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A note on tax analysis in a two-region model of monopolistic competition

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Abstract Takatsuka (Pap Reg Sci 93:595–617, 2014) compares the effects of ad valorem and unit taxes on firm location, within the framework of new trade theory. In the model, the unit tax is imposed at the instant of production, but the ad valorem tax is imposed at the instant of consumption. Since a portion of the good "melts away" during transportation, the actual amount of consumption (tax base) decreases from the point of production. This note presents a consistent application of taxation and clarifies how the timing of taxation characterizes the equilibrium location pattern.

Keywords Unit tax · Ad valorem tax · Monopolistic competition · Firm location

JEL Classification H2 · R3 · L1

1 Introduction

In a recent paper, Takatsuka (2014) develops an appealing model for determining the tax effects on a firm's location in the new trade theory (NTT) framework. He compares the effects of two different tax methods—namely, ad valorem tax and unit (specific) tax—on a firm's agglomeration. This study contributes to tax analysis by examining

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tax effects within the framework of two asymmetric regions that have costly tradable goods; most of the literature, on the other hand, has compared the effects of ad valorem tax and unit tax within a single-country framework.[1]

One of the key factors that characterize the equilibrium location pattern is the iceberg transportation cost: more than one unit of traded good must be shipped for one unit to reach the other region. The role of transportation costs in firms' agglomeration is well known, but the new subject of discussion when the NTT model includes the tax policy is the timing of taxation. Since the amount of tradable goods at the instant of production differs from the amount of goods that arrives at the instant of consumption, the timing of taxation—that is, whether the government imposes a tax on the amount of production or the amount of consumption—exerts a significant influence on the equilibrium.

In his analysis, Takatsuka (2014) assumes that the unit tax is imposed at the instant of production, and that the ad valorem tax is imposed at the instant of consumption. Under this setup, the tax base is smaller in the case of the ad valorem tax, meaning that the analysis mixes the two effects on the equilibrium location: (1) the familiar tax effects seen by changing the power to control the price, and (2) the timing effects that originate from the different tax base amounts.

Assuming the timing of taxation is the same for the unit tax and the ad valorem tax, this note proposes an alternative model that restores Takatsuka's results. Our finding is that if the firm bears the burden of taxation while the consumers incur the transportation costs, then the location pattern has no relation to the timing of taxation.

2 Model

The economy presented by Takatsuka (2014) consists of two regions, $i = N, S$, and two sectors, $j = M, A$. The amount of labor in region N and S is given by L and L^*, respectively, where the asterisk denotes the variables in region S. The labor is freely mobile between the two sectors in a region, but is immobile across the regions. The total amount of labor in the economy is fixed at $\bar{L}$. The regional asymmetry is captured by the amount of labor between the two regions, and $0.5 < \theta \equiv L/\bar{L} < 1$ is assumed. Sector M consists of a continuum of firms that produce differentiated products under IRS technology and within a monopolistic competition market. Sector A produces a homogenous product under CRS technology and a perfectly competitive market. The government taxes the firms in sector M, but firms in sector A are not taxed.

A worker's preference is given by

$$U = \mu \ln M + A, \quad \text{where} \quad M \equiv \left[\int_0^{n^w} m(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

defined over the consumption of a private numeraire good A and differentiated products. In (1), n^w is the number of varieties in sector M, and $m(i)$ is the consumption

[1] See Keen (1998) for a general review.

of the differentiated product i. $\sigma > 1$ denotes the elasticity of substitution between any two differentiated products produced in sector M, and $\mu \in (0, 1)$ represents the consumer's weight for differentiated products.

The worker owns a unit of labor and supplies it inelastically. In sector M, each firm uses a marginal input of m units of labor and a fixed input of f units of labor. The wage rate in region S is normalized by $w^* = 1$, and the wage in region N is denoted by w. In the production in sector A, one unit of product is produced by one unit of labor. Hence, the price of the good in sector A is given by $p_A = w$ and $p_A^* = w^* = 1$.

The profit of each firm in sector M is given by

$$\pi = p_{NN}x_{NN} + p_{NS}x_{NS} - c(x_{NN}, x_{NS}), \tag{2}$$

$$\pi^* = p_{SS}x_{SS} + p_{SN}x_{SN} - c^*(x_{SS}, x_{SN}), \tag{3}$$

where p_{rs} and x_{rs} are, respectively, the consumer price and the supply within region s of differentiated goods made in region r. The differentiated products are tradable with a certain transportation cost. The transportation cost is characterized by Samuelson's iceberg cost, in which $\tau_M \in (1, \infty)$ units of good in sector M must be shipped for one unit to reach the other region. Transportation within a region is assumed to be costless.

In Takatsuka (2014), the total costs of producing differentiated products in regions N and S are given, respectively, as

$$c(x_{NN}, x_{NS}) = fw + (mw + t_u + t_a p_{NN})x_{NN} + (mw\tau_M + t_u\tau_M + t_a p_{NS})x_{NS}, \tag{4}$$

$$c(x_{SS}, x_{SN}) = fw^* + (mw^* + t_u + t_a p_{SS})x_{SS} + (mw^*\tau_M + t_u\tau_M + t_a p_{SN})x_{SN}, \tag{5}$$

where t_u and t_a are the unit tax and ad valorem tax imposed by the national government in this economy, respectively. The tax is imposed uniformly between the two regions. The first term in (4) and (5) represent the fixed cost, and the second term shows the sum of the variable cost and tax payment associated with the products supplied for consumers in the home region. The third term is designed to represent the sum of the variable cost and tax payment associated with the products supplied for consumers in the other region. For example, in (4), the amount of $\tau_M x_{NS}$ is produced in region N to export the product to region S, and the firm pays the unit tax, $t_u\tau_M x_{NS}$, with the tax rate of t_u.[2] Note that the amount of product that reaches the consumers in region S is x_{NS}, thus implying that the firm is taxed when it is produced in region N (i.e., before the product reaches consumers in region S). The third term suggests otherwise,

[2] The transportation costs in the model correspond partially to packing and transportation/delivery expenses as found in general corporate accounting. However, they are not absolutely identical: the transportation cost in the economic model is broadly interpreted, and includes factors that are associated with non-contiguity, language barriers, exchange rate barriers, insecurity, and other plausible bilateral characteristics. Meanwhile, in corporate accounting, it is defined as the direct expenses a firm incurs when it transfers its inventory or other assets to another location, which is mainly associated with the distance. There have been several attempts to measure the transportation cost, alongside extensive interpretations. See, for instance, Eaton and Kortum (2002) and Anderson and Wincoop (2003, 2004).

however. The term given by $t_a p_{NS} x_{NS}$ is the ad valorem tax payment by the firm in region N. Amount x_{NS} reaches the consumers and the ad valorem tax is imposed on the sales amount. This setting treats the timing of the two tax methods differently: the unit tax is imposed at the instant of production (not at the time the consumers buy the product), while the ad valorem tax is imposed at the instant of consumption (not at the instant of production).

Under this setting, Takatsuka (2014) derives the following market-clearing conditions, which lead him to derive some lemmas and propositions.

$$(1-t_a)\left[\frac{\frac{p_A}{w}L}{n+n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}\phi_M}+\frac{\frac{p_A^*}{w}L^*\phi_M}{n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}+n\phi_M}\right]=\frac{f\sigma}{\mu}, \tag{6}$$

$$(1-t_a)\left[\frac{\frac{p_A}{w^*}L\phi_M}{n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}+n^*\phi_M}+\frac{\frac{p_A^*}{w^*}L^*}{n^*+n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}\phi_M}\right]=\frac{f\sigma}{\mu}, \tag{7}$$

where $MC_u \equiv mw + t_u$, $MC_u^* \equiv mw^* + t_u$, and $\phi_M \equiv \tau_M^{1-\sigma}$.

3 A proposed modification

In this section, we first present two models in which the ad valorem tax and the unit tax are imposed at the instant of either production or consumption. The two models show that the key equations obtained in Takatsuka (2014)—namely, (6) and (7)—no longer survive, and that his results depend on the timing of taxation. We then propose an alternative model that completely replicates the results derived by Takatsuka.

3.1 Taxation on production

When the tax is imposed at the instant of production, the cost function is defined as follows.

$$c(x_{NN}, x_{NS}) = fw + (mw + t_u + t_a p_{NN})x_{NN} + (mw + t_u + t_a p_{NS})\tau_M x_{NS}, \tag{8}$$

$$c(x_{SS}, x_{SN}) = fw^* + (mw^* + t_u + t_a p_{SS})x_{SS} + (mw^* + t_u + t_a p_{SN})\tau_M x_{SN}. \tag{9}$$

The critical variation of (8) and (9) from (4) and (5) is the term given by $t_a p_{NS} \tau_M x_{NS}$ and $t_a p_{SN} \tau_M x_{SN}$. The unit tax and ad valorem tax are now imposed when the firms produce the differentiated product (before the product “melts” in the process of transportation). In this case, the prices are set as

$$p_{NN} = \frac{(mw+t_u)\sigma}{(1-t_a)(\sigma-1)}, \quad p_{SS} = \frac{(mw^*+t_u)\sigma}{(1-t_a)(\sigma-1)},$$

$$p_{NS} = \frac{(mw+t_u)\sigma}{(1-\tau_M t_a)(\sigma-1)}, \quad p_{SN} = \frac{(mw^*+t_u)\sigma}{(1-\tau_M t_a)(\sigma-1)}.$$

The price levels differ from those derived in Takatsuka (2014); thus, the market-clearing conditions also differ.

$$\left[\frac{\left(\frac{1}{1-t_a}\right)^{-\sigma}\frac{p_A}{w}L}{\left(\frac{1}{1-t_a}\right)^{1-\sigma}n+\left(\frac{1}{1-\tau_M t_a}\right)^{1-\sigma}n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}\phi_M}\right] + \left[\frac{\left(\frac{1}{1-\tau_M t_a}\right)^{-\sigma}\frac{p_A^*}{w}L^*\phi_M}{\left(\frac{1}{1-t_a}\right)^{1-\sigma}n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}+\left(\frac{1}{1-\tau_M t_a}\right)^{1-\sigma}n\phi_M}\right]=\frac{f\sigma}{\mu}, \tag{10}$$

$$\left[\frac{\left(\frac{1}{1-\tau_M t_a}\right)^{-\sigma}\frac{p_A}{w^*}L\phi_M}{\left(\frac{1}{1-t_a}\right)^{1-\sigma}n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}+\left(\frac{1}{1-\tau_M t_a}\right)^{1-\sigma}n^*\phi_M}\right] + \left[\frac{\left(\frac{1}{1-t_a}\right)^{-\sigma}\frac{p_A^*}{w^*}L^*}{\left(\frac{1}{1-t_a}\right)^{1-\sigma}n^*+\left(\frac{1}{1-\tau_M t_a}\right)^{1-\sigma}n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}\phi_M}\right]=\frac{f\sigma}{\mu}. \tag{11}$$

(10) and (11) do not coincide with (6) and (7), thus suggesting that the results derived in Takatsuka (2014) will not hold without some revision.

If the unit tax and ad valorem tax are imposed on production, the timing of the ad valorem tax will now differ from that seen in Takatsuka's setting. In this case, the transportation cost is included in the ad valorem tax base, which means that the firms will pay a higher tax compared to the tax paid when the ad valorem tax is imposed after the shipment. This reduces the marginal revenue from export, which is the main explanation for why our results differ from Takatsuka's (2014).

3.2 Taxation on consumption

When the tax is imposed at the time of consumption, the cost function is given by

$$c(x_{NN}, x_{NS}) = fw + (mw + t_u + t_a p_{NN})x_{NN} + (mw\tau_M + t_u + t_a p_{NS})x_{NS}, \tag{12}$$

$$c(x_{SS}, x_{SN}) = fw^* + (mw^* + t_u + t_a p_{SS})x_{SS} + (mw^*\tau_M + t_u + t_a p_{SN})x_{SN}. \tag{13}$$

The amount of production is $\tau_M x_{rs}$, but the amount of the product that reaches the consumers is x_{rs}. Hence, the firm's tax payments in the case of the ad valorem tax and the unit tax are given by $t_u x_{rs}$ and $t_a p_{rs} x_{rs}$, respectively. In this case, the firms set their prices at

$$p_{NN} = \frac{(mw + t_u)\sigma}{(1 - t_a)(\sigma - 1)}, \quad p_{SS} = \frac{(mw^* + t_u)\sigma}{(1 - t_a)(\sigma - 1)},$$
$$p_{NS} = \frac{(mw\tau_M + t_u)\sigma}{(1 - t_a)(\sigma - 1)}, \quad p_{SN} = \frac{(mw^*\tau_M + t_u)\sigma}{(1 - t_a)(\sigma - 1)}.$$

Using these equations, the market-clearing conditions are given by

$$(1 - t_a)\left[\frac{\frac{p_A}{w}L}{n + n^*\left(\frac{mw^*\tau_M + t_u}{mw + t_u}\right)^{1-\sigma}\phi_M} + \frac{\frac{p_A^*}{w}L^*\phi_M}{n^*\left(\frac{mw^* + t_u}{mw\tau_M + t_u}\right)^{1-\sigma} + n\phi_M}\right] = \frac{f\sigma}{\mu}, \tag{14}$$

$$(1 - t_a)\left[\frac{\frac{p_A}{w^*}L\phi_M}{n\left(\frac{mw + t_u}{mw^*\tau_M + t_u}\right)^{1-\sigma} + n^*\phi_M} + \frac{\frac{p_A^*}{w^*}L^*}{n^* + n\left(\frac{mw\tau_M + t_u}{mw^* + t_u}\right)^{1-\sigma}\phi_M}\right] = \frac{f\sigma}{\mu}. \tag{15}$$

These conditions differ from (6) and (7), indicating that Takatsuka's results regarding the equilibrium location pattern do not hold without modification.

If the unit tax and the ad valorem tax are imposed after the shipment, the transportation cost is deducted from the unit tax base, which is smaller than the tax base when tax is imposed before the shipment. In this case, the marginal cost for the firm is lower than the marginal cost in Takatsuka's setting, in which the transportation cost is not deducted from the unit tax base. The lower marginal cost enables the firm to set a lower price, and so the equilibrium location differs from that of Takatsuka (2014).

3.3 Alternative proposal

The iceberg transportation cost model of Takatsuka (2014) assumes that the firms produce $\tau_M(>1)$ units of product, but that only one unit of product reaches the consumers; thus, they consume only one unit of product. Instead, we assume here that the firms produce $\tau_M(>1)$ units of product and that $\tau_M(>1)$ units of product reach the consumers. However, at the instant of consumption, the consumers can consume only one unit of product. The critical feature of this setting is that, while in the Takatsuka model the firms incur the transportation cost, in our model, the consumers bear the transportation expense.[3]

[3] Under the assumption of iceberg transportation cost, it appears that the "principle of three" equivalence of gross domestic product (GDP) is not realized, without considering who incurs the transportation costs. However, the mismatch of the "principle of three" equivalence of GDP is dissolved if we implicitly assume there is a competitive transportation sector, in which the iceberg transportation cost meets the disbursements for the services provided by the transportation sector. The treatment of transport sector will be briefly discussed in the final section, referring the works incorporating activities in the transport sector.

When the consumers incur the product transportation costs, the maximization problem for consumers in region N is defined as

$$\begin{aligned} \max \quad & U = \mu \ln M + A \\ \text{s.t.} \quad & \int_0^n p(i)x(i)di + \int_0^{n^*} \tau_M p(j)x(j)dj + p_A A = I, \end{aligned}$$

where I is the income and

$$M \equiv \left[\int_0^n x(i)^{\frac{\sigma-1}{\sigma}} di + \int_0^{n^*} x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}.$$

The first-order conditions give the demand function as

$$M = \frac{\mu}{P}, \quad A = \frac{I - \mu p_A}{p_A}, \quad x(i) = \mu p_A \frac{p(i)^{-\sigma}}{P^{1-\sigma}}, \quad x(j) = \mu p_A \frac{[\tau_M p(j)]^{-\sigma}}{P^{1-\sigma}},$$

where P is the price index; additionally,

$$P \equiv \left\{ \int_0^n p(i)^{1-\sigma} di + \int_0^{n^*} [\tau_M p(j)]^{1-\sigma} dj \right\}^{\frac{1}{1-\sigma}}.$$

The demand function in region S can be derived in a similar fashion.

The profit of firms in each region is given by

$$\begin{aligned} \pi_N &= p_{NN}x_{NN} + \tau_M p_{NS}x_{NS} - c(x_{NN}, x_{NS}), \\ \pi_S &= p_{SS}x_{SS} + \tau_M p_{SN}x_{SN} - c(x_{SS}, x_{SN}), \end{aligned}$$

where

$$c(x_{NN}, x_{NS}) = fw + (mw + t_u + t_a p_{NN})x_{NN} + (mw + t_u + t_a p_{NS})\tau_M x_{NS}, \quad (16)$$

$$c(x_{SS}, x_{SN}) = fw^* + (mw^* + t_u + t_a p_{SS})x_{SS} + (mw^* + t_u + t_a p_{SN})\tau_M x_{SN}. \quad (17)$$

(16) and (17) show that the firms producing $\tau_M x_{rs}$ units sell the same amount to the consumers in other regions. In this case, the timing of taxation becomes neutral; the results are not affected by the time at which the government imposes tax.

The price levels are determined as

$$p_{NN} = p_{NS} = \frac{(mw + t_u)\sigma}{(1 - t_a)(\sigma - 1)} \quad \text{and} \quad p_{SS} = p_{SN} = \frac{(mw^* + t_u)\sigma}{(1 - t_a)(\sigma - 1)},$$

which suggests that the firms set the same price on the product for domestic consumption and the product bound for other regions. The price of the product from other regions includes the cost of transportation, however.

The market-clearing conditions are now given by

$$(1-t_a)\left[\frac{\frac{p_A}{w}L}{n+n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}\phi_M}+\frac{\frac{p_A^*}{w}L^*\phi_M}{n^*\left(\frac{MC_u^*}{MC_u}\right)^{1-\sigma}+n\phi_M}\right]=\frac{f\sigma}{\mu}, \quad (18)$$

$$(1-t_a)\left[\frac{\frac{p_A}{w^*}L\phi_M}{n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}+n^*\phi_M}+\frac{\frac{p_A^*}{w^*}L^*}{n^*+n\left(\frac{MC_u}{MC_u^*}\right)^{1-\sigma}\phi_M}\right]=\frac{f\sigma}{\mu}, \quad (19)$$

which coincide with (6) and (7), thus showing that the equilibrium location patterns fully coincide with that derived by Takatsuka (2014).

In summary, when consumers incur the transportation cost, the tax effects on the location pattern shown by Takatsuka (2014) are preserved. The separation of the tax burden and the transportation cost burden makes the timing of taxation irrelevant; thus, it is critical to abstract the pure effects of taxation on the location pattern.

4 Concluding remarks

Takatsuka's (2014) analysis treats the timing of taxation differently for unit tax and ad valorem tax. The present note aligns the timing and examines whether the results regarding the equilibrium location pattern still hold. The first part of this note shows that Takatsuka's results do not survive if the government imposes a tax at either the point of production or the point of consumption. In its second part, the note presents a proposed modification that replicates Takatsuka's results. The essential argument is to split the burden of transportation costs and taxation; if firms bear the burden of taxation and consumers incur the transportation costs, then the location pattern has no relation to the timing of taxation.

In this note, we present additional analysis of Takatsuka (2014) and clarify how firm location changes as the timing of taxation changes. The critical finding is that a firm's location depends on when it is taxed. In the presence of transportation costs, a tax before shipment does not deduct transportation cost from the tax base, but a tax after shipment will deduct it. This suggests that the tax burden differs depending on the timing of taxation, thus leading us to conclude that the government's choice of timing of taxation affects the equilibrium location pattern. Based on our results, we derive one clear policy implication. In addition to the standard policy options—such as taxes and subsidies—the government can use the timing of the imposition of a tax as a policy instrument by which to control firm allocation within a country. It is sometimes difficult for a financially pressed government to provide subsidies for firms' relocation, but our findings show that the government still has the option to change a firm's relocation incentive by choosing when to tax.

Before closing this note, we refer to a standard but less general assumption applied herein. To focus on the role of the timing of taxation in the model with transportation cost, the analysis in the present model simply assumes that the transportation cost is of an iceberg type. This enables us to derive clear results so as to reexamine Takatsuka's findings, but our results in Sect. 3.3 may not survive if we explicitly model the behavior in the transportation sector. The exogenous iceberg transportation cost is specified by the disappearance of a part of the product in the transportation process. In such a setting, labor employment in the transportation sector is eliminated from the model, and that labor wage is determined based on the arbitrage condition between the manufacturing sector and the agriculture sector, as in standard new economic geography (NEG) and Takatsuka models. If we explicitly account for the behavior of the transportation sector, we find that labor wages are determined by interactions among three sectors—namely, agriculture, manufacturing, and transportation. In this case, our main results would change, since labor wages would differ between our model and that of Takatsuka. This is because the labor demand in the transportation sector depends on who incurs the transportation cost. As our main focus is on clarifying the effects of the timing of taxation on a firm's relocation, however, we skip the formal formulation of the transportation sector. Nonetheless, recent studies explicitly incorporate the transportation sector, and this has received much attention in the NEG literature (Takahashi 2006; Behrens et al. 2009; Behrens and Picard 2011). Analyses of these issues are important from a practical perspective and warrant future research.

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