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Design optimization of cubic Bezier horn for ultrasonic machining

PRAVEEN KUMAR RAI, VINOD YADAVA and RABINDRA KUMAR PATEL*

Mechanical Engineering Department, Motilal Nehru National Institute of Technology Allahabad,
Prayagraj 211004, India
e-mail: rme1511@mnnit.ac.in; vinody@mnnit.ac.in; rkpatel@mnnit.ac.in

Abstract. In ultrasonic machining (USM), higher amplitude is required for higher material removal rate (MRR). The objective of this research is to develop a new horn design for high displacement amplitude for getting maximum MRR within a working condition. The profile of the horn has been optimized and designed. In this analysis, optimization procedure and finite-element method have been used for the design of horn of USM. In this, the variation of stress components along the length of the horn has been studied. In the middle, maximum stress is found due to a change in cross-section but it is within the endurance limit of the horn material. The cubic Bezier horn as compared with traditional horn is up to 19% more amplified. Finally, an optimized cubic Bezier profile for the horn has been designed, which has maximum displacement amplitude for higher MRR and safe working stresses for the horn material.

1. Introduction

In the field of manufacturing, more and more challenges are faced by the technologist and engineers with the development of technology [1–3]. Lot of new engineering materials have been developed, in recent years. These materials have many potential engineering applications. However, because of the high machining cost, application has been limited. A cost-effective advance machining process is required for these materials. Ultrasonic machining (USM) is one of the advanced machining processes. Figure 1 shows the set-up of an ultrasonic machine. The ability of cutting of USM depends initially on the design ability of horn (also known as tool holder or concentrator) [4–8].

In the modal analysis, Seah *et al* [9] have suggested three methods of simulation for obtaining the frequencies. These results are correlated with experimentally measured frequency. They investigated the design of efficient concentrators for the conventional USM process by employing finite-element analysis (FEA). Because of low stresses generated in the conical horn, they suggested that the chances of failure by the fatigue loading are negligible. If radius of the curvature of the corner is small, because of the stress concentration they also suggested that the stepped horn encounter is dangerous. This stress concentration leads to overheating and formation of cracks. For high amplification application of horns, one should not use the stepped horn. For any significant change in the frequency obtained, they also found that the addition of

neither tool nor the hollow tube is important along a longitudinal axis of the horn. Amin *et al* [10] have suggested a design of concentrators for the conventional USM process by employing the FEA. They have also studied the amplification and the stresses on the horn domain. For obtaining the maximum magnification factor, optimization procedure is followed. After the safe working stresses for getting a higher material removal rate (MRR), the concentrator material study has been elaborated. They also show that the concentrator profile should be cylindrical in shape at the lower (tool) end and conical shape at the upper (transducer) end. Yadava and Deoghare [11] have analysed by employing the finite-element method (FEM) and the design of concentrators (horns) for the rotary ultrasonic machine within working condition of the conventional ultrasonic machine. They also show that for the same boundary conditions and material properties the amplification factor is more for the rotary ultrasonic machine as compared with the conventional ultrasonic machine. They also show that the stresses obtained for the other conditions are more than the stress obtained for the resonance. Rosca *et al* [12] suggested the design of ultrasonic horn, which is useful in manufacturing process for USM. On the workpiece, high-amplitude waves are injected. To establish the frequency at which optimal wave transmission occurs, the dimensions of both the ultrasound transducer and tool are important. In this paper, the optimal design of horn has been obtained. These parameters vary in an integrated set-up as required. Thoe *et al* [13] suggest that the machining rate is maximum for the optimum static load and dependent on tool configuration (e.g. shape and area of cross-section), mean grit size and amplitude.
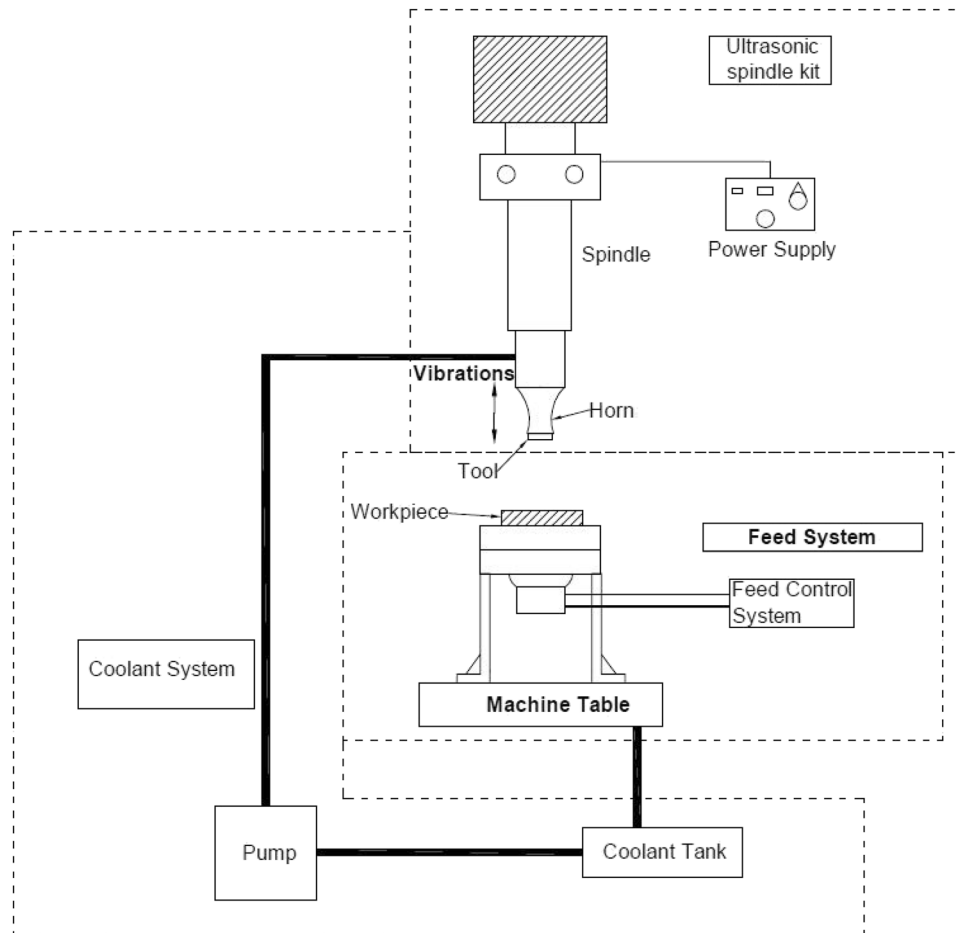
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*For correspondence

Figure 1. Ultrasonic machine set up.

The rotary ultrasonic machine performance over conventional ultrasonic machine may be explained by the combined effects of rolling contact between the embedded grains on the workpiece/tool, sliding contact between free abrasive grains/workpiece and indentation of workpiece surface. To maximize the MRR and for a resonant USM system, the tool and horn design play an important role. Wang *et al* [14] have suggested the new horn design for high amplification at the lower end of horn. A design procedure using multi-objective optimization algorithm and FEM has been utilized for optimizing the displacement amplitude of the horn. Comparison of experimentally measured working frequency between the fabricated and designed horn validates the results. Jagadish and Ray [15] have suggested the stepped rectangular horn by the FEM. In this study, the horn material is taken as a titanium alloy. For finding the amplitude of vibration, stress components and natural frequency over the horn domain, the FEA has been used.

From this literature review, to get low value of stresses, maximum amplitude of vibration and the resonance condition, some shaping of horn design has been done. During the process, it is observed that designing of concentrator for ultrasonic machine has been done for the exponential,

cylindrical and stepped shapes. This work has been done theoretically and experimentally using a conventional ultrasonic machine for finding the MRR. The literature review shows that for the conventional ultrasonic machine design of horn of various Bezier profiles, optimization has not been done by employing the FEM. In this paper, the main aim is to design the horn with optimized Bezier profile for the conventional USM within the working
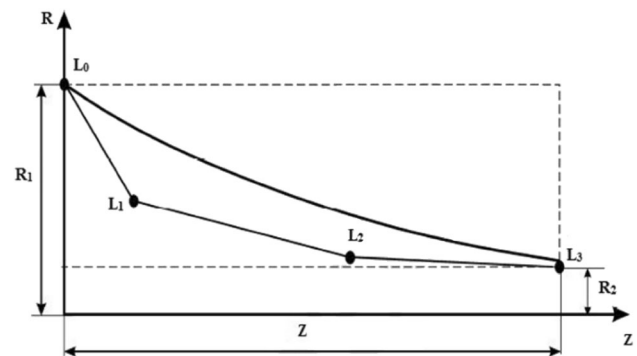

Figure 2. Cubic Bezier profile and its four control points L_0, L_1, L_2 and L_3.
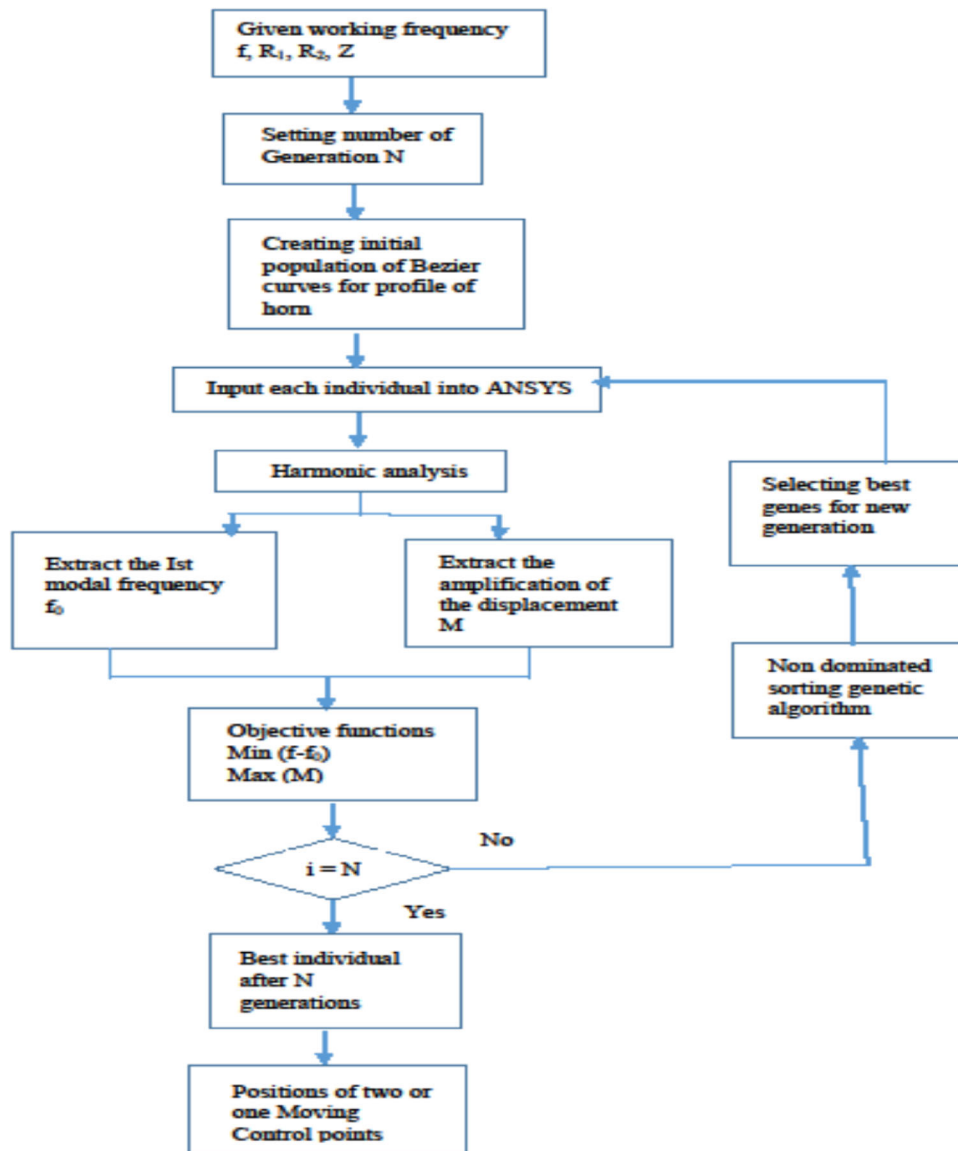
Figure 3. Optimization flowchart.

condition. In this paper, FEM-based analysis has been done to find the stress components, resonance frequency and axial amplitude over the domain for the USM using optimized Bezier profile. All results are validated with the results available in the literature.

2. Horn design

The design of horn with Bezier profile for high amplification is based on optimization procedure. The control points of cubic Bezier curve profile are optimized by parameters to meet the requirement of higher displacement amplitude. The curve of cubic Bezier horn is determined by four-point Bezier polygene L_0–L_3 as shown in figure 2. As described by Zeid [16], the last and first points, L_3 and L_0, respectively, define

the polygon for cubic Bezier profile curve. At the end and the beginning of the curve, the tangent vector has the same direction as the last and first polygons span, respectively. The cubic Bezier parametric curve is given by Eq. (1) [16]:

$$P(u) = \begin{bmatrix} (1-u)^3 & 3u(1-u)^2 & 3u^2(1-u) & u^3 \end{bmatrix} \begin{bmatrix} P_{L_0} \\ P_{L_1} \\ P_{L_2} \\ P_{L_3} \end{bmatrix}$$

$$0 \le u \le 1$$

$$(1)$$

where u is a parameter and P_{Li} is position vector of point L_i.

Points L_1 and L_2 are optimized for cubic Bezier to move in the design space enclosed by the dashed rectangle in figure 2. Positions of cubic Bezier points L_0 and L_3 are

Table 1. Merits and demerits aspects of several commonly used horn profiles [14].

Horn profile	Merits	Demerits
Stepped	High amplification	Stress concentration is high
Exponential	Stress distribution is smooth	Low amplification
Conical	Easy to design and fabricate	Low amplification
Catenoidal	Stress distribution is smooth	Moderate amplification
Gaussian	High amplification	Complicated design
Bezier	Highest amplification, stress distribution is smooth, asymmetric	Complicated design, high cost

fixed, respectively, by the specified front and back horn radius R_1 and R_2. The horn is assumed to be axisymmetric. An optimality procedure is developed and the flowchart is shown in figure 3. The non-dominated sorting genetic algorithm (NDSGA) [17] is applied for solving constraints. In each generation of genetic algorithm (GA) [18] the following are performed:

$$\text{Min} \quad f - f_0$$
$$\text{Max} \quad M = \frac{u_{L_3}}{u_{L_0}} \qquad (2)$$

where f_0 is the first axial modal frequency of the population of each generation of the horn. M is the amplification defined by the ratio of axial displacement at lower end to upper end of the horn. To solve the two objective optimization problems the fast non-dominated sorting approach [17] is used. In the sorting procedure, the concept of Pareto dominance [19] is utilized to evaluate fitness or assigning selection probability to the solution. Table 1 shows the merits and demerits of several commonly used horn profiles.

3. Analysis

3.1 *FEA*

Analysis has been done using the ANSYS software, which is flexible and the most powerful tool available for the FEM-based analysis. It is known very well that the FEM analysis is a suitable method for the amplitude, equivalent stress and resonance frequency of the USM horn of any dimensions. In the FEM, the major use of modelling a general structural simulation includes the geometry, meshing, boundary conditions, element type, real constants and material properties [20–22]. The analysis may begin, after these factors have been given. These factors are used for modal and harmonic analysis. The USM horn is manufactured from steel AISI (4063) with the material properties presented in table 2.

3.2 *Modal analysis*

Modal analysis allows the design to vibrate at a specified frequency or at resonance. For other dynamic analyses, it

helps in calculation of solutions because structure vibration characteristics determine different types of dynamic load and responses. Always perform a modal analysis first. Modal analysis is a linear analysis [23, 24]. In the modal analysis, several number of modes are available. Several numbers of modes are shown in figure 4 at different resonance frequencies.

3.3 *Harmonic analysis*

Harmonic analysis is the technique to find a steady-state response of the cubic Bezier horn to harmonic load of known frequency, where input sinusoidal loads are the forces. To solve the sinusoidal equation of motion, however, in this case, full method is used for the present ultrasonic Bezier horn.

3.4 *Mesh generation*

According to the basic principle of the FEM theory, if the mesh element size is reduced, the accuracy of result will improve. If the size of the mesh element is very small, the model will approach the optimum solution.

3.5 *Boundary condition*

Steel AISI (4063) is selected as the USM horn material. The properties of the material are available in table 2.

Table 2. Specifications of the horns using material steel AISI (4063) for USM.

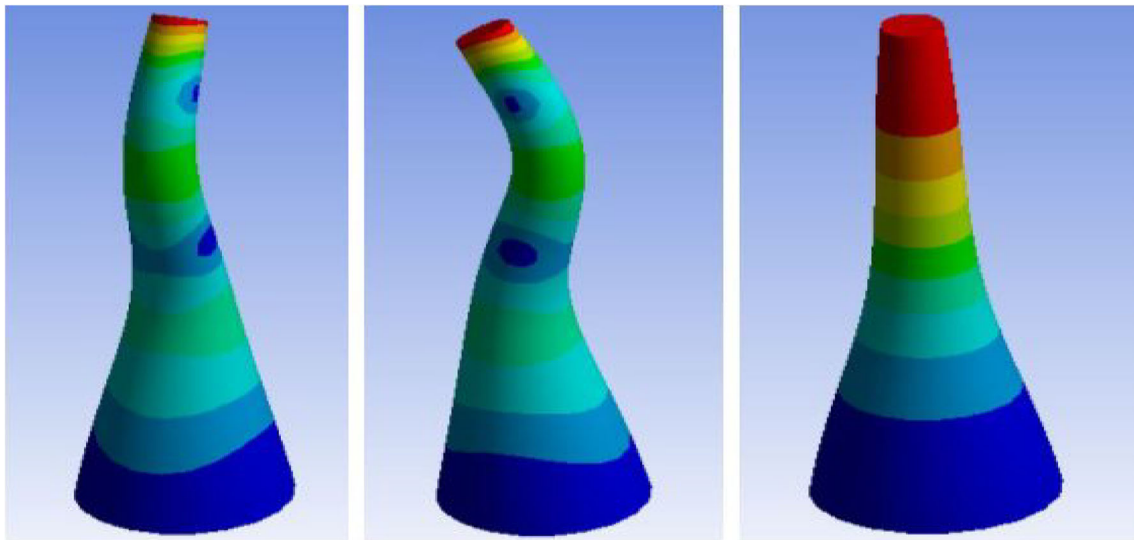| Parameters	Value
Elastic modulus E	200 GPa
Poisson's ratio μ	0.33
Mass density ρ	7800 kg/m^3
Endurance strength σ_{eq}	733 MPa
Allowable endurance limit	520 MPa
Resonance frequency range	20–25 kHz
Amplitude transducer	15 μm
Diameter of upper end horn	40 mm
Diameter of lower end horn	10 mm
Cubic Bezier horn length	124.44 mm

Figure 4. (a) Ist Mode at 20600 Hz (b) IInd Mode at 22230 Hz (c) IIIrd Mode at 23600 Hz.

Boundary condition is applied at the end of the transducer. The output of the piezoelectric transducer has been applied as the input (15-μm amplitude) at the large end of cubic Bezier horn [23–27].

4. Results and discussion

In this work, for calculation of the stress components, displacement and natural frequency, ANSYS is used. For meshing the horn domain, ANSYS APDL is used as a pre-processor. The output of ANSYS software is the co-ordinates of the nodes and connectivity of the elements, and these data have been used as the input. For presenting the results in the graphical form, MS ORIGIN software is used. The results are validated with those of Amin *et al* [10]. Later, the stresses, displacement and natural frequency are calculated for the cubic Bezier horn.

4.1 *Model validation*

The present FEM model has been validated by comparing the result of Amin *et al* [10] for displacement and stresses by taking all conditions to be the same as given in Ref. [10]. The domain is axisymmetric about Z-axis as shown in figure 5. The initial displacement of 15 μm has been given on top end DC. The problem has been solved using FEM by considering all the given boundary conditions. The axial displacement along the axis is calculated and compared with the result given in Ref. [10] as shown in figure 6. The nature of variation of the curve is similar to that in Ref. [10]. The average error in this comparison is 2.4% for the cubic Bezier horn. The magnification factor and stress obtained from present FEM model of cubic Bezier horn and corresponding value by literature [10] are very close. The
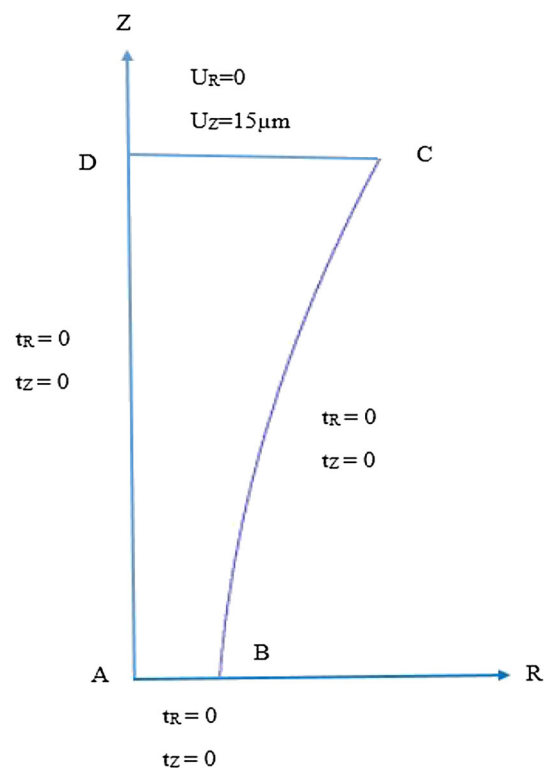

Figure 5. Domain of cubic Bezier profile horn.

corresponding error is 5.5%, which shows the accuracy of present FEM model.

4.2 *Effect of stresses along the horn axis*

The magnification factor is maximum for the cubic Bezier horn and the corresponding value is 7.34. Figure 7 shows
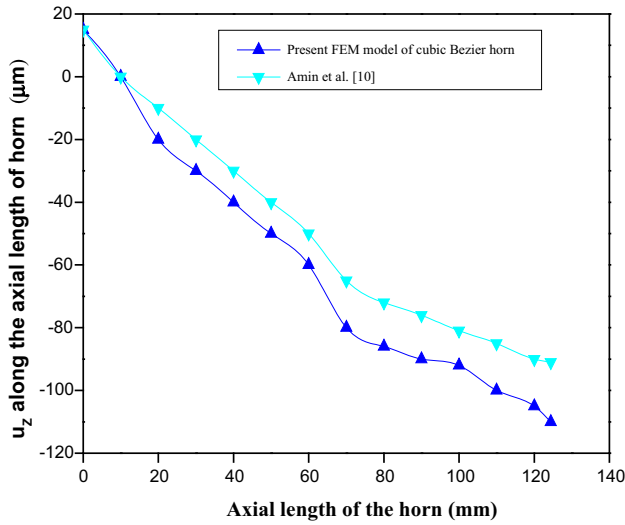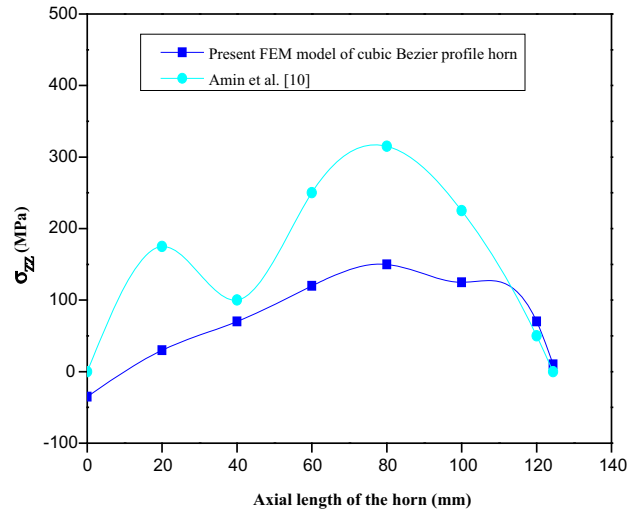
Figure 6. Model validation.
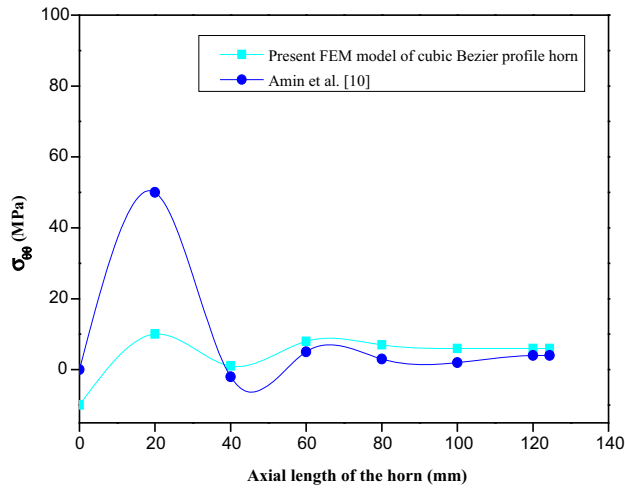

Figure 7. Circumferential stress ($\sigma_{\theta\theta}$) variation along the axial length.

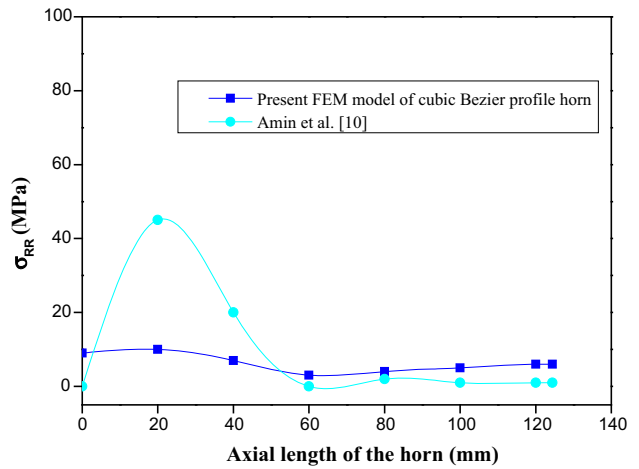**Figure 8.** Axial stress (σ_{ZZ}) variation along the axial length.

Figure 9. Radial stress (σ_{RR}) variation along the axial length.

the minimum circumferential stress for the cubic Bezier horn. The variation of the circumferential stress is similar to that of Amin *et al* [10]. Figure 8 shows the variation of the axial stress; the value of axial stress is minimum for the optimized cubic Bezier horn, the stress is within the allowable endurance limit and the horn is safe. Also, the maximum axial stress variation along the axial length of the horn is larger in comparison with circumferential stress variation. Figure 9 shows radial stress variation along axial length of the horn and the stress variation is lower in comparison with axial stress variation. Figure 9 shows that the radial stress is minimum for the cubic Bezier horn and maximum for the literature result and also the result is validated by Amin *et al* [10]. Optimized cubic Bezier horn is safe within the horn domain. Figure 10 shows shear stress variation along axial length of horn. Value of shear

stress is minimum for the cubic Bezier horn and the horn is safe. Due to the low value of shear stress, there is no distortion within the horn and also in figure 10 the variation of the shear stress is similar to that of Amin *et al* [10]. Figure 11 shows equivalent stress variation along the length of horn and literature stress variation is maximum as compared with the present cubic Bezier horn. The maximum value of equivalent stress is for the cubic Bezier horn; its value is 155.31 MPa and it is much lower than the allowable endurance limit. The stress generated over the domain of the horn is within the allowable endurance limit of the material, so the designed cubic Bezier horn is safe.

4.3 *Experimental verification*

Experimental verification has been done by figure 12. For comparison, the input parameter has been taken to be the same as that of Wang *et al* [14]. It is found that the present
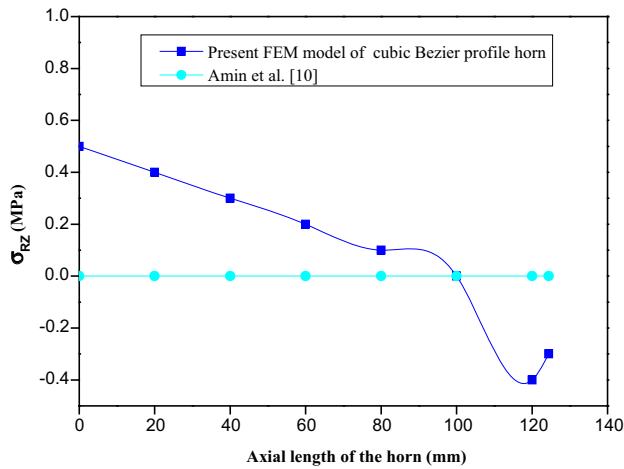
Figure 10. Shear stress variation (σ_{RZ}) along the axial length.
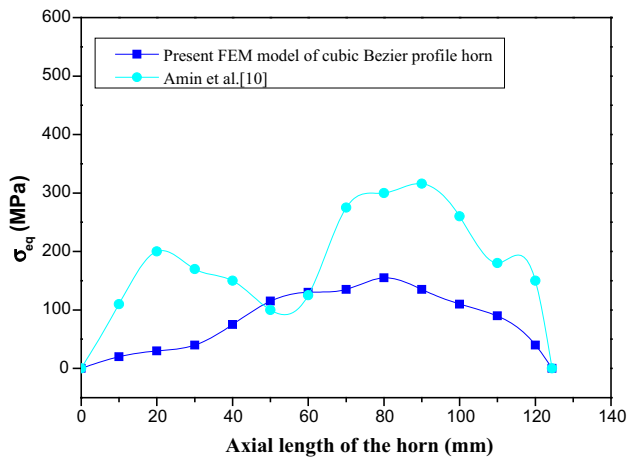

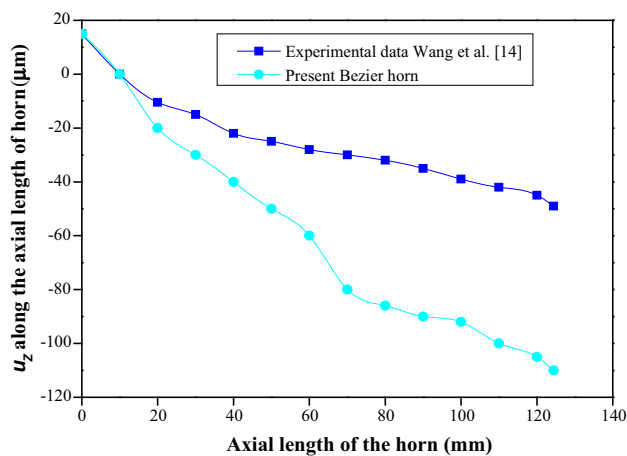**Figure 11.** Equivalent stress (σ_{eq}) variation along the axial length.

Figure 12. Experimental validation of amplitude.

Bezier horn has 55.5% more amplitude as compared with Wang *et al* [14]. Figure 12 shows a trend of the present result similar to that given in the literature [14] and the

Table 3. Ease/complexity in manufacture aspects of several commonly used horn profiles [23].

Horn profile	Ease/complexity
Stepped	Easy to manufacture
Exponential	Complicated to manufacture
Conical	Easy to manufacture
Catenoidal	Easy to manufacture
Gaussian	Complicated to manufacture
Bezier	Complicated to manufacture

magnification factor has been found to be more in the present Bezier horn. Through the experimental investigation, researchers have found that for high amplification the MRR is high as given in Rozenberg *et al* [1]. In the present work, amplification has been found to be more so the MRR would be high for the Bezier horn profile. Table 3 shows the ease/complexity in manufacture aspects of several commonly used horns profile.

5. Conclusions

In this present work, the optimized design of cubic Bezier profile horn for USM by the FEM have been considered. The stress components, natural frequency and magnification factor induced within the domain of optimized cubic Bezier profile horn have been calculated.

1. FEM and multiobjective optimization algorithm are developed to optimizing the displacement amplification of the cubic Bezier profile horn.
2. Value of the natural frequency of cubic Bezier horn is 23.6 kHz. For the same boundary conditions and material properties, the amplitude is more for the optimized cubic Bezier horn as compared with the result from literature.
3. The maximum equivalent stress for optimized cubic Bezier horn is 155.71 MPa at the middle of horn towards lower end.
4. The maximum equivalent stress is lower than the allowable endurance strength (520 MPa). Optimized cubic Bezier shape of the horn has not failed by yielding or by fatigue.
5. At the lower end of the horn, stresses generated are almost zero because it is free to move. The induced shear stress is almost zero, so there is no distortion taking place within the horn and also stresses are linear in radial direction.
6. It is concluded that the present Bezier horn has obtained 55.5% more amplitude as compared with experimental horn in literature.

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