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Convergence and permanence of a delayed Nicholson's Blowflies model with feedback control

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Abstract In this paper, we study a generalized Nicholson's Blowflies model with feedback control and multiple time-varying delays. Under proper conditions, we employ a novel proof to establish some criteria to guarantee the global exponential convergence and permanence of this model. Moreover, we give two examples to illustrate our main results.

Keywords Feedback control · Time-varying delays · Convergence · Permanence · Nicholson's Blowflies model

Mathematics Subject Classification (2000) 34C25 · 34K13

1 Introduction

As we known, Nicholson's blowflies model belongs to a class of biological systems and it (or its analogue equation) has been attracted more attention because of its extensively realistic significance. In particular, there have been extensive results on Nicholson's a blowflied model in the literature, which were paced on the most important dynamic behaviors of this model such as existence of positive solutions, persistence, permanence, oscillation and stability. We refer the reader to [1–7] and the references cited therein.

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Recently, Wang and Fang [8] proposed and studied the following discrete and continuous Nicholson's blowflies models with feedback control:

$$\begin{cases} x(n+1) = x(n)\exp\{-\delta(n) + p(n)e^{-\alpha(n)x(n)} - c(n)u(n)\}, \\ \Delta u(n) = -a(n)u(n) + b(n)x(n-m), \end{cases} \tag{1.1}$$

and

$$\begin{cases} x'(t) = x(t)(-\delta(t) + p(t)e^{-\alpha(t)x(t)} - c(t)u(t)), \\ u'(t) = -a(t)u(t) + b(t)x(t-\tau). \end{cases} \tag{1.2}$$

where $\delta(t)$, $p(t)$, $c(t)$, $a(t)$, $b(t)$ are all continuous functions bounded above and below by positive constants and τ is a positive constant. By developing some new analysis technique, they showed that feedback control variable has no influence on the permanence of the above system (1.1). However, they did not investigate the persistent property of the system (1.2). On the other hand, in the real world, the delays in differential equations of population and ecology problems are usually time-varying. Thus, it is worthwhile continuing to investigate the convergence and permanence of Nicholson's blowflies models with feedback control and time-varying delays.

In this paper we consider the following generalized Nicholson's blowflies model with feedback control and multiple time-varying delays:

$$\begin{cases} x_1'(t) = -\alpha(t)x_1(t) + \beta(t)x_1(t-\tau_1(t))e^{-\gamma(t)x_1(t-\tau_2(t))} \\ \qquad\qquad - c(t)x_1(t)x_2(t-\eta(t)), \\ x_2'(t) = -\lambda(t)x_2(t) + b(t)x_1(t-\delta(t)). \end{cases} \tag{1.3}$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $b(t)$ and $\lambda(t)$ are all continuous functions bounded above and below by positive constants, $c(t) > 0$, $\tau_1(t) \geq 0$, $\tau_2(t) \geq 0$, $\eta(t) \geq 0$ and $\delta(t) \geq 0$ are bounded continuous functions. Obviously, (1.2) is a special case of (1.3) with $\tau_1(t) \equiv \tau_2(t) \equiv 0$, $\eta(t) \equiv 0$ and $\delta(t) \equiv const > 0$.

Throughout this paper, given a bounded continuous function g defined on R, let g^i and g^s be defined as

$$g^i = \inf_{t\in R} g(t), \qquad g^s = \sup_{t\in R} g(t).$$

Denote by $R^2(R^2_+)$ the set of all (nonnegative) real vectors. Let $C = \prod_{i=1}^2 C([-r_i, 0], R^1)$ be the continuous functions space equipped with the usual supremum norm $||\cdot||$, and let $C_+ = \prod_{i=1}^2 C([-r_i, 0], R^1_+)$, where $r_1 = \max\{\tau_1^s, \tau_2^s, \delta^s\}$ and $r_2 = \eta^s$. If $x_i(t)$ is defined on $[t_0 - r_i, \sigma)$ with $t_0, \sigma \in R^1$ and $i \in I := \{1, 2\}$, then we define $x_t \in C$ as $x_t = (x_t^1, x_t^2)$ where $x_t^i(\theta) = x_i(t+\theta)$ for all $\theta \in [-r_i, 0]$ and $i \in I$.

Due to the biological interpretation of model (1.3), we just consider initial conditions associated with system (1.3) are of the form:

$$x_{t_0} = \varphi,\ \varphi = (\varphi_1, \varphi_2) \in C_+ \quad \text{and} \quad \varphi_1(0) > 0, \varphi_2(0) > 0. \tag{1.4}$$

Define a continuous map $f: R^1 \times \prod_{i=1}^2 C([-r_i, 0], R^1_+) \to R^2$ by setting

$$f_1(t,\varphi) = -\alpha(t)\varphi_1(0) + \beta(t)\varphi_1(-\tau_1(t))e^{-\gamma(t)\varphi_1(-\tau_2(t))} - c(t)\varphi_1(0)\varphi_2(-\eta(t))$$

and

$$f_2(t,\varphi) = -\lambda(t)\varphi_2(0) + b(t)\varphi_1(-\delta(t)).$$

Then, f is a locally Lipschitze map with respect to $\varphi \in C_+$, which ensure the existence and uniqueness of the solution of (1.3) with admissible initial conditions (1.4).

We write $x_t(t_0,\varphi)(x(t;t_0,\varphi))$ for a solution of the admissible initial value problem (1.3) and (1.4) with $x_{t_0}(t_0,\varphi) = \varphi$ and $t_0 \in R$. Also, let $[t_0, \eta(\varphi))$ be the maximal right-interval of existence of $x_t(t_0,\varphi)$.

The remaining part of this paper is organized as follows. In Sect. 2, we shall give some notations and preliminary results. In Sect. 3, we shall derive new sufficient conditions for checking the global exponential convergence and permanence of system (1.2). In Sect. 4, we shall give some examples and remarks to illustrate our results obtained in the previous sections.

2 Preliminary results

Now we state several Definitions and Lemmas which play important roles in the proof of main results.

Definition 2.1 System (1.3)–(1.4) is said to be permanent, if there are positive constants m_i and M_i such that

$$m_i \le \liminf_{t\to+\infty} x_i(t;t_0,\varphi) \le \limsup_{t\to+\infty} x_i(t;t_0,\varphi) \le M_i, \quad \text{for all } i = 1,2.$$

Lemma 2.1 [9] If $a > 0, b > 0$ and $\frac{dx}{dt} \ge b - ax$, when $t \ge t^*$ and $x(t^*) > 0$, we have

$$\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}.$$

If $a > 0, b > 0$ and $\frac{dx}{dt} \le b - ax$, when $t \ge t^*$ and $x(t^*) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}.$$

Lemma 2.2 Let $\tau_1(t) \equiv \tau_2(t)$ for all $t \in R$. Then, the solution $x_t(t_0,\varphi) \in C_+$ for all $t \in [t_0, \eta(\varphi))$, the set of $\{x_t(t_0,\varphi) : t \in [t_0, \eta(\varphi))\}$ is bounded, and $\eta(\varphi) = +\infty$. Moreover, $x_i(t;t_0,\varphi) > 0$ for all $t \ge t_0, i = 1,2$.

Proof Since $\varphi \in C_+$, using Theorem 5.2.1 in [10, p. 81], we have $x_t(t_0,\varphi) \in C_+$ for all $t \in [t_0, \eta(\varphi))$. Let $x(t) = (x_1(t), x_2(t)) = x(t;t_0,\varphi)$. Integrating the second equation of (1.3) from t_0 to t, we have

$$x_2(t) = e^{-\int_{t_0}^t \lambda(u)du} x_2(t_0) + e^{-\int_{t_0}^t \lambda(u)du} \int_{t_0}^t e^{\int_{t_0}^s \lambda(v)dv} b(s)x_1(s-\delta(s))ds, \quad (2.1)$$

for all $t \in [t_0, \eta(\varphi))$. It follows from $x_2(t_0) = \varphi_2(0) > 0$ that $x_2(t) > 0$ for all $t \in [t_0, \eta(\varphi))$. Again from (1.3) and $\sup_{u \geq 0} ue^{-\gamma^i u} = \frac{1}{\gamma^i e}$, we get

$$\begin{aligned} x_1'(t) &= -(\alpha(t) + c(t)x_2(t - \eta(t)))x_1(t) + \beta(t)x_1(t - \tau_1(t))e^{-\gamma(t)x_1(t-\tau_2(t))} \\ &\leq -\alpha(t)x_1(t) + \beta(t)x_1(t - \tau_1(t))e^{-\gamma(t)x_1(t-\tau_2(t))} \\ &\leq -\alpha^i x_1(t) + \frac{\beta^s}{\gamma^i e}, \end{aligned} \tag{2.2}$$

which, together with $x_1(t_0) = \varphi_1(0) > 0$, implies that

$$\begin{aligned} x_1(t) &= e^{-\int_{t_0}^t \rho(u)du} x_1(t_0) \\ &\quad + e^{-\int_{t_0}^t \rho(u)du} \int_{t_0}^t e^{\int_{t_0}^s \rho(v)dv} \beta(s)x_1(s - \tau_1(s))e^{-\gamma(s)x_s(s-\tau_2(s))} ds \\ &> 0, \quad \text{where } t \in [t_0, \eta(\varphi)), \rho(u) = \alpha(u) + c(u)x_2(u - \eta(u)), \end{aligned} \tag{2.3}$$

and

$$x_1(t) \leq e^{-\alpha^i(t-t_0)} x_1(t_0) + \frac{\beta^s}{\gamma^i \alpha^i e}(1 - e^{-\alpha^i(t-t_0)}), \quad \text{where } t \in [t_0, \eta(\varphi)). \tag{2.4}$$

Therefore, $x_1(t)$ is bounded on $[t_0, \eta(\varphi))$. From (2.1), we obtain that $x_2(t)$ is also bounded on $[t_0, \eta(\varphi))$. According to Theorem 2.3.1 in [11], we can easily obtain $\eta(\varphi) = +\infty$. This ends the proof of Lemma 2.2. □

Lemma 2.3 Let $\alpha^i > \beta^s$. Then, the solution $x_t(t_0, \varphi) \in C_+$ for all $t \in [t_0, \eta(\varphi))$, the set of $\{x_t(t_0, \varphi) : t \in [t_0, \eta(\varphi))\}$ is bounded, and $\eta(\varphi) = +\infty$. Moreover, $x_i(t; t_0, \varphi) > 0$ for all $t \geq t_0, i = 1, 2$.

Proof By a carbon copy of the proof of Lemma 2.2, we obtain that the $x_t(t_0, \varphi) \in C_+$, and $x_i(t; t_0, \varphi) > 0$ for all $t \in [t_0, \eta(\varphi)), i = 1, 2$.

We now show that $x_1(t)$ is bounded on $[t_0, \eta(\varphi))$. Define a continuous function $\Gamma(u)$ by setting

$$\Gamma(u) = -[\alpha^i - u] + \beta^s e^{ur_1}, \quad u \in [0, 1].$$

Then, we have

$$\Gamma(0) = -\alpha^i + \beta^s < 0,$$

which implies that there exist two constants $\eta > 0$ and $\sigma \in (0, \lambda^i) \cap (0, 1]$ such that

$$\Gamma(\sigma) = -[\alpha^i - \sigma] + \beta^s e^{\sigma r_1} < -\eta < 0. \tag{2.5}$$

We consider the Lyapunov functional

$$V(t) = x_1(t)e^{\sigma t}. \tag{2.6}$$

Calculating the derivative of $V(t)$ along the solution $x(t)$ of (1.3), we have

$$\begin{aligned} V'(t) &\le -\alpha(t)x_1(t)e^{\sigma t} + \beta(t)x_1(t-\tau_1(t))e^{-\gamma(t)x_1(t-\tau_2(t))}e^{\sigma t} \\ &\quad - c(t)x_1(t)x_2(t-\eta(t))e^{\sigma t} + \sigma x_1(t)e^{\sigma t} \\ &\le (\sigma-\alpha(t))x_1(t)e^{\sigma t} + \beta(t)x_1(t-\tau_1(t))e^{\sigma t}, \quad \text{for all } t \in (t_0, \eta(\varphi)). \end{aligned} \tag{2.7}$$

We claim that

$$V(t) = x_1(t)e^{\sigma t} < e^{\sigma t_0}\Big(\max_{t\in[-r_0,t_0]} |\varphi(t) - x^*(t)| + 1\Big) := M_1 \quad \text{for all } t \in (t_0, \eta(\varphi)). \tag{2.8}$$

Contrarily, there must exist $\eta(\varphi) > t_* > t_0$ such that

$$V(t_*) = M_1 \quad \text{and} \quad V(t) < M_1 \quad \text{for all } t \in [t_0 - r_1, t_*), \tag{2.9}$$

which implies that

$$V(t_*) - M_1 = 0 \quad \text{and} \quad V(t) - M_1 < 0 \quad \text{for all } t \in [t_0 - r_1, t_*). \tag{2.10}$$

Together with (2.7) and (2.10), we obtain

$$\begin{aligned} 0 &\le (V(t_*) - M_1)' \\ &= V'(t_*) \\ &\le (\sigma - \alpha(t_*))x_1(t_*)e^{\sigma t_*} + \beta(t_*)x_1(t_* - \tau_1(t_*))e^{\sigma t_*} \\ &= (\sigma - \alpha(t_*))x_1(t_*)e^{\sigma t_*} + \beta(t_*)x_1(t_* - \tau_1(t_*))e^{\sigma(t_*-\tau_1(t_*))}e^{\sigma\tau_1(t_*)} \\ &\le [(\sigma - \alpha^i) + \beta^s e^{\sigma r_1}]M_1. \end{aligned} \tag{2.11}$$

Thus,

$$0 \le (\sigma - \alpha^i) + \beta^s e^{\sigma r_1},$$

which contradicts with (2.5). Hence, (2.8) holds. It follows that

$$x_1(t) < M_1 e^{-\sigma t} \quad \text{for all } t \in (t_0, \eta(\varphi)). \tag{2.12}$$

Thus, $x_1(t)$ is bounded on $[t_0, \eta(\varphi))$. From (2.1), we obtain that $x_2(t)$ is bounded on $[t_0, \eta(\varphi))$. Again from Theorem 3.1 in [11], we can easily obtain $\eta(\varphi) = +\infty$. This ends the proof of Lemma 2.3. □

3 Main results

Theorem 3.1 *Assume that* $\alpha^i > \beta^s$, *then, the solution* $x(t; t_0, \varphi)$ *of* (1.3) *and* (1.4) *converges exponentially to* $(0, 0)$ *as* $t \to +\infty$.

Proof Since $\alpha^i > \beta^s$, by a similar argument as in the proof in Lemma 2.3, we get

$$x_1(t) < M_1 e^{-\sigma t} \quad \text{for all } t > t_0. \tag{3.1}$$

Integrating the second equation of (1.3) from T_0 to $t (\geq T_0 = t_0 + \max\{r_1, r_2 + 1\})$, by (3.1), we get

$$\begin{aligned}
x_2(t) &= e^{-\int_{T_0}^t \lambda(u)du} x_2(T_0) + \int_{T_0}^t e^{-\int_s^t \lambda(v)dv} b(s) x_1(s - \delta(s)) ds \\
&\leq x_2(T_0) e^{\lambda^i T_0} e^{-\lambda^i t} + b^s M_1 \int_{T_0}^t e^{\lambda^i (s-t)} b(s) e^{-\sigma(s-\delta(s))} ds \\
&\leq x_2(T_0) e^{\lambda^i T_0} e^{-\lambda^i t} + \frac{b^s M_1 e^{\sigma r_1}}{\lambda^i - \sigma} e^{-\lambda^i t} (e^{(\lambda^i - \sigma)t} - e^{(\lambda^i - \sigma) T_0}) \\
&\leq \left[x_2(T_0) e^{\lambda^i T_0} e^{-(\lambda^i - \sigma)t} + \frac{b^s M_1 e^{\sigma r_1}}{\lambda^i - \sigma} \right] e^{-\sigma t} \\
&\leq M_2 e^{-\sigma t},
\end{aligned} \tag{3.2}$$

where $M_2 = x_2(T_0) e^{\lambda^i T_0} + \frac{b^s M_1 e^{\sigma r_1}}{\lambda^i - \sigma}$. It follows from (3.1) and (3.2) that the solution $x(t; t_0, \varphi)$ of (1.3) and (1.4) converges exponentially to $(0, 0)$ as $t \to +\infty$. This completes the proof of Theorem 3.1. □

Theorem 3.2 *Let*

$$\inf_{t \in R} \frac{\beta(t)}{\alpha(t)} > 1, \qquad \lim_{t \to +\infty} c(t) = 0, \quad \text{and} \quad \tau_1(t) \equiv \tau_2(t) \quad \text{for all } t \in R, \tag{3.3}$$

then system (1.3)–(1.4) *is permanent.*

Proof From Lemma 2.2, we obtain that the set of $\{x_t(t_0, \varphi) : t \in [t_0, +\infty)\}$ is bounded and there exist positive constants K_1 and K_2 such that

$$0 < x_1(t) \leq K_1, \qquad 0 < x_2(t) \leq K_2, \quad \text{for all } t > t_0. \tag{3.4}$$

It follows that

$$\limsup_{t \to +\infty} x_1(t) \leq K_1, \qquad \limsup_{t \to +\infty} x_2(t) \leq K_2. \tag{3.5}$$

We next prove that there exists a positive constant k_1 such that

$$\liminf_{t \to +\infty} x_1(t) \geq k_1. \tag{3.6}$$

Suppose, for the sake of contradiction, $\liminf_{t \to +\infty} x_1(t) = 0$. For each $t \geq t_0$, we define

$$m(t) = \max\left\{ \xi : \xi \leq t, x_1(\xi) = \min_{t_0 \leq s \leq t} x_1(s) \right\}$$

Observe that $m(t) \to +\infty$ as $t \to +\infty$ and

$$\lim_{t\to+\infty} x_1(m(t)) = 0. \tag{3.7}$$

However, $x_1(m(t)) = \min_{t_0\le s\le t} x_1(s)$, and so $x_1'(m(t)) \le 0$. According to the fact that $\tau_1(t) \equiv \tau_2(t)$ for all $t \in R$, we have

$$\begin{aligned}
0 &\ge x_1'(m(t))\\
&= -\alpha(m(t))x_1(m(t)) + \beta(m(t))x_1(m(t) - \tau_1(m(t)))e^{-\gamma(m(t))x_1(m(t)-\tau_1(m(t)))}\\
&\quad - c(m(t))x_1(m(t))x_2(m(t) - \eta(m(t)))
\end{aligned}$$

and consequently,

$$\begin{aligned}
&\alpha(m(t))x_1(m(t)) + c(m(t))x_1(m(t))x_2(m(t) - \eta(m(t)))\\
&\quad \ge \beta(m(t))x_1(m(t) - \tau_1(m(t)))e^{-\gamma(m(t))x_1(m(t)-\tau_1(m(t)))}\\
&\quad \ge \beta(m(t))x_1(m(t) - \tau_1(m(t)))e^{-\gamma^s x_1(m(t)-\tau_1(m(t)))}.
\end{aligned} \tag{3.8}$$

This, together with (3.7) implies that

$$\lim_{t\to+\infty} x_1(m(t) - \tau_1(m(t))) = 0. \tag{3.9}$$

Now we select a sequence $\{t_n\}_{n=1}^{+\infty}$ such that

$$\lim_{n\to+\infty} t_n = +\infty, \qquad \lim_{n\to+\infty} \alpha(m(t_n)) = \alpha^*, \qquad \lim_{n\to+\infty} \beta(m(t_n)) = \beta^*. \tag{3.10}$$

In view of (3.8), we get

$$\begin{aligned}
&\alpha(m(t_n)) + c(m(t_n))x_2(m(t_n) - \eta(m(t_n)))\\
&\quad \ge \beta(m(t_n))\frac{x_1(m(t_n) - \tau_1(m(t_n)))e^{-\gamma^s x_1(m(t_n)-\tau_1(m(t_n)))}}{x_1(m(t_n))}\\
&\quad \ge \beta(m(t_n))\frac{x_1(m(t_n) - \tau_1(m(t_n)))e^{-\gamma^s x_1(m(t_n)-\tau_1(m(t_n)))}}{x_1(m(t_n) - \tau_1(m(t_n)))}\\
&\quad = \beta(m(t_n))e^{-\gamma^s x_1(m(t_n)-\tau_1(m(t_n)))}.
\end{aligned} \tag{3.11}$$

Letting $n \to +\infty$, (3.10) and (3.11) imply that

$$\lim_{n\to+\infty} \frac{\beta(m(t_n))}{\alpha(m(t_n))} = \frac{\beta^*}{\alpha^*} \le 1,$$

which contradicts with (3.3). Hence, (3.6) holds. It follows that there exists a $T_1 > 0$ such that $x_1(t) > \frac{k_1}{2}$ for all $t > T_1$, which combing with the second equation of system (1.3) leads to

$$x_2'(t) \ge -\lambda^s x_2 + b^i \frac{k_1}{2}, \quad \text{for all } t > T_1.$$

Applying the first part of Lemma 2.1 into the above inequality, it follows that

$$\liminf_{t\to+\infty} x_2(t) \geq \frac{b^i \frac{k_1}{2}}{\lambda^s} := k_2. \tag{3.12}$$

This completes the proof of Theorem 3.2. □

4 Examples and remarks

In this section, we give two examples to demonstrate the results obtained in previous sections.

Example 4.1 Consider the following Nicholson's Blowflies model with feedback control and multiple time-varying delays:

$$\begin{cases} x_1'(t) = -(10+\cos^2 t + |\sin t|)x_1(t) \\ \qquad + (1+\cos^4 t)x_1(t - e^{|\arctan t|})e^{-x_1(t-e^{2|\arctan t|})} \\ \qquad - (100+\cos^2 t + |\sin t|)x_1(t)x_2(t - e^{3|\arctan t|}), \\ x_2'(t) = -(1+\cos^4 t + |\sin t|)x_2(t) \\ \qquad + (1000+\cos^2 t + |\sin t|)x_1(t - e^{3|\arctan t|}). \end{cases} \tag{4.1}$$

Then, $\tau_1(t) = e^{|\arctan t|}$, $\tau_2(t) = e^{2|\arctan t|}$, $\eta(t) = e^{3|\arctan t|}$, $\delta(t) = e^{3|\arctan t|}$, $r = r_1 = r_2 = e^{\frac{3\pi}{2}}$, $\alpha^i \geq 10 > 3 \geq \beta^s$. It follows that the Nicholson's Blowflies model (4.1) satisfies all the conditions in Theorem 3.1. Hence, from Theorem 3.1, for system (4.1), if $\varphi \in \{\varphi \in C_+ : \varphi_1(0) > 0, \varphi_2(0) > 0\}$, then $x(t; t_0, \varphi)$ converges exponentially to $(0, 0)$ as $t \to +\infty$.

Remark 4.1 It is clear that system (4.1) is a Nicholson's Blowflies model with feedback control and multiple time-varying delays, and the time-varying in this system are not constants. Therefore, all the results in [7, 9] and the references therein cannot be applicable to prove that all the solutions of (4.1) with admissible initial conditions converge exponentially to $(0, 0)$. This implies that the results of this paper are new and they complement previously known results.

Example 4.2 Consider the following Nicholson's Blowflies model with feedback control and multiple time-varying delays:

$$\begin{cases} x_1'(t) = -(1+\cos^2 t + |\sin t|)x_1(t) \\ \qquad + (21+\cos^4 t)x_1(t - e^{2|\arctan t|})e^{-x_1(t-e^{2|\arctan t|})} \\ \qquad - \dfrac{1+t^2}{10+t^4}x_1(t)x_2(t - e^{3|\arctan t|}), \\ x_2'(t) = -(1+\cos^4 t + |\sin t|)x_2(t) + (1000+\cos^2 t + |\sin t|)x_1(t - e^{3|\arctan t|}). \end{cases} \tag{4.2}$$

Then, $\tau_1(t)=\tau_2(t)=e^{2|\arctan t|}$, $r=r_1=r_2=e^{\frac{3\pi}{2}}$,

$$\inf_{t\in R}\frac{\beta(t)}{\alpha(t)}=\inf_{t\in R}\frac{21+\cos^4 t}{1+\cos^2 t+|\sin t|}\geq 7>1, \qquad \lim_{t\to+\infty}c(t)=\lim_{t\to+\infty}\frac{1+t^2}{10+t^4}=0,$$

which imply that the Nicholson's Blowflies model (4.2) satisfies all the conditions in Theorem 3.2. Hence, from Theorem 3.2, the system (4.2)–(1.4) is permanent.

Remark 4.2 Since [8] did not investigate the persistent property of the system (1.2). Moreover, (1.2) is a special case of (1.3). This implies that Theorem 3.2 extends and improves the corresponding results in [8].

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