-\beta)+1]}}. \tag{30}$$

When the specifications ω_c and ϕ_m are given, the parameters of the controller can be obtained according to (28) and (30).

4 Simulation example

In this section, a numerical experiment is presented to show the effectiveness of the Newton iterative algorithm for the second-order system by using the impulse response. For comparison, we also present numerical results of a multi-point identification algorithm for the second-order system based on the step response tests in [16].

Consider the following second-order system:

$$G_1(s) = \frac{6s+10}{(s+1)(s+3)}.$$

According to (2), the transfer function $G_1(s)$ can be rewritten as

$$G_1(s) = \frac{6s+10}{(s+1)(s+3)} = \frac{2}{s+1} + \frac{4}{s+3}.$$

Table 1 The parameter estimates and errors ($\sigma^2 = 0^2$)

k	a_1	a_2	a_3	a_4	δ
1	1.6318	3.2773	2.1797	4.7007	0.1825
2	1.2800	3.0899	1.8964	4.6708	0.1351
3	1.1148	3.0863	1.9051	4.4794	0.0930
4	1.0592	3.0889	1.9438	4.3267	0.0636
5	1.0302	3.0887	1.9720	4.2208	0.0441
10	0.9916	3.0794	2.0312	4.0124	0.0158
15	0.9937	3.0704	2.0488	3.9677	0.0168
20	1.0002	3.0634	2.0583	3.9520	0.0180
30	1.0110	3.0533	2.0704	3.9358	0.0200
40	1.0182	3.0471	2.0782	3.9260	0.0217
50	1.0227	3.0431	2.0833	3.9197	0.0229
True values	1.0000	3.0000	2.0000	4.0000	

Table 2 The parameter estimates and errors ($\sigma^2 = 0.05^2$)

k	a_1	a_2	a_3	a_4	δ
1	1.6507	3.3063	2.1892	4.7345	0.1908
2	1.3205	3.1194	1.9166	4.7179	0.1460
3	1.1481	3.1125	1.9098	4.5537	0.1079
4	1.0894	3.1167	1.9451	4.4132	0.0807
5	1.0587	3.1184	1.9712	4.3160	0.0628
10	1.0128	3.1174	2.0218	4.1304	0.0324
15	1.0081	3.1150	2.0318	4.0974	0.0282
20	1.0088	3.1131	2.0350	4.0899	0.0272
30	1.0113	3.1104	2.0377	4.0856	0.0265
40	1.0130	3.1087	2.0392	4.0836	0.0261
50	1.0139	3.1076	2.0400	4.0825	0.0259
True values	1.0000	3.0000	2.0000	4.0000	

In simulation, the measured data are generated by the impulse response experiment. The input signal is an impulse signal and the disturbance signal $\{v(t)\}$ is an uncorrelated noise sequence with zero mean and variance $\sigma^2 = 0$ and $\sigma^2 = 0.05^2$.

Case 1: The parameter estimates using the Newton iterative algorithm

Utilizing the Newton iterative algorithm to estimate the parameters of the second-order system, the parameter estimates and their estimation errors are shown in Tables 1 and 2 and the estimation error $\delta := \|\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}\|/\|\boldsymbol{\theta}\|$ versus k is shown in Fig. 1.

In practical industrial processes, the disturbances exist widely. We add noise with zero mean and the variance $\sigma^2 = 0.05^2$ to the output data. The estimated model of the second-order system by the Newton iterative algorithm is given by

$$G_2(s) = \frac{6.1225(s + 1.7115)}{(s + 1.0139)(s + 3.1076)}.$$

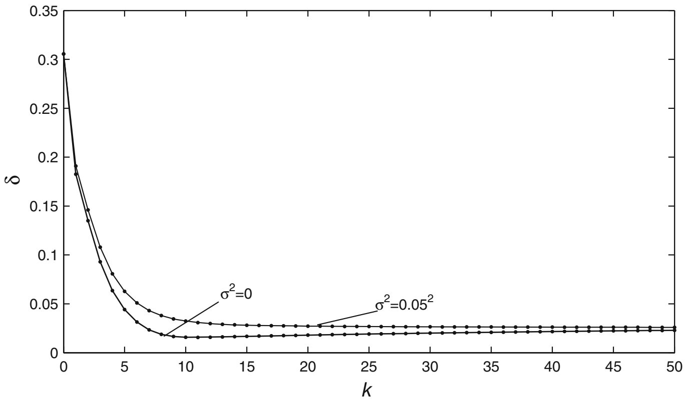

Fig. 1 The estimation error δ versus k

Table 3 The measured data of the step response test ($\sigma^2 = 0.05^2$)

t_i	$y(t_i)$	t_i	$y(t_i)$	t_i	$y(t_i)$	t_i	$y(t_i)$
0.2	4.83	1.2	13.52	2.2	15.81	3.2	16.40
0.4	7.97	1.4	14.16	2.4	16.00	3.4	16.35
0.6	10.04	1.6	14.49	2.6	16.12	3.6	16.39
0.8	11.58	1.8	14.95	2.8	16.08	4.0	16.52
1.0	12.63	2.0	15.57	3.0	16.37	5.0	16.61

Case 2: The comparison with the multi-point method

For comparison, the multi-point identification method based on the step response test in [16] is used to compare the performance of the proposed algorithm. We apply a step signal with the amplitude 5 ($r(t) = 5$, $t \geq 0$) to the input port of the second-order system and take a disturbance signal $\{v(t)\}$ to be an uncorrelated noise sequence with variance $\sigma^2 = 0.05^2$. The variance is the same as the impulse experiment by the Newton iterative algorithm. The measured data of the step responses is shown in Table 3.

The main idea of the multi-point identification method in [16] is that choosing some special points data of the dynamical process constructs algebraic equations and solves these equations to compute the parameters of the transfer function. In order to enhance the accuracy of the parameters estimation, we take three groups data to estimate the parameters of the second-order system model.

Consider the second-order system model is described as

$$G(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)}, \; T_1 < T_2, \; T_3 \neq T_1, \; T_3 \neq T_2,$$

where K is the gain, T_1, T_2 and T_3 are the time constants. The gain $K = 3.332$ is estimated by the steady-sate value. In the simulation, the special points are located at t_i, $2t_i$ and $3t_i$ ($t_i = 0.2, 0.4, 0.6$). The parameters to be estimated are obtained by the

Table 4 The special points data and the parameter estimates ($\sigma^2 = 0.05^2$)

t_i	$y(t_i)$	$y(2t_i)$	$y(3t_i)$	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$
0.2	4.83	7.97	10.04	0.4304	2.4981	1.0050
0.4	7.97	11.58	13.57	0.2385	0.8376	0.7007
0.6	10.04	13.52	14.95	0.4590	1.3432	0.7636
Parameter estimates				0.3760	1.5596	0.8231
True values				0.3333	1.0000	0.6000

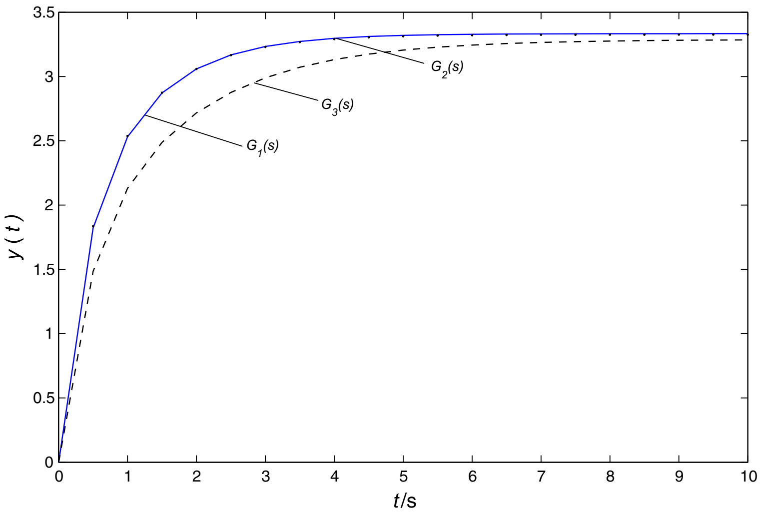

Fig. 2 The unit-step responses curves

average parameter estimation values. The data of the special points and the parameter estimates are shown in Table 4.

The estimated model of the second-order system by the multi-point identification method is given by

$$G_3(s) = \frac{3.332(0.8231s + 1)}{(0.3760s + 1)(1.5596s + 1)} = \frac{4.6799(s + 1.2149)}{(s + 2.6957)(s + 0.6412)}.$$

Case 3: The comparison of the step responses

In order to test the accuracy of the estimated models using the Newton iterative algorithm and the multi-point identification method, we take a unit-step signal as an input signal to the system. The unit-step responses curves of $G_1(s)$, $G_2(s)$ and $G_3(s)$ are shown in Fig. 2. The solid-line is the response curve of the true model, the dot-line is the response curve of the estimated model by the Newton iterative identification method and the dash-line is the response curve of the estimated model by the multi-point identification method. From the unit-step response curves, we can see that the unit-step response curve of $G_2(s)$ are closer to that of $G_1(s)$ than $G_3(s)$. It means that the Newton iterative algorithm is more effective than the multi-point identification method.

Case 4: The controller design

After obtaining the model of the second-order system, we design a proportional derivative controller to achieve the desired dynamical performance of the system. Because the estimated model using the Newton iterative algorithm has higher accuracy, we take $G_2(s)$ as the plant to be controlled.

Suppose that $\omega_c = 300$ rad/s and $\phi_m = \pi/6$. The proportional derivative controller can be designed according to the above specifications. Utilizing (28) and (30) gives the transfer function of the proportional derivative controller

$$C(s) = 25.4382\left(1 - \frac{0.0056s}{0.0001s + 1}\right).$$

Then, the open-loop transfer function of the system is

$$G_k(s) = G_2(s)C(s) = 25.4382\left(1 - \frac{0.0056s}{0.0001s + 1}\right) \times \frac{6.1225(s + 1.7115)}{(s + 1.0139)(s + 3.1076)}.$$

The bode diagrams of $G_2(s)$ and $G_k(s)$ are shown in Fig. 3, where the solid-line is the bode diagram of $G_k(s)$ and the dot-line is the bode diagram of $G_2(s)$. As is observed from Fig. 3, the crossover frequency and phase margin specifications can be fulfilled in the control of the proportional derivative controller and the frequency bandwidth of the system is broaden. In practical process control, the closed-loop systems have more advantages than the open-loop systems. So, we construct a closed-loop system and apply a unit-step signal to the input port of the closed-loop system. The step response curve is shown in Fig. 4.

From the simulation results, we can draw the following conclusions.

- The parameter estimation errors obtained by the Newton iterative algorithm decrease gradually with the increasing of iteration k and some minor fluctuation exists – see the estimation errors in the last columns of Tables 1 and 2. As the variance of the noise decreases, the parameter estimation errors given by the proposed Newton iterative algorithm become smaller gradually. For a deterministic system with the noise variance $\sigma^2 = 0$, the estimated parameters of the second-order system are very close to the true parameter values.
- For comparison, the Newton iterative algorithm and the multi-point identification method are compared to show the effectiveness of the proposed method in this paper. The estimated model of the system by the Newton iterative algorithm has higher accuracy than the multi-point identification method. For the systems with disturbances, the parameter estimation errors become large. Moreover, the gain is estimated by the system steady-state value when we use the multi-point identification method. But in practical industrial processes, the exact steady-state values are not easy to obtain because of disturbances. For the proposed Newton iterative identification algorithm based on the impulse response experiment, it can avoid estimating the gain according to the steady-state value. The drawback of the proposed Newton iterative algorithm is that the selection of the initial value can affect the results of the parameter estimation. So, the initial values should be at the vicinity of the true values when using the Newton iterative algorithm.

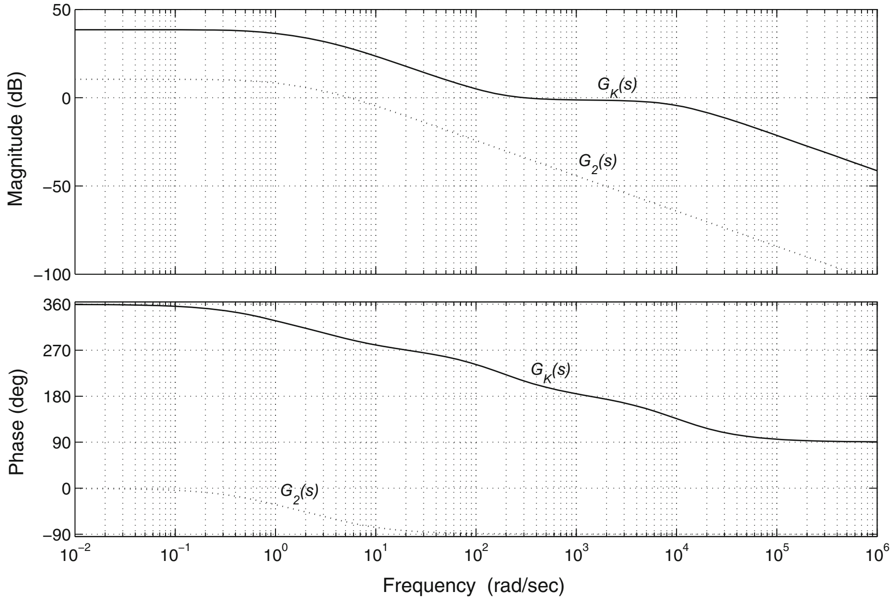

Fig. 3 The bode diagrams of $G_2(s)$ and $G_k(s)$

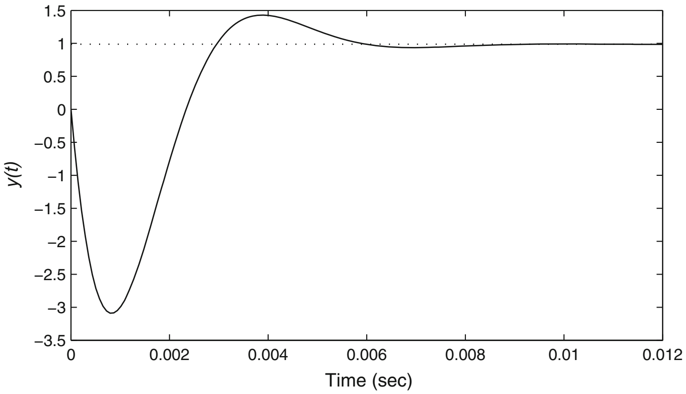

Fig. 4 The step response of the closed-loop system

– The proportional derivative controller can be designed according to the proposed controller design method. Once the specifications of the gain crossover frequency ω_c and the phase margin ϕ_m are given, a proportional derivative controller can be obtained. The Bode diagrams of the control plant transfer function and the open-loop system transfer function show that the specifications can be fulfilled by the proportional derivative controller. The step response curve shows that the closed-loop second-order system has good dynamic performance and stability. The settling time is very short in the control of the proportional derivative controller by the proposed controller design method. The output response of the closed-loop can reach the setting value quickly.

5 Conclusions

This paper presents a Newton iterative estimation algorithm to estimate the parameters of a second-order system with different poles and one zero. Using the partial fraction method, the transfer function of the second-order system is rewritten as the sum of the partial fractions for avoiding utilizing the steady-state values of systems to estimate the gain. This method can reduce the impact on the estimation accuracy because of the existence of disturbances. After obtaining the estimated model, a controller design method is proposed based on the specifications of the gain crossover frequency and the phase margin. The simulation results show that the parameters of the second-order system can be estimated by the proposed method and the dynamic and stability performance is good in the control of the designed proportional derivative controller.

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