A two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation

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Abstract. The theory of thermoelasticity with energy dissipation is employed to study plane waves in a fibre-reinforced anisotropic thermoelastic half-space. We apply a thermal shock on the surface of the half-space which is taken to be traction free. The problem is solved numerically using a finite element method. Moreover, the numerical solutions of the non-dimensional governing partial differential equations of the problem are shown graphically. Comparisons are made with the results predicted by Green–Naghdi theory of the two types (GNII without energy dissipation) and (GNIII with energy dissipation). We found that the reinforcement has great effect on the distribution of field quantities. Results carried out in this paper can be used to design various fibre-reinforced anisotropic thermoelastic elements under thermal load to meet special engineering requirements.

Keywords. Fibre-reinforced; finite element method; thermoelasticity with energy dissipation.

1. Introduction

Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The mechanical behaviour of many fibre-reinforced composite materials is adequately modelled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites the fibres are usually arranged in parallel straight lines. The characteristic property of a reinforced composite is that its components act together as a single anisotropic unit as long as they remain in the elastic condition. Sengupta & Nath (2001) discussed the problem of surface waves in fibre-reinforced anisotropic elastic media. Singh (2001) showed that, for wave propagation in fibre-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. Hashin & Rosen (1964) gave the elastic moduli for fibre-reinforced materials. In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible.

The theory of couple thermoelasticity was extended by Lord & Shulman (1967) and Green & Lindsay (1972) by including the thermal relaxation time in constitutive relations. The theory was extended for anisotropic body by Dhaliwal & Sherief (1980). Green & Naghdi (1993) proposed a new generalized thermoelasticity theory by including the thermal-displacement gradient among the independent constitutive variables. An important feature of this theory, which is not present in other thermoelasticity theories, is that it does not accommodate dissipation of thermal energy. The relevant fundamental aspects of this theory are contained in Grenn & Naghdi (1991; 1992). Singh (2006) studied the wave propagation in thermally conducting linear fibre-reinforced composite materials with one relaxation time.

The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. A numerical solution technique is used to calculate the solution of general problems. For this reason the finite element method is chosen.

The Finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. On the other hand, the finite element method in different generalized thermoelastic problems has been applied by many authors (see for instance Tian *et al* 2006; Abbas 2007; Abbas & Abd-alla 2008; Abbas & Othman 2009; Youssef & Abbas 2007).

In the present work, the Green and Naghdi theory is applied to study the influence of reinforcement on the total deformation body and the interaction with each other. Furthermore, the problem is solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement and the stress components are represented graphically. As an application of the results carried out in this paper, we can design various fibre-reinforced anisotropic thermoelastic elements under thermal load to meet special engineering requirements.

2. Basic equations and formulation

According to Green & Nagdhi (1993) and Singh (2006), the linear equations governing thermoelastic interactions in homogeneous anisotropic solid in the absence of body force and heat sources are as follows:

$$\tau_{ij,j} = \rho \ddot{u}_i, \quad i, j = 1, 2, 3, \tag{1}$$

$$K^* T_{,ij} + K_{ij} \dot{T}_{,ij} = \rho c_e \ddot{T} + T_o \beta_{ij} \ddot{u}_{i,j}, \quad i, j = 1, 2, 3. \tag{2}$$

The constitutive equation for a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector **a** will be as follows (Belfield *et al* (1983):

$$
\begin{aligned}
\tau_{ij} = {} & \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha \left(a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk} \right) \\
& + 2 \left(\mu_L - \mu_T \right) \left(a_i a_k e_{kj} + a_j a_k e_{ki} \right) \\
& + \beta\, a_k a_m\, e_{km} a_i a_j - \beta_{ij} \left(T - T_0 \right) \delta_{ij}, \quad i, j, k, m = 1, 2, 3,
\end{aligned}
\tag{3}
$$

where ρ is the mass density; u_i the displacement vectors components; e_{ij} the strain tensor; τ_{ij} the stress tensor; T the temperature change of a material particle; T_o the reference uniform temperature of the body; β_{ij} the thermal elastic coupling tensor; c_e the specific heat at constant strain; K_{ij} the thermal conductivity; K^* the material characteristic of the theory; λ, μ_T are elastic parameters; α, β, $(\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters and $\mathbf{a} \equiv (a_1, a_2, a_3)$,

$a_1^2 + a_2^2 + a_3^2 = 1$. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

We consider the problem of a fibre-reinforced anisotropic half-space ($x_1 \geq 0$) All the considered functions will depend on the time t and the coordinates x_1 and x_2 Thus, the displacement vector u_i will have the components

$$u = u_1 = u(x_1, x_2, t), \quad v = u_2 = v(x_1, x_2, t), \quad w = u_3 = 0. \tag{4}$$

We choose the fibre-direction as $\mathbf{a} \equiv (1, 0, 0)$ so that the preferred direction is the x_1 axis and Eqs. (1), (2) and (3) are simplified to be

$$\tau_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u}{\partial x_1} + (\lambda + \alpha)\frac{\partial v}{\partial x_2} - \beta_{11}(T - T_o), \tag{5}$$

$$\tau_{22} = (\lambda + 2\mu_T)\frac{\partial v}{\partial x_2} + (\lambda + \alpha)\frac{\partial u}{\partial x_1} - \beta_{22}(T - T_o), \tag{6}$$

$$\tau_{12} = \mu_L\left(\frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_2}\right), \tag{7}$$

$$A_{11}\frac{\partial^2 u}{\partial x_1^2} + A_{12}\frac{\partial^2 v}{\partial x_1 \partial x_2} + A_{13}\frac{\partial^2 u}{\partial x_2^2} - \beta_{11}\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$A_{22}\frac{\partial^2 v}{\partial x_2^2} + A_{12}\frac{\partial^2 u}{\partial x_1 \partial x_2} + A_{13}\frac{\partial^2 v}{\partial x_1^2} - \beta_{22}\frac{\partial T}{\partial x_2} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{9}$$

$$\left(K^* + K_{11}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x_1^2} + \left(K^* + K_{22}\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x_2^2} = \rho c_e \frac{\partial^2 T}{\partial t^2} + T_o\frac{\partial^2}{\partial t^2}\left(\beta_{11}\frac{\partial u}{\partial x_1} + \beta_{22}\frac{\partial v}{\partial x_2}\right), \tag{10}$$

with

$$A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \ A_{12} = \alpha + \lambda, \ A_{13} = \mu_L, \ A_{22} = \lambda + 2\mu_T,$$

$$\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11} + (\lambda + \alpha)\alpha_{22}, \ \beta_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22},$$

where α_{11}, α_{22} are coefficients of linear thermal expansion. For convenience, the following non-dimensional variables are used:

$$(x', y', u', v') = c\chi(x, y, u, v), \quad t' = c^2\chi t, \quad T' = \frac{\beta_{11}(T - T_0)}{\rho c^2},$$

$$(\tau'_{11}, \tau'_{22}, \tau'_{12}) = \frac{1}{\rho c^2}(\tau_{11}, \tau_{22}, \tau_{12}), \quad c^2 = \frac{A_{11}}{\rho}, \quad \chi = \frac{\rho c_e}{K_{11}}. \tag{11}$$

In terms of the non-dimensional quantities defined in Eqs. (11), the above governing equations reduce to (dropping the dashed for convenience)

$$\tau_{11} = \frac{\partial u}{\partial x_1} + B_1\frac{\partial v}{\partial x_2} - T, \tag{12}$$

$$\tau_{22} = B_1\frac{\partial u}{\partial x_1} + B_2\frac{\partial v}{\partial x_2} - B_3 T, \tag{13}$$

$$\tau_{12} = B_4 \left(\frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_2} \right), \tag{14}$$

$$\frac{\partial^2 u}{\partial x_1^2} + (B_1 + B_4) \frac{\partial^2 v}{\partial x_1 \partial x_2} + B_4 \frac{\partial^2 u}{\partial x_2^2} - \frac{\partial T}{\partial x_1} = \frac{\partial^2 u}{\partial t^2}, \tag{15}$$

$$B_2 \frac{\partial^2 v}{\partial x_2^2} + (B_1 + B_4) \frac{\partial^2 u}{\partial x_1 \partial x_2} + B_4 \frac{\partial^2 v}{\partial x_1^2} - B_3 \frac{\partial T}{\partial x_2} = \frac{\partial^2 v}{\partial t^2}, \tag{16}$$

$$\left(\varepsilon_2 + \varepsilon_3 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial x_1^2} + \left(\varepsilon_2 + \varepsilon_3 \frac{K_{22}}{K_{11}} \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial x_2^2} = \frac{\partial^2 T}{\partial t^2} + \frac{\partial^2}{\partial t^2} \left(\varepsilon_0 \frac{\partial u}{\partial x_1} + \varepsilon_1 \frac{\partial v}{\partial x_2} \right), \tag{17}$$

where

$$(B_1, B_2, B_4) = \frac{1}{A_{11}} (A_{12}, A_{22}, A_{13}), \ B_3 = \frac{\beta_{22}}{\beta_{11}}, \ (\varepsilon_0, \varepsilon_1) = \frac{T_o \beta_{11}}{A_{11} \rho c_e} (\beta_{11}, \beta_{22}),$$

$$(\varepsilon_2, \varepsilon_3) = \frac{1}{\rho c_e} \left(\frac{K^*}{c^2}, K_{11} \chi \right).$$

3. Finite element formulation

In this section, the governing equations of generalized thermoelasticity are summarized, followed by the corresponding finite element equations. In the finite element method, the eight-node isoparametric, quadrilateral element is used for displacement components and temperature calculations. The displacement component $\{u\}$ and the temperature T are related to the corresponding nodal values $\{u^e\}$ and $\{T^e\}$ by

$$\{u\} = [N] \{u^e\} \ , \ \{T\} = [N'] \{T^e\}, \tag{18}$$

where $[N]$ and $[N']$ are shape functions which are given by

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & .... & N_n & 0 \\ 0 & N_1 & 0 & N_2 & .... & 0 & N_n \end{bmatrix} \ , \ [N'] = \{ N_1 \ N_2 \ ... \ N_n \}, \tag{19}$$

where n denotes the number of nodes in the grid. According to the geometrical equation as the following:

$$e_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right). \tag{20}$$

As well as $T' = T_{,i}$, it yields

$$\{e\} = [D_1] \{u^e\}, \ T' = [D_2] \{T^e\}. \tag{21}$$

The variational form of the above equation is

$$\delta \{e\} = [D_1] \delta \{u^e\}, \ \delta T' = [D_2] \delta \{T^e\}. \tag{22}$$

If the coordinates x_1 and x_2 are considered then $[D_1]$ and $[D_2]$ have the following forms, respectively:

$$[D_1] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x_1} & 0 & \dfrac{\partial N_2}{\partial x_1} & 0 & \cdots & \dfrac{\partial N_n}{\partial x_1} & 0 \\[2ex] 0 & \dfrac{\partial N_1}{\partial x_2} & 0 & \dfrac{\partial N_2}{\partial x_2} & \cdots & 0 & \dfrac{\partial N_n}{\partial x_2} \\[2ex] \dfrac{\partial N_1}{\partial x_2} & \dfrac{\partial N_1}{\partial x_1} & \dfrac{\partial N_2}{\partial x_2} & \dfrac{\partial N_2}{\partial x_1} & \cdots & \dfrac{\partial N_n}{\partial x_2} & \dfrac{\partial N_n}{\partial x_1} \end{bmatrix}, \tag{23}$$

$$[D_2] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x_1} & \dfrac{\partial N_2}{\partial x_1} & \cdots & \dfrac{\partial N_n}{\partial x_1} \\[2ex] \dfrac{\partial N_1}{\partial x_2} & \dfrac{\partial N_2}{\partial x_2} & \cdots & \dfrac{\partial N_n}{\partial x_2} \end{bmatrix}. \tag{24}$$

The virtual displacement principle of domain V and its boundary A can be written as

$$\int_V \left(\delta \{e\}^T \{\tau_{ij}\} + \delta \{T'\} \left[K^* T_{,i} + K_{ii} \dot{T}_{,i} \right] \right) dV$$
$$+ \int_V \left(\delta \{u^e\} \rho \ddot{u}_i + \delta \{T^e\} \left[\rho c_e \dot{T} + T_o \beta_{ij} \ddot{u}_{i,j} \right] \right) dV = \int_A \left(\delta \{u^e\}^T \{\overline{\tau}\} + \delta \{T^e\} \overline{T} \right) dA, \tag{25}$$

where $\{\overline{\tau}\}$ represents the components of the traction vector, and $\overline{T}$ represents the heat flux. In equation (25), the governing equations are multiplied by weighting functions and then are integrated over the spatial domain V with the boundary A. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to equations (15), (16) and (17) can be obtained as

$$\sum_{e=1}^{n} \left(\begin{bmatrix} M_{11}^e & 0 & 0 \\ 0 & M_{22}^e & 0 \\ M_{31}^e & M_{32}^e & M_{33}^e \end{bmatrix} \begin{Bmatrix} \ddot{u}^e \\ \ddot{v}^e \\ \ddot{T}^e \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{33}^e \end{bmatrix} \begin{Bmatrix} \dot{u}^e \\ \dot{v}^e \\ \dot{T}^e \end{Bmatrix} \right.$$
$$\left. + \begin{bmatrix} K_{11}^e & K_{12}^e & K_{13}^e \\ K_{21}^e & K_{22}^e & K_{23}^e \\ 0 & 0 & K_{33}^e \end{bmatrix} \begin{Bmatrix} u^e \\ v^e \\ T^e \end{Bmatrix} = \begin{Bmatrix} F_1^e \\ F_2^e \\ F_3^e \end{Bmatrix} \right), \tag{26}$$

where the coefficients in equation (26) are given below:

$$M_{11}^e = \int_V [N]^T [N] \, dV, \qquad M_{22}^e = \int_V [N]^T [N] \, dV,$$
$$M_{31}^e = \int_V \varepsilon_0 [N]^T \left[\frac{\partial N}{\partial x_1} \right] dV, \quad M_{32}^e = \int_V \varepsilon_1 [N]^T \left[\frac{\partial N}{\partial x_2} \right] dV,$$
$$M_{33}^e = \int_V [N]^T [N] \, dV, \qquad C_{33}^e = \int_V \varepsilon_3 [N]^T \left[\frac{\partial N}{\partial x_1} + \frac{K_{22}}{K_{11}} \frac{\partial N}{\partial x_2} \right] dV,$$

$$K_{11}^e = \int_V \left(\left[\frac{\partial N}{\partial x_1} \right]^T \left[\frac{\partial N}{\partial x_1} \right] + B_4 \left[\frac{\partial N}{\partial x_2} \right]^T \left[\frac{\partial N}{\partial x_2} \right] \right) dV, \qquad K_{13}^e = \int_V - \left[\frac{\partial N}{\partial x_1} \right]^T [N] \, dV,$$

$$K_{12}^e = \int_V (B_1 + B_4) \left[\frac{\partial N}{\partial x_2} \right]^T \left[\frac{\partial N}{\partial x_1} \right] dV, \qquad K_{23}^e = \int_V -B_3 \left[\frac{\partial N}{\partial x_2} \right]^T [N] \, dV,$$

$$K_{21}^e = \int_V (B_1 + B_4) \left[\frac{\partial N}{\partial x_1} \right]^T \left[\frac{\partial N}{\partial x_2} \right] dV, \qquad F_1^e = \int_A [N]^T \, \bar{\tau}_x \, dA,$$

$$K_{22}^e = \int_V \left(B_2 \left[\frac{\partial N}{\partial x_2} \right]^T \left[\frac{\partial N}{\partial x_2} \right] + B_4 \left[\frac{\partial N}{\partial x_1} \right]^T \left[\frac{\partial N}{\partial x_1} \right] \right) dV, \quad F_2^e = \int_A [N]^T \, \bar{\tau}_y \, dA,$$

$$K_{33}^e = \int_V \varepsilon_2 \left(\left[\frac{\partial N}{\partial x_1} \right]^T \left[\frac{\partial N}{\partial x_1} \right] + \left[\frac{\partial N}{\partial x_2} \right]^T \left[\frac{\partial N}{\partial x_2} \right] \right) dV, \qquad F_3^e = \int_A [N]^T \, \bar{q} \, dA.$$

Symbolically, the discretized equations of equations (26) can be written as

$$M\ddot{d} + C\dot{d} + Kd = F^{ext}, \tag{27}$$

where M, C, K and F represent the mass, damping, stiffness matrices and external force vectors, respectively and $d = [u \; v \; T]^t$. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method (see Zienkiewicz & Taylor 2000; Reddy 1993; Cook *et al* 1989).
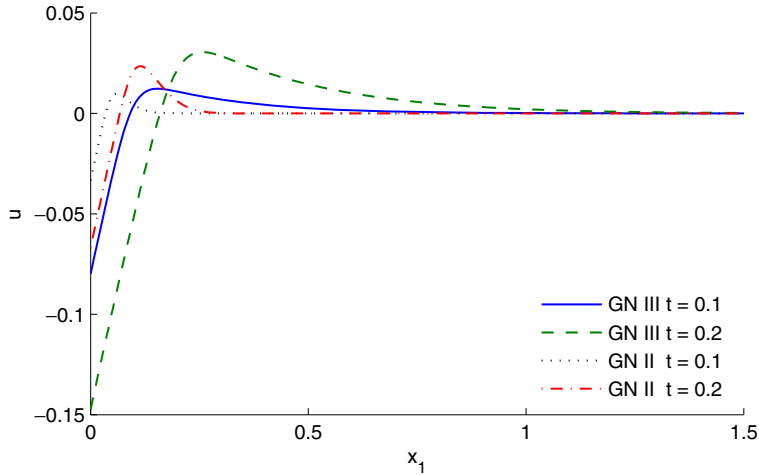
4. Application

We consider the problem of a half-space Ψ, which is defined as follows:

$$\Psi = \{(x_1, x_2, x_3) : 0 \le x_1 < \infty, -\infty < x_2 < \infty, -\infty < x_3 < \infty\}.$$
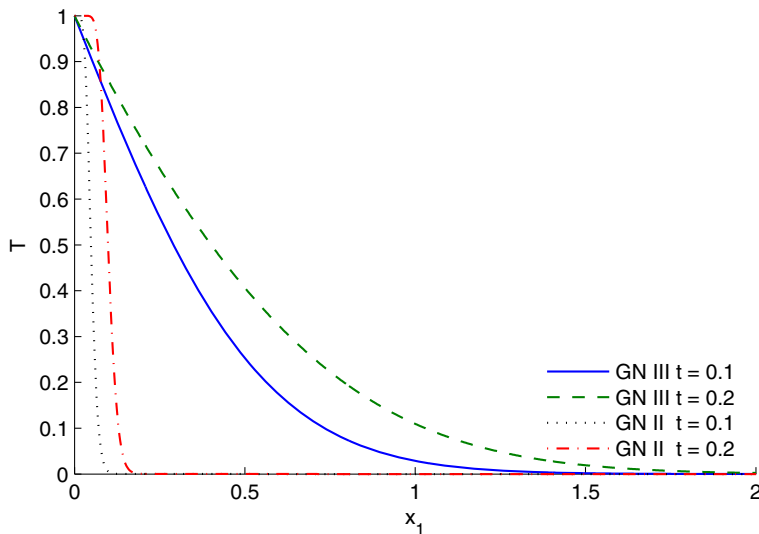
**Figure 1.** The temperature distribution for different values of t at $x_2 = 1$.

Figure 2. Horizontal displacement distribution u for different values of t at $x_2 = 1$.

The surface of the half-space is taken to be traction free, and the thermal shock $g(x_2, t)$ applied on the surface at $x_1 = 0$ is taken of the form

$$g(x_2, t) = T_1 H(t) H(l - |x_2|),\qquad (28)$$

where H is the Heaviside unit step function and T_1 is a constant. This means that heat is applied on the surface of the half-space on a narrow band of width $2l$ surrounding the x_2-axis to keep it at temperature T_1, while the rest of the surface is kept at zero temperature. Assuming the following initial conditions:

$$u = v = T = 0 \quad t = 0, \quad \dot{u} = \dot{v} = \dot{T} = 0 \quad t = 0.\qquad (29)$$
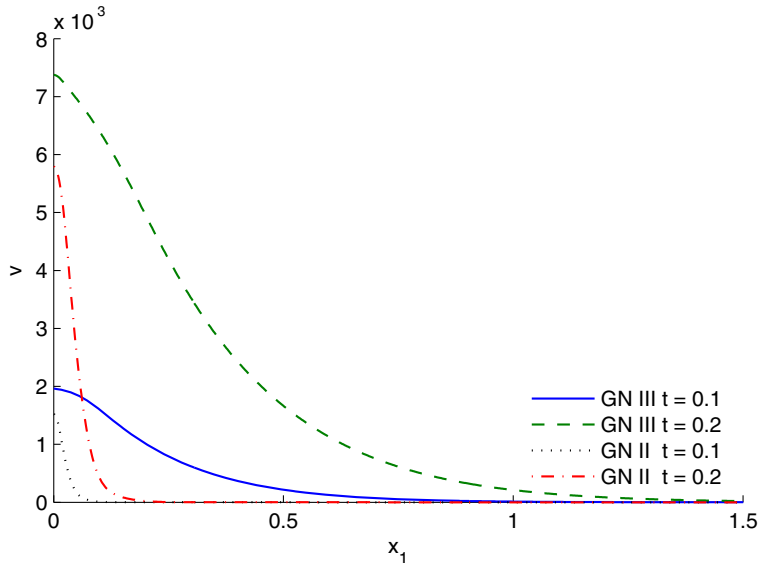
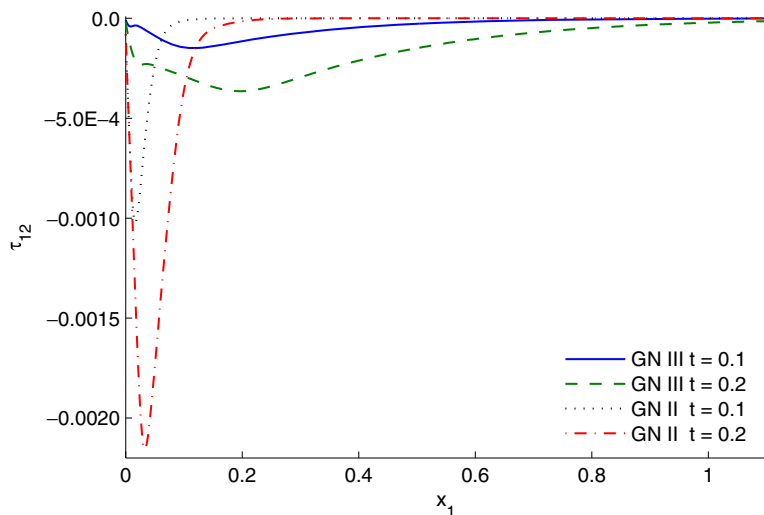**Figure 3.** Vertical displacement distribution v for different values of t at $x_2 = 1$.
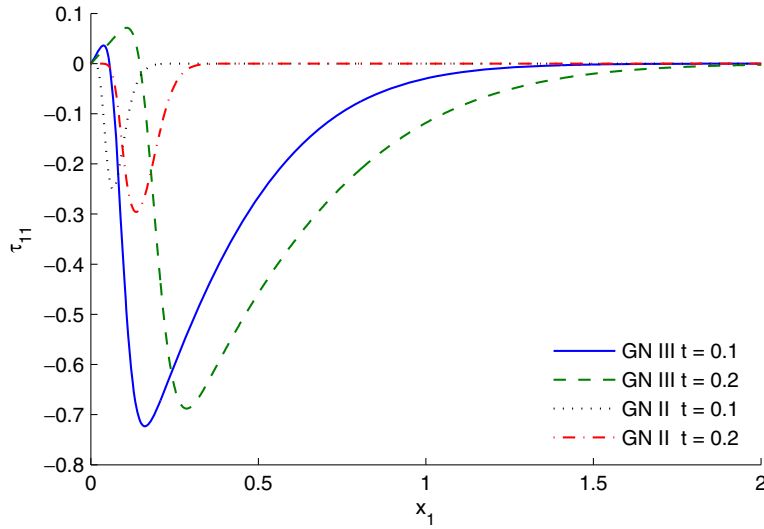
Figure 4. The distribution of stress component τ_{11} for different values of t at $x_2 = 1$.

5. Numerical example

To study the effect of reinforcement on wave propagation, we use the following physical constants for generalized fibre-reinforced thermoelastic materials.

$\rho = 2660 \text{ kg/m}^3$, $\lambda = 5.65 \times 10^{10} \text{ N/m}^2$, $\mu_T = 2.46 \times 10^{10} \text{ N/m}^2$, $\mu_L = 5.66 \times 10^{10} \text{ N/m}^2$,
$\alpha = -1.28 \times 10^{10} \text{ N/m}^2$, $\beta = 220.90 \times 10^{10} \text{ N/m}^2$, $\alpha_{11} = 0.017 \times 10^{-4} \text{ deg}^{-1}$,
$\alpha_{22} = 0.015 \times 10^{-4} \text{ deg}^{-1}$, $c_e = 0.787 \times 10^3 \text{J kg}^{-1} \text{deg}^{-1}$, $T_o = 293 \, k$,
$K_{11} = 0.0921 \times 10^3 \, J \, \text{m}^{-1} \text{s}^{-1} \text{deg}^{-1}$, $K_{22} = 0.0963 \times 10^3 \text{J m}^{-1} \text{s}^{-1} \text{deg}^{-1}$,
$T_1 = 1, l = 1$, $\varepsilon_2 = 0.25$.

**Figure 5.** The distribution of stress component τ_{12} for different values of t at $x_2 = 1$.
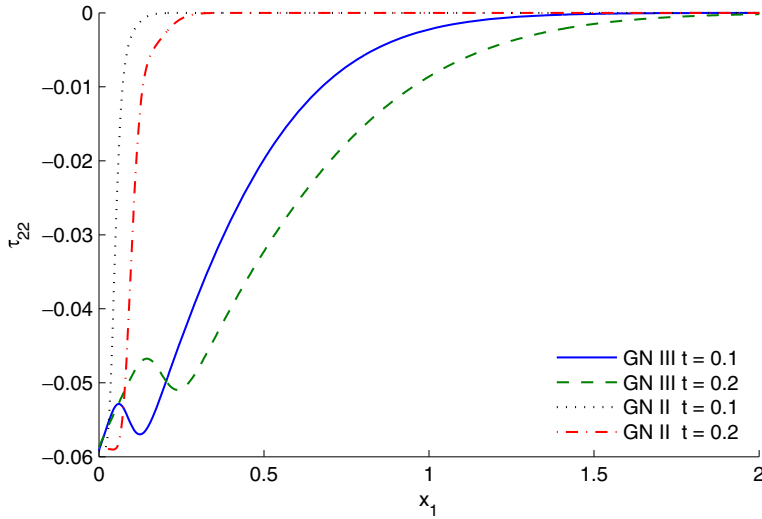
Figure 6. The distribution of stress component τ_{22} for different values of t at $x_2 = 1$.

Due to the geometrical shape and thermal boundary condition symmetries of the half-space, the problem can be treated as plane strain problem. Only half of the half-space need to be considered because of the symmetries. Calculated results for the response of the non-dimensional temperature T, displacement components u, v and stress components τ_{11}, τ_{12} and τ_{22} of the half of the half-space are shown in figures 1–12. These figures represent the solution obtained using the Green–Naghdi theory of type II ($\varepsilon_3 = 0$) and type III ($\varepsilon_3 = 1$). The Green–Naghdi theory developed a theory of thermoelasticity without energy dissipation whose temperature evolution
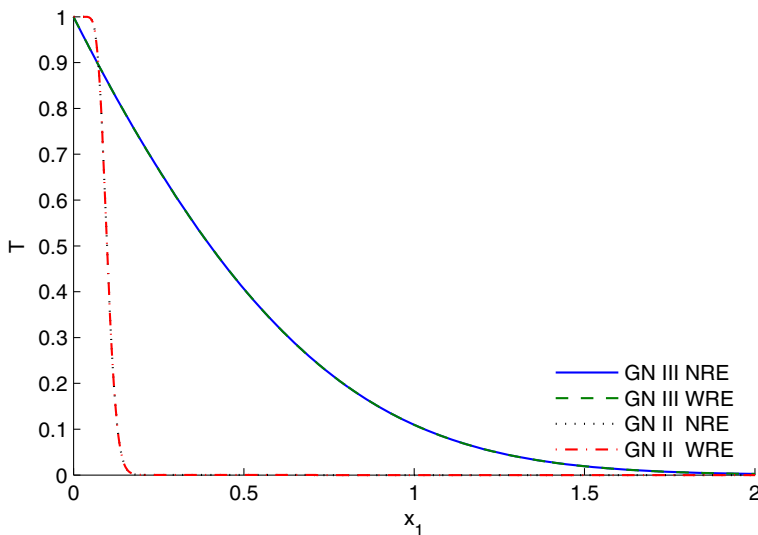

Figure 7. The temperature distribution at $t = 0.2$ and $x_2 = 1$.
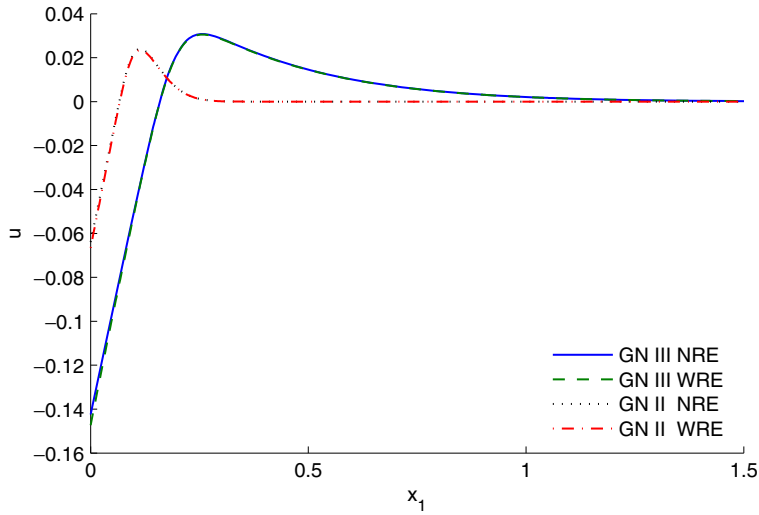
Figure 8. Horizontal displacement distribution u at $t = 0.2$ and $x_2 = 1$.

equation is hyperbolic. Among others, the introduction of a new internal variable, i.e., the thermal displacements with ($\varepsilon_3 = 0$) leads to the theory without energy dissipation.

It can be found from figures 1–12 that, the physical quantities depend not only on time and space, but also depend on the characteristic parameter of the Green–Naghdi theory. Figures 1–6 are drawn to give comparison of the results obtained for displacements, temperature and stresses for two models of Green and Naghdi, namely GN model II and GN model III, against the thickness x_1 for different values of t at $x_2 = 1$. Figure 1 shows the variation of temperature with x_1 and it indicates that temperature field has maximum value at the boundary and then decreases to zero. Figure 2 shows the variation of horizontal displacement with respect to x_1 at two different
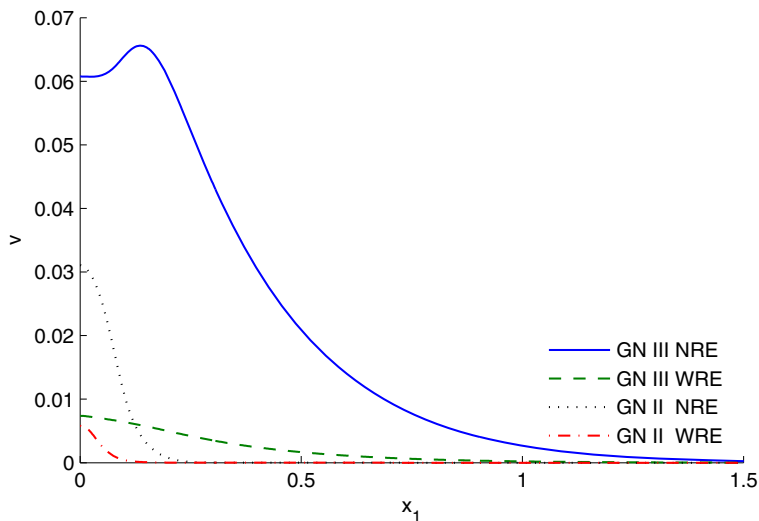

**Figure 9.** Vertical displacement distribution v at $t = 0.2$ and $x_2 = 1$.
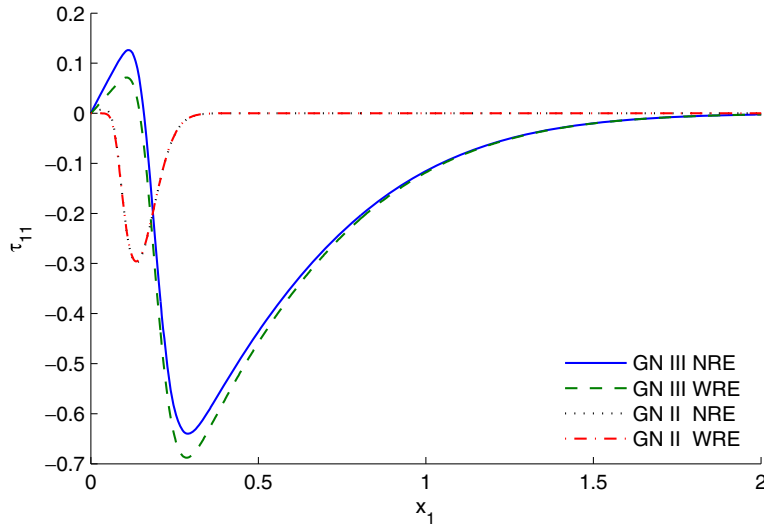
Figure 10. The distribution of stress component τ_{11} at $t = 0.2$ and $x_2 = 1$.

times and it indicates that when the surface of the half-space is taken to be traction free, and the thermal shock applied on the surface, the displacement at $t = 0.1$ shows a negative value at the boundary of the half space and it attains a stationary maximum value after some distance. Finally it decreases to zero value. The similar nature of variation is observed at $t = 0.2$. Figure 3 exhibits the space variation of vertical displacement for different values of time in which we observed that, significant difference in the value of displacement is noticed for the different value of t for the two types. We can also see that the maximum point of the displacement increases for the type (GNIII). Figures 4–6 show the variations of stress field with respect to x_1 for different values of
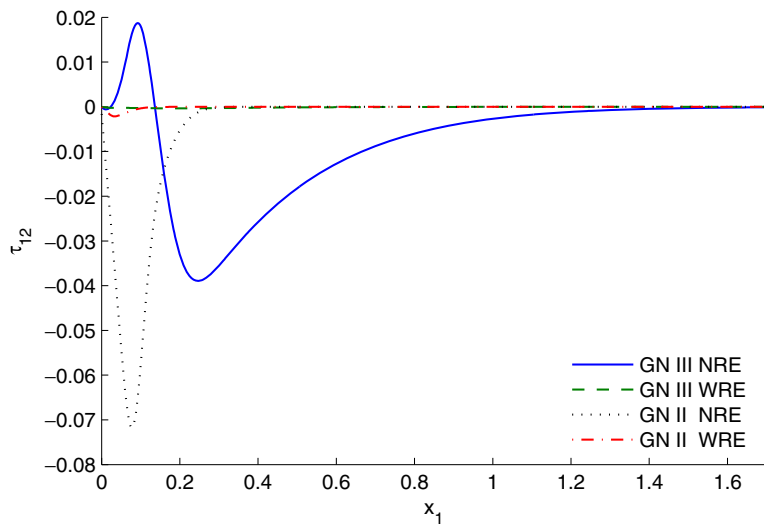
Figure 11. The distribution of stress component τ_{12} at $t = 0.2$ and $x_2 = 1$.
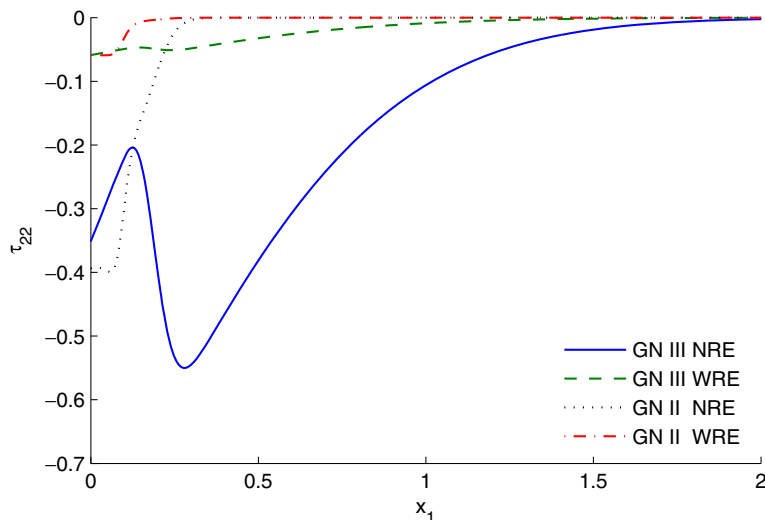
Figure 12. The distribution of stress component τ_{22} at $t = 0.2$ and $x_2 = 1$.

time in which we observed that, the stresses τ_{11} and τ_{12} are zero at $x_1 = 0$ which satisfies the boundary conditions of the problem.

Figures 7–12 show the variation of the physical quantities with space x_1 at $t = 0.2$ under two types with reinforcement (WRE) and without reinforcement (NRE i.e., $\alpha = 0$, $\beta = 0$ and $\mu_L - \mu_T = 0$). Also, the comparisons are made with the results predicted by these at $t = 0.2$ for two models of Green and Naghdi, namely GN model II and GN model III. It is easy to see from figures 7 and 8 that the reinforcement has very small effect on the temperature and horizontal displacement while, it has a significant effect on the other field quantities as expected (x_1-axis is the fibre-direction). Figures 9–12 show the fibre-reinforced has a high strength.

6. Conclusion

In this paper, we have investigated the solution of two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space based on Green–Naghdi theory by using the finite element method. The differences of the field quantities predicted by the GN theory of two types are remarkable in the presence and absence of reinforcement. When $k \to 0 \, (\varepsilon_3 = 0)$, Eq. (26)c reduces to the heat conduction equation under GN (type II) theory. The reinforcement has a great effect on the distribution of field quantities. The results carried out in this paper can be used to design various fibre-reinforced anisotropic thermoelastic elements under thermal load to meet special engineering requirements.

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