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Determining large deflections in rectangular combined loaded cantilever beams made of non-linear Ludwick type material by means of different arc length assumptions

IBRAHIM EREN

Yildiz Technical University, Mechanical Engineering Department, 34349, Besiktas, Istanbul, Turkey
e-mail: er@yildiz.edu.tr

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Abstract. In this study, large deflection of cantilever beams of Ludwick type material subjected to a combined loading consisting of a uniformly distributed load and one vertical concentrated load at the free end was investigated. In calculations, both material and geometrical non-linearity have been considered. Horizontal and vertical deflections magnitudes were calculated throughout Euler–Bernoulli curvature-moment relationship assuming different arc lengths. Vertical deflections were calculated by using Runge–Kutta method. More simple and easily understandable results have been obtained compared to the previous studies about the issue and compatible values have been obtained for most of the compared values.

Keywords. Large deflections; material non-linearity; geometrical non-linearity.

1. Introduction

Large deflections under variable loads in bearer systems is a popular subject on which many studies were conducted. Due to the importance of this subject, studies are being done on the issue. In many cases encountered in different engineering issues, results are adequately approximate. However, well known curvature is not linear in bending and thus, real material does not have linear stress–strain relationship. When this fact is considered, deflections can not be determined by analytical methods, instead numerical and approximate methods should be employed. Large deflection in uniform and non-uniform, concentrated and combined loaded linear elastical cantilever beams have been investigated in earlier studies (Bisshopp & Drucker 1945; Scott *et al* 1955; Lau 1982; Rao & Rao 1986; Baker 1993; Lee *et al* 1993; Frisch-Fay 1962; Fertis 1999). Prathap and Varadan (1976) had calculated large deflections in cantilever beams made of non-linear Ramberg–Osgood type material on which concentrated load effected on the free end. Same problem had been solved for cantilever beams on which moment effected on the free end by Varadan & Joseph (1987). Large deflections in Ludwick type non-linear cantilever beams on which concentrated load effected on the free end, had been investigated by Lewis & Monasa (1981). Lewis and Monasa (1982) had solved the same problem for cantilever beams on which moment effected on the free end.

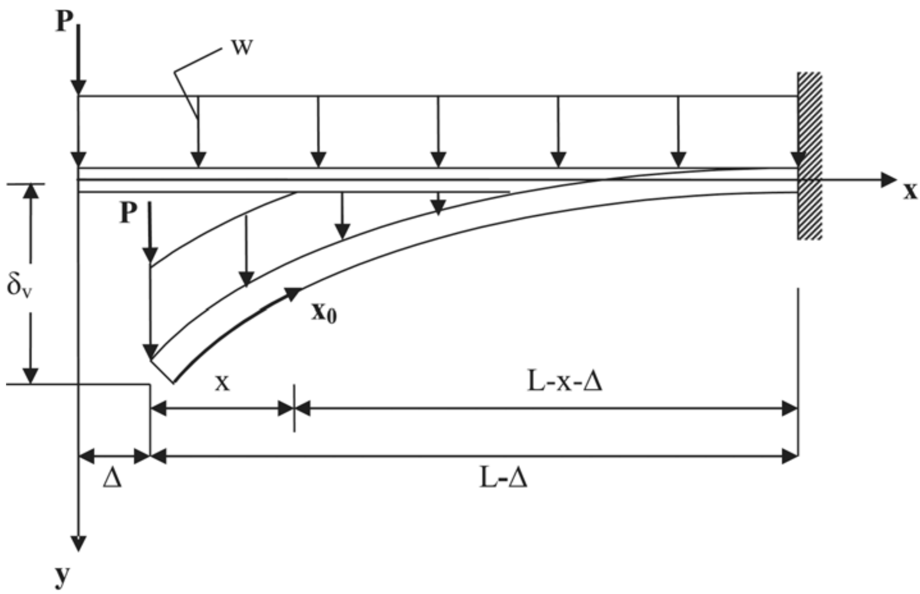

Figure 1. Cantilever beam subjected to combined load.

Lo and Gupta (1978) had calculated large deflections in rectangular beams by assuming logarithmical material stress–strain relationship beyond elastic limit. Lee (2002) had calculated large deflections in cantilever beams made of Ludwick type non-linear material on which uniformly distributed load and concentrated load effected on the free end.

2. Combined loaded cantilever beam

In this study, large deflections have been calculated in non-linear Ludwick type rectangular combined loaded cantilever beams by means of different arc length assumptions, as shown in figure 1. P is the concentrated load effecting on the free end, L is the length of cantilever beam, w is the uniformly distributed load of intensity, x_o, arc length, Δ, horizontal deflection magnitude, δv, vertical deflection magnitude at the free end. Mathematica 5·2 software has been employed for calculations.

3. Formulation and solution

Moment expression, depending on uniformly distributed load of intensity w, is given below for figure 1.

$$M = \frac{wx}{2}x_0 + Px. \tag{1}$$

Assumptions made by Fertis for arc length and horizontal deflection functions are given below:

$$x_0 = x + \Delta(x),\ \text{Fertis (1999)}, \tag{2}$$

$$\Delta(x) = \Delta,\ \text{Fertis (1999)}, \tag{3}$$

$$\Delta(x) = \frac{\Delta x}{(L - \Delta)},\ \text{Fertis (1999)}. \tag{4}$$

If (2) and (3) are written in the moment expression,

$$M = \frac{wx}{2}(x + \Delta) + Px \tag{5}$$

is obtained.

$$P = wL,\ \text{Lee (2002)} \tag{6}$$

is an assumption.

$$\sigma = B\varepsilon^{\frac{1}{n}}, \text{(Lewis \& Monosa 1982)}. \tag{7}$$

This expression is stress–strain relationship for Ludwick type material. Here, σ is stress, ε is strain, B and n is constant depending on material properties.

Euler–Bernoulli curvature-moment relationship is shown below:

$$\kappa = \frac{y(x)}{(1 + (y'(x))^2)^{\frac{3}{2}}} = \frac{M^n}{K_n}, \text{(Lewis \& Monosa 1981)}. \tag{8}$$

Here, κ is the curvature and K_n is a constant depending on material properties

$$K_n = \frac{n^n b^n h^{2n+1} B^n}{2^{n+1}(1 + 2n)^n P^n}, \text{(Lewis \& Monosa 1981)} \tag{9}$$

b is the width and h is the height of the rectangular cross section. If moment expression found for the first arc length assumption and previous equations are employed, curvature equation can be stated as below:

$$\kappa = \frac{L^{n+1}}{K_n} \times \frac{1}{L}\left(\frac{x}{2L}\left(\frac{x}{L} + \frac{\Delta}{L} + 2\right)\right)^n, \tag{10}$$

curvature equation is written according to $\frac{L^{n+1}}{K_n}$ dimensionless value.

If both sides of the curvature equation in (8) are integrated;

$$\int \kappa dx + C_1 = \frac{y'(x)}{(1 + (y'(x))^2)^{\frac{1}{2}}} \tag{11}$$

is obtained.

$$\int \kappa dx + C_1 = G,\ \text{Fertis (1999)} \tag{12}$$

$$y'(x) = \frac{G}{(1 - (G)^2)^{\frac{1}{2}}},\ \text{Fertis (1999)} \tag{13}$$

$$\frac{x}{L} = \bar{x}, \frac{\Delta}{L} = \delta_h \tag{14}$$

If these equations are converted to dimensionless units and thus,

$$G = \frac{2^{-n}\frac{L^{n+1}}{K_n}\bar{x}(\bar{x}(\bar{x}+(2+\delta_h)))^n\left(1+\frac{\bar{x}}{2+\delta_h}\right)_2^{-n}F1\left[1+n,-n;2+n;-\frac{\bar{x}}{2+\delta_h}\right]}{(1+n)}+C_1 \quad (15)$$

is obtained.

$$\bar{x} = 1-\delta_h;$$

$y'(1-\delta_h)$ value is taken as zero as a boundary condition, C_1 integration constant is obtained as shown below:

$$C_1 = -\frac{\left(\frac{3}{2}\right)^n\frac{L^{n+1}}{K_n}(1-\delta_h)^{n+1}\left(1+\frac{1-\delta_h}{2+\delta_h}\right)_2^{-n}F1\left[1+n,-n;2+n;-\frac{1-\delta_h}{2+\delta_h}\right]}{(1+n)} \quad (16)$$

Finally,

$$G = \frac{2^{-n}\frac{L^{n+1}}{K_n}\bar{x}(\bar{x}(\bar{x}+(2+\delta_h)))^n\left(1+\frac{\bar{x}}{2+\delta_h}\right)_2^{-n}F1\left[1+n,-n;2+n;-\frac{\bar{x}}{2+\delta_h}\right]}{(1+n)} \pm \frac{\left(\frac{3}{2}\right)^n\frac{L^{n+1}}{K_n}(1-\delta_h)^{n+1}\left(1+\frac{1-\delta_h}{2+\delta_h}\right)_2^{-n}F1\left[1+n,-n;2+n;-\frac{1-\delta_h}{2+\delta_h}\right]}{(1+n)} \quad (17)$$

is obtained.

Arc length equation is,

$$\int_0^{(L-\Delta)}\sqrt{(1+(y'(x))^2)} = L, \text{ (Lewis \& Monosa 1982)} \quad (18)$$

$$\int_0^{(L-\Delta)}\sqrt{\left(1+\left(\frac{G}{(1-(G)^2)^{\frac{1}{2}}}\right)^2\right)} = L \quad (19)$$

If G expression is written in the arc length equation and 3 to 1 rule of Simpson is integrated, then δ_h horizontal deflection could be calculated by finding root of the equation obtained with Newton method. This process is repeated depending on different values of n and dimensionless L^{n+1}/K_n ratio.

If, $y(1-\delta_h) = 0$ boundary condition that can be written from (13) and (17) considered, $y(\bar{x})$ interpolation function is obtained by the help of Runge–Kutta method. In this function, $y(0)$ value for $\bar{x} = 0$ gives the vertical deflection at the free end of the beam as a dimensionless δ_v/L value.

New curvature expression is written as below if the previous problem is solved by employing x_0 arc length assumption which is selected using different horizontal deflection functions stated in (2) and (4).

$$\kappa = \frac{L^{n+1}}{K_n}\times\frac{1}{L}\left(\frac{x}{2L}\left(\frac{x}{L}+\frac{x}{L}\frac{\frac{\Delta}{L}}{\left(1-\frac{\Delta}{L}\right)}+2\right)\right)^n \quad (20)$$

G is found as below when dimensionless conversion made, previous curvature and appropriate equations are employed:

$$G = \frac{2^{-n}\frac{L^{n+1}}{K_n}\bar{x}\left(-\frac{\bar{x}(\bar{x}-2(-1+\delta_h))}{(-1+\delta_h)}\right)^n\left(1+\frac{\bar{x}}{2-2\delta_h}\right)^{-n}}{(1+n)} \times {}_2F1\left[1+n, -n; 2+n; -\frac{\bar{x}}{2-2\delta_h}\right] + C_1 \tag{21}$$

When $y'(1-\delta_h) = 0$ boundary condition is employed in $\bar{x} = 1-\delta_h$, C_1 integration constant becomes,

$$C_1 = -\frac{2^{-n}\frac{L^{n+1}}{K_n}\left(1+\frac{1-\delta_h}{2-2\delta_h}\right)^{-n}(1-\delta_h)\left(-\frac{(1-\delta_h)(1-2(-1+\delta_h)-\delta_h)}{(-1+\delta_h)}\right)^n}{(1+n)} \times {}_2F1\left[1+n, -n; 2+n; -\frac{(1-\delta_h)}{(2-2\delta_h)}\right] \tag{22}$$

thus,

$$G = \frac{2^{-n}\frac{L^{n+1}}{K_n}\bar{x}\left(-\frac{\bar{x}(\bar{x}-2(-1+\delta_h))}{(-1+\delta_h)}\right)^n\left(1+\frac{\bar{x}}{2-2\delta_h}\right)^{-n}}{(1+n)} \times {}_2F1\left[1+n, -n; 2+n; -\frac{\bar{x}}{2-2\delta_h}\right] \pm \frac{2^{-n}\frac{L^{n+1}}{K_n}\left(1+\frac{1-\delta_h}{2-2\delta_h}\right)^{-n}(1-\delta_h)\left(-\frac{(1-\delta_h)(1-2(-1+\delta_h)-\delta_h)}{(-1+\delta_h)}\right)^n}{(1+n)} \times {}_2F1\left[1+n, -n, 2+n, -\frac{(1-\delta_h)}{(2-2\delta_h)}\right] \tag{23}$$

is obtained.

δ_h horizontal deflection values are calculated by finding root of the arc length equation, which is written according to G expression. This calculation is made by utilising Simpson rule and Newton method. This process is repeated depending on different values of n and dimensionless L^{n+1}/K_n ratio.

If, $y(1-\delta_h) = 0$ boundary condition that can be written from (13) and (17) considered, $y(\bar{x})$ interpolation function is obtained by the help of Runge–Kutta method. In this function, $y(0)$ value for $\bar{x} = 0$ gives the vertical deflection at the free end of the beam as a dimensionless δ_v/L value.

4. Results and discussion

In tables 1 and 2, dimensionless horizontal and vertical deflection values, which were calculated for two different x_o arc length assumptions, are tabulated depending on L^{n+1}/K_n ve n values. Besides, for $n = 2{\cdot}16$ as given in (Lewis & Monosa 1982) (7) equation, deflection in cantilever beams made of Ludwick type non-linear annealed copper material which were loaded as shown in figure 1, has been given in comparison with the result of Lee (2002).

Table 1. Large deflection values of cantilever beams made of Ludwick type material subjected to a combined load assuming to be $x_0 = x + \Delta$, Eren (2006).

x_0	n	δ	L^{n+1}/K_n 0·25	0·5	0·75	1	2	3	4	5	6	7	8	9	10
$x_0 = x + \Delta$	1	δ_h	0·007	0·028	0·059	0·094	0·235	0·341	0·418	0·475	0·519	0·554	0·583	0·607	0·627
		δ_v/L	0·113	0·218	0·31	0·388	0·587	0·684	0·738	0·772	0·797	0·817	0·835	0·851	0·86
	2	δ_h	0·008	0·029	0·057	0·087	0·191	0·266	0·32	0·362	0·396	0·423	0·447	0·467	0·484
		δ_v/L	0·12	0·226	0·314	0·384	0·552	0·637	0·687	0·721	0·746	0·765	0·779	0·792	0·802
	2·16*	δ_h	0·008	0·03	0·058	0·088	0·191	0·265	0·318	0·36	0·393	0·421	0·444	0·464	0·482
		δ_v/L	0·123	0·231	0·319	0·388	0·555	0·639	0·69	0·724	0·749	0·768	0·784	0·796	0·807
	2·16	δ_h	0·008	0·03	0·058	0·087	0·188	0·26	0·312	0·352	0·384	0·411	0·433	0·453	0·47
		δ_v/L	0·122	0·229	0·317	0·386	0·55	0·632	0·682	0·716	0·74	0·759	0·774	0·786	0·796
	3	δ_h	0·01	0·035	0·063	0·092	0·18	0·239	0·282	0·315	0·342	0·364	0·383	0·399	0·414
		δ_v/L	0·137	0·25	0·336	0·4	0·546	0·618	0·663	0·693	0·716	0·734	0·749	0·76	0·771
	4	δ_h	0·014	0·044	0·074	0·101	0·179	0·229	0·264	0·292	0·314	0·333	0·349	0·362	0·375
		δ_v/L	0·163	0·283	0·364	0·422	0·549	0·612	0·651	0·679	0·699	0·716	0·729	0·74	0·749
	5	δ_h	0·02	0·055	0·087	0·112	0·182	0·225	0·256	0·28	0·299	0·315	0·328	0·34	0·351
		δ_v/L	0·196	0·319	0·395	0·447	0·557	0·612	0·646	0·671	0·689	0·704	0·716	0·727	0·735

*References are the values in Lee (2002). Assumptions made for x_o are not valid for these calculations.

Table 2. Large deflection values of cantilever beams made of Ludwick type material subjected to a combined load assuming to be $x_0 = x + \Delta\left(\frac{x}{L-\Delta}\right)$, Eren (2006).

			L^{n+1}/K_n												
x_0	n	δ	0·25	0·5	0·75	1	2	3	4	5	6	7	8	9	10
$x_0 = x + \Delta(x/L - \Delta)$	1	δ_h	0·007	0·028	0·058	0·093	0·229	0·332	0·406	0·462	0·505	0·54	0·569	0·593	0·613
		δ_v/L	0·113	0·217	0·309	0·386	0·581	0·678	0·733	0·768	0·793	0·812	0·827	0·842	0·855
	2	δ_h	0·008	0·029	0·056	0·085	0·186	0·258	0·311	0·352	0·385	0·412	0·435	0·454	0·472
		δ_v/L	0·12	0·226	0·312	0·381	0·547	0·631	0·681	0·716	0·741	0·76	0·775	0·788	0·798
	2·16*	δ_h	0·008	0·03	0·058	0·088	0·191	0·265	0·318	0·36	0·393	0·421	0·444	0·464	0·482
		δ_v/L	0·123	0·231	0·319	0·388	0·555	0·639	0·69	0·724	0·749	0·768	0·784	0·796	0·807
	2·16	δ_h	0·008	0·029	0·057	0·085	0·183	0·252	0·303	0·342	0·374	0·4	0·422	0·441	0·458
		δ_v/L	0·122	0·229	0·315	0·383	0·545	0·626	0·676	0·71	0·735	0·754	0·769	0·782	0·792
	3	δ_h	0·01	0·034	0·063	0·09	0·176	0·233	0·275	0·307	0·333	0·355	0·373	0·39	0·404
		δ_v/L	0·137	0·249	0·333	0·396	0·54	0·612	0·657	0·688	0·711	0·729	0·744	0·756	0·766
	4	δ_h	0·014	0·043	0·073	0·099	0·175	0·223	0·258	0·285	0·307	0·325	0·341	0·354	0·366
		δ_v/L	0·163	0·281	0·362	0·419	0·545	0·607	0·646	0·673	0·694	0·711	0·724	0·735	0·745
	5	δ_h	0·02	0·055	0·085	0·11	0·179	0·221	0·251	0·274	0·292	0·308	0·322	0·333	0·344
		δ_v/L	0·196	0·317	0·392	0·443	0·553	0·607	0·642	0·666	0·685	0·7	0·712	0·722	0·731

*References are the values in Lee (2002). Assumptions made for x_o are not valid for these calculations.

Table 3. The effect of different arc length assumptions on the cantilever beam made of Ludwick type material under combined load.

			L^{n+1}/K_n												
x_0	n	δ	0·25	0·5	0·75	1	2	3	4	5	6	7	8	9	10
$x_0 = x + \Delta$	2·16*	δ_h	0·008	0·03	0·058	0·088	0·191	0·265	0·318	0·36	0·393	0·421	0·444	0·464	0·482
	2·16	δ_h	0·008	0·03	0·058	0·087	0·188	0·26	0·312	0·352	0·384	0·411	0·433	0·453	0·47
	Deviation (%)		0·00	0·00	0·00	1·14	1·57	1·89	1·89	2·22	2·29	2·38	2·48	2·37	2·49
	2·16*	δ_v/L	0·123	0·231	0·319	0·388	0·555	0·639	0·69	0·724	0·749	0·768	0·784	0·796	0·807
	2·16	δ_v/L	0·122	0·229	0·317	0·386	0·55	0·632	0·682	0·716	0·74	0·759	0·774	0·786	0·796
	Deviation (%)		0·81	0·87	0·63	0·52	0·90	1·10	1·16	1·10	1·20	1·17	1·28	1·26	1·36
$x_0 = x + \Delta(x/L - \Delta)$	2·16*	δ_h	0·008	0·03	0·058	0·088	0·191	0·265	0·318	0·36	0·393	0·421	0·444	0·464	0·482
	2·16	δ_h	0·008	0·029	0·057	0·085	0·183	0·252	0·303	0·342	0·374	0·4	0·422	0·441	0·458
	Deviation (%)		0·00	3·33	1·72	3·41	4·19	4·91	4·72	5·00	4·83	4·99	4·95	4·96	4·98
	2·16*	δ_v/L	0·123	0·231	0·319	0·388	0·555	0·639	0·69	0·724	0·749	0·768	0·784	0·796	0·807
	2·16	δ_v/L	0·122	0·229	0·315	0·383	0·545	0·626	0·676	0·71	0·735	0·754	0·769	0·782	0·792
	Deviation (%)		0·81	0·87	1·25	1·29	1·80	2·03	2·03	1·93	1·87	1·82	1·91	1·76	1·86

*References are the values in Lee (2002). Assumptions made for x_o are not valid for these calculations.

Table 4. The effect of different 'n' parameter assumptions on the cantilever beam made of Ludwick type material under combined load..

			L^{n+1}/K_n												
x_0	n	δ	0·25	0·5	0·75	1	2	3	4	5	6	7	8	9	10
$x_0 = x + \Delta$	1	δ_h	0·007	0·028	0·059	0·094	0·235	0·341	0·418	0·475	0·519	0·554	0·583	0·607	0·627
	2·16	δ_h	0·008	0·03	0·058	0·087	0·188	0·26	0·312	0·352	0·384	0·411	0·433	0·453	0·47
	Deviation (%)		12·50	6·67	−1·72	−8·05	−25·00	−31·15	−33·97	−34·94	−35·16	−34·79	−34·64	−34·00	−33·40
	1	δ_h	0·007	0·028	0·059	0·094	0·235	0·341	0·418	0·475	0·519	0·554	0·583	0·607	0·627
	5	δ_h	0·02	0·055	0·087	0·112	0·182	0·225	0·256	0·28	0·299	0·315	0·328	0·34	0·351
	Deviation (%)		65	49·09	32·18	16·07	−29·1	−51·6	−63·3	−69·6	−73·6	−75·9	−77·7	−78·5	−78·6
	1	δ_v/L	0·113	0·218	0·31	0·388	0·587	0·684	0·738	0·772	0·797	0·817	0·835	0·851	0·86
	2·16	δ_v/L	0·122	0·229	0·317	0·386	0·55	0·632	0·682	0·716	0·74	0·759	0·774	0·786	0·796
	Deviation (%)		7·38	4·80	2·21	−0·52	−6·73	−8·23	−8·21	−7·82	−7·70	−7·64	−7·88	−8·27	−8·04
	1	δ_v/L	0·113	0·218	0·31	0·388	0·587	0·684	0·738	0·772	0·797	0·817	0·835	0·851	0·86
	5	δ_v/L	0·196	0·319	0·395	0·447	0·557	0·612	0·646	0·671	0·689	0·704	0·716	0·727	0·735
	Deviation (%)		42·35	31·66	21·52	13·2	−5·39	−11·8	−14·2	−15·1	−15·7	−16·1	−16·6	−17·1	−17
$x_0 = x + \Delta(x/L - \Delta)$	1	δ_h	0·007	0·028	0·058	0·093	0·229	0·332	0·406	0·462	0·505	0·54	0·569	0·593	0·613
	2·16	δ_h	0·008	0·029	0·057	0·085	0·183	0·252	0·303	0·342	0·374	0·4	0·422	0·441	0·458
	Deviation (%)		12·50	3·45	−1·75	−9·41	−25·14	−31·75	−33·99	−35·09	−35·03	−35·00	−34·83	−34·47	−33·84
	1	δ_h	0·007	0·028	0·058	0·093	0·229	0·332	0·406	0·462	0·505	0·54	0·569	0·593	0·613
	5	δ_h	0·02	0·055	0·085	0·11	0·179	0·221	0·251	0·274	0·292	0·308	0·322	0·333	0·344
	Deviation (%)		65	49·09	31·76	15·45	−27·9	−50·2	−61·8	−68·6	−72·9	−75·3	−76·7	−78·1	−78·2
	1	δ_v/L	0·113	0·217	0·309	0·386	0·581	0·678	0·733	0·768	0·793	0·812	0·827	0·842	0·855
	2·16	δ_v/L	0·122	0·229	0·315	0·383	0·545	0·626	0·676	0·71	0·735	0·754	0·769	0·782	0·792
	Deviation (%)		7·38	5·24	1·90	−0·78	−6·61	−8·31	−8·43	−8·17	−7·89	−7·69	−7·54	−7·67	−7·95
	1	δ_v/L	0·113	0·217	0·309	0·386	0·581	0·678	0·733	0·768	0·793	0·812	0·827	0·842	0·855
	5	δ_v/L	0·196	0·317	0·392	0·443	0·553	0·607	0·642	0·666	0·685	0·7	0·712	0·722	0·731
	Deviation (%)		42·35	31·55	21·17	12·87	−5·06	−11·7	−14·2	−15·3	−15·8	−16	−16·2	−16·6	−17

When table 3 is analysed, deviation between horizontal deflections is in the 0 % and 2.49 % range depending on $\frac{L^{n+1}}{K_n}$ dimensionless value. This range is calculated for Ludwick type annealed copper material if $x_0 = x + \Delta$ assumption is made for horizontal deflection. $n = 2.16$ for this condition as stated in reference Lee (2002). If we analyse the vertical deflection at the free end deviation from the reference Lee (2002) is in the 0.52–1.52 % range. If we assume that $x_0 = x + \Delta\left(\frac{x}{L-\Delta}\right)$ then, horizontal deflection deviation is between 0 and 5% and vertical deflection deviation is between 0.81 and 2.03% compared to the reference Lee (2002) values.

In table 4, deflection values for different n and x_o values and deviations from deflection values for $n = 1$ (linear) are shown. Here, deviation is not a mistake instead, a statement showing the change in percentage. In table 4, for $\frac{L^{n+1}}{K_n}$ values where deviation is positive, vertical and horizontal deflections increase with increasing n and if deviation is negative vertical and horizontal deflections decrease with increasing n.

5. Conclusion

It should be seen that, deviation (%) for horizontal deflection values is greater than vertical deflection values for both x_o assumption. It can be concluded that $x_0 = x + \Delta$ assumption is more applicable for these calculations because this assumption gives smaller deviations than x_o assumption for vertical and horizontal deflection values and also it gives smaller values than the values obtained by Lee (2002).

If x_0 arc length is employed with close assumptions as shown previously, deflection calculation is simplified. In such cases, $y'(x)$ expression could be obtained by making x_0 assumption and employing curvature. Thus, deflection calculations become possible for different loading and joint structures by employing the mentioned solution method which is also employed for solving many other problems.

Bottom line, method employed by Fertis (1999) which is based on 2 different x_0 arc length gives much more simplified, easier and understandable solution than the method employed by Lee (2002) for calculation of large deflections in combined loaded, cantilever beams that has Ludwick type stress–strain relationship.

List of symbols

b	width of the rectangular cross section
B	Constant depending on Ludwick type material properties
h	height of the rectangular cross section
K_n	Constant depending on Ludwick type material properties
L	Length of cantilever beam
M	Bending moment
n	Constant depending on Ludwick type material properties
P	Concentrated load
w	Uniformly distributed load of intensity
x_o	Arc length
Δ	Horizontal deflection magnitude
$\Delta(x)$	Horizontal deflection function

δv Vertical deflection magnitude at the free end
σ Stress
ε Strain
κ Curvature

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