



# Economy-wide rebound and the returns on investment in energy efficiency

Andrew Jarvis · Carey W King

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**Abstract** In this paper we re-examine the relationship between global Gross Domestic Product (GDP), Primary Energy Use (PEU) and Economic Energy Efficiency (EEE) to explore how investment in energy efficiency causes rebound in energy use at the global scale. Assuming GDP is a measure of final useful work, we construct and fit a biophysics-inspired nonlinear dynamic model to global GDP, PEU and EEE data from 1900—2018 and use it to estimate how energy efficiency investments relate to output growth and hence economy-wide rebound effects. We illustrate the effects of future deployment of enhanced energy efficiency investments using two scenarios through to 2100. The first maximizes GDP growth, requiring energy efficiency investment to rise ~twofold. Here there is no decrease in PEU growth because economy-wide rebound effects dominate. The second scenario minimizes PEU growth by increasing energy efficiency investment ~3.5 fold. Here PEU and GDP growth are near fully decoupled and rebound effects are minimal, although this results in a long run, zero output growth regime. We argue it is this latter regime that is compatible with the deployment of enhanced energy efficiency to meet

climate objectives. However, while output growth maximising regimes prevail, efficiency-led pledges on energy use and emissions reduction appear at risk of failure at the global scale.

**Keywords** Energy efficiency · Economy-wide rebound · Return on investment · Productive structure · Final useful work

## Introduction

Improving energy efficiency has re-emerged as an important policy focus internationally. It comprises approximately 40 percent of the Nationally Determined Contributions (NDCs) currently pledged under the Paris climate agreement (UNEP, 2021) while COP28 ratified an objective to double to growth rate of energy efficiency globally by 2030. Alongside this, energy efficiency improvements are also proposed for addressing slowing output growth in developed economies (e.g. BEIS, 2018), with global energy supply shocks having acted to amplified this need. Resolving how climate, growth and energy security objectives align has become central to debates surrounding the possibility of attaining so-called ‘green growth’ (OECD, 2011), where energy efficiency improvements are used to partially decouple economic outputs from primary energy inputs.

There are mounting concerns over the green growth narrative, largely driven by the widespread

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A. Jarvis (✉)  
Lancaster University, Lancaster, UK  
e-mail: a.jarvis@lancs.ac.uk

C. W. King  
University of Texas, Austin, USA  
e-mail: careyking@mail.utexas.edu

appreciation of the growth imperative at the centre of government and business, and how this agenda might undermine any desire to reduce society's environmental footprint. A significant factor in this debate is an increasing appreciation of rebound and backfire effects, or Jevons Paradox, which emphasise that energy efficiency improvements can lead to output growth that in turn drives input growth, rather than input degrowth (Jevons, 1866). This appears to be especially true as both time and length scales increase (Bruns et al., 2021), presumably as more of the relevant feedback processes take effect. However, understanding of these economy-wide, long run feedbacks appears limited and in need of further research (Saunders et al., 2021).

Most of the economic analyses underpinning climate scenarios implicitly assume that rebound effects are small, and hence the users of these models conclude increasing energy efficiency can play a central role in reducing greenhouse gas emissions via reductions in energy use (Riahi et al., 2017). Some go further and argue energy efficiency should be a central focus for climate policy (Grübler et al., 2018). Although numerous estimates of rebound effects suggest they are not large enough to undermine policy objectives on energy efficiency (Gillingham et al., 2013, 2016), these tend to be made on relatively short time and length scales, or with models that assume energy is a minor factor of output (Brockway et al., 2021). In contrast, recent estimates using models that attempt to capture large-scale and long-term energy feedbacks tend to indicate rebounds are significant, and that policy focused on using energy efficiency improvements to reduce energy inputs run a significant risk of failure (Stern, 2017; Bruns et al., 2021; Kander et al., 2020; Brockway et al., 2021). In addition, our collective experience of the industrial era is that, over time, machines have indeed become more efficient while global society has continually increased its rate of energy consumption.

The risks around economy-wide rebound effects are probably best represented through the dynamic relationship between Primary Energy Use (PEU) inputs and Gross Domestic Product (GDP) outputs, particularly at the global scale where our collective efforts to manage climate ultimately plays out. However, the debate over the roles energy and energy efficiency play in determining economic output is long and contested, with no clear picture having yet

emerged (Brockway et al., 2018; Kalimeris et al., 2014; Stern, 2011). On the one hand there are those who point out the relatively small fraction of production costs imposed by energy mean that energy plays a minor role in growth (Dennison, 1979; Grubb et al., 2018; Newberry, 2003). On the other hand, there are those who emphasise how energy use necessarily underpins all activity, including that of economies (Ayres & Warr, 2009; Garrett, 2011; King, 2021; Kümmel, 2011; Sakai et al., 2019; Soddy, 1926).

A common approach employed in energy analysis is to partition economic output between primary energy and efficiency through the identity  $GDP = EEE \times PEU$ , where EEE is referred to as the Economic Energy Efficiency, and its inverse the energy intensity of the economy (Kander et al., 2020). Both EEE and energy intensity have been used extensively in energy efficiency research, especially in macroscopic national/global scale studies, even though there is a general acceptance that the relationship between EEE and energy efficiency is confounded by numerous non-energy factors (Saunders et al., 2021).

Although subjective judgements are deeply embedded throughout the accounting that underpins all GDP data, because the aim is to attempt to capture the annual production of real economic value, despite all its failings, GDP data are designed to capture the valued physical activity in economic systems, even if that activity is often highly dematerialised/information rich. For this physical activity to be valued it needs to be in some sense useful, even if at times we might struggle to see this utility through the deep complexity shrouding the economy. This line of arguing is consistent with the emerging conclusion that GDP parallels the rate useful work is performed by the economy (Garrett, 2011; Serrenho et al., 2016; Warr et al., 2010).

If GDP is proportional to the rate of useful work is performed by the economy, then  $GDP = EEE \times PEU$  is more than simply an identity, with EEE now being proportional to the thermodynamic energy efficiency of the economy when translating primary energy flows into the rate at which final useful work is performed. This thermodynamic view of the economy suggests that the only additional inputs and outputs we need to consider are those of matter and information. As we will see below, these directly impact the energy efficiency of the economy in ways that need not be considered omissions from an analysis that

focuses exclusively on GDP, PEU and EEE data, especially at the global scale where economic interactions are internalised.

There is, however, an important difference between how analysts traditionally quantify energy efficiency and the efficiency being measured in EEE. If GDP is indeed proportional to final useful work, then EEE includes all relevant energy transformations of the economy, whereas contemporary energy efficiency estimates invariably exclude transformations downstream of the provision of ‘energy services’. For example, the provision of heat as an energy service to the car-making sector does not describe the productive structure (cars) created by use of that heat. Likewise, the provision of transport fuels to power these cars falls upstream of the valued final useful work they perform. Therefore, rather than viewing EEE as corrupted by additional inputs and outputs, it could be argued that it is genuinely system-spanning, providing GDP is indeed measuring the rate final useful work is being done.

Rather than attempt to add further empirical evidence to underpin any linear scaling between GDP and the rate of final useful work, we instead assume this relationship as a central tenet of this paper in order to explore the interpretation of economy-wide rebound dynamics under this assumption. By doing so, we build on both the empirical evidence to date and the theoretical position that society necessarily values useful work, even if the accountancy process tracking this valuation might be somewhat flawed and the definition of what is useful is again shrouded in deep complexity.

The final useful work of the economy is to perform work on itself and its surroundings (Garrett, 2011). Although this work takes many forms, the outcome is necessarily the spatial ordering of matter leading to the creation of structures. Because the work involved is, by definition, judged to be useful,<sup>1</sup> these structures must also be in some sense useful or productive. The only definition of productive afforded here is structure that is able to facilitate future flows of final useful energy and hence work. These productive structures

are principally space-filling resource acquisition, distribution, and end use networks (Jarvis et al., 2015) and the ordering of matter in their creation encodes information, which goes some way to explaining why, although we might attribute GDP to useful physical activity, this activity is not solely material, but necessarily involves a significant role for information. Although we identify these productive structures as being physical entities (space-filling networks), they are not simply physical capital, but are instead mixtures of both human and produced capital.

We draw two important conclusions from the preceding paragraphs that translate into two further central assumptions in this work. The first conclusion is that the investment of final useful work into the creation of productive structure can be partitioned into either increasing the flow of primary energy into the system, or increasing the internal efficiency of that system when transducing primary energy into flows of final useful work (output=efficiency x input). Thus, the productive structures giving rise to output can be seen as having two traits, their ability to demand/supply inputs, and the efficiency with which these inputs are translated from input to output. It is this realisation that enables an analysis of rebound effects because we can now consider the impacts of incremental increases in investment of final useful work in energy efficiency on the growth rates of primary energy inputs. The second conclusion is that, by definition, all output of final useful work is being invested into the creation of productive structure. Put another way, there is no room in this framework for final useful work to be simply ‘consumed’ in ways that have no bearing on productive structure creation and hence future flows of final useful work. If there is consumption of energy, it is through the fraction of the primary energy flow not translated into final useful work and hence not valued i.e. all waste energy deriving from systemic inefficiency. In time even the stock of useful work decays, reminding us that productive structure is simply a locally-ordered slowing of the rate of free energy dissipation and natural drive to increase entropy.

The ‘all valued output is invested’ paradigm employed here appears to challenge a central tenet of orthodox economics, that people are consumers investing to maximise that consumption. However, not only is our ‘all valued output is invested’ paradigm a logical extension of the productive

<sup>1</sup> Here *useful* does not mean good, but simply the physical ability to do work. Likewise, *productive* also does not mean good, but simply the ability to facilitate the future flows of final useful work.

consumption literature (Steger, 2002), it is also consistent with accounting procedures that include inventories for the creation of human capital (World Bank, 2021). These inventories suggest human capital stocks are large enough to require the reclassification of consumption into investment into human capital. Of course, this paradigm also requires us to redefine the turnover timescales for stocks of productive structure, which traditionally are defined as having a >3 year cut off in national accounts (Chester et al., 2024), but in reality can exist on any timescale from seconds to centuries or more. It also requires us to recognise that shorter-lived productive structures can be and are intermediaries in the creation of longer-lived productive structures. For example, the coffee we drink has initial effects lasting only hours, but it can help leverage effects that can last a lifetime.

Because productive structures are necessarily space-filling networks, the economy can be viewed as inhabiting a spatial domain (Jarvis et al., 2015). If the efficiency of these networks is captured through an energy efficiency metric then, by definition, the inputs of primary energy must be a metric largely devoid of efficiency traits, thereby only describing the size of the domain being occupied by the networks comprising the economy. As a result, we will often refer to primary energy flow as a surrogate for size of the domain occupied by the global economy. One way of conceptualising this is through acknowledging that energy resources are distributed throughout the earth system and hence are captured by the economy growing into them to occupy that space. If primary energy flows relate to the size of the economy, then energy efficiency relates to the orientation, information, and complexity of the structures within this domain. We assume this also applies to raising efficiency through raising the exergy value of the primary energy portfolio, through say increasing the proportion of direct electricity production.

In this paper we develop a model based on the premise that GDP measures the rate the economy performs useful work on itself and its surroundings. We then assume this useful work is invested exclusively into the creation of productive structures that affect both the rate primary energy enters the economy and the efficiency with which this is transduced into useful work. Sect. "The model" describes this model, and Sect. "Model calibration and interpretation" describes the data analysis that derives parameter values for the

model using global GDP and PEU data 1900—2018. Sect. "Rebound effects" applies the model to estimate the magnitude of rebound effects associated with differing levels of investment in energy efficiency. Sect. "Cautions and limitations" highlights some concerns and gaps in our understanding, while Sect. "Conclusions" concludes.

## The model

### Core model assumptions

A1: GDP is a proxy for the rate the economy performs final useful work on itself and its surroundings, and thus GDP and useful work are linearly related.

A2: Productive structures have two distinct traits that can be developed independently through the investment of output: their thermodynamic efficiency, and their size and hence ability to capture primary energy.

A3: The final useful work being performed by the economy is invested exclusively into the creation of productive structure.

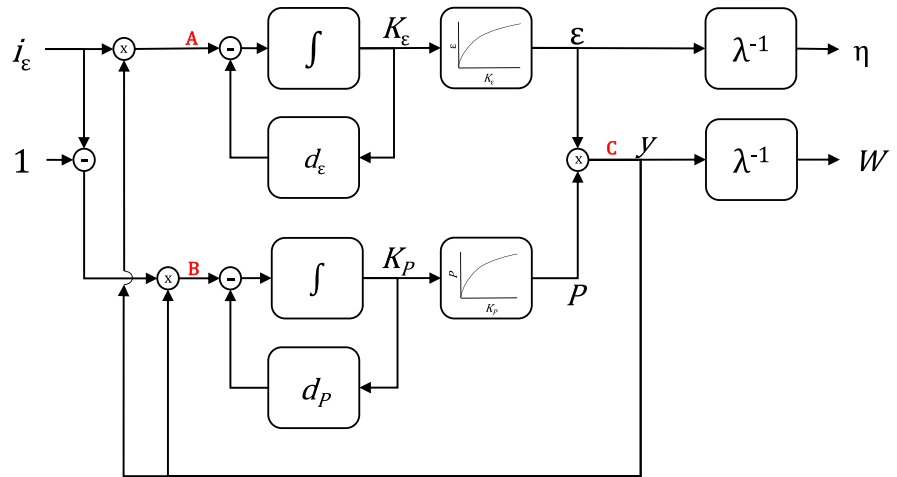
### Model structure

The system diagram for the model is given in Fig. 1. We start describing our model by recasting the identity  $GDP = EEE \times PEU$  into its energy physics equivalent. If the annual flow of primary energy into the economy is  $P$ , then the work done by this energy input,  $W$ , is given by

$$W = \eta P \quad (1)$$

where  $\eta$  is the thermodynamic energy efficiency of the economy at converting the flow of primary energy into final useful work. We assume (A1)  $GDP = y = \lambda W$ , where  $\lambda$  converts from final useful work into the equivalent monetary unit and is assumed constant providing GDP is expressed in constant units. Now  $EEE = \varepsilon = \lambda \eta$ . For completeness we note  $(1 - \eta)P$  is the rate primary energy is wasted when traversing the economy, although the rate the economy sheds waste heat is marginally higher than this if we also take into account the subsequent decay of its structures.

**Fig. 1** The system diagram for the model detailed in Sect. "The model" (see Eqs. 1–3 for definitions). The ROI Eqs. (4a,b) are derived from the forward path gains A → C (ROI<sub>ε</sub>) and B → C (ROI<sub>p</sub>). ∫ represents time integration from initial conditions



For familiarity, and because we exploit output data measured in monetary units, we retain monetary units hereon. Hopefully, this should also assist those who would rather see  $GDP = EEE \times PEU$  only as an identity and hence view our analysis as a heuristic account of economy-wide rebound effects associated with EEE rather than energy efficiency.

We assume (A2) that the two factors,  $\epsilon$  and  $P$ , which determine output,  $y$ , are each the product of the accumulation of investments of final useful work into productive structures. Because  $\lambda$  converts between the flow of final useful work and its monetary equivalent in GDP, then  $\lambda$  also converts between an accumulated stock of final useful work and the equivalent capital value of that stock,  $K$ . Therefore, assuming final useful work can only be invested in the creation of productive structures (A3), the net change in the stock of capital invested characterizing the energy efficiency of productive structure,  $K_\epsilon$ , is given by the first order conservation

$$\dot{K}_\epsilon = i_\epsilon y - d_\epsilon K_\epsilon, \tag{2a}$$

where  $i_\epsilon$  is the proportion of final useful work invested into developing the energy efficiency of productive structures, and  $d_\epsilon$  is the representative decay rate for these structures. Likewise, the net change in the stock of capital attributed to filling space and hence capturing primary energy,  $K_p$ , is given by the first order conservation,

$$\dot{K}_p = (1 - i_\epsilon)y - d_p K_p \tag{2b}$$

where, again,  $d_p$  is the representative decay rate for the associated productive structures.

National account methodologies artificially impose a ~3 year cut-off between definitions of capital stocks and consumer goods (Chester et al., 2024), and it is this that artificially gives rise to consumption simply because it cannot be attached to the creation of short-lived productive assets. It is clear from everyday experience that the economy is awash with <3 year productive structures yielding future returns (e.g., a short-lived toy enables child’s brain to development that might last a lifetime), even if we currently believe we consume these structures. From this argument  $d_{\epsilon,p}$  represents the full spectrum of turnover timescales in the economy and not simply the >3 year structures. As a result, we estimate  $d_{\epsilon,p}$  as free parameters rather than pre-assuming their value as would be the norm. We also expect  $d_{\epsilon,p}$  to be larger than would be traditionally employed in capital accounting because it necessarily includes the effects of <3 year structural turnover.

We close our framework by relating the capital stocks,  $K_\epsilon$  and  $K_p$ , to the primary energy use and energy efficiency traits for which they were invested. We anticipate return to scale effects associated with increasing the quantity of productive structure dedicated to either trait such that

$$\epsilon = a_\epsilon K_\epsilon^{b_\epsilon} \tag{3a}$$

$$P = a_p K_p^{b_p} \tag{3b}$$

where  $a_{\epsilon,p}$  and  $b_{\epsilon,p}$  are scaling parameters.



For primary energy use, a range of mechanisms can be invoked for return to scale effects. Firstly, there is the obvious tendency to pick the lower hanging fruit first when harvesting energy resources, hence requiring ever increasing investments to access future units of supply. Given energy resources are spatially distributed in the earth system, another way of seeing this effect is that distribution costs necessarily increase over time as the economy expands and energy resources become more distant from where they are converted into final useful work (Jarvis, 2018; Jarvis et al., 2015). This argument extends to energy demand since increasing the number of end-use service units in a system can also be seen to expand the domain of that system and hence the mean path length of distribution networks (Banavar et al., 1999). Finally, the metabolic scaling literature indicates that the interface across which energy flows into a system can have a lower dimensional scale to the system itself, and this necessarily creates sub-linear scaling effects (Ballesteros et al., 2018; Banavar et al., 2010).

Efficiency should also experience declining returns to scale because energetically more efficient systems tend to be structurally more complex (Ruzzenenti & Basosi, 2008), requiring higher levels of investment to find, make and maintain future configurations. Specifically, finding more efficient, information rich configurations of the economy becomes progressively harder as the probability of discovering these ‘better’ (lower entropy, more structured) configurations declines. These diminishing returns are exacerbated by the fact that any particular configuration represents significant lock-in of investments, such that the search for better configurations is always restricted by what Kauffman (2002) would refer to as “the adjacent possible”.

Through combining Eqs. (1), (3a) and (3b) we get  $y = a_\epsilon a_P K_\epsilon^{b_\epsilon} K_P^{b_P}$ , which shares some structural similarity to orthodox, nonlinear two factor aggregate production functions. However, rather than seeing labour, capital and total factor productivity as factors driving output, the net accumulation of investments of final useful work in the creation of productive structure are partitioned into the fully endogenous evolution of thermodynamic energy efficiency and size/primary energy acquisition which, we argue, are fundamental traits of the associated networks. These networks necessarily include people who, like machines and

buildings, both fill space and determine the efficiency productive structures.

#### Returns on investment, growth and rebounds

The scaling relationships in Eq. (3a,b) are fundamental to our analysis because at any point in time they provide differential sensitivities of output returns to inputs of investment. This provides society with choices to affect output growth through how it invests in either using more primary energy, or becoming more efficient with that energy, as defined by  $i_\epsilon$ . The aim initially will be to find what pattern of investment is consistent with observations of GDP, PEU and EEE, 1900–2018, and having done so to interpret this pattern retrospectively through exploring the associated returns on these investments. We define Return On Investment (ROI) as the cumulative additional output,  $\Sigma\Delta y$ , from an incremental annual investment of output to increase the stock associated with either primary energy or efficiency,  $\Delta y_K$ . From Eqs. (2) and (3) and the associated forward path gains in Fig. 1, we derive the ROIs for investments in either  $K_\epsilon$  or  $K_P$  as

$$\text{ROI}_\epsilon = \frac{\Sigma\Delta y}{\Delta y_{K_\epsilon}} = \frac{b_\epsilon a_\epsilon K_\epsilon^{(b_\epsilon-1)} P}{d_\epsilon} \quad (4a)$$

and

$$\text{ROI}_P = \frac{\Sigma\Delta y}{\Delta y_{K_P}} = \frac{b_P a_P K_P^{(b_P-1)} \epsilon}{d_P} \quad (4b)$$

This definition for ROI predicts all future returns to output from a unit investment in either  $K_\epsilon$  or  $K_P$  if all states persist at their current level for the expected lifetime of the investment (Jarvis, 2018). Of course, these states will not stay constant, and so this definition of ROI is only the current view of the future performance of an investment if the economy remained at its current level. Although limited, this is probably the best any investor could hope for in practice given their inability to predict the future path of the global economy accurately.

Following Bruns et al., (2021), we explore rebound effects in relation to primary energy use growth rates following an increase in energy efficiency investment. From Eq. (4a,b) and Fig. 1, the ROI of either primary energy use or energy efficiency is nonlinear in their

stock and linear in their co-factors (either energy efficiency or primary energy use, respectively). This means the output growth rate is increased by switching investment away from the most limiting factor through varying  $i_e$ . It is this zero-sum switching of investment that accounts for observed variations in the growth rates of PEU, GDP and EEE 1900–2018 in the model. Our model contrasts with orthodox growth models in which growth is endogenously controlled through the level of investment of output in produced capital relative to consumption. Although the balance of investment between energy efficiency and levels of primary energy use is an unfamiliar way of viewing investment practice, energy crises and the emerging attempts to manage climate risk have acted to raise awareness of these factors. Furthermore, in theory, attempts to maximise output growth should cause the associated growth factors to become fully co-limiting (von Neumann, 1937), whether these are familiar traits such as human and produced capital, or our less familiar stocks of work associated with the productive structures determining efficiency and primary energy use. We explore this in Sect. "Model calibration and interpretation", using the convergence of the ROI's as a test for this.

The nonlinear behaviour of the ROIs also means that switching investment from primary energy to energy efficiency induces nonlinear effects on primary energy use and hence there is potential for nonlinear rebound effects. These effects are, by definition, economy-wide. Increasing investment in energy efficiency means investment in primary energy inputs are simultaneously proportionally reduced. As a result, the growth rate of primary energy use declines near instantaneously, while the energy efficiency growth is boosted. We call this rapid degrowth in primary energy use following an increase in energy efficiency investment the 'feedforward' effect of the energy efficiency investment (see Figs. 1 and 2).

The feedforward increase in the growth rate of energy efficiency leads to energy efficiency increasing and hence a progressive increase in the ROI of primary energy use,  $ROI_p$  (Eq. 4b). Depending on the state of the system, this causes primary energy use regrowth. Figure 1 reveals this to be a feedback effect of output adjustment. Although the magnitude of this feedback effect is state dependent, without exception it acts to increase primary energy use, i.e. it is a rebound effect (Fig. 2). We call this the 'feedback

(rebound)' effect of the energy efficiency investment (Fig. 2). Again, both the feedforward and the feedback (rebound) effects of the increase in energy efficiency investment are economy-wide and nonlinear.

The remainder of the paper calibrates Eqs. 1–3 using a unified 1900–2018 global GDP, PEU and EEE dataset, and then looks to interpret the result with a specific focus on rebound effects. We end by using the calibrated model to explore two end-member scenarios through to 2100 with an eye on the implications of these forecasts for current energy, economic and climate policy. The first scenario maximises GDP growth, the second minimises PEU and hence by implication emissions growth.

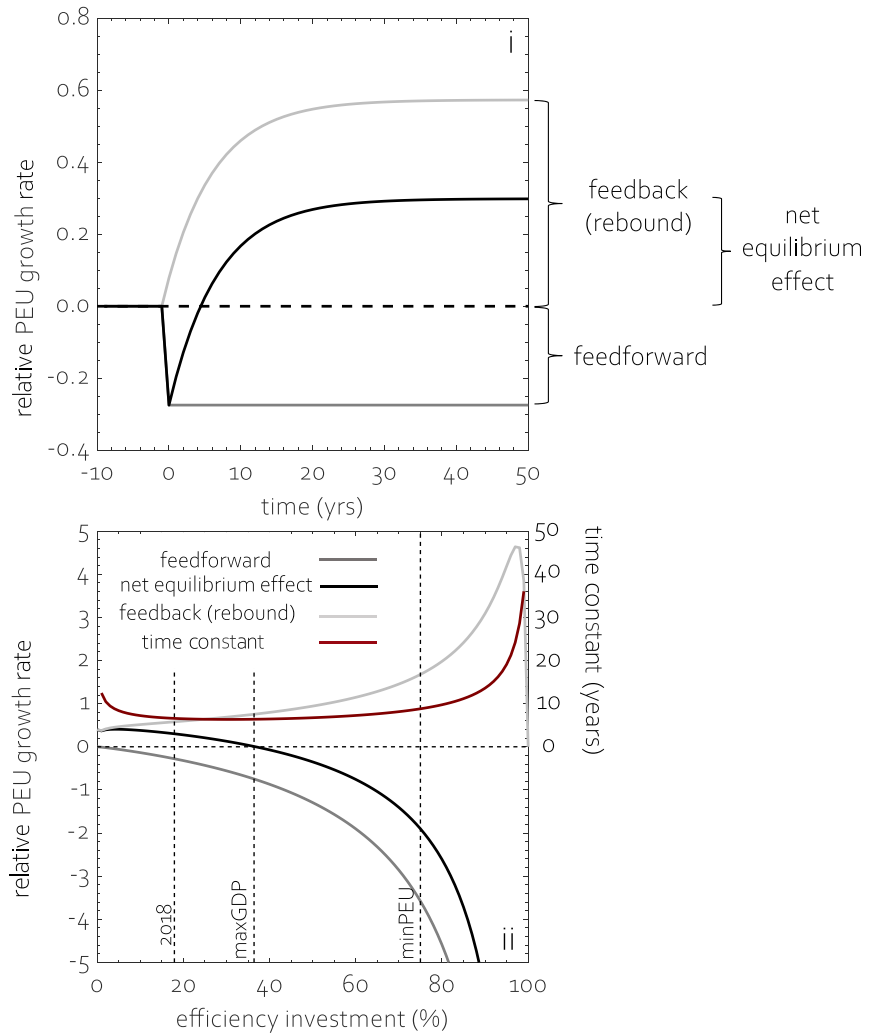
## Model calibration and interpretation

### GDP and PEU data 1900–2018

Given the range of available GDP and PEU observations, and the sensitivity of regression results to the particulars of these data, we have elected to produce a single, homogeneous PEU and GDP series which blends the public domain series listed in Table 1. The eight global GDP series used in this study are a compilation of available, reputable inflation adjusted (constant) data. To reconcile the fact that these data do not have a consistent base unit and compilation method, all GDP series were linearly scaled to the World Bank (WB) constant (2010) market exchange rate (MER) series. This only serves to homogenise units and give each series equal weight when averaging. Similarly, the four global PEU series were linearly scaled to the International Energy Agency (IEA) data, again to reconcile unit differences and methods of compilation.

All analysis is based on the annual averages of the eight GDP and four PEU series listed in Table 1. The final PEU, EEE and GDP series are shown in Fig. 3i along with their associated relative growth rates in Fig. 3ii. Both GDP and PEU grow throughout the period whereas EEE is somewhat stagnant up until the 1970's, after which time it grows steadily. The emergence of growth in EEE after the 1970's results in a relative decoupling of GDP and PEU growth. Prior to this, GDP and PEU growth was approximately 1:1. GDP and PEU growth peaked in the 1950's and 60's and the increases in EEE post 1970

**Fig. 2 i** Changes PEU relative growth rate following a small ( $10^{-6}$ ) perturbation in investment in EEE applied to the 2018 state of the global economy. This is partitioned into its feedforward and feedback (rebound) components. Changes in PEU growth are expressed relative to a 1%/yr change in EEE growth. **ii** The relationship between the feedforward, feedback (rebound) and net equilibrium responses shown in i. given different background levels of efficiency investment. Backfire states are given by net growth > 0. Also shown is the time constant for the feedback (rebound) response



appear to be correlated with steady declines in GDP and PEU growth. Currently, GDP growth appears to be comprised of near equal measures of EEE and PEU growth (Fig. 3ii).

Fitting and model performance

We fit Eqs. 1, 2, and 3 to the log of the EEE, PEU and GDP data shown in Fig. 3i using a standard Levenberg–Marquardt non-linear least squares algorithm, minimising autocorrelation in the model residuals assuming these to be AR(1).

As raised earlier,  $i_{\epsilon}$ , or the relative proportion of final useful work invested in energy efficiency, represents the decision variable in the analysis. Therefore, the aim is to capture how this changes over time. When fitting to

the 1900–2018 data we find this proportion changes significantly either side of World-War 2 (WW2; see Fig. 2.iii), with the pre WW2 regime characterised by high levels of investment in efficiency, and the post WW2 regime prioritising investments in primary energy. Specifically, it is this shift in investment from efficiency to primary energy post WW2, in conjunction with the associated ROI’s of these investments, that accounts for the rapid acceleration of output in the 1950’s and 60’s (see below). We parameterise this change to bring it within the fitting process assuming the following smooth transition

$$i_{\epsilon}(t) = i_{\epsilon 1} + \frac{i_{\epsilon 2} - i_{\epsilon 1}}{1 + e^{-k(t-t_{1/2})}} \tag{5}$$



**Table 1** Global Primary Energy Use (PEU) and constant Gross Domestic Product (GDP) data sources used in this study

Variable	Cover	Source (as of 21/08/2019)
GDP		
World Bank <i>GWP</i> (PPP 2011 USD)	1990—2018	<a href="https://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD">https://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD</a>
World Bank <i>GWP</i> (MER 2011 USD)	1960—2018	<a href="https://data.worldbank.org/indicator/ny.GDP.mktp.kd">https://data.worldbank.org/indicator/ny.GDP.mktp.kd</a>
United Nations (2010 USD)	1970—2017	<a href="https://unstats.un.org/unsd/amaapi/api/file/6">https://unstats.un.org/unsd/amaapi/api/file/6</a>
(Expenditure PPP 2011 USD)	1950—2017	<a href="http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/53">http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/53</a>
(Output PPP 2011 USD)	1950—2017	<a href="http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/54">http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/54</a>
(National-accounts 2011 USD)	1950—2017	<a href="http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/47">http://febpwt.webhosting.rug.nl/Dmn/Templates/Execute/47</a>
Maddison ( <i>CGWP</i> 2011 USD)	1900—2016	<a href="https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx">https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx</a>
Maddison ( <i>RGWP</i> 2011 USD)	1900—2016	<a href="https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx">https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx</a>
PEU		
International Energy Agency ( $\text{EJ yr}^{-1}$ )	1970—2016	<a href="https://webstore.iea.org/world-energy-balances-2018">https://webstore.iea.org/world-energy-balances-2018</a>
British Petroleum ( $\text{Mtoe yr}^{-1}$ )	1965—2018	<a href="https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/xlsx/energy-economics/statistical-review/bp-stats-review-2019-all-data.xlsx">https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/xlsx/energy-economics/statistical-review/bp-stats-review-2019-all-data.xlsx</a>
International Institute Applied Sys- tems Analysis ( $\text{EJ yr}^{-1}$ )	1900—2014	<a href="http://www.iiasa.ac.at/web/home/research/researchPrograms/TransitiontoNewTechnologies/PFUDB.en.html">http://www.iiasa.ac.at/web/home/research/researchPrograms/TransitiontoNewTechnologies/PFUDB.en.html</a>
Energy Information Administration ( $\text{TBtu yr}^{-1}$ )	1980—2016	<a href="https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.01">https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.01</a>

where  $i_{\epsilon 1}$  is the level of efficiency investment before the transition,  $i_{\epsilon 2}$  is the level after the transition,  $t_{1/2}$  is the year associated with a 50% change and  $k$  is the rate constant for that change.

The raw error series appear to be non-constant variance pre v. post WW2. As a result, we weight the errors by dividing by their pre and post WW2 standard deviations prior to decorrelating and minimising. The minimised errors are shown in Fig. 4i.

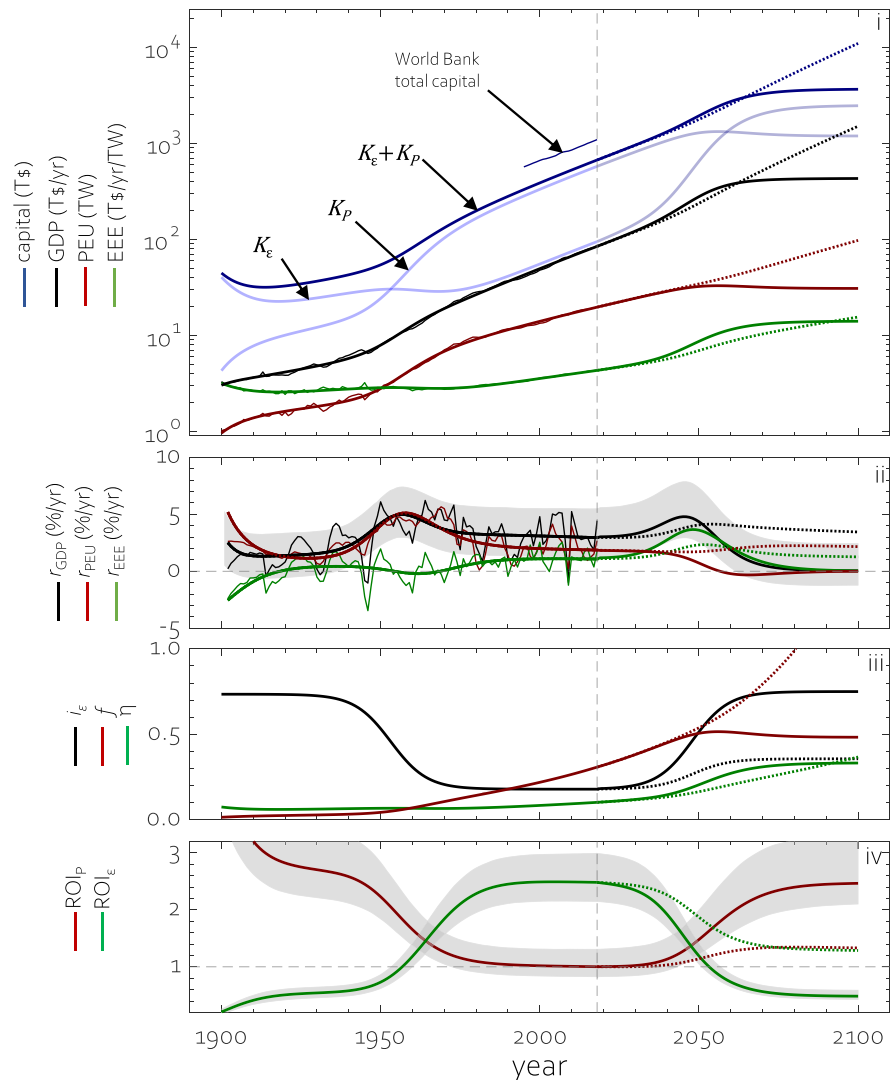
Despite having 12 free parameters (Table 2), four of which simply characterize variations in  $i_{\epsilon}$ , the model converges and the parameter-error space suggests uniqueness in the optimised parameter values (Fig. 4iii). The unfiltered model residuals give a mean absolute error of just 3.46% for GDP, 3.26% for PEU and 2.07% for EEE. All three series of residuals have zero mean (Fig. 4ii), but are significantly auto and cross-correlated. The AR(1) pre-filtering removes all significant short-run autocorrelation from the residual series, but some significant longer run autocorrelation was apparent suggesting the presence of longer cycles

in these annual data. The weighted residuals appear to be near constant variance (Fig. 4i), and each passes an Anderson–Darling test for normality ( $P < 0.05$ ). The estimated parameters are given in Table 2 along with an estimate of their 90th percentile ranges. 90th percentile ranges are used hereon.

#### Depreciation rates

We estimate the depreciation rates for  $K_{\epsilon}$  and  $K_p$  to be 13.00 (10.07—15.93) %/yr and 9.02 (6.27—11.77) %/yr respectively. These are higher than one would expect for economy-wide capital, which is generally ascribed aggregate depreciation rates in the range 3 to 5%/yr (Chester et al., 2024). We reconcile this difference by pointing out that all output is necessarily being invested in our framework and, as such, what would traditionally be considered as consumption is behaving as short-lived productive structure. If aggregate depreciation rates represent the first moment of the inverse of the turnover timescale of capital pools

**Fig. 3** **i** Absolute values and **ii** relative growth rates of total capital, Gross Domestic Product (GDP), Primary Energy Use (PEU) and Economic Energy Efficiency (EEE). All dollar values expressed in constant 2010 dollars. Thin lines are data and thick are the fitted model states 1900—2018. Post 2018 the dashed lines are the maximum GDP growth scenario (maxGDP), while the unbroken lines are the minimum PEU growth-scenario (minPEU; see text). **iii** The estimated investment fraction of GDP into efficiency capital ( $i_\epsilon$ ). Also shown are the estimates of the fraction of available space occupied by the economy ( $f$ ) and thermodynamic efficiency ( $\eta$ ) of the economy (see text). **iv** Returns On Investment (ROI) estimated from Eq. (4a and b). Uncertainty envelopes are 90th percentile ranges



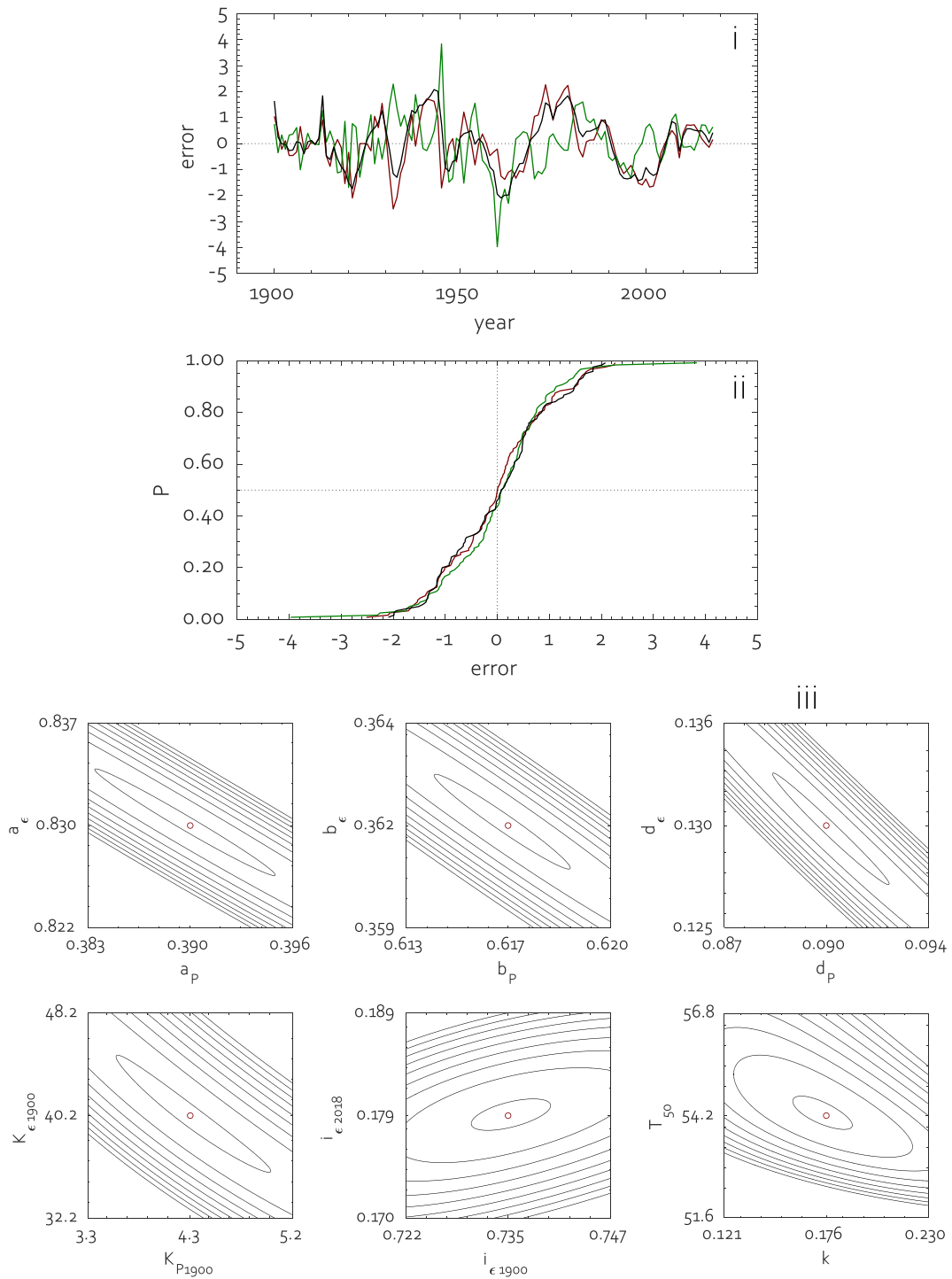
(Chester et al., 2024), then the turnover timescales for  $K_\epsilon$  and  $K_p$  are 7.69 (6.73–8.96) yrs and 11.09 (9.23–13.57) yrs respectively. That  $K_\epsilon$  is shorter lived on average than  $K_p$  is in line with efficiency closely aligning with the more transient, informational states of productive structure.

Our model predicts total capital as  $K_T = K_\epsilon + K_p$ , which is the accumulation of investment of final useful work into overall productive structure, less decay. Comparing this with World Bank total capital (World Bank, 2021) we find our estimate is 71% that of the World Bank figure (Fig. 3i). We suggest that this difference is largely the product of the estimated decay rates of productive structure, which are significantly larger than what might be assumed for either the

produced or human capital comprising the World Bank total.

### Scaling relationships

Figure 5 shows the two estimated scaling relationships for Eq. (3a,b). As predicted, the observed scaling is sub-linear for both primary energy use and efficiency at  $\epsilon \propto K_\epsilon^{0.36}$  and  $P \propto K_p^{0.62}$  (Table 2; Fig. 5). However, the sum of the two scaling exponents is close to one, at 0.98 (0.96–1.00), indicating GDP output is somewhat linear in total capital (Fig. 5), even if it is highly nonlinear in each factor. This estimated net linearity is perhaps not surprising given the global economy grew consistently throughout the 118 years the model



**Fig. 4** **i** The model error series for the relative growth rates of GDP (black), PEU (red) and EEE (green). **ii** The cumulative probability of the reduced model errors. **iii** The sum of square

error response surface as a function of the model parameters for selected parameter combinations. Local minima shown in red

**Table 2** Model parameter estimates. Uncertainties as 90th percentiles

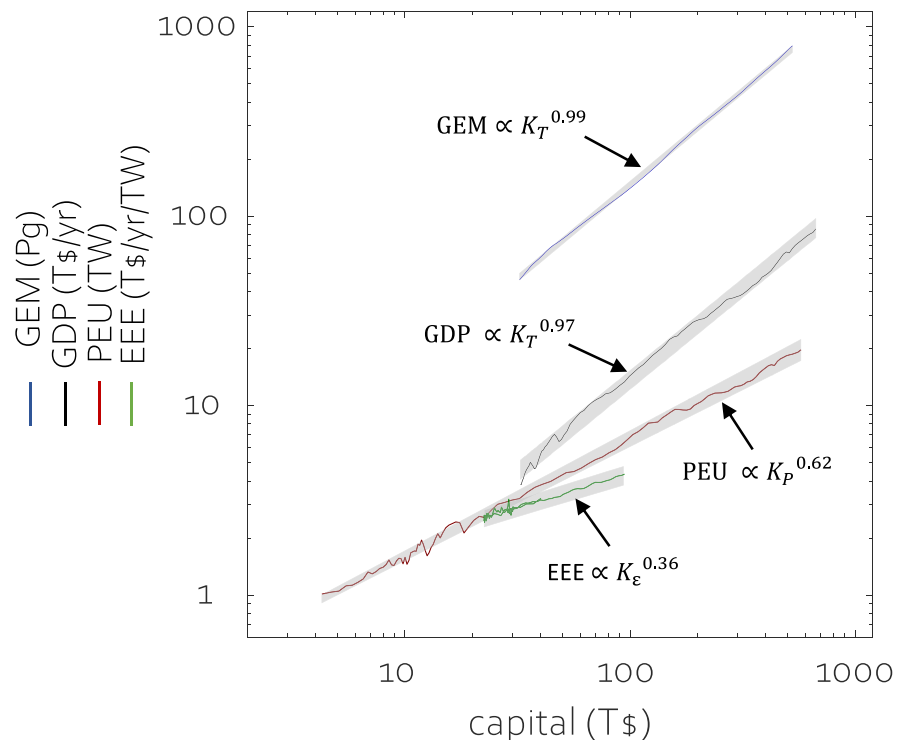
Parameter	Units	Value
$a_\epsilon$	T\$/yr/TW	0.83 (0.71—0.95)
$a_p$	TW	0.39 (0.36—0.42)
$b_\epsilon$		0.36 (0.34—0.38)
$b_p$		0.62 (0.59—0.65)
$d_\epsilon$	%/yr	13.00 (10.07—15.93)
$d_p$	%/yr	9.02 (6.27—11.77)
$K_{\epsilon 1900}$	T\$	40.2 (32.2—48.2)
$K_{p 1900}$	T\$	4.3 (3.8—4.8)
$i_{\epsilon 1}$		0.27 (0.22—0.32)
$i_{\epsilon 2}$		0.82 (0.81—0.84)
$k$	%/yr	0.18 (0.16—0.19)
$t_{1/2}$	yr	1954.2 (1953.5—1954.9)

was constrained on, and that such behaviour is to be expected in a system so focused on maintaining output growth, where there must be strong selective pressure to develop constant return to scale productive structures such that growth is maintained. We also find that our total capital scales near linearly with GDP ( $\text{GDP} \propto K_T^b$ ;  $b = 0.97 \pm 0.03$ ; Fig. 5). We note

constant aggregate returns to scale across factors of production is also invariably assumed in orthodox macroeconomic models (Krugman & Wells, 2015).

If useful work is expended to create productive structures and these structures are necessarily space-filling networks, then because  $K_p$  is, by definition, devoid of efficiency, it should align with the size of the space occupied by the economy. If so,  $K_p$  describes the size of the spatial domain of both the supply side (primary energy resources being harvested) and the demand side (the domain occupied by the sum of all units of demand). The scaling  $P \propto K_p^{0.62}$  signals the penalties associated with increasing domain size are significant, penalties due to, among other things, increasing the mean path length of distribution networks within the economy (Jarvis et al., 2015). However,  $\epsilon \propto K_\epsilon^{0.36}$  signals the penalties on increasing efficiency are approximately twice that of primary energy, underscoring the difficulties associated with finding and developing more efficient networks. Perhaps more importantly though is the fact that this scaling on efficiency again appears just large enough to raise the economy to near linear scaling overall such that growth is maintained. This again underscores the importance of efficiency

**Fig. 5** The relationships between capital and Global Economic Mass (GEM 1900 – 2010; Krausmann et al., 2017), GDP, PEU and EEE. Scaling relationships for PEU and EEE taken from Table 2. Scaling relationships for GEM and GDP are regression fits to log transformed data where  $K_T = K_\epsilon + K_p$ . Uncertainty envelopes are 90th percentile ranges



improvements in maintaining output growth, highlighting the risk of rebound effects (as discussed in Sect. "Rebound effects").

As discussed earlier, productive structures are constructed using final useful work to arrange materials into low probability configurations. This means we should expect to see a relationship between measures of Global Economic Mass (GEM) and our measures of final useful work. If the amount of work required to add and maintain a unit of mass in the global economy is conserved, we should see a linear relationship between GEM and the total capital stock or accumulated final useful work. Figure 5 shows the relationship between our total capital estimates,  $K_T$ , and the GEM estimated by Krausmann et al., (2017) 1900–2010. This relationship is strongly linear ( $GEM \propto K_T^b; b = 0.99 \pm 0.01$ ), giving a mass to capital ratio of  $1.42 \pm 0.23 \text{ kg}/(2010\$)$  for the 110 years covered. Assuming  $\lambda = 1.4 \text{ (2010\$)/MJ}$  (see below), we estimate that a gram of the global economy requires, on average,  $0.51 \pm 0.12 \text{ kJ}$  of final useful work to accrete it into productive structure. Not surprisingly, we arrive at a very similar result if we regress the annual change in final useful work,  $GDP/\lambda$ , on the annual change in GEM. We take the apparent stationarity of the mass density of total capital as evidence in support of our framework.

### Historical narrative

Despite the possibility of near constant growth, what we observe in historical data are relatively radical variations in the growth rate of GDP on a broad range of timescales (Fig. 3ii). We identify three growth regimes. Pre-WW2 output growth is low and volatile. Post WW2 and pre-1970 is marked by a sustained period of increasing output growth in what has become known as the Great Acceleration (Steffen et al., 2015). Finally, the post-1970 era is characterised by modest deceleration of output driven by the secular stagnation of developed economies (Summers, 2015). Our model offers the following account for these regimes.

Pre-WW2, investment in efficiency attracts approximately 70% of the total output (Fig. 3iii). However, the returns on these efficiency investments are poor ( $ROI_\epsilon < 1$ ; Fig. 3iv) such that total capital is actually falling or stagnant throughout the pre-WW2 period (Fig. 3i). It is interesting to note that this period of

shrinking or stagnating total capital and low overall returns on investment is correlated with the era of extreme volatility in output growth, the great recessions/depressions and two world wars. Such volatility would not be helped by the higher turnover rates associated with the efficiency-orientated capital which was dominating the global economy at this time (Fig. 3i).

In contrast to  $ROI_\epsilon$ , pre-WW2  $ROI_p \approx 2.5$  (Fig. 3iv), so shifting investment into physical expansion represents a significant opportunity to increase output growth. Although investors appear slow to realise this, unsurprisingly investment in PEU eventually rises significantly from the 1940's onwards (Fig. 3iii) and, as a result, the global economy experiences rapidly increasing output growth (Fig. 3ii) in what might be referred to as a wave of globalisation given the space-filling character of this investment. Here, increases in output growth are supported almost exclusively by the rate of increase in PEU investment allied to the relatively large ROI of these investments (Fig. 3iii and iv). However, exploiting this opportunity also undermines the returns of this strategy, and by the 1960's these returns approach those for efficiency (Fig. 3iv). Somehow this state must have been experienced in a very real way because the fraction of investment into PEU stops increasing as  $ROI_\epsilon$  and  $ROI_p$  approach parity in the 1960's (Fig. 3iii and iv) and the system transiently experiences a maximum growth state (Fig. 3ii) where neither  $ROI_\epsilon$  and  $ROI_p$  are limiting.

By the 1970's,  $ROI_\epsilon$  exceeds that of  $ROI_p$  and, not surprisingly, the fraction of investment into PEU flattens off, prompting modest increases in efficiency growth leading to modest relative decoupling of PEU and GDP growth (Fig. 3ii and iii; Cserekyei et al., 2016). Because of the persistently high fraction of investment in PEU post 1960s, the ROI of these investments has fallen consistently since that time (Fig. 3iv), mirroring observed slow declines in the values for ROIs of fossil energy sources (Brockway et al., 2019; Hall et al., 2014; King et al., 2015). Energy ROIs are usually defined in terms of returns of primary energy per unit primary energy invested, not final output returns on a unit of final output investment (Brockway et al., 2019; Jarvis, 2018). That aside, declining energy ROI is partially ascribed to the depletion of the more accessible forms of energy (Hall et al., 2014), and this is consistent with

the account we offered for the sublinear scaling of PEU investments because it implies increasing mean path length of the associated energy acquisition networks (Jarvis, 2018). Currently,  $ROI_p \sim 1$ , a condition that has persisted for at least the last two decades (Fig. 3iv). Because this implies little or no growth resulting from investment in primary energy, we identify this as a possible cause for any secular stagnation currently experienced in developed economies. In contrast,  $ROI_\epsilon > 2$  (Fig. 3iv), which we argue helps explain the emerging interest in efficiency-led investments as a means of tackling secular stagnation, over and above any desire to reduce the environmental burden of the economy.

### Rebound effects

Because of the nonlinear dynamics of the model, we explore rebound effects through both simulation and local linearisation. We start by applying a small ( $10^{-6}$ ) single year, impulse perturbation to the fraction of investment in energy efficiency,  $\Delta i_\epsilon$ , at different equilibrium levels of energy efficiency investment,  $i_\epsilon$ . We then analyse the subsequent dynamic adjustment of PEU relative growth rate,  $\Delta r_p$  (Bruns et al., 2021) as if driven by the simultaneous perturbation in the efficiency growth rate,  $\Delta r_\epsilon$ . For ease of interpretation the results are presented as if a unit increase in  $\Delta r_\epsilon$  was the perturbation.

Figure 2i shows the output response for  $\Delta r_p$  when assuming  $i_\epsilon = 18\%$ , the value associated with 2018. From Fig. 2i we see that, despite the nonlinearity of the model, the PEU growth dynamic is locally linear and comprised of a simple two-phase response. This applies across all background equilibria for  $i_\epsilon$ . The first phase dynamic is the near-instantaneous reduction in PEU growth driven by the marginal shift in output investment away from PEU to efficiency. We refer to this as the feedforward effect of energy efficiency investment. The second phase dynamic is the lagged recovery in PEU growth driven by increasing returns to PEU, in turn driven by increasing efficiency (Eq. 4b). We refer to this as the feedback (rebound) effect of energy efficiency investment. For 2018 we estimate the feedforward effect to be a 28% reduction in PEU growth per unit efficiency growth, and the feedback effect a 58% increase in PEU growth in equilibrium. This gives a net effect of +30% i.e. a

30% backfire if the efficiency investment was intended to reduce PEU growth. The time constant for the feedback response at this level of efficiency investment is 6.4 years, leading to a full effect in  $\sim 30$  years (Fig. 2i). These estimates of present-day rebound are significantly more than the economy-wide rebounds reviewed by Brockway et al., (2021), but close to the rebound estimates of Sakai et al., (2019) using an explicit energy-economy model and Bruns et al., (2021) using a vector autoregression analysis of PEU and GDP data. They are also consistent with the theoretical result from the King (2022) biophysical stock and flow consistent macroeconomic growth model.

Figure 2ii shows the feedforward and feedback rebound responses corresponding to differing equilibrium levels of efficiency investment. Even in a relatively simple model like the one developed here, this response is highly nonlinear, suggesting economy-wide rebound effects are context specific. At low levels of efficiency investment, feedforward effects are small and feedback effects large, hence backfire dominates the net effect because  $ROI_\epsilon > ROI_p$  (Eq. 4a). At  $i_\epsilon = 36\%$  growth is maximised,  $ROI_\epsilon = ROI_p$ , and the net effect (feedforward + feedback) is zero in equilibrium (Fig. 2ii). However, above this threshold further increases in efficiency investment mean feedforward effects start to dominate such that the net effect on PEU growth becomes negative because  $ROI_\epsilon < ROI_p$ . Figure 2ii also shows that the time constant for the feedback rebound response is relatively independent of the level of efficiency investment at  $\sim 7$  years until you approach very low/high levels of investment where it rises significantly.

### Two future scenarios

If the global economy was under leveraged on primary energy pre-WW2, then it appears heavily over leveraged currently, given returns to efficiency appear more than twice those of returns to primary energy in 2018 (Fig. 3iv). Furthermore, because  $ROI_p \approx 1$  currently, all output growth in the economy, including growth in PEU, must actually be derived from current levels of efficiency investments. To explore the extent efficiency can be leveraged to further support output growth, we search for a pattern of efficiency investment that maximises output growth 2018–2100 (the maxGDP scenario). A constraint on this scenario is the efficiency investment cannot be changed



any faster than it has historically in line with Eq. (5). Because we are simply trying to elucidate the role of enhanced efficiency in fostering output growth and input degrowth, we make no attempt to include the myriad of possible additional future constraints on the global economy, such as increasing climate damages which are discussed qualitatively in Sect. "Cautions and limitations". We do, however, explore constraints on thermodynamic efficiency and size below.

From Figs. 2ii and 3iii we see that maximising GDP growth requires doubling investment in efficiency, from 18% of output in 2018 to 36% by 2050. This causes the ROIs for both efficiency and PEU to converge in the long run (Fig. 3iv) as would be expected in such a scenario where both factors become co-limiting. This highlights how doubling efficiency investment could become a central component to maintaining, or even boosting, output growth in the coming decades. Investors might be becoming increasingly aware of the growth-enhancing effect of efficiency investment, and that this awareness is a motive behind the so-called 'green growth' agenda. Under the 'stocktake' carried out for COP28, an objective of "doubling energy efficiency improvements by 2030" (UNFCCC, 2023) was declared in support of the UNFCCC's objectives to honour the Paris Agreement. Although the COP28 objective on energy efficiency growth is somewhat ambiguous, it appears more consistent with our maxGDP scenario suggesting it is attempting to honour both climate and economic objectives. Figure 3ii shows that the transition to doubling of present-day efficiency investment causes increases in efficiency and output growth but with no degrowth in primary energy use. However, in the long run this maxGDP scenario boosts returns to primary energy investment leading to its growth rate rising post 2060. As a result, even though it may not be UNFCCC's intention, their double-efficiency-growth scenario appears to be more in support of output growth objectives than it does degrowth in primary energy, and hence emissions.

Even though there appears to be a substantial risk that economy-wide feedbacks might lead to climate policy on energy efficiency investment backfiring, this does not mean energy efficiency cannot be deployed in the battle against climate change. In the extreme, investing exclusively in energy efficiency would starve investment in PEU, and this would cause energy-based emissions to decline to zero,

although it would also mean no output and ultimately no economy because PEU is required for economic activity. This does suggest however that the nonlinearities in the system could be exploited to find a scenario where there is a functioning economy and yet efficiency improvements have led to reductions in primary energy use and hence emissions growth. As a result, we look for the pattern of efficiency investment that minimises PEU growth across the remainder of the century (the minPEU scenario) without shrinking output. For this we require energy efficiency investment to rise to 75% of output by 2070 in the model (Fig. 3iii). At this level of efficiency investment, feedforward effects significantly outweigh feedback rebound effects (Fig. 2ii) and, as a result, between now and 2050 PEU growth is declining while GDP growth is rising, driven exclusively through efficiency growth (Fig. 3ii). By ~2050 the ROIs for both PEU and efficiency converge, only transiently, and GDP growth is again at a maximum, even though PEU growth is declining. As a result, it could be argued that this pre-2050 regime honours both growth and climate objectives. Post ~2060, PEU even goes into (slight) degrowth while both efficiency and GDP are still growing. However, output growth rapidly becomes limited by the ROI of efficiency (Fig. 3iv) such that by ~2080 all growth is extinguished. At this level of efficiency investment, the response time of the equilibration of the feedback effect is about twice that currently (Fig. 2ii).

The model contains no hard limits on either domain size or thermodynamic efficiency; there are just significant diminishing returns to scale on each. In reality, the economy is subject to physical limits from both the physical size of its planetary home, the resources therein, and the thermodynamic limits on efficiency any system can ultimately achieve. To explore these hard limits we attempt to reconstruct, albeit speculatively, estimates of both thermodynamic efficiency,  $\eta$ , and the fraction of available space occupied by the economy,  $f$ . Jarvis (2018) and Warr et al., (2010) estimate that the global economy is currently somewhere near 10% efficient at translating primary energy into final useful work, while Ritchie and Roser (2013) speculate that humans have appropriated approximately 30% of the available space on earth. We assume these as our 2018 initial conditions and apply the observed/simulated relative growth rates for EEE and PEU before/after 2018 to reconstruct both

$\eta$  and  $f$  pre/post 2018. Assuming  $\eta = 0.1$  in 2018 is equivalent to assuming  $\lambda = 1.4$  (2010\$)/MJ for the global economy in Eq. (1), which compares to Serrenho et al.'s, (2016) estimate of  $\lambda = 1.2$  (2010\$)/MJ for Portugal and Warr et al.'s (2010) estimate of  $\lambda = 0.8$  (2010\$)/MJ for the US.

Figure 3iii indicates that, for the maxGDP scenario,  $\eta$  rises to just short of 40% by 2100 and is still growing, with the economy running out of physical space (i.e.,  $f > 100\%$ ) by around 2080. For the minPEU scenario, efficiency and filled space stabilise at  $\sim 35\%$  and  $\sim 50\%$  respectively by 2100 (Fig. 3iii) i.e. the spatial footprint of the economy is  $\sim 15\%$  larger than today, but efficiency has risen more than three-fold. The available portfolio of energy saving technologies appears substantial (Grübler et al., 2018), as does the opportunity to exploit artificial intelligence to co-ordinate the selection, development and running of more efficient configurations of the economy. As a result, significant increases in present-day efficiency appear within reach, even though we may also be describing an economy too complex for people to engage with. Carnot also tells us the thermodynamic limit on efficiency will be substantially less than 100% and, just like running out of physical space, this represents a hard boundary. Thus, we might suspect the maxGDP result is physically unachievable through to 2100 due to both the spatial and efficiency constraints, whereas the minPEU scenario might only be constrained on efficiency. Any approach to hard boundaries in either size or efficiency would be experienced through additional, rapid decreases in returns to scale and hence ROIs.

### Cautions and limitations

Given all models are wrong, it is foolish to pretend that the one we have developed here fully explains the complex reality of the global economy. That said, it rests on a clear thermodynamic foundation and is constrained by appropriate global data that it explains with a degree of parsimony. We believe it is useful because it explains tendencies of the data over long timescales of multiple decades. However, there are weaknesses in the model that should be articulated. We identify the following:

- a. GDP does not equal final useful work, even if there is good reason to assume it might provide a useful proxy for it. In addition to the obvious concerns over whether GDP measures what it purports to, the fact that debt/credit flows are included in its measurement may partially decouple it from real time final useful work. Furthermore, some final useful work that is not directly monetised e.g. housework, while other monetised activities do not have an obvious link to quantities of useful work e.g. art sales. Add to this the fact that GDP includes expenditures on the smashing up and rebuilding of productive structure through military campaigns certainly complicates matters, although the protection and clearance of productive structures by destructive structures is a feature of the ecosystems that have often inspired our model development. We look to the exergy/energy economics community to provide further supporting evidence in this area as it has done to date.
- b. While the discrete stocks of aggregate final useful work,  $K_\epsilon$  and  $K_p$ , have meaning in the context of complex systems, it could well be that we will struggle to disaggregate specific data along these lines. For example, how much of the final useful work that goes into creating a train can be attributed to the efficiency of the train at doing useful work (part of  $K_\epsilon$ ) versus the degree to which the train undertakes activities that move mass and energy over the domain occupied by society (part of  $K_p$ )? We believe the answer to this does not lie in trying to better understand the behaviour of individual artefacts, but instead can only be assessed through a wider understanding of how networks of productive structures are created and function.
- c. For practical reasons our model treats efficiency and domain size as independent. However, as discussed earlier, the filling of space necessarily increases the mean path length of the networks underpinning productive structure, creating an inverse relationship between efficiency and space that we have not accounted for explicitly. One can see this interaction more generally. Efficiency is an intensive property of productive structure, while space-filling is extensive. As a result, expanding the domain of the economy necessarily dilutes efficiency investments, something we

have again not accounted for. We see this as the next iteration of the model framework.

- d. As with all modelling studies, we have left out a host of additional effects in our attempt to elucidate economy-wide rebound effects. Important omissions in this space include the effects of climate-society feedbacks and rapid transitions in the global energy mix to attempt to manage climate risks. However, we anticipate these effects will be expressible in future developments of the framework. For example, large climate damages translate to higher decay rates,  $d_e$  and  $d_p$ , for productive structures. A transition to renewables should increase  $\eta$  as exergy values of primary energy rise due to direct electricity production, although this likely imposes significant additional space requirements. Because our framework fully closes at the global scale, it might not contribute much insight into data at regional and country scales.

Although these concerns are significant, we do not believe they are terminal, and hold that our results and conclusions still offer valuable insights into economy-wide rebound effects. We also point to the fact that although many of the concepts we have developed are familiar, particularly to those working on complex, thermodynamically open systems, the framework we have presented is new, even as it borrows heavily from other disciplines. In light of this newness, we anticipate future research, including by us, will address these concerns.

## Conclusions

It appears the response of energy use to energy efficiency investment could be highly nonlinear and potentially counterintuitive. The question motivating this research was whether the current efficiency-led green growth narrative was a fallacy in relation to climate objectives. Our conclusion is that, as currently practised, it most likely is. If growth remains the objective of economic policy and practice then our analysis indicates modest efficiency improvements will become central to achieving this objective. Under this investment regime, economy-wide rebound effects look likely to swallow up any of the planned climate dividends attached to efficiency investments. If we were genuinely interested in using efficiency

improvements to play a credible role in our collective attempts to avoid dangerous climate change, we need to explore radically higher efficiency investment regimes, because these appear much less prone to rebound effects. This strategy, however, would also be associated with implicitly abandoning output growth as a long-term objective, even though in the medium term that growth could be significantly enhanced by this strategy. A radically higher efficiency investment strategy also cannot be seen as problem free as it acts to substantially raise  $ROI_p$ , while recreating some of the low growth, high volatility conditions that prevailed pre-WW2.

Just as it did in the 1970's, the 2022 energy crises reminded us that we are fundamentally supported by the flows of energy and matter from nature on through our economy to where they become useful. Any rethinking this motivates should not simply focus on dampening turbulent energy markets, for a similar recalibration on energy is needed to help us better engage with the task of reducing energy-related emissions. If we are to rethink the role of energy in our lives it also feels appropriate that we recast the models we use to resolve the spaghetti of economic interactions that often frustrate our understanding. We take our efforts here as our first approximation of this.

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**Data availability** All data and code are available from the corresponding author on request.

## Declarations

**Conflicts of interest** The authors declare no conflicts of interest associated with this research.

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