Anomalous Magnetic Moment of Muon*

Measurements Confront Calculations

Amol Dighe

Recently, the 'Muon g-2' experiment at Fermilab announced a new measurement of the muon's anomalous magnetic moment with better than one-in-a-million accuracy. While confirming the earlier measurement at the Brookhaven National Lab, it has also strengthened the discrepancy with the theoretical calculations, which have also been performed with the same level of accuracy. We shall discuss the physics behind the anomalous magnetic moment of the muon and some theoretical and experimental aspects of its determination. We shall also touch upon what this discrepancy, if confirmed, would mean for our knowledge of particle physics.

1. Magnetic Moment and Gyromagnetic Ratio

The magnetic moment of a particle tells us how it responds to an external magnetic field. In the presence of an external magnetic field \vec{B} , the torque on a particle with magnetic moment $\vec{\mu}$ is

$$\vec{\tau} = \vec{\mu} \times \vec{B} \ . \tag{1}$$

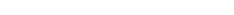
The magnetic moment is closely related with the angular momentum of the particle. For example, in classical electromagnetism, a particle with mass m and charge q, moving in a circle with a speed \vec{v} , has an orbital angular momentum $\vec{L} = m \ \vec{r} \times \vec{v}$, and a magnetic moment $\vec{\mu} = (q/2) \ \vec{r} \times \vec{v}$. As a result,

$$\vec{\mu} = \frac{q}{2m}\vec{L} \ . \tag{2}$$

However, once we include quantum mechanics, we know that ele-

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mentary particles like electrons also have spin angular momentum  $\vec{S}$ . Hence, they can have a nonzero magnetic moment even when not moving. For elementary particles like electrons, this is called their intrinsic magnetic moment  $\vec{\mu}_S$ . While the directions of  $\vec{S}$  and  $\vec{\mu}_S$  are parallel (or antiparallel) to each other, their magnitudes do not follow (2).

In general, we have

$$\vec{\mu}_S = g \frac{q}{2m} \vec{S} \ , \tag{3}$$

where g is termed as the 'gyromagnetic ratio'.

When atoms are kept in an external magnetic field  $\vec{B}$ , the energy levels corresponding to the same principal quantum number n as well as the same orbital quantum number  $\ell$  split<sup>1</sup>. As a result, the number of distinct energy levels of the electron increases. In order to explain this observed increase in the number of energy levels, Goudsmit and Uhlenbeck [3] proposed the idea of an electron having two spin polarizations so that the spin of an electron is 1/2, i.e.,  $S_z = \pm \hbar/2$ . The splitting due to the Paschen–Back effect is proportional to  $\mu_z |\vec{B}|$ . The measurements of atomic spectra yield  $\mu_z \approx \pm e\hbar/(2m)$ , so that the corresponding gyromagnetic ratio in (3) for spin angular momentum becomes  $g \approx 2.2$ 

The reason for  $g \approx 2$  for electron becomes clear from relativistic quantum mechanics. In this formalism, we understand electrons as solutions  $\psi$  of the Dirac equation [4]

$$\gamma^{\alpha} (i \hbar \partial_{\alpha} - e A_{\alpha}) \psi = m c \psi , \qquad (4)$$

where  $\gamma^{\alpha}$  (with  $\alpha=0,1,2,3$ ) are  $4\times 4$  Dirac matrices, and  $A_{\alpha}\equiv (\phi/c,-\vec{A})$  is the 4-vector that combines the electromagnetic scalar and vector potentials. The nature of this solution  $\psi$  is a 4-component object, called spinor, which describes a particle and and its antiparticle, with  $S_z=\pm\hbar/2$  for each. The magnetic moment of this solution may be obtained by evaluating the interaction of  $\psi$  with the magnetic field  $\vec{B}=\nabla\times\vec{A}$ . The calculation gives  $\mu_z\approx\pm e\hbar/(2m)$ , that is, g=2.

In relativistic quantum mechanics, the gyromagnetic ratio of all spin-1/2 particles like electron is thus g = 2. However, quan-

<sup>&</sup>lt;sup>1</sup>The nature of this splitting depends on the strength of  $\vec{B}$ . When  $\vec{B}$  is small, this is called the Zeeman effect [1], while when it is larger than the typical magnetic field inside the atom itself, it is called the Paschen-Back effect [2].

<sup>&</sup>lt;sup>2</sup>It is said that Heisenberg had asked Goudsmit, "What have you done with the factor of 2?"

tum field theory gives rise to a deviation from this, leading to a nonzero value for g - 2, called the anomalous magnetic moment.

#### 2. Anomalous Magnetic Moment

The mathematical formulation of quantum mechanics is based on the Hibert space, where particles can interact with each other, but the number of particles does not change. In order to describe phenomena that involve the creation or annihilation of particles, we need quantum field theory (QFT), whose formulation is based on the Fock space, wherein the number of particles can change. This allows us to describe the decays of particles, interconversion of mass and energy, as well as interactions among particles. In QFT, all particles are fluctuations in quantum fields defined over the whole spacetime. Real particles are the fluctuations that we can experimentally observe and track. However, there also exists 'virtual' particles that cannot be observed independently, though their effects can be observed through their interactions with real particles. In QFT, even the vacuum with apparently 'nothing' is actually filled with virtual particles that keep on getting created and destroyed all the time, as long as the short time for which they survive<sup>3</sup> satisfies the uncertainty principle  $\Delta E \cdot \Delta t \sim \hbar$ .

For example, the quantum theory of electromagnetism, i.e., quantum electrodynamics (QED), allows an electron to emit or absorb a photon. Two electrons can then interact via one electron emitting a photon and the other absorbing it. The amplitudes of such processes can be calculated using Feynman diagrams like that shown in *Figure* 1. The virtual photon that is exchanged between these two electrons is then the carrier of the electromagnetic force.

# 2.1 Magnetic Moment of Electron

In the language of QFT, the magnetic moment of an electron originates from the term that describes the amplitude of interaction of an electron with an external photon. The simplest Feynman diagram for such a process is shown on the left in *Figure 2*. The

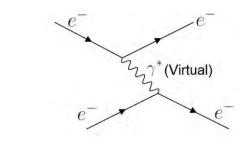
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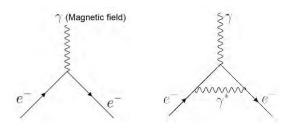
<sup>3</sup>While this is a handwaving statement, QFT provides a well-defined mathematical formulation for this.

**Figure 1.** The Feynman diagram representing an interaction between two electrons due to the exchange of a virtual photon. The virtual particles in all figures are marked with a superscript \*.

**Figure 2.** The diagram on the left represents the simplest 'tree level' process that leads to  $g_e = 2$ . The diagram on the right represents the 'one-loop' process giving the leading contribution to the anomalous magnetic moment of the electron.

The electron can emit a virtual photon before its interaction with the external photon and reabsorb the same virtual photon after the interaction. This gives rise to what is called a one-loop process.





calculation of this diagram indeed gives the gyromagnetic ratio of the electron to be  $g_e = 2$ .

However, QED allows, rather mandates, that the process shown in the right panel of *Figure* 2 also take place. That is, the electron can emit a virtual photon before its interaction with the external photon and reabsorb the same virtual photon after the interaction. This gives rise to what is called a one-loop process. This process results in an effective change in the interaction strength of the electron with the external photon, thus changing the electron's magnetic moment. This leads to the 'anomalous magnetic moment' of the electron, or the nonzero value of  $g_e - 2$ .

The contribution of the one-loop process above is, in fact, the major correction to the electron magnetic moment. It can be calculated and quantified in terms of the gyromagnetic ratio to be [5]

$$g_e - 2 \approx \frac{\alpha}{\pi} \,,$$
 (5)

where  $\alpha$  is the fine-structure constant,  $\alpha \approx 1/137$ . The anomalous magnetic moment was first experimentally confirmed by Kusch and Foley [6] by precision measurements of atomic spectra.

However, this is not all since there can even be QED processes

with multiple virtual photons and even virtual fermions, emitted and absorbed. One has to include the contributions of the 'higherorder' diagrams.

Further, QED is a part of the Standard Model (SM) of particle physics. The SM also includes the weak and strong nuclear interactions of particles. Processes involving virtual particles and their interactions with the electron via strong and weak forces also contributes to the magnetic moment of the electron. Note that, though electrons and photons themselves do not undergo strong interactions, photons interact with quarks, which in turn experience strong interactions. All such processes contribute to the anomalous magnetic moment.

Higher-order processes with multiple loops may involve combinations of the weak, strong, and electromagnetic interactions. As the number of loops increases, the number of contributing processes also increases. Fortunately, the actual contributions of these processes keep on decreasing so that the series expressed in powers of the coupling constants converges, and we can have a prediction to verify with measurements. The value of  $g_e$  has been calculated with a precision of one in  $10^{10}$ . The theoretical prediction is

$$g_e(\text{th}) = 2.002\ 319\ 304\ 3632\ (1528)\ ,$$
 (6)

where the numbers inside the bracket refer to the uncertainty in the final four digits.

The measurement of magnetic moment is possible by using (1) and the relation  $\vec{\tau} = d\vec{S}/dt$  (which is valid in non-relativistic scenarios), to get

$$\frac{d\vec{S}}{dt} = g \, \frac{q}{2m} \, \vec{S} \times \vec{B} \,. \tag{7}$$

This implies that in a uniform external magnetic field  $\vec{B}$ , the spin of the electron precesses about the magnetic field with a constant 'Larmor' frequency

$$\omega_{\text{Larmor}} = g \; \frac{q}{2m} \; |\vec{B}| \; , \tag{8}$$

QED is a part of the Standard Model (SM) of particle physics. The SM also includes the weak and strong nuclear interactions of particles. without any change in its magnitude.

The measurement of the spin precession frequency then leads to a measurement of g, or effectively, the measurement of the magnetic moment  $\vec{\mu}$ . The value of  $g_e$  has been measured to an even better level of accuracy than its theoretical calculation in (6). The measurement,

$$g_e(\exp) = 2.002\ 319\ 304\ 3614\ (0056)$$
, (9)

performed by comparing the spin precession frequency of electrons in a Penning trap with their cyclotron frequency [7], matches perfectly with the theoretical calculations in (6) within uncertainties. This is perhaps the most precise matching between the theory and experiment in any branch of science, and is a strong evidence that QFT and the SM describe our world reliably.

# 3. Anomalous Magnetic Moment of Muon

Muons are elementary particles in the SM with spin-1/2.

Muons are elementary particles in the SM with spin-1/2. As far as strong, weak and electromagnetic interactions are concerned, they behave exactly the same as electrons. However, they are about 210 times heavier than electrons. They were discovered in cosmic rays in the early 20th century. They can be easily produced at particle colliders as the decay products of charged pions, which in turn can be produced in copious amounts by the collisions of accelerated proton beams on a target. Muons are now finding applications in muon tomography for non-destructive observations of the interiors of large objects like volcanoes and pyramids. Since the measured anomalous magnetic moment of the electron has matched perfectly with the predictions, it is worthwhile asking if we can do the same with the magnetic moment of the muon. Since the calculation of this quantity needs detailed knowledge of particles and interactions in the SM, this would serve as a precision test of our knowledge of the SM.

Let us now discuss the calculation and measurement of the anomalous magnetic moment of muon,  $g_{\mu} - 2$ .

# **3.1** Calculation of $g_{\mu} - 2$

In the SM, the couplings of electrons and muons with all particles are identical, except for the Yukawa couplings with Higgs. Even the Yukawa couplings for these two particles are quite small. As a result, the major processes contributing to  $g_{\mu}$  are the same as those contributing to  $g_{e}$ , with  $e^{-}$  replaced by  $\mu^{-}$  in the description given in Section 2.1. However, since the amplitudes of some of these processes are mass-dependent, their contribution to  $g_{\mu}$  is larger. As a result, it is expected that the effect of such processes could be observable in the measurement of  $g_{\mu} - 2$  even at a lower precision.

The lowest-order correction to  $g_{\mu}$  is identical to the one for  $g_e$ :

$$g_{\mu} - 2 = \frac{\alpha}{\pi} + O(\alpha^2) , \qquad (10)$$

which is due to the one-loop process similar to the one shown in the right panel of *Figure* 2. Every additional loop in QED gives a contribution that has an additional factor of  $(\alpha/\pi)$ . As of now, QED contributions have been calculated to a precision of  $O[(\alpha/\pi)^5]$ . The contributions to  $g_\mu$  from weak interactions have also been calculated to a precision of much better than  $O(10^{-9})$ . For the electromagnetic and weak interactions, the perturbative expansion in powers of their respective coupling constants converge; hence the calculations can be truncated confidently after a few powers of the coupling constant.

This is no longer the case when strong interactions enter the picture. As mentioned earlier, photons can interact with quarks, and the quarks can further interact via the exchange of gluons, as mandated by the theory of strong interactions, quantum chromodynamics (QCD). The coupling constant of QCD is large at energies of the order of the muon mass. As a result, the perturbative expansions do not converge, and other methods are needed for reliable calculations. One of the methods—sometimes called the 'pheno' method—uses measurements from scattering experiments involving similar strong interaction processes to determine the QCD contribution to  $g_{\mu}$ . The other method is ab-initio non-

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perturbative calculations using lattice gauge theory.

The calculations are quite involved, and very few specialized groups around the world are capable of performing them at this time. Some such groups came together in 2020 for a so-called 'Theory Initiative', where the value of  $g_{\mu}$ , calculated using their collective expertise, was determined to be [8]

$$g_{\mu}(th) = 2.002 \ 331 \ 836 \ 20 \ (86) \ .$$
 (11)

Here, the numbers inside brackets refer to the uncertainty in the last two digits. Note that this is already a prediction of  $g_{\mu}$  to an accuracy of a few parts in a billion. The major uncertainty comes from the calculation of QCD processes discussed above.

The anomalous magnetic moment of muon is often expressed in terms of the fractional deviation of  $g_{\mu}$  from 2, i.e., the quantity

$$a_{\mu} \equiv (g_{\mu} - 2)/2$$
 (12)

One may write the above theoretical prediction of  $a_{\mu}$  as

$$a_u(\text{th}) = (116, 591, 810 \pm 43) \times 10^{-11}$$
 (13)

## **3.2** *Measurement of* $g_{\mu}$ – 2

The measurement of the anomalous magnetic moment of muon has a long history. The first  $a_{\mu}$  measurement was made in 1957 with an accuracy of about 30% [9]. Then there were a series of experiments at CERN that brought down the uncertainty to 8 in a million, by 1976. The E821 experiment conducted at Brookhaven National laboratory during 1990–2001 had made the most precise measurement of  $a_{\mu}$  so far, to about 0.5 in a million [10]. The 'Muon g-2' experiment at Fermilab is the latest step in this precision quest.

As compared to the electron, the main problem with measuring the magnetic moment of muon is that muon is unstable—it decays with a half-life of  $\tau_{\mu} \approx 2.2$  micro-seconds. This may not leave us with enough time for trapping the muon and trying to see its spin precess. However, the special theory of relativity comes to

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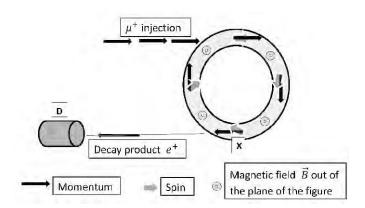


Figure 3. A rough sketch of muons in a storage ring, with momentum and spin vectors precessing at different speeds due to the anomalous magnetic moment. The detector D counts the positrons coming from the decays of muons when they are at the position X.

the rescue, providing time dilation that increases the lifetime of muon travelling with speed v, as measured in the lab:

$$\tau_{\text{lab}} = \frac{\tau_{\mu}}{\sqrt{1 - v^2/c^2}} \gg \tau_{\mu} ,$$
(14)

if  $v \approx c$ . Here,  $\tau_{\mu}$  is the lifetime of muon at rest. Thus, in order to make the muons survive longer, they have to be kept moving fast. This can be achieved by producing a collimated muon beam of fixed energy and having it go around in a ring of appropriate radius by applying a constant magnetic field  $\vec{B}$  perpendicular to its motion. A typical setup is as shown in *Figure* 3.

As a result of the muon magnetic moment, the muon spin precesses around the magnetic field<sup>4</sup>  $\vec{B}$ , in addition to changing direction along with momentum. The relative angular frequency vector for muon spin compared to the muon momentum is given by

$$\vec{\omega}_a = \frac{q}{m_\mu} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{v} \cdot \vec{B}) \frac{\vec{v}}{c^2} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{v} \times \vec{E}}{c^2} \right] , \tag{15}$$

where  $\gamma$  is the Lorentz factor, and  $\vec{E}$  is any electric field present. In the experimental set-up described above,  $\vec{v} \perp \vec{B}$ , so we have  $\vec{v} \cdot \vec{B} = 0$ , and the second term vanishes. Also, when the energy is chosen such that  $a_{\mu} \approx 1/(\gamma^2 - 1)$ , the third term can also be minimized. This 'magic energy' corresponds to  $\gamma \approx 29.3$ , or

<sup>4</sup>Clearly, such a precise experiment would need a precise magnetic field. The Fermilab experiment used the same magnet as was used in the earlier Brookhaven experiment. In fact, the delicate but large magnet of about 7 m diameter was transported from Brookhaven to Chicago on a barge, coming south along the East coast of the USA and then going north up the Mississippi river to avoid road travel as much as possible.

 $E \approx 3.094$  GeV. This is the energy of the muon beam used in the 'Muon g-2' experiment at Fermilab. The magnetic field is chosen to be  $|\vec{B}| = 1.45$  T, so that the radius of the ring is a manageable R = 7.112 m.

The experiment was performed with positive muons  $(\mu^+)$ . The direction of the muon spin is determined by exploiting its decay to a positron  $(e^+)$  and two neutrinos  $(\nu_e)$  and  $\bar{\nu}_\mu$ . While the neutrinos cannot be detected, the positron is preferentially emitted in the direction of the muon spin. Focusing on a large number of muons decaying at a given location, the direction of muon spin at that location may be determined statistically by the fraction of positrons emitted in a particular direction. The spin precession would be reflected in the modulation of this number of positrons as a function of time.

The detector D shown in *Figure* 3 counted the number of electrons produced by muons decaying when they are at position X. The time modulation of this number was fitted to a function that may be written approximately as<sup>5</sup>

$$N(t) = N_0 e^{-t/(\gamma \tau_{\mu})} \left[ 1 + A \cos(|\vec{\omega}_a|t + \phi_0) \right]. \tag{16}$$

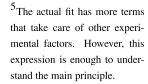
The exponential decay is due to the decaying muons, whose number decreases as they keep on going around in the ring. The modulation is due to the anomalous magnetic moment, with the angular frequency dependence on  $a_{\mu}$  as given in (15).

The measurement of the anomalous magnetic moment, based on the data analyzed till April 2021 by the Fermilab Muon g-2 collaboration, was [11]

$$a_{\mu}(\exp, \text{Fermilab}) = (116, 592, 040 \pm 54) \times 10^{-11}$$
. (17)

When combined with the earlier measurement at the Brookhaven experiment [10], which was almost as precise, the value of  $a_{\mu}$  would be

$$a_{\mu}(\exp) = (116, 592, 061 \pm 41) \times 10^{-11}$$
. (18)



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#### 4. The Discrepancy Between Calculation and Measurement

The calculated value of  $a_{\mu}$  in (13) and its measurement in (18) are not consistent with each other. Indeed,

$$\Delta a_u \equiv a_u(\exp) - a_u(\th) = (256 \pm 59) \times 10^{-11}$$
, (19)

which differs from zero by a statistical significance of about  $4.2\sigma$ . This means that the probability that the result is due to statistical fluctuations alone is less than 60 in a million, almost negligible. The Fermilab experiment is expected to analyze much more data in the next few years, reducing the uncertainty by a factor of almost 4. It is thus possible that the statistical significance of the discrepancy may become much stronger very soon.

What would such a discrepancy mean? One exciting possibility is that there are some processes beyond the Standard Model of particle physics that contribute to  $g_{\mu}$  (or equivalently, to  $a_{\mu}$ ) that have not been included in the calculations. This would be an indication that there may be new particles or new forces of nature that we are not aware of yet. Many such possibilities of physics beyond the SM are being explored. Most of them involve particles that are so heavy that they cannot be produced as real particles even at the high-energy colliders that we have at present. However, they may exist as virtual particles and make their presence felt through 'quantum corrections', i.e., contributions to the loop diagrams in QFT.

It is always prudent not to jump to conclusions, and check our calculations as well as measurements thoroughly. For example, though the comparison above has been with the theoretical prediction [8] that most of the experts agree on, the question of uncertainties, especially in the QCD contributions, has not been completely settled yet. Indeed, recently there has been a calculation [12] of these processes using lattice gauge theory, which gives a result much closer to the measurement. Resolving these theoretical issues is very crucial, especially because the result would have important consequences for our understanding of particle physics.

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Even on the experimental side, while the experiment at Fermilab was independent of the one at Brookhaven, they used the same magnet, and hence any systematic error due to the magnet will be common to both the experiments. This possibility, though remote, can only be tested if experiments are carried out that use completely different techniques to determine the same quantity. Two experiments, one at J-PARC [13] in Japan, that uses higher energy muons and a stronger but compact magnet, and the MUonE project [14] at CERN that aims to measure the differential cross section of  $\mu e \rightarrow \mu e$  to unprecedented precision, are expected to come up in the next decade, which may put to rest any doubts on the measurement side.

#### 5. Conclusion

New discoveries in science sometimes announce themselves boldly. But sometimes, they are hidden in the details and can only be revealed by tedious calculations and careful experiments.

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New discoveries in science sometimes announce themselves boldly—they appear as a clear deficit in the number of neutrinos coming from the Sun, or as a peak corresponding to the Z-boson mass in  $e^+e^-$  colliders. But sometimes, they are hidden in the details and can only be revealed by tedious calculations and careful experiments.

The matching between the calculation and measurement of the anomalous magnetic moment of the electron, to better than one part in a billion, gave us strong evidence for quantum field theory on the basis of which the Standard Model of particle physics is constructed. The mismatch between the calculation and measurement of the anomalous magnetic moment of the muon, at the level of one part in a million, may give us strong evidence that there is something beyond the Standard Model, just waiting to be discovered.

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