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REGULAR ARTICLE

Early Understanding of Cardinal Number Value: Semiotic, Social, and Pragmatic Dimensions in a Case Study with a Child from 2 to 3 Years Old

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Abstract
Studies on cardinality date back many years, and it remains a current research issue today. Indeed, despite the many findings on the topic, there is still controversy surrounding when and how children understand cardinality. The main studies on this understanding employs the "how many" question in situations involving the quantification of objects, analyzing the relationship between counting and cardinality. In the present study, we argue that it is essential to consider how cardinality is used in the context in which it arises, including the in- depth consideration of semiotic, social, and pragmatic dimensions, in order to fully comprehend the topic. We analyze in microgenesis the interaction between a two-year-old girl and her mother when they are playing a game requiring the understanding and use of cardinality through five sessions conducted over the course of a year. Our findings suggest that, in the proposed situation, cardinal understanding develops slowly and gradually requires an integrated body of resources (such as gestures and the use of objects). In highlighting the role of semiotics and social interactions in the development of cardinal understanding, this research underscores the fundamental role that early education should play in its development.

Keywords Cardinality · Adult-child interactions · Gestures · Uses of objects · Semiotic systems

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Introduction

Conceptualizing Cardinality

Understanding of cardinal value has long been a central topic in the study of number development. It is mostly accepted that children's understanding of cardinality is a key foundational support for later mathematic development (Bugden and Ansari 2011; Chu et al. 2015; De Smedt and Gilmore 2011; Geary 2018; among others).

Piaget (1952) was one of the pioneers in the study on number development, and in highlighting the importance of cardinality for the understanding of numbers. His definition of cardinality was based on the comparison of collections of equal quantities (logic of conservation) so, according to him, children could only know about cardinality at 7–8 years, in the stage of concrete operations.

Maybe Piaget was too strong in his proposal (Bryant 1997), so his conception gave rise to the idea that understanding of cardinality occurs much earlier, at 2–3 years. Gelman and Gallistel (1978) proposed cardinality as a fundamental principle for counting: following different criteria and different kinds of experiments from Piaget's, Gelman and Gallistel considered that cardinality refers to the comprehension that the last number word said indicates the numerical value of the collection. So, understanding cardinality means knowing that the last counted word refers both to the set as a whole and to the cardinality of the set; if you understand cardinality, you know the meaning of the number words and their relation with their unique and specific quantities. Cardinality principle connects counting to exact, cardinal numbers (Carey 2004; Cheung et al. 2017; Negen and Sarnecka 2015).

Perhaps, unlike the criteria followed by Piaget, Gelman & Gallistel considered less demanding or even too weak requirements (Bryant 1997), so that the nature of the relationship between counting and cardinality has been widely discussed in the context of when children start to understand that counting is an activity carried out to achieve meaningful results (i.e., a cardinal value) rather than an isolated, procedural action.

The most widely used task to assess children's early understanding of the relationship between counting and cardinality is the use of the "how many" question and "give-a-number" tasks in situations involving the quantification of objects (Gelman and Gallistel 1978; Gelman and Meck 1986; Hall 1988; Le Corre and Carey 2007; Lee 2016; Rodríguez et al. 2013; Sarnecka and Carey 2008; Schaeffer et al. 1974; Van Marle et al. 2014; Wynn 1990). Regarding this topic, some authors propose that the first four number words are mastered in order, one at a time. The child's progress on this front is called their number-knower level, and these children are then considered cardinal-principal-knowers (Carey 2009; Sarnecka and Lee 2009).

Nevertheless, the fact that a child answers the question "how many" with the last counted word or another number word is not enough to ensure that he or she comprehends that this number word represents the total amount of the collection. Although producing a cardinal number tag can be considered a first level in the understanding of cardinality (Brissaud 1992; Wynn 1992), this production does not constitute strong evidence of a broad understanding of this dimension. Such an answer seems to be insufficient to determine whether children are really aware of the cardinality of a collection (Bermejo 1996; Bermejo and Lago 1990). Also, recent studies highlight the importance of taking into account contextual factors, suggesting that understanding numerical values according to the numerical sequence could happen in a much less strict way than proposed by the cardinal-knowers model, calling into question the appropriateness of overly generalized descriptors for the main landmarks in early number development (Rodríguez and Scheuer 2015; Martí et al. 2013).

In fact, there is controversy surrounding when and how children understand cardinality, especially if we take into account that learning cardinality takes time, and children learn it at different speeds. A review of most of the studies performed on cardinality suggests that no result or conclusion fits *all* children or suits *all* number tasks or situations.

Pragmatic Aspects in Cardinality Understanding

Among all the dimensions that can contribute to cardinal development, emphasis should be placed on the importance of pragmatic aspects. Knowing how to count (properly assign number tags to objects) does not mean that a child knows about cardinality; furthermore, knowing about cardinality does not mean that a child knows how to *use* this information to solve a real-world problem. In fact, one of the greatest difficulties in mathematics lies in using procedures and concepts in a pragmatic way.

Understanding of cardinality has many meaningful *uses* in pragmatic terms, including cardinal *uses of objects*. The nature of the objects cannot be neglected when we consider cardinal activities. The first counting experiences – like most experiences in life – include artifacts (we count "things"). Artifacts have public meanings based on their cultural history and public or conventional uses in daily life. In the case of cardinality, the flexibility of the concept must also be considered: underlying its comprehension is a semiotic system – the number system – that must be flexible to accommodate diverse situations related to the material world as referent (i.e., the word "five" can refer to infinite sets of different kinds of objects more or less familiar, or even actions).

Semiotic Mediation in Understanding of Cardinality

In everyday life, objects and their diverse pragmatic uses are often introduced to the child by an adult (and many concepts) along with their rules. Adults interact with children through semiotic systems (gestures, the use of objects, language) (Rodríguez and Moro 1999) and highly varied performances.

In this regard, understanding the development of *cardinal* uses of objects must include, first, comprehension of the primary conventional uses of the objects in question (their function), and, second, comprehension of the semiotic systems used during the adult-child interaction, in order to begin to comprehend how children develop new ideas (about cardinality) supported by previous ones (e.g., different types of uses of objects). Thus, to analyze the development of cardinality understanding, it is crucial to consider the uses of objects and other semiotic systems that support development and learning processes.

In addition to the use of objects, another semiotic system that could support cardinal development is the gesture system. For example, children begin to use pointing gestures at the age of 11 or 12 months to provide information to the other in a prosocial and cooperative attitude with the other (Liszkowski et al. 2006). In the context of numerical activities, these gestures are transformed and conform to new cognitive resources or goals. In a counting activity, children can touch objects to control the elements they have already counted and differentiate them from those they have yet to count, or to associate numeric words with counted objects (Graham 1999; Saxe and Kaplan 1981; see also Clark 2005, on pointing in house plans).

This topic leads us to another important discussion: if touch-pointing objects is so relevant for counting activities, how do children make this transition from segmenting and counting units to a cardinal reference? There must be a transition when the child makes a shift from counting situations in terms of separate entities to a mental reconstruction of the situation in which a number word refers to *all* the entities as an unit, differentiating between counting and cardinal reference (Hall 1988). This idea highlights a key aspect: the need to consider units to conceive of cardinality and, therefore, the need for *segmentation* of the objects (their uniqueness or discretization) (Rodríguez 2012; Rodríguez and Scheuer 2015). In any counting situation, identifying which objects are "countable" (Fuson 1988) is imperative for the child to be able to assign them the proper number tags (one, two, three) and extract their cardinal value (three).

In the present study, we consider cardinality to be a complex construction and thus highlight the importance of social, pragmatic, and semiotic aspects of this construction. Although cardinality is a widely researched topic in studies about numbers, there is a gap in the literature regarding *how* children move from the comprehension of entities as units to the comprehension of units as *a whole set* when interacting with an adult who helps them through communicative mediators.

We argue that rather than considering quantification tasks, it is essential to consider the uses of cardinality within the context in which it arises, including through in-depth consideration of the material world in question. To this end it is important to conduct studies that identify not only quantification, but also the meaningful and flexible use of cardinality to adapt to different everyday situations. Analyzing the mediators used by adults during the interaction is likewise important to fully comprehend children's development. We cannot neglect the fact that in everyday life it is the adult who presents the material world and new concepts to the child using semiotic systems.

For all these reasons, it seems necessary to adopt a pragmatic-semiotic position (Rodríguez et al. 2018) to study early understanding of cardinality. From this perspective, studies about cardinal understanding need to include the influence of other semiotic systems through which the child may receive support at an early age, when interacting with others in functional contexts. Further, studies about cardinal understanding in children should be conducted in everyday environments, such as homes and schools, and through longitudinal studies to assess developmental trajectories.

Our objective in this paper was to analyze the semiotic systems used in the interactions between a child and an adult that may have supported the child's development of cardinality, in a game situation involving objects that can be used cardinally (i.e., dice with dots). To this end, we sought to study:

1) The progress of the child's understanding of the cardinal value of a set and its use in a game played by the child with the help of an adult.
2) The semiotic mediators used by the dyad:

   2.1) By the child: in the *quantification* of dots (gestures and language).

2.2) By the adult: non-verbal and verbal communicative mediators used during the interaction (during *quantification* and the *advancement of the horse*).

Method

The objective of this paper is to determine how the understanding of the cardinality of representations of small sets of items (up to five units corresponding to the dots on a die) develops in a child between the ages of 24 and 36 months during interaction with a familiar adult. This question was studied in a triadic game situation (child-adult-objects) involving cardinal understanding and its use in a context of play. We proposed a game situation that consisted of two parts: quantification and use of quantitative information. The *quantification* part could be considered as a counting situation (in line with "how many" tasks), once the dyad had counted the dots, they would go to the following part of the game: advancing the horse according to the number of dots. We call this second section of the game *use of quantitative information*. We considered this second part of the game as the one that represents symbolic use of cardinality. For this consideration, we consider DeLoache's approach on symbolic representations in spatial domains (DeLoache 1991, 1995). Although DeLoache employed direct representations of its referents (photographs, or drawing), and we used a much more abstract representations (representations of quantities – dots), we share with DeLoache her conception that the symbolic use of external representation implies the transfer of its meaning to another situation. The ages were chosen taking into account the previous literature on cardinality presented above.

Given the abstract nature of the representations we used, and assuming that understanding of cardinality is progressive, this study is based on the perspective outlined in Rodríguez and Moro (1999) on the pragmatics of the object. We analyzed the *development* and *use* of cardinality through a longitudinal case study. We considered this method to be more genuinely apt to capture development. In addition, we studied the sort of semiotic mediators (non-verbal and verbal) used by the dyad that may have supported the child in this process. We analyzed in microgenesis the interaction between the child and the adult for observing the process as it occurred.

Participants

The participants were a girl and an adult, her mother. The family had a medium-high socioeconomic status and lived in Barcelona, Spain. The adult, who was 35 years old, had a college degree (in child education). The child was not yet attending school. The mother reported never having played with dice with her, nor did she report doing so during the study.

Materials and Procedures

The materials were chosen based on those used in an earlier study (Martí et al. 2013). The objects used were: (1) a 4 cm six-faced red wood die, whose faces showed no, one, two, three, four, or five green dots arranged in an irregular manner, different from the dots on a conventional die; (2) a 10 × 150 cm strip of green EVA foam representing a path segmented with black lines into 10 squares, each with a yellow oval in the center; (3) a 10 cm tall replica horse; and (4) a bottle cap used as the horse's "food bowl." (See Fig. 1).

The game consisted in rolling the die and advancing the horse as many squares up the path ("steps") as dots shown on the die until the horse reached the "food." The goal of the game was to "feed the horse" by getting it to the "bowl" located at the end of the path. After explaining to the adult what the game consisted of and providing the objects, the researcher invited her to "play with her daughter as she would normally do, respecting, as far as possible, the rules of the game."

Five sessions were recorded, separated by three-month intervals, at their home, when the girl was 24, 27, 30, 33, and 36 months old (S1, S2, S3, S4, and S5). In the first session, the mother signed a form provided by the researcher, giving her consent for the study and the use of images and data for academic purposes. Each session lasted approximately 5 min.

Categories of Analysis

Each video was analyzed with the ELAN media annotator (EUDICO Linguistic Annotator, 2011, version 4.6.1), according to the codes below. We separately analyzed the semiotic systems used by the adult (A) and the child (C) and segmented the sessions according to the moments of the game: *quantification* and *advancement of the horse*.

I. First analysis. The progress of the child's understanding of the cardinal value of a set and its use in the played game (Objective 1).

For this part of the analysis, we took into account the I. 1) *answer types of C during quantification* and I. 2) *types of uses of objects during quantification and advancement of the horse* (see Table 1).

II. Second analysis. The semiotic mediators used by the dyad (Objective 2)

II. 1) *Semiotic mediators used by C during the quantification of dots*

For this analysis, we took into account the semiotic modes of expression used by C to count the dots (Table 2).

Fig. 1 Material (path, horse, wood die and "food bowl")

Table 1 Codes for the First analysis: The progress of C's understanding of the cardinal value of a set and its use in a game (C = Child)

I. First analysis: The progress of C's understanding of the cardinal value of a set and its use in a game.	
I.1) Answer types during quantification:	
Code name	Code description
1) Fixed number	C repeatedly expressed the same numerical value on different occasions to quantify the dots, e.g., regardless of the face of the die showing, she might say that there were "three dots".
2) Error > 1	C expressed a numerical value with an error greater than one (plus or minus) in relation to the dots actually shown, e.g., C might touch three dots, but then touch two of them again, while saying the numerical words "one, two, three, *four, five*."
3) Error = 1	C expressed a numerical value with an error of one (plus or minus) in relation to the dots actually shown on the face of the die, e.g., C might touch two dots, but then touch one of them again, while saying the numerical words "one, two, *three*" when the face of the die showed two.
4) Partially correct	C expressed the number correctly in one mode of expression (oral or gestures) but incorrectly in the other, e.g., C might touch two dots, one at a time, while saying the number words "five and seven" or say number words in the right sequence, but touch the dots indiscriminately.
5) Correct response	C expressed the number correctly in one or both modes of expression (oral or gesture), e.g., C might say three for three dots or count "one, two, three" or hold up three fingers.
I.2) Uses of the objects during quantification and advancement of the horse:	
Code name	Code description
1) Non--conventional	C used the objects (die, horse, path, food) ignoring their functions in the context of the game, e.g., C might use the die as a "seat" for the horse or as "food" for the horse.
2) Protoconventi-onal	The use of the object was close to the conventional one. During *quantification*, the protoconventional uses were the answer types *fixed number*, *error > 1, error = 1* and *partially correct*. During *advancement of the horse*, C generally seemed to understand the basic rules of the game (counting dots before advancing the horse), but the horse was not used in accordance with the exact cardinal value of the collections shown on the die, e.g., C might advance the horse to the food or C might advance the horse one step more or less along the path than indicated by the number of dots on the die or the number of dots reported by C during the previous *quantification*.
3) Conventional	During *quantification*, the response was congruent with the number of dots shown on the top face of the die (correct response). During *advancement of the horse*, the advancement was congruent with the cardinal value of the dots on the die. The conventional uses of the horse constituted the *cardinal use* of the quantitative information.

II. 2) *Verbal and non-verbal semiotic mediators used by A*

For this part of the analysis, we considered the semiotic mediators used by A that may have helped C in her comprehension of the task. We analyzed the verbal and non-verbal communicative mediators used by A (see Table 3).

Codes of Verbal communicative mediators are based on Van de Pol et al. 2010 (review of scaffolding strategies). Codes of gestures are composed of *ostensive gestures* (Rodríguez et al. 2015), *placing* (Clark 2003), *pointing* (Butterworth 1998; Liszkowski et al. 2006; Rodríguez and Moro 1999), and *symbolic gestures* (Palacios and Rodríguez 2014).

Table 2 Codes for the Second analysis (part 1): Semiotic mediators used by C during the quantification of dots (C=Child)

II. Second analysis: The semiotic mediators used by the dyad	
II.1) By C: During the quantification of dots	
Code name	Code description
1) Oral	C verbally expressed the number of dots.
2) Symbolic-numeric gesture	C indicated the number of dots on the die by holding up her fingers, without oral expression.
3) Mixed (oral + pointing gesture, or oral + symbolic-numeric gesture)	C expressed the number both, orally and with gestures – through an immediate pointing gesture, where C would touch the dots, or through a symbolic-numeric gesture, where C would show numerical information with her fingers.

It should be noted that A interventions were not always exclusively verbal, but rather were often accompanied by gestures and/or uses of objects (nonverbal communicative mediators). This analysis aimed to identify and analyze the gestures and uses of objects made by A that took place during some types of verbal productions (see Table 3).

Results

In this section, we analyze the semiotic systems used in the interactions between C and A that may have supported C's development of cardinality. Results are presented in relation to our objectives, presented in the Introduction section.

Objective 1. Progress of the child's understanding of the cardinal value of a set in a game with an adult

Answer Types during Quantification

The answer types observed during quantification in S1 and S2 were as follows: *fixed number*, when C expressed the same numerical value for each face of the die; and *error* > *1*, when C was off by one (plus or minus) in relation to the numerical value indicated on the die. In S1, C reported the value "three" in all situations when quantifying the dots on the faces of the die (see Fig. 2). It should be noted that this value (three) was the value reported by A in the demonstration she made at the beginning of the first session. In S5, the protoconventional uses found were the answer types: *partially correct*, when C expressed the numerical value in mixed fashion (oral and gestures) and was wrong in only one mode of expression, and *error* = *1*, when C was off by one with regard to the value indicated on the die.

Thus, in S1, C expressed the number of dots orally, but erroneously; from S2 on, she also performed counting (through mixed expressions, with immediate pointing gestures); and, especially from S4 on, C began to evaluate the faces of the die correctly using sudden oral expressions and symbolic-numeric gestures. It should be noted that the sudden evaluations (i.e., expressions not preceded by counting) that C performed

Table 3 Codes for the Second analysis (part 2): The semiotic mediators used by the dyad - Verbal and non-verbal semiotic mediators used by A during the interaction with C (A = Adult; C = Child)

II. Second analysis: The semiotic mediators used by the dyad	
II.2) By A: During quantification and advancement of the horse	
- Verbal communicative mediators	
Code name	Code description
1) Affective assistance	Verbal mediators linked to positive assessments of C's uses of objects and frustration control. This included *error acceptance* (following non-conventional or protoconventional uses of objects by C, A would not correct her, but rather accept that use) and *positive evaluation* (after a conventional use of an object by C, A would confirm or positively evaluate the use).
2) Cognitive assistance	When referring to the structure of the game, to promote cognitive structuring in C. This included: *presentation* (A would present the material, rule or objective of the game), *instruction* (A would encourage C to quantify the dots or advance the horse), *error prevention* (A would warn C not to make non-conventional or protoconventional uses of the objects), and *correction* (A would correct C's performance, considering it to be non-conventional or protoconventional).
3) Metacognitive assistance	Questions seemingly intended to encourage C to reflect on her own performance. This included: *calls for recapitulation* (after quantification, A would raise the recapitulation of the cardinal value (total value) of the dots, connecting it with the advancement of the horse, e.g., by asking after quantification, "Then, how many steps is the horse going to advance?"); *calls for ratification* (in response to a conventional use, A would again question the use, seemingly with the intention of confirming that C was aware of the use she had made, e.g., saying, "Three. Are you sure?"); and *calls for self-regulation* (after a non-conventional or protoconventional use, A would question C about it, e.g., asking "Are you sure that was three?").
- Nonverbal communicative mediators	
Code name	Code description
Gestures	*1) Ostensive:* sign and referent coincide, that is, they are the same, e.g., A might offer or show the die to C. *2) Placing:* positioning of the object to suggest or invite the other to use it conventionally, in a way that suggests its use, e.g., A might place the die near C inviting her to evaluate it. *3) Pointing:* maintenance of a relation of proximity with the referent. Three types of pointing gestures were distinguished: (1) *distant pointing*, when the person did not touch the indicated object; (2) *immediate pointing*, when she touched the indicated object once; and (3) *multiple pointing*, when she touched the indicated object multiple times (e.g., touching the same dot or same square on the path more than once). *4) Symbolic:* implies a degree of absence of the referent, e.g., A might wave her empty fingers over the bottle cap meaning "put food on the plate."
Uses of objects	*1) Distant demonstration (DD):* A presented herself as a model, using the object according to its function, e.g., quantifying the dots or advancing the horse along the path. *2) Immediate demonstration (ID):* A directly guided C's body, introducing her to the object's use, e.g., guiding C's hand during the horse's advancement. *3) Inhibition of non-conventional uses (INCU):* using her hand, A prevented C from performing non-conventional uses, e.g., taking C's hand or stopping C in the advancement when it exceeded the number of dots. This usually took place in the *error prevention* and *correction* categories of the second analysis.

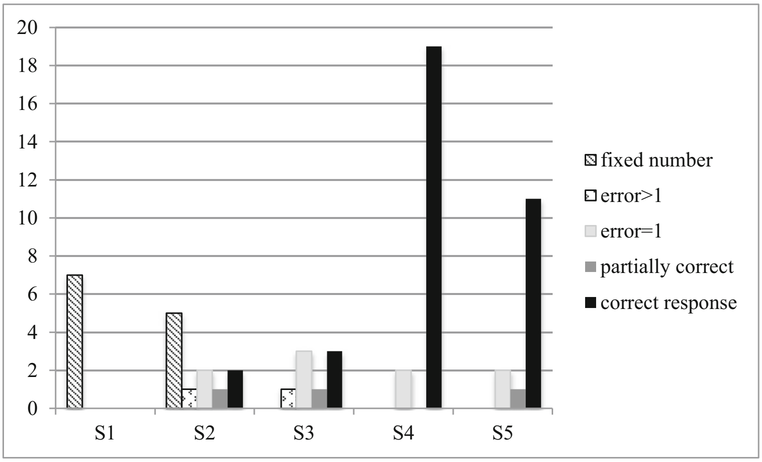

Fig. 2 C's answer types when quantifying the dots by session (Objective 1)

occurred with faces one, two, and three only (face one from S2 on, and face two and three from S4 on). With faces four and five, C always used mixed expressions accompanied by immediate pointing gestures.

Because the game was a free game, the occurrence of the faces of the die was not predefined. Also, by S5, C was able to correctly quantify *all* faces. Her first correct quantifications, in S2, occurred with faces one and three (although C also made mistakes regarding these faces in that session). From S4 on, we observed only correct responses for the blank face and faces one and two, i.e., from S4 on, we no longer detected any errors with regard to these faces. In S4 and S5, we found both errors and correct responses with faces three, four, and five. Thus, the correct quantification of the dots occurred progressively: conventional uses with the blank face and faces one and two stabilized before the correct response with faces three, four and five.

Uses of the Objects during Quantification and Advancement of the Horse

Now we will present the main results on C's progress in understanding cardinal values in the proposed situation in terms of uses of objects. (See Fig. 3).

According to our data, the conventional uses of the horse (what we considered to be the cardinal use of the object) were concentrated in S4 and S5. There was one exception: an occurrence in S2 (following correction of A with face one). In contrast, the conventional uses of the die (or correct quantification of dots) occurred sooner, beginning in S2.

Non-conventional uses in the quantification of dots were infrequent, occurring only in S2 and S3 (C tried to pick up the horse after rolling the die, but before quantifying the dots, one and three times, respectively). Non-conventional uses were more frequent during the advancement of the horse than during quantification. Non-conventional uses occurred mostly in the first three sessions: C moved the horse off the path, advanced the die instead of the horse, and attempted to remove or move the path.

Protoconventional uses in the advancement of the horse occurred in all sessions. There were also notable changes throughout the sessions. In the first three sessions, the

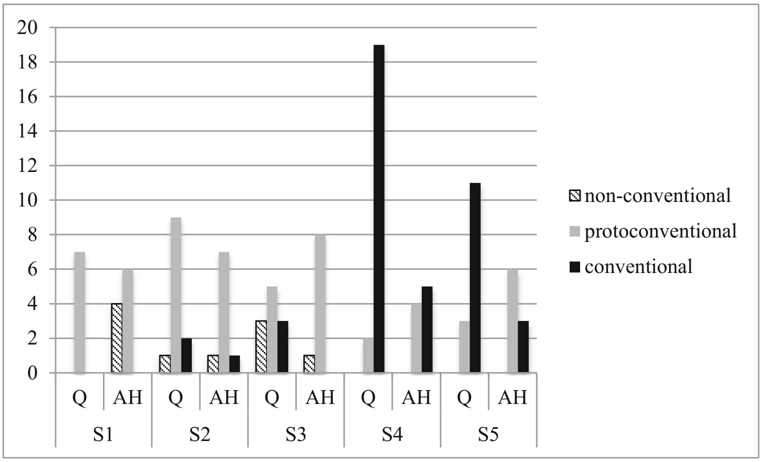

Fig. 3 Uses of the objects by C during quantification of the dots (Q) and advancement of the horse (AH) by session (Objective 1)

protoconventional uses consisted mainly in moving the horse directly to the food (continuous advances).

With regard to the faces of the die, conventional advancement of the horse took place only with the blank face and faces one and two. Also, a developmental change was observed: the conventional uses with face one began to take place in S2 and with face two, in S4. In S5, we found conventional uses with faces one and two.

Objective 2: The semiotic mediators used by the dyad

Semiotic Modes of Expression Used in the Quantification of Dots, by C (Objective 2.1)

In this analysis, we aimed to answer "*Which semiotic modes of expression does the child use to count the dots?*". We will show how C quantified the dots of the die in order to identify how she developed from comprehension of the units (segmentation) to comprehension of the set of dots as a whole (cardinality).

In S1 and S2, oral expression was predominant (although all responses were erroneous in S1). It should be noted that in S1, C neither pointed nor made symbolic-numeric gestures. In S2 and S3, C expressed herself orally – and correctly – only when reporting face one. From S2 on, we began to observe mixed expressions (immediate pointing gesture with oral expression, when C touched the dots accompanying the gestures with numerical words). Such mixed expressions were very common in S3, but only with immediate pointing gestures, as in S2.

In S4 and S5, C used predominantly oral mediators (correctly), but also used mixed expressions. Strikingly, symbolic-numeric gestures appeared for the first time in S4, revealing a clear gap between immediate pointing gestures (from S2 on) and symbolic-numeric gestures, which would be more complex (see Fig. 4).

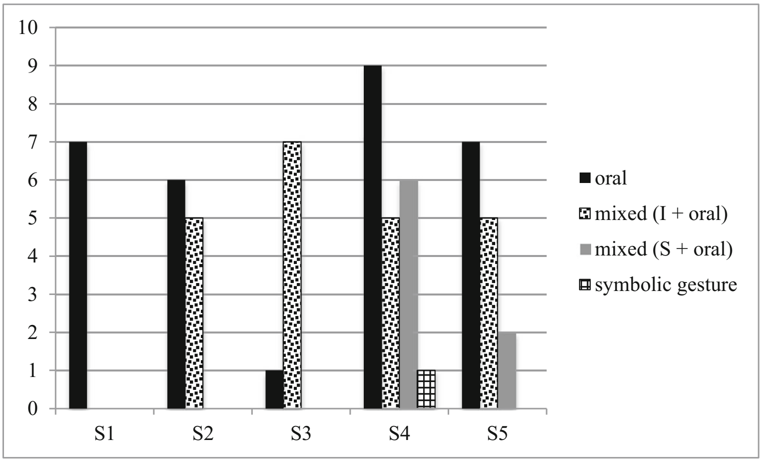

Fig. 4 Semiotic modes of expression in the quantification of dots by C by session: mixed (I + oral) = immediate pointing gesture + oral expression; mixed (S + oral) = symbolic-numeric gesture + oral expression (Objective 2.1)

Semiotic Mediators Used (Verbal and Non-verbal Communicative Mediators), by A (Objective 2.2)

During quantification, up to S3, A responded to the majority of C's errors in quantifying the dots with error acceptance, although she also used corrections, as well as metacognitive mediators, such as calls for self-regulation. In general, both A's error acceptances and corrections were followed by calls for recapitulation, wherein A tried to emphasize the cardinality, connecting the expressed numerical value (correct or not) with the advancement of the horse (see Observation 1).

Observation 1: Call for Recapitulation by a (S3, 14″)

A: "Come on; let's go for the die. Let's count the dots. [A takes C's hand to walk with her to the die.] Count them. C: one... two... three... four... and five... [Evaluating face four, C produces immediate pointing gestures to touch the dots, but repeats the gesture on one of them.] A: Hmm, let's see... Well, *how many steps* does the horse have to walk?"

From S3 on, some of A's corrections were followed by correct responses from C. In S4 and S5, when C gave more correct responses in quantification, A recurrently used metacognitive verbal mediators, which seemed to encourage C to reflect on her performance. These were cases in which A seemed to want to be sure C was aware of the quantification she had made or knew the total (cardinal) value of the collection. She usually asked "how many?" after the correct quantifications. Only from S4 on did C respond to A's calls for ratification correctly. Thus, only from 33 months of age (S4), did C, after quantifying the dots correctly, also correctly report the cardinal values of the collections (see Observation 2).

Observation 2: Information on the Cardinal Value of the Collection Reported by C after Correct Quantification (S5, 16″)

A: "Come on, get the die! [A points at the die from a distance.] How many dots are there? Count them, count the dots. Count them. Count them. C: one... two... three... and four. [Evaluating face four, C correctly makes immediate pointing gestures to each dot.] A: Good job! So, *how many squares does the horse have to advance*? *How many dots were there*? [A picks up the die and performs an ostensive gesture.] C: Four! A: Great!"

It was only from S4 on that C quantified the dots correctly, reported the cardinal value, and advanced the horse correctly, according to the reported value.

The main gestures used by A in the quantification of the dots were pointing gestures (distant, immediate, and multiple; total = 33), especially immediate ones to indicate the dots that C should count. A was particularly active in trying to prompt C to count (e.g., How many dots do we have? [A says, touching a dot.] How many dots are there? [A points to each dot.] (S2, 4″).

A performed pointing gestures mostly in S2, whereas, from S3 on, the use of these gestures tended to decline. This is probably because, beginning in S2, C generally performed pointing gestures to count the dots. Recall that C performed pointing gestures from S2 on. In S1 she did not use immediate pointing gestures. Attention should also be drawn to the symbolic gestures (total = seven) used by A from S1 on, but mainly in S4 and S5. These gestures were used, above all, to put "food" on the horse's plate, but from S4 on, they were used to show values by holding up fingers (once in S4; twice in S5). C's symbolic-numerical gestures (total = nine) took place in S4 and S5. A performed ostensive gestures (total = eight), as well as placing gestures (total = nine), when offering or moving the die closer to C (see Fig. 5).

A also performed distant demonstrations (DDs), in S1 and S2, when quantifying the dots. The inhibitions of non-conventional uses (INCUs), when A corrected or prevented non-conventional or protoconventional uses by C, had disappeared by S4.

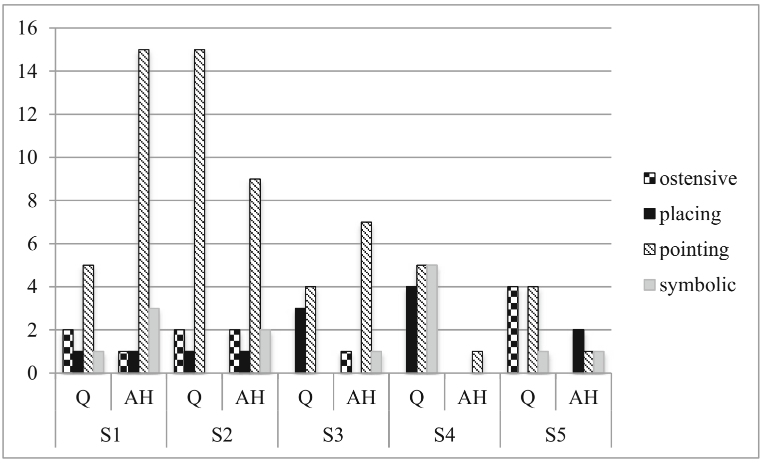

Fig. 5 Gestures performed by A during quantification of the dots (Q) and advancement of the horse (AH) by session (Objective 2.2)

In S4 and S5, A performed exclusively verbal corrections and preventions on a few occasions (see Fig. 6).

During the advancement of the horse, the cognitive assistance was more diverse than during quantification of the dots. In S1 and S2, A frequently performed complementary uses. Specifically, A would accompany the first steps taken by C to advance the horse with numerical words, saying them rhythmically and slowly. It should be noted that from S3 on, A's instructions were usually accompanied by requests for C to advance the horse, which she herself accompanied with numerical words, in a way that encouraged C to self-regulate, controlling the number of steps taken herself. It is worth emphasizing that during the advancement of the horse, A frequently resorted to a symbolic scenario in her instruction: "the horse has to go forward to reach its food because it is hungry."

The main gestures used by A during the advancement of the horse were pointing gestures directed at the horse and to emphasize the squares on the path (immediate, multiple, and distant; total = 33). In S1, these gestures were more frequent, and they had practically disappeared by S4, when C advanced correctly, respecting the segments of the path (see Fig. 6).

A's demonstrations of the advancement of the horse were distant (DD), when A advanced the horse by herself, and immediate (ID), when A guided C's hand to advance the horse. IDs took place until S2. It should be noted that the IDs performed by A occurred only during advancement of the horse in S1, i.e., in the part of the game that seemed to involve greater difficulty for C. No IDs were found during quantification.

Also, inhibitions of non-conventional uses (INCUs) (total = 16) occurred until S3 (during quantification). They occurred along with the corrections and preventions to avoid the advancement of the horse beyond the number indicated by the dots, as well as to prevent C from taking objects improperly. From S4 on, the few corrections made were verbal: A no longer took C's hands to prevent advancement of the horse (see Fig. 6).

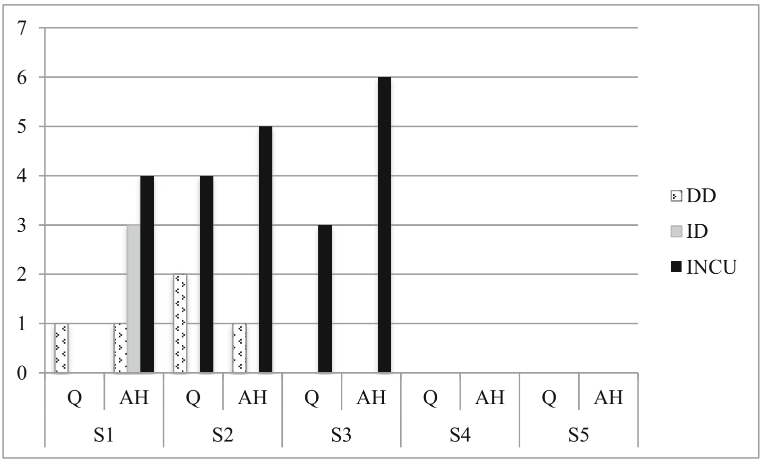

Fig. 6 Uses of objects by A during quantification (Q) and advancement of the horse (AH): DD = distant demonstration; ID = immediate demonstration; INCU = inhibition of non-conventional uses (Objective 2.2)

Discussion

This study highlights the multimodal nature of the communication established by the dyad (oral language, gestures, and uses of objects) as being necessary for constructing an early understanding of cardinality in a pragmatic context. Our results show that A used various semiotic systems (gestures, uses of objects, language) to help C in understanding and using cardinal information. C also used various semiotic resources during the interaction.

In general, our research showed that the development of cardinal understanding is related to factors such as the age of the girl, the magnitude of the collections, and the cognitive demands of the situation, which, above all, depends on the role of the adult helping the girl with *what to count*, *why*, and *what portion of reality* has to be segmented in order to be counted.

In general, our studies also confirmed that, although children need to master and understand a series of principles for counting (Gelman and Gallistel 1978), they need to have a very clear understanding of *why* they are counting, what they are going to do with the information gained (Briars and Siegler 1984; Fuson 1988). We observed that C's first attempts at quantification were *oral.* Faced with the question "how many...?," C could "think" of numerical words, differentiating them from other words in her vocabulary. In the first session, C gave number-word responses in a non-reflective pattern, i.e., she simply answered the "how many" question with whatever number word came to mind. The nature of some of the responses given in this case provides additional support for the notion that this number word does not have a cardinal reference. According to our study, C initially gave number-word responses with no cardinal reference and later came to understand the cardinality reference of the last counted word.

In this line, our results agree with those reported by Hall (1988), Bermejo (1996), Bermejo and Lago (1990), regarding children first learning only last-word responses with little reference to cardinality. According to those authors, for 2-year-olds, a "how many" question functions predominantly as a directive to say a number word (though not the last counted word). The "how many" question begins to function as a directive to count for 3-year-olds and then reverts back for 4-year-olds producing an even split between the two responses. This age pattern suggests that it is important for children to learn both responses to the "how many" question. Despite our results regarding quantification processes (before 33 months of age, C could correctly quantify the dots, but could not use this information to correctly move the horse), our study led us to conclude that understanding of cardinality can only be assumed when the child can *use* the cardinal number correctly in pragmatic terms, not only in quantification activities. This need for additional understanding clearly indicates that the count-to-cardinal transition is important, but still too primitive to assume a child understands cardinal-reference.

This study offered insight into details of these processes between the ages of 2 and 3 years, in the presence of adult assistance. The adult usually separated counting from cardinal value: the mother would encourage the girl to count the dots and tell her *how many dots* there were. Further, if C made a cardinal mistake, A would ask her to certify her answer with a new counting process. This action may have led the girl to consider using counting processes as a way to obtain cardinal values on some occasions. Also, in some instances, in the first sessions, after C counted, A informed her of the cardinal value through recapitulation of the last word counted by the girl. This underscores the influence of the adult assistance in the child's understanding of the last word counted and early understanding of cardinality. Thus, at first, accurate counting seemed to be independent of the last-word relationship and cardinality; this relationship required the assistance of the adult.

Overall, adult assistance was diverse: it occurred in both verbal and nonverbal forms, it was more or less direct, and it seemed to influence both the understanding of the units and the regulation of the advancement along the path. The adult gave numerous and diverse types of verbal assistance (Van de Pol et al. 2010): *affective*, by giving C emotional support, using various strategies to keep her interested in the game and its requirements (e.g., accepting her quantification mistakes); *cognitive*, by encouraging organized and justified explanations of the structure of the activity; and *metacognitive*, by giving C opportunities to reflect on her own performance. All this was done in a way that seemed to gradually adjust to the girl's actions over the course of the sessions. The adult's efforts to transfer control, her adjustment to the symbolic nature of the situation, and the girl's increasing self-regulation were also very obvious.

Especially during the advancement of the horse, C needed to show her ability to transfer the cardinal information about the dots to another referent situation. We are aware of the particular cognitive demand of our task and materials: on the one hand, in order to use the information and advance the horse properly, C had to previously quantify the number of dots accurately, which is complex mainly with collections of more than three (Carey 2009; Ceulemans et al. 2012; Huang et al. 2010; Wynn 1990, 1992). On the other hand, while the transference is more direct and perceptive when it comes to spatial information (e.g., position of objects in a photograph and real space; see: DeLoache 1991, 1995; DeLoache and Burns 1994; Salsa 2013), in our case it is a more complex transference including two kinds of objects: the dots (of a discrete nature) and the squares of the path.

It should likewise be noted that advancing the horse according to the information on the die requires regulating the horse along the entire path. It is interesting to note that, even though the layout of the path (linear, non-circular, and with added ovals) could be conducive to the treatment of the segments as units (Siegler and Ramani 2008), in the present study this fact did not seem to contribute significantly to facilitating the advancement: C showed difficulty in properly segmenting the squares, which could partly explain her difficulties.

Although the girl's difficulty in segmenting the squares of the path could be related to the understanding of numbers themselves (i.e., the consideration that numbers imply discreet units), we believe this difficulty also occurred because the understanding of the symbolic scenario presented in the game (the horse and its food) was very important; it guided the actions of the girl and was imperative in the understanding of the rules of the game and of the cardinal use of the information.

Recall that C, especially up to S3, took the horse directly to the "food" to accomplish the goal of the game (thus showing a lack of inhibition). Giving food to the horse seemed to be the central objective and indicated that this was a very familiar activity for the girl based on a well-established understanding of the symbolic uses of objects. These uses of the horse underscored that, in reality, the situation we proposed was only possible because C already had some degree of conventional-symbolic understanding of the proposed objects. It was evidenced that this understanding allowed her to access the symbolic scenario in which the object/replica horse represented the animal horse that "jumped"

(Palacios and Rodríguez 2014) along a "path," because it was "hungry" and had to get to the "food." Adult assistance was essential in this regard, as A frequently situated the game as a symbolic scenario where the horse was always "starving" and eager to reach its "food."

So, throughout the study, one could say that C first understood the purpose of the game to be "feeding the horse" (to which end, she insisted on continuously advancing the horse, to take it to the "food," up to S3). She then gradually came to understand the rules of the game until, finally, she was able to regulate the horse's advancement through a cardinal reference. In general, the greatest difficulty in advancing the horse did not seem to be due solely to the difficulty of self-regulation linked to cardinality, but also to the understanding of the rules of the game. The introduction of the rules (or the conditions for the horse to eat: i.e., that the dots had to be quantified before giving food to the horse) was clearly the first difficulty for the girl. The difficulty of regulation with the cardinal information appeared later.

According to our findings, we argue, first, that when C started counting, she used gestures to segment the material world into units. We can thus further argue that pointing gestures are a previous semiotic system that is important to the development of cardinal understanding. In fact, the performance of the segmentation of the dot units underlines the importance the dyad gave to segmentation for counting and the comprehension of a genuine concept of a number (Fuson 1988; Rodríguez and Scheuer 2015). Second, subitizing, or the sudden apprehension of quantities (Benoit et al. 2004; Dehaene and Cohen 1994), occurred subsequent to the evaluation of the quantities by counting, at least in the investigated situation and age. The fact that C was finally able to mentally and suddenly represent quantities (according to our study, from 33 months of age) was based on her previous performance of counting actions using immediate pointing gestures. Our findings allow us to infer that subitizing is the sudden realization of the cardinal values of collections, something that is slowly built (according to our findings, at least six months after quantification through gestures and depending on the magnitude of the collection). Third, we agree that counting and determining how many elements there are in a set is not sufficient to assure that a child understands the cardinality of the collection. Understanding of cardinality can only be assumed when the child can *use* the cardinal number correctly, in pragmatic terms. Finally, fourth, in the processes of quantifying and using cardinal information, interaction with an adult plays a fundamental role.

Conclusions

Based on our research, we can say that the development of cardinal understanding in the proposed situation happens slowly and gradually and requires an integrated body of resources. We believe that our results show the development of cardinal understanding, through the "braiding" of previous semiotic systems with more recent ones, so that the first numerical understanding links to previous semiotic systems shared with the adult. It was not difficult for the girl to understand the symbolic scenario in which the horse was placed. The same applies to the degree of understanding and use of different types of gestures, from the ostensive to the symbolic, through the indexical. We feel that these systems, insofar as they support each other, fostered the girl's understanding of the cardinal reference of the quantities. Nevertheless, we would like to address some limitations of the study and point to future lines of research.

We are aware of the difficulty of determining the effectiveness of the investigated assistance. Despite our findings, we cannot say, in a causal manner, that the assistance given by the adult produced the understanding of the cardinal value nor that the girl succeeded in the game because of it. The number of case studies should be expanded to further analyze the trends observed and deepen the analysis to accurately identify patterns of actions. In addition, it is important to note the time limits of each session. If the session had lasted longer, the interaction would have continued and we would have more information about the interaction established.

Although our research did not have as an objective to propose methods of intervention, our data show the fundamental role of early educational involvement in cardinality development. We would like to emphasize what should be the starting point in interventions in this regard. We consider that the understanding of how children develop handling numerical situations, and how semiotic mediators may support it, is the best communicative and educational opportunity to adjust the early semiotic systems to make them conducive to developing mathematical understanding. We think that considering this idea and studying in depth the development process allows us to intervene intentionally. In our view, in highlighting the role of social interactions in the development of cardinal understanding, this research raises the fundamental role that early education should play in the development of this understanding. Although this issue is a broadly researched topic, it seems that the difficulties related to the teaching of its concepts are still a current issue, especially given the difficulty children have, throughout their school life, learning mathematics.

Also, despite the extensive material available on cardinal understanding in children, there is little research on how children *develop* this understanding. We believe that much remains to be explored about the role of the adult in sharing conventional meanings and rules with the child through semiotic systems used by both, which could contribute to the development of cardinal number value. In addition, more research with ecological validity in significant situations for children is needed.

Finally, we would like to emphasize the importance of conducting longitudinal studies and microgenetic analysis to reveal the social, communicative and material-cultural niche of infancy, in times that the mainstream psychology of early development seems to misrepresent such fundamental elements (Ratcliff et al. 2018).

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Compliance with Ethical Standards

Conflict of Interest All authors Sílvia Cavalcante, Cintia Rodríguez and Eduardo Martí declare that they have no conflict of interest.

Ethical Approval All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards.

Informed Consent Informed consent was obtained from all individual participants included in the study.

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