



Quantized control for nonhomogeneous Markovian jump T-S fuzzy systems with missing measurements

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Abstract

In this paper, in terms of the T-S fuzzy technique, the quantization control designs are resolved for a class of nonhomogeneous Markov jump systems (MJSs) with partially unknown transition probabilities. Different from the previous research, the transition probabilities are time-variant and not known exactly in the MJSs. Particularly in a network environment, it is considered that the effects of data packet dropouts and the occurrence of signal quantization simultaneously emerge in the closed-loop circuit. Furthermore, based on a fuzzy Lyapunov function and a set of linear matrix inequalities, one can achieve the desired H_∞ performance and the sufficient conditions such that the corresponding closed-loop system is stochastically stable. By the cone complementarity linearisation (CCL) procedure, a sequential minimization problem is tackled efficiently to gain the solutions of the dynamic output feedback controller (DOFC). Finally, the validity of the suggested technique is showed via a simulation example.

Keywords Nonhomogeneous Markovian jump systems · T-S fuzzy · Quantization control

1 Introduction

In the past few decades, MJSs are a kind of specific stochastic dynamic systems, which has a wide range of employments in networked control systems, aerospace, power, and manufacturing. Many realistic complicated systems [1] may suffer unpredictable abrupt changes in parameters and structures, which is frequently caused by maintenances or failures of the components, environmental disturbances and so on. MJSs have been introduced as powerful and appropriate tool to describe such complex situations. The network control systems [2, 3] are typical examples which would be modeled via MJSs [4], and network delays and packet dropouts can be supposed by Markov processes. Note that in different periods packet

losses and delays [5–7] are not in accord. For some significant results on this subject, we can consult the reports in [8–11] and the references therein. However, it is generally true that the existence of exponential distribution of the jump time brings about many restrictions on application in MJSs. And as a result of constant transition rates, the conclusions achieved from the MJSs are conservative in nature. In the entire operation region the transition probabilities are time-varying. Different from the MJSs, a time variant matrix of transition probabilities is the greatest feature of nonhomogeneous MJSs (NMJSs). The mentioned works can be approximately split into nonlinear MJSs and linear ones [12]. Clearly, without loss of generality, it is well identified that the nonlinear MJSs normally have higher usability.

On the other research front, numerous practical models and systems contain complex uncertainties and nonlinearities, which takes the control design and the analysis of systems into straitened circumstances [13]. Based on the stochastic set stabilization, both the strategy consensus and the control of output tracking are presented [14]. The set stability of equivalent stochastic system with probabilities time delays is investigated in terms of the matrix of state transition probabilities [15]. Due to appearance of T-S fuzzy technique, the effective method has been employed to describe complicated nonlinear systems in accordance with

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a set of “IF-THEN” rules. Consequently, based on T-S fuzzy systems, some representative results of filtering [16], control issues [17], stabilization and stability are gained, for example, [18–25] and the references therein. Very recently, the researchers had paid close attention to fuzzy MJSs. By some presented slack matrices, they separate Lyapunov function matrices from system variables such that the design of controller and the analysis of stability are addressed in [26]. However, in a lot of the gained results about MJSs, assuming that transition probabilities are exactly known and time invariant, a homogeneous Markov chain or Markov process comes into being. These assumptions are invalid in some actual circumstances. It is considered that a polytope set is employed to represent the feature of time-varying transition rate in such situations. We can assess the numerical values in some operating point in spite of the matrices of transition probabilities are not fully known. The polytope is a convex set such that it is described to handle the nonhomogeneous fuzzy MJSs (NFMJSs) accompanied by time-varying transition probabilities. Therefore, in [27], a few of novel plan methods are farther enhanced for NFMJSs.

In addition, in the field of control systems, the issue on saturating quantization measurements has been proved to be a hot theme in recent years. Under the circumstances of network, real-valued signals of the controller and ones of the model are always projected into piecewise-constant signals before transmission in the closed-loop circuit. In some literatures, the results have been obtained toward this direction, for example [19]. In [28], in the special systems the state-feedback controller was proposed. However, in realistic example the state-feedback possesses the critical shortcoming that the states of system are always hard to achieve or can not be obtained on account of method, final cost, etc [29–33]. What’s more, the measurements of sensor and the instructions output of controller require new quantized techniques when the signals are sent via networks. Therefore, inspired by the aforementioned work, we want to construct a DOFC for NFMJSs with unmeasurement states.

Motivated by the above discussion, in this paper, we investigate the quantized control design for a class of NMJSs with partly unknown and time variant transition probabilities via T-S fuzzy technique. The main contributions of this paper are as below: i) In the uplink and downlink, the effects of data packet dropouts and signal quantization are considered simultaneously. In terms of the T-S fuzzy method and the approach of parameter dependent Lyapunov function, we have achieved a sufficient conditions which equip the NMJSs with required performance as well as stochastic stability. ii) In our work, a key characteristic is that we employ the conception of nonhomogeneous Markov process in which transition probabilities are time-variant. iii) Along with the development of researches, the

desired DOFC has been demonstrated. Meanwhile, the CCL procedure is applied to the DOFC solutions.

The remainder of this article is listed as follows: The system description and preliminaries are expressed in Section 2. The main results of the studied problem for NFMJSs are derivated under data missing and time variant transition probabilities in Section 3. Section 4 presents a numerical example and we formulate the conclusion of this paper in Section 5.

Notation In this paper, suppose a complete probability space (Ω, F, Pr) in which Pr , F and Ω denotes the probability measure defined over F , σ - algebra and the sample space, respectively. The symmetry term is denoted by symbol $(*)$. The matrix transposition is signified by superscript “ T ”. The square integrable space on $[0, \infty)$ is indicated by $l_2[0, \infty)$. The expectation of α is represented by $E\{\alpha\}$. The expectation of α conditional on β is indicated by $E\{\alpha/\beta\}$. The notation $P > 0$ (≥ 0) means that under real symmetric structure it is positive definite (semi-definite). $\|M\|$ refers to the norm of a matrix. $|\cdot|$ refers to the Euclidean norm of a vector and the norm of conventional $l_2[0, \infty)$ is defined by $\|\cdot\|_2$. If the dimensions of matrices are not clearly regulated, the compatible dimensions are assumed.

2 Problem formulations

In this section, the T-S fuzzy model is considered. It is a non-linear discrete-time system on probability space (Ω, F, Pr) , which may be denoted via the fuzzy model.

2.1 T-S fuzzy model

The i -th rule of T-S fuzzy MJSs (FMJSs):

Rule i : If $\vartheta_1(t)$ is $\bar{\lambda}_{i1}$ and \dots and $\vartheta_\ell(t)$ is $\bar{\lambda}_{i\ell}$ then

$$\begin{cases} x(t+1) = A_i(r_t)x(t) + B_i(r_t)u(t) + E_i(r_t)\varpi(t) \\ z(t) = C_i(r_t)x(t) + D_i(r_t)u(t) + F_i(r_t)\varpi(t) \\ y(t) = G_i(r_t)x(t) \quad i \in \bar{N}, \end{cases} \quad (1)$$

where $x(t) \in R^{n_x \times 1}$ is the state vector; $u(t) \in R^{n_u \times 1}$ is the control input vector; $y(t) \in R^{n_y \times 1}$ is the measured output vector; $z(t) \in R^{n_z \times 1}$ is the controlled output vector; $\varpi(t) \in l_2[0, \infty)$ are external disturbances and $\varpi(t) \in R^{n_\varpi \times 1}$. $A_i(r_t)$, $B_i(r_t)$, $C_i(r_t)$, $D_i(r_t)$, $E_i(r_t)$, $F_i(r_t)$, $G_i(r_t)$ are constant matrices with appropriate dimensions; $\bar{\lambda}_{ij}$ ($j = 1, 2, \dots, \ell$) represents the membership grade of $\vartheta_\ell(t)$; $[\vartheta_1(t), \dots, \vartheta_\ell(t)]^T$ are known premise variables. $\bar{N} = (1, 2, \dots, \lambda)$, λ is the number of rules.

The final FMJSs system is listed as follows:

$$\begin{cases} x(t+1) = \sum_{i=1}^{\lambda} h_i [A_i(r_t)x(t) + B_i(r_t)u(t) + E_i(r_t)\varpi(t)] \\ z(t) = \sum_{i=1}^{\lambda} h_i [C_i(r_t)x(t) + D_i(r_t)u(t) + F_i(r_t)\varpi(t)] \\ y(t) = \sum_{i=1}^{\lambda} h_i [G_i(r_t)x(t)], \end{cases} \tag{2}$$

where we consider for all t : $\epsilon_i(\vartheta(t)) = \prod_{j=1}^{\ell} \bar{\lambda}_{ij}(\vartheta_j(t))$,

($i = 1, \dots, \lambda$) is the membership function of the model with the i -th rule. In this paper, we assume $\epsilon_i(\vartheta(t)) \geq 0$, $\sum_{i=1}^{\lambda} \epsilon_i(\vartheta(t)) > 0$, $h_i(\vartheta(t)) = \frac{\epsilon_i(\vartheta(t))}{\sum_{i=1}^{\lambda} \epsilon_i(\vartheta(t))}$, then

$h_i(\vartheta(t)) \geq 0$, $\sum_{i=1}^{\lambda} h_i(\vartheta(t)) = 1$. In what follows, we write $h_i(\vartheta(t))$ by h_i for brevity.

The Markov chain is represented by $\{r_t, t \geq 0\}$ which takes values in the space $\mathfrak{S} = \{1, 2, \dots, \omega\}$. The matrix of transition probability is $\Lambda(t) = \{\pi_{mn}(t)\}$, $m, n \in \mathfrak{S}$. From mode m at time t to mode n at time $t + 1$, the transition probability is denoted by $\pi_{mn}(t) = Pr(r_{t+1} = n | r_t = m)$, and $\pi_{mn}(t) \geq 0, \forall m, n \in \mathfrak{S}, \sum_{n=1}^{\omega} \pi_{mn}(t) = 1$. In the system (2), the time-variant matrix of transition probability $\Lambda(t) = \{\pi_{mn}(t)\}$ is proposed as a polytope P_{Λ} . $P_{\Lambda} = \{\Lambda(t) = \sum_{\tau=1}^{\kappa} \zeta_{\tau}(t)\Lambda^{(\tau)}(t), \sum_{\tau=1}^{\kappa} \zeta_{\tau}(t) = 1, 0 \leq \zeta_{\tau}(t) \leq 1\}$, where the vertices of P_{Λ} are denoted by $\Lambda^{(\tau)}(t), \tau = 1, 2, \dots, \kappa$, and κ is the number of the chosen vertices. $\Lambda^{(\tau)}(t)$ includes some elements which are partially unknown or uncertain, namely, the matrices of transition probability possesses

incomplete transition characterization. We consider $\mathfrak{S} = \mathfrak{S}_k^m + \mathfrak{S}_{uk}^m, \forall m \in \mathfrak{S}$, where

$$\begin{aligned} \mathfrak{S}_k^m &= (n : \pi_{mn} \text{ is known}) \\ \mathfrak{S}_{uk}^m &= (n : \pi_{mn} \text{ is unknown}), \forall m \in \mathfrak{S}. \end{aligned} \tag{3}$$

Also, we define $\mathfrak{S}_k^m = (\varphi_1^m, \dots, \varphi_v^m), \forall 1 \leq v \leq \omega, \forall m \in \mathfrak{S}$. Where φ_v^m is the v -th known element in the m -th row of matrix $\Lambda(t)$.

2.2 Output feedback controller

In this paper, in terms on the T-S fuzzy model (2), the DOFC is constructed:

Rule i : If $\vartheta_1(t)$ is $\bar{\lambda}_{i1}$ and \dots and $\vartheta_{\ell}(t)$ is $\bar{\lambda}_{i\ell}$ then

$$\begin{cases} \eta_c(t+1) = A_i^c(r_t)\eta_c(t) + B_i^c(r_t)y^c(t) \\ u^c(t) = C_i^c(r_t)\eta_c(t), \end{cases} \tag{4}$$

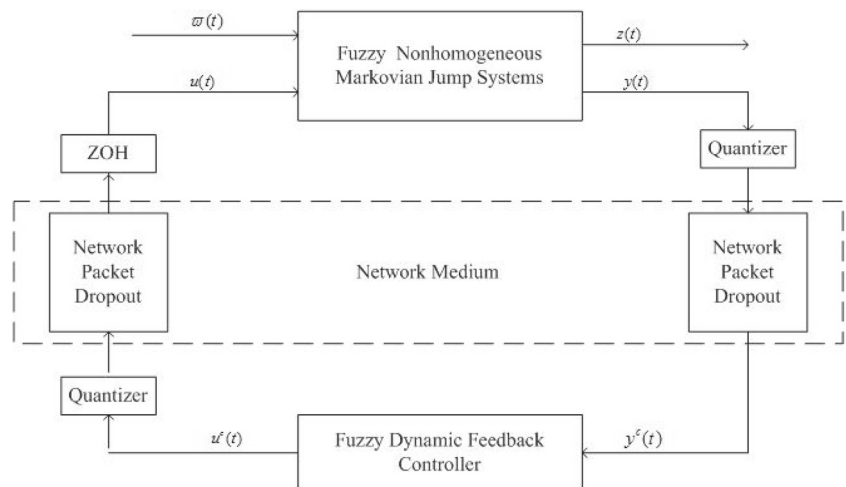
where $\eta_c(t) \in R^{n_{\eta} \times 1}, u^c(t) \in R^{n_u \times 1}$ and $y^c(t) \in R^{n_y \times 1}$ respectively denote the state of the controller, the output of the controller, the input of the controller. $A_i^c(r_t), B_i^c(r_t), C_i^c(r_t)$ are the gains to be determined matrices with appropriate dimensions. Then the DOFC is described as follows:

$$\begin{cases} \eta_c(t+1) = \sum_{i=1}^{\lambda} h_i [A_i^c(r_t)\eta_c(t) + B_i^c(r_t)y^c(t)] \\ u^c(t) = \sum_{i=1}^{\lambda} h_i C_i^c(r_t)\eta_c(t). \end{cases} \tag{5}$$

2.3 Quantization and unreliable communication links

From Fig. 1, we note that the NFMJSs are quantized in the environment of network with the unreliable links. Before the signal is conveyed in the digital channel, the output measurements $y(t)$ and the output $u^c(t)$ of the controller are quantized respectively in the network. The system (2) is

Fig. 1 Plant flow chart



subordinate to logarithmic quantizer $q_m(\cdot) = q_{r_t}(\cdot)$ which is characterized by

$$q_m(\cdot) = \left[q_m^{(1)}(\cdot) q_m^{(2)}(\cdot) \dots q_m^{(n_y)}(\cdot) \right]^T, m \in \mathfrak{S}, \tag{6}$$

where $q_m^{(n)}(\cdot)$ is assumed to be symmetric. $q_m^{(n)}(y_n(t)) = -q_m^{(n)}(-y_n(t)), n = 1, \dots, n_y$. For $m \in \mathfrak{S}$, the set of quantification levels of $q_m^{(n)}(\cdot)$ is represented by $\Upsilon_n = \left\{ \pm \eta_{\bar{h}}^{(m,n)} \mid \eta_{\bar{h}}^{(m,n)} = (\rho^{(m,n)})^{\bar{h}} \cdot \eta_{(0)}^{(m,n)}, \bar{h} = \pm 1, \pm 2, \dots \right\} \cup \left\{ \eta_{(0)}^{(m,n)} \right\} \cup \{0\}, 0 < \rho^{(m,n)} < 1, \left\{ \eta_{(0)}^{(m,n)} \right\} > 0$, where $\rho^{(m,n)}$ denotes the quantizer density of the subquantizer $q_m^{(n)}(\cdot)$ and $\eta_{(0)}^{(m,n)}$ are the initial values for subquantizer $q_m^{(n)}(\cdot)$. The quantizer $q_m^{(n)}(\cdot)$ is represented as follows:

$$q_m^{(n)}(y_n(t)) = \begin{cases} \eta_{\bar{h}}^{(m,n)}, & \text{if } \frac{\eta_{\bar{h}}^{(m,n)}}{1+\delta^{(m,n)}} < y_n(t) < \frac{\eta_{\bar{h}}^{(m,n)}}{1-\delta^{(m,n)}} \\ 0, & \text{if } y_n(t) = 0 \\ -q_m^{(n)}(-y_n(t)), & \text{if } y_n(t) < 0, \end{cases} \tag{7}$$

where $\delta^{(m,n)} = (1 - \rho^{(m,n)}) / (1 + \rho^{(m,n)})$ are the parameters of the quantizer. Based on [34], the logarithmic quantizer (7) may be described by

$$q_m(y(t)) = (I_{n_y} + \Delta_{(m,n_y)})y(t), \tag{8}$$

where $\Delta_{(m,n_y)} = \text{diag} \{ \delta^{(m,1)}, \dots, \delta^{(m,n_y)} \}, 0 < \Delta_{(m,n_y)} < I_{n_y}$. In the same way,

$$q_m(u^c(t)) = (I_{n_u} + \Delta_{(m,n_u)})u^c(t). \tag{9}$$

From Fig. 1, in the closed-loop circuit, it can be seen that data missing randomly occurs in the network. Thus $q_m(y(t)) \neq y^c(t), q_m(u^c(t)) \neq u(t)$. In light of the application of the stochastic technique, the aforementioned phenomenon is denoted as follows

$$\begin{cases} y^c(t) = \alpha(t)q_m(y(t)) = \alpha(t)(I_{n_y} + \Delta_{(m,n_y)})y(t) \\ u(t) = \beta(t)q_m(u^c(t)) = \beta(t)(I_{n_u} + \Delta_{(m,n_u)})u^c(t), \end{cases} \tag{10}$$

where $\alpha(t)$ and $\beta(t)$ fulfill Bernoulli random distribution. The $\alpha(t)$ is applied to denote the data dropout of the downlink and the $\beta(t)$ is applied to denote the data dropout of the uplink. Consider $\alpha(t), \beta(t)$ as following $Pr\{\alpha(t) = 1\} = E\{\alpha(t)\} = \bar{\alpha}, Pr\{\alpha(t) = 0\} = 1 - \bar{\alpha}, Pr\{\beta(t) = 1\} = E\{\beta(t)\} = \bar{\beta}, Pr\{\beta(t) = 0\} = 1 - \bar{\beta}$, where $\bar{\alpha} \in [0, 1]$ and $\bar{\beta} \in [0, 1]$ are constants. Suppose $\alpha(t) = \bar{\alpha} + \tilde{\alpha}(t), \beta(t) = \bar{\beta} + \tilde{\beta}(t)$, then $E\{\tilde{\alpha}(t)\tilde{\alpha}(t)\} = \bar{\alpha}(1 - \bar{\alpha}), E\{\tilde{\beta}(t)\tilde{\beta}(t)\} = \bar{\beta}(1 - \bar{\beta})$. According to the Eq. 10, one can obtain

$$\begin{cases} \eta_c(t+1) = \sum_{i=1}^{\lambda} h_i [A_i^c \eta_c(t) \\ + \alpha(t) B_i^c (I_{n_y} + \Delta_{(m,n_y)}) y(t)] \\ u^c(t) = \sum_{i=1}^{\lambda} h_i C_i^c \eta_c(t). \end{cases} \tag{11}$$

Combining Eqs. 2–11, the closed loop system is obtain as follows

$$\begin{cases} \bar{\xi}(t+1) = \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} h_i h_j [A_{ij}(r_t) \bar{\xi}(t) + \Xi_i(r_t) \varpi(t)] \\ z(t) = \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} h_i h_j [C_{ij}(r_t) \bar{\xi}(t) + F_i(r_t) \varpi(t)], \end{cases} \tag{12}$$

$$\text{where } A_{ij}(r_t) = \begin{bmatrix} A_i(r_t) \\ \alpha(t) B_j^c(r_t) (I_{n_y} + \Delta_{(m,n_y)}) G_i(r_t) \\ \beta(t) B_i(r_t) (I_{n_u} + \Delta_{(m,n_u)}) C_j^c(r_t) \\ A_j^c(r_t) \end{bmatrix},$$

$$C_{ij}(r_t) = \begin{bmatrix} C_i(r_t) & \beta(t) D_i(r_t) (I_{n_u} + \Delta_{(m,n_u)}) C_j^c(r_t) \end{bmatrix},$$

$$\bar{\xi}(t) = \begin{bmatrix} x(t) \\ \eta_c(t) \end{bmatrix}, \Xi_i(r_t) = \begin{bmatrix} E_i(r_t) \\ 0 \end{bmatrix}.$$

2.4 Definition and Lemma

Definition 1 [35] The closed-loop system (12) with $\varpi(t) \equiv 0$ is considered to be stochastically stable, for any initial condition $\bar{\xi}(0) \in R^n$ and $r_0 \in \mathfrak{S}$, if there exists a matrix $W > 0$ such that the following condition holds

$$E \left\{ \sum_{t=0}^{\infty} |\bar{\xi}(t)|^2 \mid_{(\bar{\xi}(0), r_0)} \right\} < \bar{\xi}^T(0) W \bar{\xi}(0).$$

Definition 2 [36] For a given constant $\gamma > 0$, under zero initial condition the system (12) with an H_{∞} performance γ is considered to be stochastically stable, if under $\varpi(t) \equiv 0$ it is stochastically stable, then for all nonzero $\varpi(t) \in l_2[0, \infty)$ the following condition holds

$$E \left\{ \sum_{t=0}^{\infty} |z(t)|^2 \right\} \leq \gamma^2 \|\varpi\|_2^2.$$

Lemma 1 [34] Suppose that M, N and T are real matrices with appropriate dimensions and $T^T T \leq I$, then for any scalar $\varepsilon > 0$, one can have

$$MTN + N^T T^T M^T \leq \varepsilon^{-1} M M^T + \varepsilon N^T N. \tag{13}$$

Lemma 2 [37] If the following conditions are founded

$$M_{ii} < 0, i = 1, 2, \dots, \lambda. \tag{14}$$

$$\frac{1}{\lambda - 1} M_{ii} + \frac{1}{2} (M_{il} + M_{li}) < 0, i \neq l, i, l = 1, 2, \dots, \lambda. \tag{15}$$

Then we have the following inequality

$$\sum_{i=1}^{\lambda} \sum_{l=1}^{\lambda} h_i h_l M_{il} < 0. \tag{16}$$

3 Main results

Theorem 1 For a supposed disturbance attenuation le-vel $\gamma > 0$, the closed-loop system (12) is stochastically stable and the controller gains are solvable if there exist positive definite matrices $P_l(m)$, $m \in \mathfrak{S}$, ($l = 1, \dots, \lambda$) fulfilling

$$(A_{ij}^1)^T \tilde{P}_l(n) A_{ij}^1 + (A_{ij}^2)^T \tilde{P}_l(n) A_{ij}^2 - P_i(m) < 0, \quad (17)$$

$$\Gamma_{ijl\tau}^1(m) = \begin{bmatrix} -\varphi_l^{-1} & 0 & \mathfrak{N}_v^m \hat{A}_{ij} \\ 0 & -I & \sqrt{\prod_v^m} \hat{C}_{ij} \\ * & * & -\prod_v^m Q_i \end{bmatrix} < 0, n \in \mathfrak{S}_k, \quad (18)$$

$$\Gamma_{ijl\tau}^2(m) = \begin{bmatrix} -\hat{P}_l^{-1}(n) & 0 & \hat{A}_{ij} \\ 0 & -I & \hat{C}_{ij} \\ * & * & -Q_i \end{bmatrix} < 0, n \in \mathfrak{S}_{uk}, \quad (19)$$

where

$$-\varphi_l^{-1} = \text{diag} \{ -\hat{P}_l^{-1}(\varphi_1^m) \dots -\hat{P}_l^{-1}(\varphi_v^m) \},$$

$$\mathfrak{N}_k^m = [\sqrt{\pi_{m\varphi_1^m}(\tau)} I \dots \sqrt{\pi_{m\varphi_v^m}(\tau)} I]^T,$$

$$\tilde{P}_l(n) = \sum_{n=1}^{\omega} \pi_{mn}(\tau) P_l(n), \prod_v^m = \sum_{n \in \mathfrak{S}_k^m} \pi_{mn}(\tau),$$

$$\hat{C}_{ij} = \begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix}, \hat{A}_{ij} = \begin{bmatrix} A_{ij}^1 & \Xi_i \\ A_{ij}^2 & 0 \end{bmatrix},$$

$$\hat{P}_l(n) = \begin{bmatrix} P_l(n) & 0 \\ 0 & P_l(n) \end{bmatrix}, Q_i = \begin{bmatrix} P_i(m) & 0 \\ 0 & \gamma^2 I \end{bmatrix},$$

$$A_{ij}^1 = \begin{bmatrix} A_i(m) \\ \bar{\alpha} B_j^c(m)(I_{n_y} + \Delta_{(m,n_y)}) G_i(m) \\ \bar{\beta} B_i(m)(I_{n_u} + \Delta_{(m,n_u)}) C_j^c(m) \\ A_j^c(m) \end{bmatrix},$$

$$A_{ij}^2 = \begin{bmatrix} 0 \\ \sqrt{\bar{\alpha}(1-\bar{\alpha})} B_j^c(m)(I_{n_y} + \Delta_{(m,n_y)}) G_i(m) \\ \sqrt{\bar{\beta}(1-\bar{\beta})} B_i(m)(I_{n_u} + \Delta_{(m,n_u)}) C_j^c(m) \\ 0 \end{bmatrix},$$

$$C_{ij}^1 = [C_i(m) \ \bar{\beta} D_i(m)(I_{n_u} + \Delta_{(m,n_u)}) C_j^c(m)],$$

$$C_{ij}^2 = [0 \ \sqrt{\bar{\beta}(1-\bar{\beta})} D_i(m)(I_{n_u} + \Delta_{(m,n_u)}) C_j^c(m)].$$

Proof Considering $\varpi(t) \equiv 0$, the system (12) is proved to be stochastically stable. For system (12), we define the following Lyapunov function and assume $r_t = m$ at time instant t . $V(t, m) = \bar{\xi}^T(t) \left[\sum_{i=1}^{\lambda} h_i P_i(m) \right] \bar{\xi}(t)$, where

$P_i(m) > 0$, supposing $h_l^+ = h_l^+(\vartheta(t + 1))$, $\zeta_{\tau}(t) = \zeta_{\tau}$, one have

$$\begin{aligned} E \{ \Delta V(t, r_t) \} &= E \{ V(t + 1, r_{t+1}) | \bar{\xi}(t), r_t \} - V(t, r_t) \\ &\leq \bar{\xi}^T(t) \sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{\tau=1}^{\kappa} h_i h_j \zeta_{\tau} \left[(A_{ij}^1)^T \tilde{P}_l(n) A_{ij}^1 \right. \\ &\quad \left. + (A_{ij}^2)^T \tilde{P}_l(n) A_{ij}^2 - P_i(m) \right] \bar{\xi}(t). \end{aligned} \quad (20)$$

According to Eq. 17, we can have the system (12) is stoch-astically stable.

Let $\Psi = \sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{\tau=1}^{\kappa} h_i h_j \zeta_{\tau} [(A_{ij}^1)^T \tilde{P}_l(n) A_{ij}^1 + (A_{ij}^2)^T \tilde{P}_l(n) A_{ij}^2 - P_i(m)]$, From $\hat{\lambda}_{\min}(-\Psi) |\bar{\xi}(t)|^2 \leq \bar{\xi}^T(t) (-\Psi) \bar{\xi}(t) \leq \hat{\lambda}_{\max}(-\Psi) |\bar{\xi}(t)|^2$, one can obtain $E \{ \Delta V(t, r_t) \} \leq -\hat{\lambda}_{\min}(-\Psi) \bar{\xi}^T(t) \bar{\xi}(t)$, which implies $E \left\{ \sum_{t=0}^{\infty} |\bar{\xi}(t)|^2 \right\} \leq \bar{\xi}^T(0) (\hat{\lambda}_{\min}(-\Psi))^{-1} \sum_{i=1}^{\lambda} h_i P_i(m) \bar{\xi}(0)$.

Let $W = (\hat{\lambda}_{\min}(-\Psi))^{-1} \sum_{i=1}^{\lambda} h_i P_i(m)$, from Eq. 17 achieve $\Psi < 0$ and $W > 0$. Therefore, in light of Definition 1, we obtain that the system (12) is stochastically stable. When the zero initial condition exists we will consider the H_{∞} performance in the following section. The index on H_{∞} performance is as follows

$$\begin{aligned} J &= E \left\{ z^T(t) z(t) | \Theta(t), r_t \right\} - \gamma^2 \varpi^T(t) \varpi(t) \\ &\quad + E \left\{ V(t + 1, r_{t+1}) | \Theta(t), r_t \right\} - V(t, r_t). \end{aligned} \quad (21)$$

Let $\Theta(k) = \begin{bmatrix} \bar{\xi}(k) \\ \varpi(k) \end{bmatrix}$, and we obtain

$$\begin{aligned} J &= E \left\{ z^T(t) z(t) | \Theta(t), r_t \right\} - \gamma^2 \varpi^T(t) \varpi(t) \\ &\quad + E \left\{ V(t + 1, r_{t+1}) | \Theta(t), r_t \right\} - V(t, r_t) \\ &= E \left\{ \Theta^T(t) \sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{s=1}^{\lambda} \sum_{o=1}^{\lambda} h_i h_j h_s h_o \right. \\ &\quad \left([A_{ij} \ \Xi_i]^T \left[\sum_{\tau=1}^{\kappa} \zeta_{\tau} \sum_{n=1}^{\omega} \pi_{mn}(\tau) P_l(n) \right] [A_{so} \ \Xi_s] \right) \Theta(t) \left. \right\} \\ &\quad - \gamma^2 \varpi^T(t) \varpi(t) - \bar{\xi}^T(t) \left[\sum_{i=1}^{\lambda} h_i P_i(m) \right] \bar{\xi}(t) \\ &\quad + E \left\{ \Theta^T(t) \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{s=1}^{\lambda} \sum_{o=1}^{\lambda} h_i h_j h_s h_o \right. \\ &\quad \left. \left([C_{ij} \ F_i]^T [C_{so} \ F_s] \right) \Theta(t) \right\} \end{aligned}$$

$$\begin{aligned} &\leq \Theta^T(t) \sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{\tau=1}^{\kappa} h_i h_j \zeta_{\tau} \\ &\quad \left\{ \begin{bmatrix} (A_{ij}^1)^T \tilde{P}_l(n) A_{ij}^1 + (A_{ij}^2)^T \tilde{P}_l(n) A_{ij}^2 & (A_{ij}^1)^T \tilde{P}_l(n) \mathcal{E}_i \\ \mathcal{E}_i^T \tilde{P}_l(n) A_{ij}^1 & \mathcal{E}_i^T \tilde{P}_l(n) \mathcal{E}_i \end{bmatrix} \right. \\ &\quad + \begin{bmatrix} (C_{ij}^1)^T C_{ij}^1 + (C_{ij}^2)^T C_{ij}^2 & (C_{ij}^1)^T F_i \\ F_i^T C_{ij}^1 & F_i^T F_i \end{bmatrix} \\ &\quad \left. - \begin{bmatrix} P_i(m) & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \Theta(t) \\ &= \Theta^T(t) \sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{\tau=1}^{\kappa} h_i h_j \zeta_{\tau} \\ &\quad \left\{ \begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix}^T \begin{bmatrix} C_{ij}^1 & F_i \\ C_{ij}^2 & 0 \end{bmatrix} + \begin{bmatrix} A_{ij}^1 & \mathcal{E}_i \\ A_{ij}^2 & 0 \end{bmatrix}^T \right. \\ &\quad \left. \begin{bmatrix} \tilde{P}_l(n) & 0 \\ 0 & \tilde{P}_l(n) \end{bmatrix} \begin{bmatrix} A_{ij}^1 & \mathcal{E}_i \\ A_{ij}^2 & 0 \end{bmatrix} - \begin{bmatrix} P_i(m) & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \Theta(t). \end{aligned}$$

From above formula, we can have

$$\begin{aligned} &\hat{A}_{ij}^T \left[\sum_{n=1}^{\omega} \pi_{mn}(\tau) \hat{P}_l(n) \right] \hat{A}_{ij} + \hat{C}_{ij}^T \hat{C}_{ij} - Q_i \\ &= \hat{A}_{ij}^T \left[\sum_{n \in \mathfrak{S}_k^m} \pi_{mn}(\tau) \hat{P}_l(n) \right] \hat{A}_{ij} \\ &\quad + \sum_{n \in \mathfrak{S}_k^m} \pi_{mn}(\tau) [\hat{C}_{ij}^T \hat{C}_{ij} - Q_i] \\ &+ \hat{A}_{ij}^T \left[\sum_{n \in \mathfrak{S}_{uk}^m} \pi_{mn}(\tau) \hat{P}_l(n) \right] \hat{A}_{ij} \\ &\quad + \sum_{n \in \mathfrak{S}_{uk}^m} \pi_{mn}(\tau) [\hat{C}_{ij}^T \hat{C}_{ij} - Q_i]. \end{aligned}$$

Then, from Schur complement, for each $n \in \mathfrak{S}_k^m$, pre- and postmultiplying by

$$\text{diag}[-\hat{P}_l^{-1}(\varphi_1^m) \dots -\hat{P}_l^{-1}(\varphi_v^m) \ I \ I],$$

one can have

$$\begin{bmatrix} -\hat{P}_l^{-1}(\varphi_1^m) & \dots & 0 & 0 & \sqrt{\pi_{m\varphi_1^m}(\tau)} \hat{A}_{ij} \\ * & \ddots & \vdots & \vdots & \vdots \\ * & * & -\hat{P}_l^{-1}(\varphi_v^m) & 0 & \sqrt{\pi_{m\varphi_v^m}(\tau)} \hat{A}_{ij} \\ * & * & * & -I & \sqrt{\prod_v^m} \hat{C}_{ij} \\ * & * & * & * & -\prod_v^m Q_i \end{bmatrix}, \quad (22)$$

$$\sum_{n \in \mathfrak{S}_{uk}^m} \pi_{mn}(\tau) \begin{bmatrix} -\hat{P}_l^{-1}(n) & 0 & \hat{A}_{ij} \\ 0 & -I & \hat{C}_{ij} \\ * & * & -Q_i \end{bmatrix}. \quad (23)$$

According to Eqs. 18 and 19, we can obtain $J \leq 0$ and $E \left\{ \sum_{t=0}^{\infty} |z(t)|^2 \right\} \leq \gamma^2 \|\varpi\|_2^2$. The proof is finished. \square

From the condition of Theorem 1, it is difficult to find the solutions of the controller due to the uncertainties. In terms of Lemma 1, the following theorem is presented.

Remark 1 Note that a common quadratic rather than the fuzzy Lyapunov function is used to obtain more conservative stable conditions. In terms of Theorem 1, based on the method of linear matrix inequalities [38], we can derive the DOFC condition in Theorem 2.

Theorem 2 For a supposed disturbance attenuation level $\gamma > 0$, the closed-loop system (12) is stochastically stable and the controller gains $A_j^c(m)$, $B_j^c(m)$, $C_j^c(m)$ ($j = 1, \dots, \lambda$) are solvable if there exist scalars $\varepsilon_q > 0$, ($q = 0, 1, 2, \dots, 2v$) and positive definite matrices $P_l(m)$, $m \in \mathfrak{S}$, ($l = 1, \dots, \lambda$), such that the following inequalities hold:

$$\Gamma_{ij\tau}^1(m) = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & 0 & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & 0 \\ * & * & H_{33} & 0 & 0 & H_{36} \\ * & * & * & H_{44} & 0 & 0 \\ * & * & * & * & H_{55} & 0 \\ * & * & * & * & * & H_{66} \end{bmatrix} < 0, \quad (24)$$

$n \in \mathfrak{S}_k$,

$$\Gamma_{ij\tau}^2(m) = \begin{bmatrix} -\hat{P}_l^{-1}(n) & 0 & \tilde{A}_{ij} & Y_1 & 0 & 0 \\ * & -I & \tilde{C}_{ij} & 0 & 0 & 0 \\ * & * & Q_1^* & 0 & 0 & Y_2 \\ * & * & * & Y_3 & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & Y_4 \end{bmatrix} < 0, \quad (25)$$

$n \in \mathfrak{S}_{uk}$,

where

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} -\varphi_l^{-1} & 0 \\ 0 & -I \end{bmatrix} + \varepsilon_0^{-1} \mathcal{L} \mathcal{L}^T,$$

$$H_{13} = \varepsilon_v^m \tilde{A}_{ij}, \ H_{23} = \sqrt{\prod_v^m} \tilde{C}_{ij}, \ H_{33} = \begin{bmatrix} \chi & 0 \\ 0 & -\prod_v^m \gamma^2 I \end{bmatrix},$$

$$\tilde{A}_{ij} = \begin{bmatrix} A_{1ij}(m) \\ A_{2ij}(m) \end{bmatrix}, \ \tilde{C}_{ij} = \begin{bmatrix} C_{1ij}(m) \\ C_{2ij}(m) \end{bmatrix},$$

$$\chi = -\prod_v^m P_i(m) + \chi_1,$$

$$\chi_1 = (\varepsilon_1 + \dots + \varepsilon_{2v-1}) \Phi_{ii}^T \Phi_{ii} + (\varepsilon_2 + \dots + \varepsilon_{2v}) R_i^T R_i,$$

$$H_{14} = \text{diag} \{ E_0 K_j(m) \ E_0 K_j(m) \ \dots \ E_0 K_j(m) \ E_0 K_j(m) \},$$

$$H_{44} = \text{diag} \{ -\varepsilon_1 (\pi_{m\varphi_1^m}(\tau))^{-1} I \ -\varepsilon_2 (\pi_{m\varphi_1^m}(\tau))^{-1} I$$

$$\dots - \varepsilon_{2v-1} (\pi_{m\varphi_v^m}(\tau))^{-1} I \ -\varepsilon_{2v} (\pi_{m\varphi_v^m}(\tau))^{-1} I \},$$

$$\mathcal{L} = \left[\sqrt{\pi_{m\varphi_1^m}(\tau)} \Omega_i^T(m) \ \sqrt{\pi_{m\varphi_1^m}(\tau)} \tilde{Q}_i^T(m) \ \dots \right]$$

$$\begin{aligned} & \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Omega_i^T(m) \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \bar{Q}_i^T(m) \sqrt{\prod_v^m \bar{\beta}} \bar{D}_i^T(m) \\ & \left. \sqrt{\prod_v^m} \sqrt{\bar{\beta}(1-\bar{\beta})} \bar{D}_i^T(m) \right]^T, \\ H_{36} = Y_2 &= \begin{bmatrix} (E_1 \bar{C}_{cj}(m))^T & 0 \\ 0 & 0 \end{bmatrix}, \\ H_{55} = -I, H_{66} = Y_4 &= \begin{bmatrix} -\varepsilon_0^{-1} I & 0 \\ 0 & -I \end{bmatrix}, \\ Y_1 &= \begin{bmatrix} E_0 K_j(m) & 0 \\ 0 & E_0 K_j(m) \end{bmatrix}, Y_3 = \begin{bmatrix} -\varepsilon_1 I & 0 \\ 0 & -\varepsilon_2 I \end{bmatrix}, \\ Q_1^* &= \begin{bmatrix} -P_i(m) + \varepsilon_1 \Phi_{li}^T \Phi_{li} + \varepsilon_2 R_i^T R_i & 0 \\ 0 & -\gamma^2 I \end{bmatrix}. \end{aligned}$$

Proof Let

$$\begin{aligned} A_{ij}^1 &= A_{1ij}(m) + \Delta A_{1ij}(m), \\ A_{ij}^2 &= A_{2ij}(m) + \Delta A_{2ij}(m), \\ C_{ij}^1 &= C_{1il}(m) + \Delta C_{1il}(m), \\ C_{ij}^2 &= C_{2il}(m) + \Delta C_{2il}(m), \\ A_{1ij}(m) &= \Lambda_i(m) + E_0 K_j(m) \Phi_i(m) \\ & \quad + \Omega_i(m) E_1 \bar{C}_{cj}(m), \\ A_{2ij}(m) &= E_0 K_j(m) R_i(m) + \bar{Q}_i(m) E_1 \bar{C}_{cj}(m), \\ C_{1il}(m) &= \psi_i(m) + \bar{\beta} \bar{D}_i(m) E_1 \bar{C}_{cj}(m), \\ C_{2il}(m) &= \sqrt{\bar{\beta}(1-\bar{\beta})} \bar{D}_i(m) E_1 \bar{C}_{cj}(m), \\ \Delta A_{1ij}(m) &= E_0 K_j(m) \Delta_{(m,n_y)} \Phi_{li}(m) \\ & \quad + \Omega_i(m) \Delta_{(m,n_u)} E_1 \bar{C}_{cj}(m), \\ \Delta A_{2ij}(m) &= E_0 K_j(m) \Delta_{(m,n_y)} R_i(m) \\ & \quad + \bar{Q}_i(m) \Delta_{(m,n_u)} E_1 \bar{C}_{cj}(m), \\ \Delta C_{1il}(m) &= \bar{\beta} \bar{D}_i(m) \Delta_{(m,n_u)} E_1 \bar{C}_{cj}(m), \\ \Delta C_{2il}(m) &= \sqrt{\bar{\beta}(1-\bar{\beta})} \bar{D}_i(m) \Delta_{(m,n_u)} E_1 \bar{C}_{cj}(m), \\ \Lambda_i(m) &= \begin{bmatrix} A_i(m) & 0 \\ 0 & 0 \end{bmatrix}, E_0 = \begin{bmatrix} 0 \\ I_{n_x \times n_x} \end{bmatrix}, E_1 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \\ \Phi_i(m) &= \begin{bmatrix} 0 & I \\ \bar{\alpha} G_i(m) & 0 \end{bmatrix}, \Phi_{li}(m) = \begin{bmatrix} 0 & 0 \\ \bar{\alpha} G_i(m) & 0 \end{bmatrix}, \\ \Omega_i(m) &= \begin{bmatrix} 0 & \bar{\beta} B_i(m) \\ 0 & 0 \end{bmatrix}, z \\ R_i(m) &= \begin{bmatrix} 0 & 0 \\ \sqrt{\bar{\alpha}(1-\bar{\alpha})} G_i(m) & 0 \end{bmatrix}, \\ \bar{Q}_i(m) &= \begin{bmatrix} 0 & \sqrt{\bar{\beta}(1-\bar{\beta})} B_i(m) \\ 0 & 0 \end{bmatrix}, \\ \psi_i(m) &= [C_i(m) \ 0], \bar{D}_i(m) = [0 \ D_i(m)], \\ K_j(m) &= \begin{bmatrix} A_j^c(m) & B_j^c(m) \\ 0 & C_j^c(m) \end{bmatrix}. \end{aligned}$$

It is shown that $\Gamma_{ijl\tau}^1(m) < 0$ in Eq. 18 is equivalent to

$$\Gamma_{ijl\tau}^1(m) = \begin{bmatrix} \Upsilon_{11} & 0 & \Upsilon_{13} \\ * & \Upsilon_{22} & \Upsilon_{23} \\ * & * & \Upsilon_{33} \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Upsilon_{11} &= \text{diag} \{ \bar{\Upsilon}_{11} \cdots \bar{\Upsilon}_{vv} \}, \\ \bar{\Upsilon}_{11} &= \begin{bmatrix} -P_l^{-1}(\varphi_1^m) & 0 \\ 0 & -P_l^{-1}(\varphi_1^m) \end{bmatrix}, \\ \bar{\Upsilon}_{vv} &= \begin{bmatrix} -P_l^{-1}(\varphi_v^m) & 0 \\ 0 & -P_l^{-1}(\varphi_v^m) \end{bmatrix}, \\ \Upsilon_{13} &= \begin{bmatrix} \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} A_{ij}^1 & \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \Xi_i(m) \\ \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} A_{ij}^2 & 0 \\ \vdots & \vdots \\ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} A_{ij}^1 & \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Xi_i(m) \\ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} A_{ij}^2 & 0 \end{bmatrix}, \\ \Upsilon_{22} &= \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix}; \Upsilon_{23} = \begin{bmatrix} \sqrt{\prod_v^m} C_{ij}^1 & \sqrt{\prod_v^m} F_i(m) \\ \sqrt{\prod_v^m} C_{ij}^2 & 0 \end{bmatrix}, \\ \Upsilon_{33} &= \begin{bmatrix} -\prod_v^m P_i(m) & 0 \\ 0 & -\prod_v^m \gamma^2 I \end{bmatrix}. \end{aligned}$$

It is noted that $\Gamma_{ijl\tau}^1(m) = \bar{\Gamma}_{ijl\tau}^1(m) + \Delta \hat{\Gamma}_{ijl\tau}^1(m)$, in which

$$\begin{aligned} \bar{\Gamma}_{ijl\tau}^1(m) &= \begin{bmatrix} \Upsilon_{11} & 0 & \tilde{\Upsilon}_{13} \\ * & \Upsilon_{22} & \tilde{\Upsilon}_{23} \\ * & * & \Upsilon_{33} \end{bmatrix}, \\ \tilde{\Upsilon}_{13} &= \begin{bmatrix} \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} A_{1ij}(m) & \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \Xi_i(m) \\ \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} A_{2ij}(m) & 0 \\ \vdots & \vdots \\ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} A_{1ij}(m) & \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Xi_i(m) \\ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} A_{2ij}(m) & 0 \end{bmatrix}, \\ \tilde{\Upsilon}_{23} &= \begin{bmatrix} \sqrt{\prod_v^m} C_{1ij}(m) & \sqrt{\prod_v^m} F_i(m) \\ \sqrt{\prod_v^m} C_{2ij}(m) & 0 \end{bmatrix}, \\ \Delta \hat{\Gamma}_{ijl\tau}^1(m) &= \begin{bmatrix} 0 \cdots 0 & \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \Delta A_{1ij}(m) & 0 \\ 0 \cdots 0 & \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \Delta A_{2ij}(m) & 0 \\ \vdots & \vdots & \vdots \\ 0 \cdots 0 & \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Delta A_{1ij}(m) & 0 \\ 0 \cdots 0 & \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Delta A_{2ij}(m) & 0 \\ 0 \cdots 0 & \sqrt{\prod_v^m} \Delta C_{1ij}(m) & 0 \\ 0 \cdots 0 & \sqrt{\prod_v^m} \Delta C_{2ij}(m) & 0 \\ * \cdots * & 0 & 0 \\ 0 \cdots 0 & 0 & 0 \end{bmatrix}. \tag{27} \end{aligned}$$

We rewrite the formula $\Delta\hat{\Gamma}_{ijl\tau}^1(m)$ as follows:

$$\Delta\hat{\Gamma}_{ijl\tau}^1(m) = \Delta\hat{\Gamma}(m) + \Delta\hat{\Gamma}_1(m) + \Delta\hat{\Gamma}_2(m) + \dots + \Delta\hat{\Gamma}_{2v-1}(m) + \Delta\hat{\Gamma}_{2v}(m).$$

$$\begin{aligned} \text{Let } \eta_0 &= \left[\sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \Omega_i^T(m) \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} \bar{Q}_i^T(m) \dots \right. \\ &\quad \left. \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \Omega_i^T(m) \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} \bar{Q}_i^T(m) \sqrt{\prod_v^m \bar{\beta}} \bar{D}_i^T(m) \right. \\ &\quad \left. \sqrt{\prod_v^m \sqrt{\bar{\beta}}(1-\bar{\beta})} \bar{D}_i^T(m) \ 0 \ 0 \right]^T, \\ \mu_0 &= [0 \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ E_1 \bar{C}_{cj}(m) \ 0], \\ \Delta\hat{\Gamma}(m) &= \eta_0 \Delta_{(m,n_u)} \mu_0 + \mu_0^T \Delta^T_{(m,n_u)} \eta_0^T, \\ \eta_1 &= \left[\sqrt{\pi_{m\varphi_1^m}^{(\tau)}} (E_0 K_j(m))^T \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ 0 \right]^T, \\ \mu_1 &= [0 \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ \Phi_{1i}(m) \ 0], \\ \Delta\hat{\Gamma}_1(m) &= \eta_1 \Delta_{(m,n_y)} \mu_1 + \mu_1^T \Delta^T_{(m,n_y)} \eta_1^T, \\ \eta_2 &= \left[0 \ \sqrt{\pi_{m\varphi_1^m}^{(\tau)}} (E_0 K_j(m))^T \ \dots \ 0 \ 0 \ 0 \ 0 \ 0 \right]^T, \\ \mu_2 &= [0 \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ R_i(m) \ 0], \\ \Delta\hat{\Gamma}_2(m) &= \eta_2 \Delta_{(m,n_y)} \mu_2 + \mu_2^T \Delta^T_{(m,n_y)} \eta_2^T, \\ &\vdots \\ \eta_{2v-1} &= \left[0 \ 0 \ \dots \ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} (E_0 K_j(m))^T \ 0 \ 0 \ 0 \ 0 \ 0 \right]^T, \\ \mu_{2v-1} &= [0 \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ \Phi_{1i}(m) \ 0], \\ \Delta\hat{\Gamma}_{2v-1}(m) &= \eta_{2v-1} \Delta_{(m,n_y)} \mu_{2v-1} \\ &\quad + \mu_{2v-1}^T \Delta^T_{(m,n_y)} \eta_{2v-1}^T, \\ \eta_{2v} &= \left[0 \ 0 \ \dots \ 0 \ \sqrt{\pi_{m\varphi_v^m}^{(\tau)}} (E_0 K_j(m))^T \ 0 \ 0 \ 0 \ 0 \right]^T, \\ \mu_{2v} &= [0 \ 0 \ \dots \ 0 \ 0 \ 0 \ 0 \ R_i(m) \ 0], \\ \Delta\hat{\Gamma}_{2v}(m) &= \eta_{2v} \Delta_{(m,n_y)} \mu_{2v} + \mu_{2v}^T \Delta^T_{(m,n_y)} \eta_{2v}^T. \end{aligned}$$

Then, by Lemma 1, one can have

$$\begin{aligned} \bar{\Gamma}_{ijl\tau}^1(m) &+ \varepsilon_0 \mu_0^T \mu_0 + \varepsilon_0^{-1} \eta_0 \eta_0^T + \varepsilon_1 \mu_1^T \mu_1 \\ &+ \varepsilon_1^{-1} \eta_1 \eta_1^T + \varepsilon_2 \mu_2^T \mu_2 + \varepsilon_2^{-1} \eta_2 \eta_2^T + \dots \\ &+ \varepsilon_{2v-1} \mu_{2v-1}^T \mu_{2v-1} + \varepsilon_{2v-1}^{-1} \eta_{2v-1} \eta_{2v-1}^T \\ &\quad + \varepsilon_{2v} \mu_{2v}^T \mu_{2v} + \varepsilon_{2v}^{-1} \eta_{2v} \eta_{2v}^T < 0, \end{aligned} \tag{28}$$

$$\Gamma_{ijl\tau}^1(m) = \bar{\Gamma}_{ijl\tau}^1(m) + Z_1 + Z_2 < 0, \tag{29}$$

where

$$\begin{aligned} Z_1 &= \text{diag}\{\varepsilon_1^{-1} (\pi_{m\varphi_1^m}^{(\tau)}) (E_0 K_j(m)) (E_0 K_j(m))^T \\ &\quad \dots \varepsilon_{2v}^{-1} (\pi_{m\varphi_v^m}^{(\tau)}) (E_0 K_j(m)) (E_0 K_j(m))^T \ 0 \\ &\quad 0 \ \varepsilon_0 (E_1 \bar{C}_{cj}(m))^T (E_1 \bar{C}_{cj}(m)) 0\}, \\ Z_2 &= \begin{bmatrix} \varepsilon_0^{-1} \xi \xi^T & 0 & 0 \\ * & \chi_1 & 0 \\ * & * & 0 \end{bmatrix}. \end{aligned}$$

We can obtain Eq. 24 by Schur complement for each $n \in \mathfrak{S}_k^m$, and in the same way one can achieve Eq. 25 for each $n \in \mathfrak{S}_{uk}^m$. The proof is finished. \square

From the conditions of Theorem 2, we are very difficult to find the solutions of the controller due to the conservative. We adopt the basis-dependent Lyapounov function in this paper. With the achieved LMI of the designed controller, it generates a non-convex condition. In order to solve the parameters of controller matrices, we use the CCL algorithm to tackle it. In terms of Lemma 2, the following theorem is given.

Remark 2 On the above proof of theorem 2, the matrix inequalities are applied to supply conveniences of mathematical derivation. At the same time, it will lead to more conservatives. One feasible method as in [39] is to present a constant matrix in order to decrease the conservative. In this paper, without loss of generality, the fuzzy Lyapunov function and the CCL algorithm are utilized to tackle the solutions.

Theorem 3 For a supposed disturbance attenuation level $\gamma > 0$, the closed-loop system (12) is stochastically stable and the controller gains $A_j^c(m)$, $B_j^c(m)$, $C_j^c(m)$ ($j = 1, \dots, \lambda$) are solvable if there exist scalars $\varepsilon_q > 0$, ($q = 1, 2, \dots, 2v$) and positive definite matrices $P_l(m)$, $L_l(m)$, $m \in \mathfrak{S}$, ($l = 1, \dots, \lambda$), such that the following inequalities hold:

$$\Gamma_{iil\tau}^1(m) < 0, n \in \mathfrak{S}_k, \tag{30}$$

$$\frac{1}{\lambda - 1} \Gamma_{iil\tau}^1(m) + \frac{1}{2} (\Gamma_{ijl\tau}^1(m) + \Gamma_{jil\tau}^1(m)) < 0, i \neq j, \tag{31}$$

$$\Gamma_{iil\tau}^2(m) < 0, n \in \mathfrak{S}_{uk}, \tag{32}$$

$$\frac{1}{\lambda - 1} \Gamma_{iil\tau}^2(m) + \frac{1}{2} (\Gamma_{ijl\tau}^2(m) + \Gamma_{jil\tau}^2(m)) < 0, i \neq j, \tag{33}$$

$$P_l(m) L_l(m) = I. \tag{34}$$

In Theorem 1 and Theorem 2, we define other relevant variables.

Proof In terms of Lemma 2, if the matrix inequalities (30)–(34) hold, then one can have the following inequality

$$\sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda} \sum_{j=1}^{\lambda} \sum_{\tau=1}^{\kappa} h_i h_j \zeta_{\tau} (\Gamma_{ijl\tau}^1(m) + \Gamma_{jil\tau}^2(m)) < 0. \tag{35}$$

The proof is completed. \square

We introduce the basic notion of the CCL algorithm. If $P_l(m) > 0$, $L_l(m) > 0$, $m \in \mathfrak{S}$, ($l = 1, \dots, \lambda$) are \bar{n}

dimensional solutions for the condition of LMI:

$$\begin{bmatrix} P_l(m) & I \\ I & L_l(m) \end{bmatrix} \geq 0, \forall m \in \mathfrak{S}, \quad (36)$$

then, $\text{tr}(\sum_m P_l(m)L_l(m)) \geq \bar{n}$, furthermore, if and only if $P_l(m)L_l(m) = I$,

$$\text{tr}\left(\sum_m P_l(m)L_l(m)\right) = \bar{n}. \quad (37)$$

In this paper, the quantized H_∞ DOFC design problem is as follows:

$$\min_{\text{tr}} \left(\sum_{l,m} P_l(m)L_l(m) \right), \quad (38)$$

subject to Eqs. 30–33 and 36.

Then the conclusions in Theorem 3 are resolvable if there have solutions such that $\min \text{tr}(\sum_{l,m} P_l(m)L_l(m)) = \lambda\bar{n}$ is subject to Eqs. 30–33 and 37. The algorithm is shown in Table 1.

Remark 3 In the previous algorithm, note that, an iteration technique is applied to tackle the minimization problem rather than the handled problem of customary nonconvex feasibility in Eq. 34. Because it is difficult to get the optimal

Table 1 Quantized control design algorithm

Programme Algorithm

Step 1: Seek a feasible set

$$(P_l^0(m), L_l^0(m), A_i^{c0}(m), B_i^{c0}(m), C_i^{c0}(m))$$

to satisfy Eqs. 30–33 and 36. Set $k = 0$.

Step 2: Solve the following issue

$$\min_{\text{tr}} (\sum_{l,m} (P_l(m)L_l^k(m) + P_l^k(m)L_l(m)))$$

s.t. Eqs. 30–33 and 36.

Step 3: The achieved variables

$$(P_l(m), L_l(m), A_i^c(m), B_i^c(m), C_i^c(m))$$

are substituted into the inequality Eqs. 24 and 25.

If the inequality Eqs. 24 and 25 are hold, with

$$|\text{tr}(\sum_{l,m} P_l(m)L_l(m)) - \lambda\bar{n}| < \bar{\delta}$$

for any sufficiently small scalar $\bar{\delta} > 0$,

then obtain the feasible solutions

$$(P_l(m), L_l(m), A_i^c(m), B_i^c(m), C_i^c(m)). \text{EXIT.}$$

Step 4: If $k > \bar{N}$, where \bar{N} is the allowed

maximum number of iterations, EXIT.

Step 5: Set $k = k + 1$,

$$(P_l^k(m), L_l^k(m), A_i^{ck}(m), B_i^{ck}(m), C_i^{ck}(m)) =$$

$(P_l(m), L_l(m), A_i^c(m), B_i^c(m), C_i^c(m))$ and go to Step 2.

values to satisfy the requirement that the stopping criterion is supposed to be verified in the minimization problem.

4 Numerical simulations

In the section, a numerical example is applied to illustrate the validity of the proposed design method. We suppose that three modes are in the discrete-time FMJSs (1), and the matrixes of parameters for the system (1) are listed as follows:

$$A_1\{1\} = \begin{bmatrix} 1.1363 & 0.1554 \\ -0.9647 & -0.0240 \end{bmatrix}, B_1\{1\} = \begin{bmatrix} 0.1055 \\ 0.0909 \end{bmatrix},$$

$$A_2\{1\} = \begin{bmatrix} 0.1919 & 0.3824 \\ 0.4814 & 0.3357 \end{bmatrix}, B_2\{1\} = \begin{bmatrix} 0.1060 \\ 0.0167 \end{bmatrix},$$

$$A_1\{2\} = \begin{bmatrix} 0.1925 & 0.1930 \\ -2.0994 & -0.1935 \end{bmatrix}, B_1\{2\} = \begin{bmatrix} 0.0742 \\ 0.1930 \end{bmatrix},$$

$$A_2\{2\} = \begin{bmatrix} 0.3711 & 0.2222 \\ -1.7779 & -0.0734 \end{bmatrix}, B_2\{2\} = \begin{bmatrix} 0.0786 \\ 0.2222 \end{bmatrix},$$

$$A_1\{3\} = \begin{bmatrix} 1.1572 & 0.125 \\ -0.328 & -0.25 \end{bmatrix}, B_1\{3\} = \begin{bmatrix} 0.42 \\ 0.031 \end{bmatrix},$$

$$A_2\{3\} = \begin{bmatrix} 0.46 & 0.32 \\ 0.039 & 0.2 \end{bmatrix}, B_2\{3\} = \begin{bmatrix} -0.031 \\ 0.418 \end{bmatrix},$$

$$E_1\{1\} = \begin{bmatrix} 0.0042 \\ -0.0036 \end{bmatrix}, E_2\{1\} = \begin{bmatrix} -0.0004 \\ -0.008 \end{bmatrix},$$

$$C_1\{1\} = [0.0017 \quad -0.0058],$$

$$C_2\{1\} = [-0.0020 \quad -0.0061],$$

$$D_1\{1\} = -0.0033, D_2\{1\} = 0.0228,$$

$$F_1\{1\} = 0, F_2\{1\} = 0,$$

$$G_1\{1\} = [-0.0826 \quad 0.5], G_2\{1\} = [-0.0502 \quad 1.0],$$

$$E_1\{1\} = E_1\{2\} = E_1\{3\}, E_2\{1\} = E_2\{2\} = E_2\{3\},$$

$$C_1\{1\} = C_1\{2\} = C_1\{3\}, C_2\{1\} = C_2\{2\} = C_2\{3\},$$

$$D_1\{1\} = D_1\{2\} = D_1\{3\}, D_2\{1\} = D_2\{2\} = D_2\{3\},$$

$$F_1\{1\} = F_1\{2\} = F_1\{3\}, F_2\{1\} = F_2\{2\} = F_2\{3\},$$

$$G_1\{1\} = G_1\{2\} = G_1\{3\}, G_2\{1\} = G_2\{2\} = G_2\{3\}.$$

We propose the design of DOFC. As opened up before our eyes in Fig. 1, under the circumstances of network, the signals of the controller and the ones of the model are always projected into piecewise-constant signals before transmission. The logarithmic quantizer (6) makes the signal $y(t)$ and the signal $u^c(t)$ quantize. $\rho^{(1,1)} = 0.6667$, $\rho^{(1,2)} = 0.7391$, $\rho^{(1,3)} = 0.6$ and $\eta_{(0)}^{(1,1)} = \eta_{(0)}^{(1,2)} = \eta_{(0)}^{(1,3)} = 0.0001$ are the selected quantizer densities. It can be calculated that $\delta^{(1,1)} = 0.4$, $\delta^{(1,2)} = 0.5$ and $\delta^{(1,3)} = 0.25$ hold. We apply the CCL algorithm and the LMIs in the

theorem 3 when $\bar{\alpha} = 0.8, \bar{\beta} = 0.8$. The DOFC gains are listed below

$$\begin{aligned}
 A_1^c\{1\} &= \begin{bmatrix} 0.0058 & 0.0116 \\ -0.0887 & 0.1160 \end{bmatrix}, \\
 A_2^c\{1\} &= \begin{bmatrix} 0.0594 & -0.0401 \\ 0.0209 & -0.0177 \end{bmatrix}, \\
 B_1^c\{1\} &= \begin{bmatrix} -0.0541 \\ 0.0199 \end{bmatrix}, B_2^c\{1\} = \begin{bmatrix} -0.0913 \\ 0.0689 \end{bmatrix}, \\
 C_1^c\{1\} &= [0.0092 \quad -0.0088], \\
 C_2^c\{1\} &= [-0.0028 \quad 0.0010], \\
 A_1^c\{2\} &= \begin{bmatrix} -0.0352 & 0.0519 \\ -0.0016 & -0.0188 \end{bmatrix}, \\
 A_2^c\{2\} &= \begin{bmatrix} 0.0143 & -0.0069 \\ -0.0091 & 0.0026 \end{bmatrix}, \\
 B_1^c\{2\} &= \begin{bmatrix} 0.0026 \\ 0.0111 \end{bmatrix}, B_2^c\{2\} = \begin{bmatrix} -0.0043 \\ 0.0067 \end{bmatrix}, \\
 C_1^c\{2\} &= [-0.0081 \quad -0.0393], \\
 C_2^c\{2\} &= [-0.0035 \quad 0.0103], \\
 A_1^c\{3\} &= \begin{bmatrix} -0.0142 & 0.0248 \\ 0.0003 & -0.0009 \end{bmatrix}, \\
 A_2^c\{3\} &= \begin{bmatrix} 0.0090 & -0.0060 \\ 0.0113 & -0.0015 \end{bmatrix}, \\
 B_1^c\{3\} &= \begin{bmatrix} -0.0041 \\ 0.0023 \end{bmatrix}, B_2^c\{3\} = \begin{bmatrix} -0.0043 \\ -0.0089 \end{bmatrix}, \\
 C_1^c\{3\} &= [0.0086 \quad -0.0276], \\
 C_2^c\{3\} &= [-0.0051 \quad 0.0075].
 \end{aligned}$$

Let $\zeta_\tau(t) = h_i(t)$. Membership functions for Rules 1, 2 and the matrix of transition probability are listed as follows

$$\begin{aligned}
 h_1(x_1(t)) &= \begin{cases} 1 & x_1(t) \leq -1 \\ 0.5 - 0.5x_1(t) - 1 & -1 \leq x_1(t) \leq 1 \\ 0 & \text{else,} \end{cases} \\
 h_2(x_1(t)) &= 1 - h_1(x_1(t)), \\
 \Pi^1 &= \begin{bmatrix} ? & ? & 0.25 \\ ? & ? & 0.2 \\ ? & ? & 0.35 \end{bmatrix}, \Pi^2 = \begin{bmatrix} ? & ? & 0.4 \\ ? & ? & 0.45 \\ ? & ? & 0.7 \end{bmatrix}, \\
 \Pi^3 &= \begin{bmatrix} ? & ? & 0.15 \\ ? & 0.75 & ? \\ ? & ? & 0.55 \end{bmatrix}, \Pi^4 = \begin{bmatrix} 0.3 & ? & ? \\ ? & ? & 0.45 \\ ? & 0.6 & ? \end{bmatrix},
 \end{aligned}$$

where, ? represents the unknown element.

Figure 2 shows that in both the uplink and the downlink the missing of random data packet is described. The external disturbance is given as $\varpi(t)=1/(2+t)$. Moreover, we suppose that $x(0) = [-1 \ 4]^T$ and $\eta_c(0) = [-2 \ 1]^T$ are the initial value of the model and the initial value of the

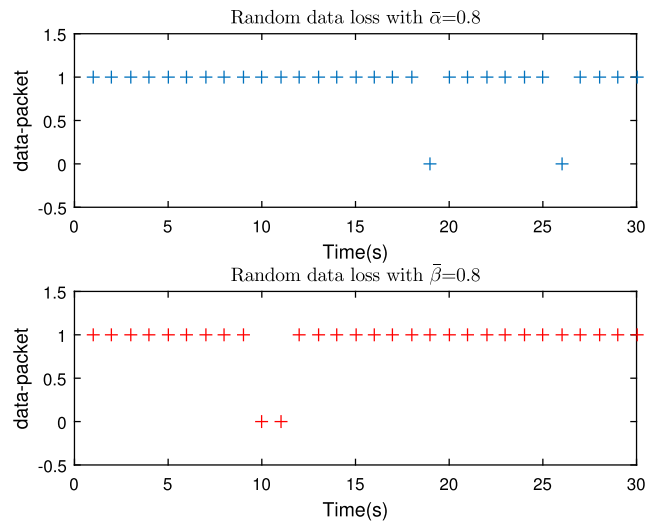


Fig. 2 The missing of random data packet

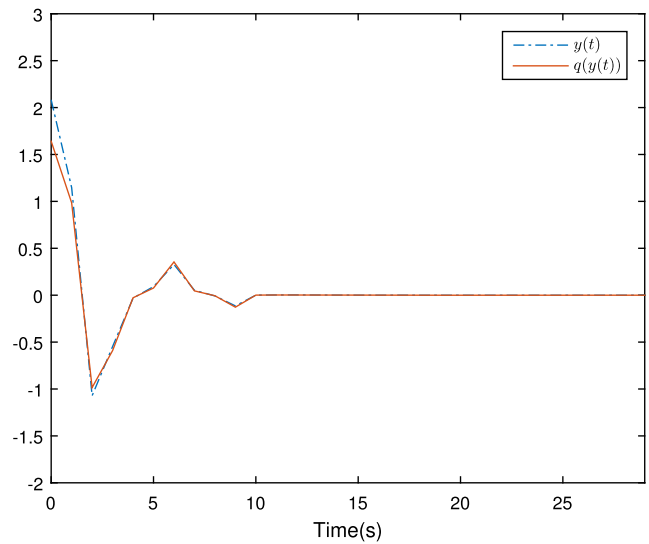


Fig. 3 Quantized signals and the output $y(t)$

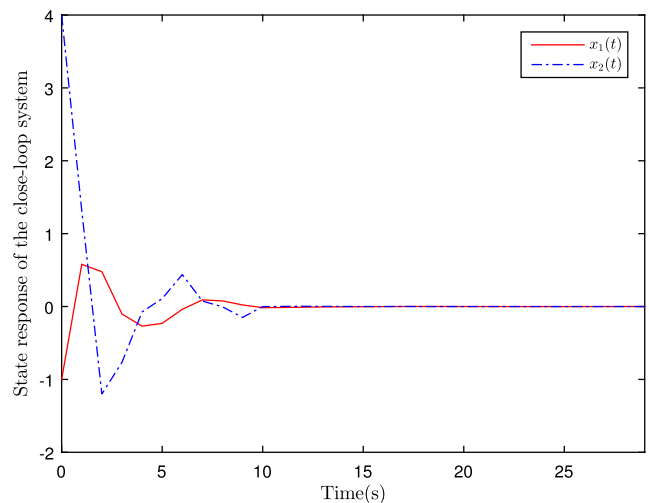


Fig. 4 The state curves of the closed-loop system

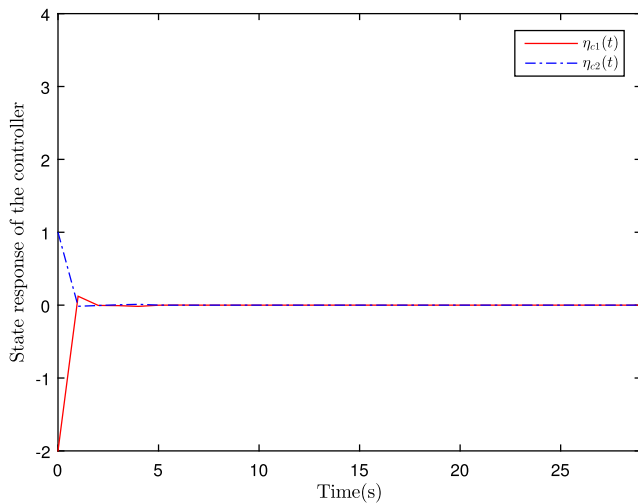


Fig. 5 The state curves of the controller

controller, respectively. The output $q_m(y(t))$ of quantized signals and the output $y(t)$ of model are displayed in Fig. 3, in which the signals are quantized well via the logarithmic quantizer. The quantized signals quickly approach zero with the passage of time, which suggests that the developed technique is applicable and correct. Figures 4 and 5 display the state curves of the closed-loop system and the state curves of the controller, respectively, which indicates the validity of the proposed method. The results indicate that the nonlinear MJSs can be effectively stabilized via the fuzzy DOFC. Furthermore, the entire model is devised perfectly along with the advantageous capacity of the controller. Perhaps more accurately, due to the condition performance the states quickly converge to the equilibrium point, which is easily noted. From Fig. 4, under the signal switching,

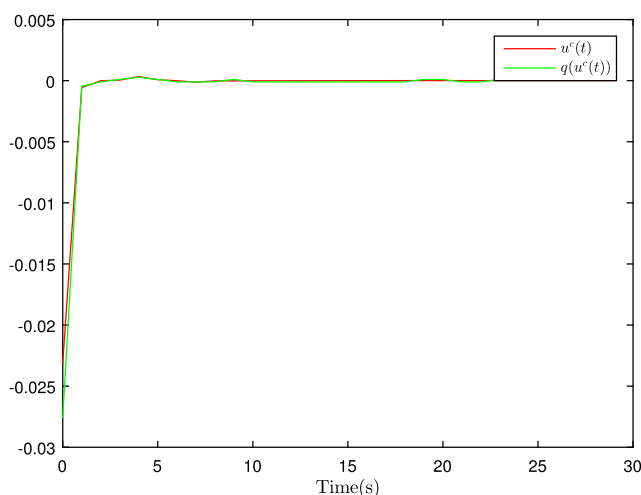


Fig. 6 Quantized signals and the output $u^c(t)$

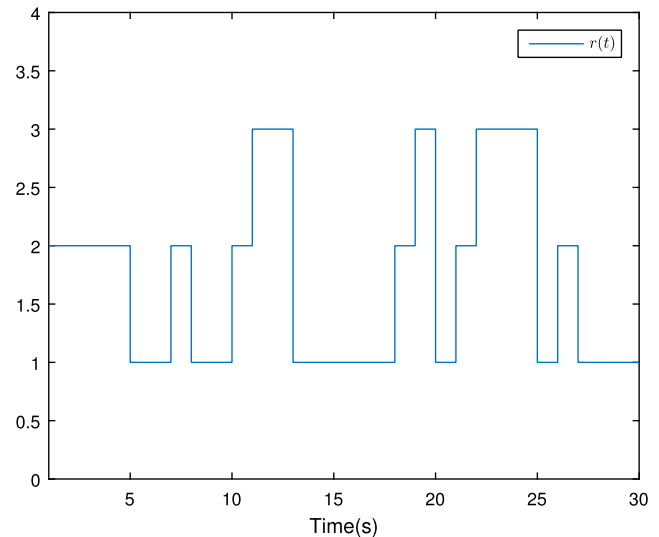


Fig. 7 the state signals of the MJSs model

it can be found that the amplitudes of the states are smaller and denser, which further means that a more ideal performance exists in the system. Further, from Figs. 4 and 5, it is observed that the states are approximating fast to zero as time k passes by. Then, we can conclude that there exists a DOFC of the form (5) such that the system (12) with H_∞ performance level is stochastically stable. The output $q_m(u^c(t))$ of quantized signals and the output $u^c(t)$ of the controller are indicated in Fig. 6. Additionally, Fig. 7 plots the state signals of the FMJSs model. The efficiency of the presented technique is illustrated by the above phenomena. Therefore, for the closed-loop system, the stochastic stability and the required H_∞ performance are ensured in this paper.

5 Conclusion

In this paper, we have dealt with the quantized control design for a class of NMJSs with partly unknown and time-variant transition probabilities by a T-S fuzzy approach. Particularly in a network environment, we considered simultaneously the effects of data packet dropouts and signal quantization in the closed-loop circuit. A sufficient condition of the stochastic stability and the required performance for the closed-loop system are presented by a fuzzy Lyapunov function. In order to deal with the solutions for the DOFC, a sequential minimization is efficiently tackled by the CCL procedure. The effectiveness of the suggested control schemes is illustrated by a simulation example. The more practical and realistic stochastic system will be followed with interest by us. Particularly, in a network environment, the fuzzy filtering with the semi-Markovian jump systems (S-MJSs) are our interest.

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