Asutosh Mukhopadhyay and his Mathematical Legacy

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In this article, we describe the life and career of the versatile genius, Asutosh Mukhopadhyay. The article offers a few glimpses of his mathematical talent and contributions to mathematics, and of his efforts to propagate and foster the study of and research in mathematics.

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1. Introduction

The legacy of Asutosh Mukhopadhyay (a. Mookerjee), known as '*Banglar Bagh*' (Bengal Tiger), can be viewed from different angles considering his accomplishments and achievements as an able administrator, unique educationist, exceptional visionary, eminent judge, grand orator, mathematical genius, tireless propagator of mathematical research, distinguished bibliophile and above all, a wonderful human being. His qualities like humility, greatness, patriotism, respect for cultural heritage and a love for literature arouse feelings of great admiration and reverence.

2. The Career of a Versatile Genius

Asutosh Mukhopadhyay was born as the eldest son of Jagattarini Devi and Gangaprasad Mukhopadhyay at Bowbazar of the then Calcutta on June 29, 1864. His mother, a pious lady, was known to be a woman of courage and strength of character. His father was a renowned doctor. He always took special care in the upbringing of his beloved son Asutosh. According to Syama Prasad Mukhopadhyay[1] (a. Mookerjee), second of the four sons of Asutosh, "Young Asutosh grew up in this somewhat grey and subdued domestic circle in which the only touch of colour was added by his great

[1] Syama Prasad Mookerjee (1901–1953) was a national leader and educational administrator. In 1951, he became the first Vice-President of CSIR.

Keywords

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passion for books, a passion which was steadily encouraged by his watchful father" [2, p.2].

Asutosh had a brilliant record as a student. He not only showed keen interest and intense love for mathematics, but he was certainly a mathematical prodigy. Even before writing the entrance examination (Class X) of Calcutta University, he had solved by himself the 348 exercises in *Exercises on Pott's Euclid*[2]. He stood second in that examination in 1879 and entered Presidency College where he studied from 1880 to 1885. In spite of being seriously ill (he suffered from chronic headaches and nearly lost his right hand to an electric shock), he could appear at the FA examination (Class XII) with tremendous mental strength and stood third in the university. Asutosh stood first in the BA examination in 1884 and received the Ishan and Vizianagaram scholarships and the Harishchandra Prize. In that year, he became the first Indian to be elected as a member of the London Mathematical Society. Again, he stood first in the MA examination in mathematics in 1885. When he obtained the MA degree in physical sciences the next year (1886), he earned the distinction of being the first student of Calcutta University to obtain dual MA degree. Soon after in the same year 1886, he received the Premchand Roychand Studentship, the coveted blue ribbon of university career. He had appeared in the studentship examination in pure mathematics, mixed mathematics and physics.

[2] This unpublished manuscript has been kept in the National Library of Kolkata as a part of the 'Asutosh Mukhopadhyay Collection'.

About this versatile genius, A Bose[3] would fondly recall [4, p.60]: "His (Asutosh's) versatility struck us with wonder. Literature, history, law, and the physical sciences claimed the attention of this ardent student as much as mathematics, although in those early years, mathematics was the subject which attracted him most and was enriched by contributions from him in the well-known Indian and foreign journals of the day." In 1887, Asutosh was appointed examiner at the MA examina-

[3] A C Bose was a mathematician, one year junior to Asutosh. He was the Controller of Examinations of Calcutta University under Asutosh's Vice-Chancellorship.

tion in mathematics, just a year after he himself had obtained a master's degree. He became the first non-European to be appointed as an examiner in a non-language subject. In 1889, he became a Fellow of the Senate of the Calcutta University and then a Member of the Syndicate; and from that time began his lifelong association with the university.

While he was pursuing his BA degree, Asutosh had also joined City College in 1883 for Bachelor of Law course. He was awarded Maharaja J M Tagore Law Gold Medal for three consecutive years from 1884 to 1886 for securing the first position every year. After obtaining his BL degree in 1887 he was enrolled as Vakil of the Calcutta High Court in 1888, with internship under the renowned barrister Rashbehari Ghosh. Asutosh continued his legal studies after he joined the High Court and obtained the degree of Doctor of Law in 1894. He was appointed P C Tagore Law Professor of Calcutta University in 1898. "This was the highest reward which a jurist in India could expect" [1, p.33]. As Tagore Law Professor he delivered 12 lectures on the 'Law of Perpetuities in British India', which were highly appreciated and which subsequently appeared as a book. Asutosh was appointed as the Judge of Calcutta High Court in 1904 at the age of 40 and continued the service till the end of 1923. He also served as the acting Chief Justice of Calcutta High Court for six months in 1920.

Asutosh was universally regarded as one of the greatest judges of British India. In his entire career as a judge, he gave more than 2000 judgments which "have left the mark of his personality on the legal history of India" [1, p.36]; many of them "are still quoted as masterpieces of judgment" [7, p.1567]. On the eve of Asutosh's farewell at Calcutta High Court, the then Advocate General said [2, p.8]:

"*In the maze and labyrinth of adjudged cases*

"You demonstrated the truth of the old saying, 'no precedents can justify an absurdity."

> *you ever walked with a firm step holding aloft the torch of justice. You demonstrated the truth of the old saying, 'no precedents can justify an absurdity'."*

Writing about his grandfather Asutosh, Sivatosh Mukhopadhyay remarked [7, p.1566], "The first phase of his career was as a votary of mathematics, the second phase as a devotee of law, and the third phase as a creator and builder of the University.". In the formative period of his life, Asutosh wanted to become a research professor of mathematics. This is evident from one of his Senate speeches delivered in 1920, in reply to the felicitation given to him on his appointment as acting Chief Justice of the Calcutta High Court. Asutosh said [2, p.4]:

> "*Nothing is dearer to me, nothing has been dearer to me than my university. I began life as a research student in mathematics when research was practically unknown in this country and the ambition of my life was to be a research professor in my university.*"

Opportunity came but in vain. Greatly impressed by the intellectual powers of Asutosh, Sir Alfred Croft, the then Director of Public Instruction of Bengal, wanted to meet the promising young mathematician after he had passed the MA examination. When Asutosh met him (1887), Sir Croft offered him an appointment in the Presidency College of Kolkata. Asutosh laid down two conditions. The first one was that he would not be transferred from Presidency College where he would carry on his research and the second was that he should have "the same status and pay as European members of the Educational Service" [1, p.8]. The authorities were willing to consider the first condition, but not the second one. As a proud patriot with high self respect, he declined the offer.

Later, Gurudas Bandyopadhyay (a. Banerjee) (1844–1918), the first Indian Vice-Chancellor of Calcutta University (1890–1891) and who was himself trained in mathematics, tried to create a Chair for Asutosh but failed. Let us quote Asutosh again [2, p.4]: "Mr Justice Gooroodass Banerjee, who was then Vice-Chancellor of this University, made a desperate attempt to create a Chair for me, but such were the times that he failed to collect even a sum which would yield a modest income of Rs 4,000 a year which was all that he and I thought would be sufficient to maintain me as a research professor. The result was that I drifted into law, but I made a determination at that time that, heaven willing, I would devote myself to the service of the university, so that in the next generation any aspiring scholar in my position might not drift into law but have full opportunities to serve the cause of letters and sciences."

"Bengal, in gaining a distinguished Judge and a great Vice-Chancellor, lost in him a still greater mathematician."
— *C V Raman*

Though Asutosh was extremely successful in his subsequent fields of work, he had to leave his mathematical research where he had shown enormous promise to contribute to the subject. Nobel Laureate C V Raman (1888–1970) rightly remarked about Asutosh [13, p.6]:

> "*Bengal, in gaining a distinguished Judge and a great Vice-Chancellor, lost in him a still greater mathematician.*"

The same type of remark came from R P Paranjpye[4] [2, p.4]. He said, "If he (Asutosh Mukhopadhyay) had made up his mind to devote himself entirely to the study of mathematics, he was sure to have secured a place in the front rank of world mathematicians".

Here, mention may be made of the close connection of Asutosh with the Indian Association for the Cultivation of Science (IACS)[5], and its founder Mahendralal Sircar (1836–1904). At the Memorial meeting of Sircar in 1904, Asutosh reminisced, "He was not only my father's

[4] Raghunath Purushottam Paranjpye (1876–1966), popularly known as 'Wrangler Paranjpye', was the first Indian to become (1899) 'Senior Wrangler' – the title bestowed on the student attaining the top rank in the undergraduate mathematics course of Cambridge University, a feat which was regarded very highly in Britain as an intellectual achievement. After his return to India (1902), R P Paranjpye became a professor of mathematics at Fergusson College, Pune, and later, its Principal. Subsequently, Paranjpye became the Vice Chancellor of Bombay University and Lucknow University. His legacy has inspired one of the earliest films in India – the (silent) documentary "The Return of Wrangler Paranjpye" by H S Bhatavdekar (1902).

[5] IACS, founded in 1876, was the first Indian institution where modern scientific research and teaching was pursued and cultivated. The work of C V Raman for which he was awarded the Nobel Prize in 1930 was carried out at IACS.

friend, but after my father's death (1889), he was more than a father to me" [3, p.294]. When Asutosh was young, he was inspired by the IACS movement, particularly by the lectures of Father Eugene Lafont who was another founding architect of IACS. In 1887, Asutosh was appointed as Honorary Lecturer of Mathematics at IACS. During 1887–91, he delivered 125 lectures on mathematics and mathematical physics [1, p.9]. He loved to teach and his lectures were of exceptionally high standard.

While performing the duty of a Judge of the Calcutta High Court, he was appointed as the Vice-Chancellor of Calcutta University 1906. The then Home Secretary H H Risley initiated the process. Apart from the brilliant career of Asutosh Mookerjee, Risley highlighted his long and intimate association with the University as a member of the Senate and Syndicate for over 16 years. In this period, he was the President of the Board of Studies in Mathematics for 11 years and represented the University in the Bengal Council from 1899 to 1903. He was an Additional Member of the Viceroy's Council representing Bengal during 1903–1904, and was a Member of the Indian Universities Commission in 1902. Risley also felt that it would be an advantage for the Vice-Chancellor to be a Judge of the High Court [1, pp.64–66; 7, p.1567].

Asutosh Mukhopadhyay served as the Vice-Chancellor of Calcutta University for five terms (from 1906–1914 and from 1921–1923) on honorary basis. He was also a member of the 1917–1919 Sadler Commission presided over by Michael Ernest Sadler which enquired into the condition, prospects and requirements of the university. Even after he ceased to be the Vice-Chancellor of Calcutta University in 1923, he was regarded as the most powerful person within the university and was looked up to for inspiration and guidance. As Lord Lytton remarked [11, p.197]:

"... in the eyes of his countrymen and in the eyes of the world, [he] represented the university so completely that for many years Sir Asutosh was in fact the university and the university was Sir Asutosh."

"... for many years Sir Asutosh was in fact the university and the university was Sir Asutosh."
— *Lord Lytton*

It was primarily due to his initiatives and personality that Calcutta University got transformed from a mere examination-holding body to a centre of excellence in post-graduate studies and research. He had the conviction that *'no University is now-a-days complete unless it is equipped with teaching faculties in all the more important branches of the sciences and the arts, and unless it provides ample opportunities for research'* [11, p.76]. He promoted higher studies in both Indian culture (with emphasis on Indian history, Indian philosophy, and the Indian languages and literatures) as well as modern science.

It is because of the assuring presence of Asutosh Mukhopadhyay that eminent barristers like Taraknath Palit, Rashbehari Ghosh and others donated money generously to Calcutta University. "The total value of such endowments amounted to nearly fifty lakhs of rupees" [2, p.13]. With these endowments Asutosh Mukopadhyay appointed as professors of the university, some of the best minds of India in various fields including P C Ray (1861–1944) and C V Raman (1888–1970). At that time, C V Raman was an accounts officer in the Indian Finance Department who used to carry out his research at the IACS during his spare time, after office hours. "Only a man of Asutosh's boldness and keen discernment could have entrusted the Palit Chair of Physics to a young scientist with a reputation yet to make and with no previous teaching record" [2, p.12]. In fact, some European members of the University Senate had objected to Raman's appointment as Palit Professor as he was not trained abroad and did not have a doctorate degree; and Asutosh had retorted, prophetically, that Raman would

"(Asutosh Mookerjee) was among the first to propose the idea that a university should be a centre of independent intellectual activity, of research, and of teaching of high quality at all levels."
– *Raghavan Narasimhan*
(The coming of age of mathematics in India, *Miscellanea Mathematica*, p.238)

Box 1.

"To another great citizen of Calcutta, a man who was most farseeing, profoundly gifted and inspired by the highest ideals, I mean the late Sir Asutosh Mookerjee, I am also under a deep debt of gratitude. Sir Asutosh ventured to ask a young and unknown Government official to throw aside the preferments of office and devote himself to the pursuit of knowledge under the aegis of the Calcutta University. This, on his part, was an act of great courage, whereas on mine it was just a case of following my own inclinations. But for the action which Sir Asutosh took, my scientific career would long ago have suffered an abrupt termination." – C V Raman (Nobel Laureate) at the civic reception by Calcutta Corporation on 26 June 1931.

certainly visit foreign universities some day *not to learn but to teach* and that he would guide many doctorate students (*Box* 1). He also appointed as lecturers, budding scientists likes S K Mitra, M N Saha, S N Bose, J C Ghosh and N R Sen, who later became famous in the world of science (*Box* 2). Some research facilities were provided, and postgraduate teaching began from 1917.

Box 2.

Sisir Kumar Mitra (1890-1963) inspired by the work of his college teacher JC Bose, pursued radio physics and became a pioneer in research on the ionosphere in India. His team obtained the first experimental evidence of the E-region of the ionosphere; he is also famous for his explanation of the Appleton ionization anomaly. His comprehensive treatise on the Upper Atmosphere was used by Russian space scientists while launching the Sputnik.

Meghnad Saha (1893-1956) laid the foundation of the theory of thermal ionisation; his 'Saha ionisation equation' is used to describe the chemical and physical conditions of stars. His work is a pillar of modern astrophysics.

Satyendra Nath Bose (1894-1974) is best known for his discovery of 'Bose-Einstein Statistics' which describes the statistical behaviour of certain types of particles (including the Higgs–Boson) which are now called 'bosons' in his honour.

Jnan Chandra Ghosh (1894-1959) one of the most illustrious students of PC Ray, is famous for the 'Ghosh Equation' in the theory of strong electrolytes. As its first Director, JC Ghosh was the architect of IIT Kharagpur, the first IIT of India.

Nikhil Ranjan Sen (1894-1963) worked mainly in relativity, cosmogony and fluid dynamics; he also did useful work in potential theory and probability. He obtained his PhD at Berlin under the German physicist Max von Laue (Nobel Laureate) and was the first Indian doctorate in relativity. He was very helpful to younger people.

As a result of the sincere efforts of Asutosh, many stalwarts in different subjects from different parts of India and abroad gathered at Calcutta University in different capacities to share their knowledge. Referring to the grand developments at the University of Calcutta under Asutosh's leadership, Lord Ronaldshay (Governor of Bengal during 1917–1922) said [2, p.15]:

> "*I had visions of a modern Nalanda growing up in this – the greatest and the most populous city of the Indian Empire.*"

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– Lord Ronaldshay

In a tribute to Asutosh in June 1964, S Radhakrishan (former President of India) rightly said [18, p.111]:

> "*Asutosh's love for learning and patronage of scholars, whatever their country, acknowledged no frontier. He believed in internationalization and humanization of all knowledge.*"

In spite of his duties as a judge and his preoccupations with the University, Asutosh continued his involvement with mathematics. In fact, he was associated with various academic institutions and learned societies. He was the Founder President of Calcutta Mathematical Society, which he established in 1908. He was also the first President of Indian Science Congress in 1914. He was elected President of Asiatic Society of Bengal in 1907. In 1908, he was awarded DSc (*Honoris Causa*) by the University of Calcutta. The British government awarded him the title C S I (Companion of the Star of India) in 1908 and the knighthood in 1911. In 1909, he was elected as the President of the Board of Trusties of the Indian Museum. He was learned in different languages like Sanskrit, Pali, French, German and Russian. He was given various titles at different times, namely, 'Saraswati' in 1910 from the Pandits of Nabadwip, 'Shastra Vachaspati' in 1912 from Dacca Saraswat Samaj, 'Sambudhagama Chakravarty' in 1914 from Cylone, and 'Bharat Martanda' in 1920.

In 1910, Asutosh was appointed the President of Imperial (now National) Library Council. Asutosh was a great bibliophile and had a personal library which is *one of the largest and most valuable collections of books in this country* [2, p.2]. His four sons donated their father's collection on March 31, 1949 to the National Library, Kolkata. The collection consists of his unpublished manuscripts, about 86,000 books and journals, many of which are valuable, rare in nature and of limited editions, from the last decades of the 19th century and the early decades of the 20th century, in different languages, Indian and foreign, on different subjects and a rich collection of reproductions of paintings. The collection contains a large number of books and papers on mathematics and physics of prominent contemporary and near contemporary scientists. The collection has been kept in a separate building as the 'Asutosh Mukhopadhyay Collection'.

One may have an idea about Asutosh's towering achievements from the speech of Deshabandhu Chittaranjan Das (1870–1925), the great national leader and legendary barrister [14, p.156]:

> "It has been said that he was a great lawyer. So indeed he was, but his greatness was greater than the greatness of a mere lawyer. It has been said that he was a great judge, I know he was a great judge, but here again his greatness was greater – far greater than the greatness of merely a great judge. It has been said that he was a great educationist. Undoubtedly he was. He was one of the foremost, and if you count the number of edu-

"It has been said that he was a great educationist. ... he was far greater than merely a great educationist. "
– Deshabandu Chittaranjan Das

> cationists all the world over I doubt whether you can come across a greater educationist than Sir Asutosh Mookerjee. But here again I stand on my original observation – he was far greater than merely a great educationist. His heart was with the nation. He was a builder. He tried to build this great Indian nation and honour it by his activities and I know many were the plans he formed, of work after his retirement."

The speech was delivered by C R Das at the condolence meeting of Calcutta Corporation after the untimely death of Asutosh on May 25, 1924.

3. Contributions to Mathematics

Asutosh began publication of research papers when he was only a first year student of FA (Class XI) at the Presidency College. His first paper was published in the reputed journal *Messenger of Mathematics*, Cambridge. This paper was actually written much earlier in 1877, at the age of 13. In just about twelve years, from 1880 to 1892, Asutosh published several other papers. Interestingly, a report by Sir Alexander Pedlar, Professor of Chemistry at Presidency College and Vice-Chancellor of Calcutta University (1904–1905), published in the *Journal of the Asiatic Society of Bengal*, states that there were 169 scientific research papers published in the journal during 1885–1895, and among them, only 14 were written by Indians. "Of the 14 papers written by the Indians, the lion's share was from the pen of Asutosh Mukhopadhyay (mathematics) and the last one was the epoch-making paper of Jagadish Chandra Bose." [3, p.291].

> "In your just admiration for all that is best in the culture of the West, do not, under any circumstances, denationalize yourselves."
> – *Asutosh Mookerjee*

Asutosh's researches led to his appointment as a Fellow of the Royal Astronomical Society and a Member of the Royal Society of Edinburgh in 1885 (at the age of

His heart was with the nation. He was a builder. He tried to build this great Indian nation..."
– *Deshabandu Chittaranjan Das*

21), a Fellow of the Royal Society of Edinburgh, a Member of the Royal Asiatic Society and a Member of the Bedford Association for the Improvement of Geometrical Teaching in 1886, a Fellow of the London Physical Society in 1887, a Fellow of the Mathematical Societies of Edinburgh and Paris in 1888, a Member of the Mathematical Society of Palermo of Sicily and a Member of Society de Physique of France in 1890, a Member of the Irish Academy in 1893, and a Fellow of the American Mathematical Society in 1900 [11, pp.3–4].

Asutosh sent 80 problems to the *Educational Times*, London. Subsequently. he sent the solutions of these questions. He also sent solutions to 155 questions submitted by others (including mathematicians like W K Clifford).

Simple proofs and elegance in the derived results, were the traits of Asutosh's mathematics, whether the problem be at an elementary level or of an advanced character. First, consider a simple rider which is one of the 348 problems solved by Asutosh in his school days:

Problem. *If two circles cut each other, then the line segment joining their centres bisects the common chord at right angles.*

Asutosh solved the problem in two different ways. One of the solutions is given below [12, pp. 27–28]:

Asutosh's Solution: Let the circles with centres C and D intersect at A and B *(Figure* 1). It is required to

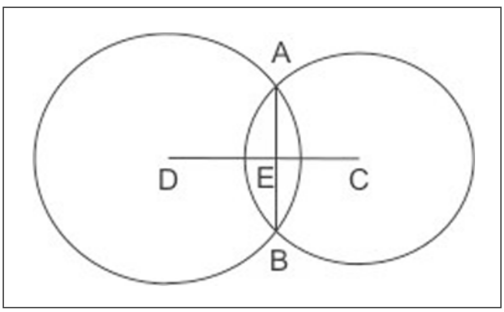


Figure 1.

prove that CD bisects AB at right angles. A, B are joined. Then a perpendicular is drawn from C on AB to intersect AB at E. Now, E and D are joined. Hence, AB is a chord to the circle with centre C, and CE is perpendicular to AB; in particular, $\angle$CEA = 1 right angle. So, CE bisects AB at right angles and we have AE = BE, i.e., E is the mid-point of AB.

Again, AB is a chord to the circle with centre D, and E is the mid-point of AB. So, DE is perpendicular to AB and hence, $\angle$DEA = 1 right angle.

So, we have, $\angle$CEA +$\angle$DEA = 1 right angle + 1 right angle = 2 right angles. Thus, DE and EC are in the same line (CD).

This completes the proof.

As an example of his elegant higher level mathematics, we consider the papers [iv] and [vii] (*Box* 3) titled, 'On the differential equation of a trajectory' and 'A general theorem on the differential equations of trajectories'. The paper [iv] was published immediately after Asutosh obtained his MA degree.

Box 3. Asutosh's Published Papers

i. Proof of Euclid I, 25, *Messenger of Mathematics*, Vol.10, pp.122–123, 1880–81.

ii. Extension of a theorem of Salmon's, *Messenger of Mathematics*, Vol.13, pp.157–160, 1883–84.

iii. A note on elliptic functions, *Quart. Jour. of Pure and Appl. Maths.*, Vol.21, pp.212–217, 1886.

iv. On the differential equations of a trajectory, *Jour. of Asiatic Soc.,* Bengal, Vol.56, Pt.II, No.1, pp.117–120, 1887.

v. On Monge's differential equation to all conics, *Jour. of Asiatic Soc.,* Bengal, Vol.56, Pt.II, No.2, pp.134–145, 1887.

vi. A memoir on plane analytic geometry, *Jour. of Asiatic Soc.,* Bengal, Vol.56. Pt.II, No.3, pp.288–349, 1887.

Box 3. Continued...

Box 3. Continued...

vii. A general theorem on the differential equations of trajectories, *Jour. of Asiatic Soc.,* Bengal, Vol.57, Pt.II, No.1, pp.72–99, 1888.

viii. On Poisson's integral, *Jour. of Asiatic Soc.,* Bengal, Vol.57, Pt.II, No.1, pp.100–106, 1888.

ix. On the differential equation of all parabolas, *Jour. of Asiatic Soc.,* Bengal, Vol.57, Pt.II, No.4, pp.316–332, 1888.

x. Remarks on Monge's differential equation to all conics, *Proc. Asiatic Soc.,* Bengal, February 1888.

xi. The geometric interpretation of Monge's differential equation to all conics, *Jour. of Asiatic Soc.,* Bengal, Vol.58, Pt.II, No.2, pp.181–185, 1889.

xii. Some applications of elliptic functions to problems of mean values, (I), *Jour. of Asiatic Soc.,* Bengal, Vol.58, Pt.II, No.2, pp.199–213, 1889.

xiii. Some applications of elliptic functions to problems of mean values, (II), *Jour. of Asiatic Soc.,* Bengal, Vol.58, Pt.II, No.2, pp.213–231, 1889.

xiv. On Clebsch's transformation of the hydrokinetic equations, *Jour. of Asiatic Soc.,* Bengal, Vol.59, Pt.II, No.1, pp.56–59, 1890.

xv. Note on Stoke's theorem and hydrokinetic circulation, *Jour. of Asiatic Soc.,* Bengal, Vol.59, Pt. II, No.1, pp.59–61, 1890.

xvi. On a curve of aberrancy, *Jour. of Asiatic Soc.,* Bengal, Vol.59 Pt.II, No.1, pp.61–63, 1890.

xvii. Mathematical Notes (Questions and Solutions), *Educational Times*, London, Vols.43 (pp.125–151), 44 (pp.144–182), 45 (pp.146–168), 1890–92.

For the above list, see [6, pp.54–55]. As mentioned by A C Bose [4] and Ganesh Prasad [5], Asutosh had written more papers that remained unpublished. These are: (a) On some definite integrals. (b) On an application of differential equations to the theory of plane cubics. (c) Researches on the number of normals common to two surfaces, two curves or a curve and a surface. (d) Application of Gauss's theory of curvature to the evaluation of double integrals.

The papers discuss the problem of determining the trajectory[6] of a system of confocal ellipses, which was first solved by the Italian mathematician Gaspare Mainardi (1800–1879) in the journal *Annli di Scienze-Mathematische e Fisiche, Tome* 1, p.251. Mainardi's problem and his own solution are as follows.

Mainardi's Problem: *Determine the oblique trajectory of a system of confocal ellipses.*

Mainardi's Solution: If the distance between the foci is $2h$, and the tangent of the angle between any given curve and the trajectory is n, then the equation of the trajectory is given by

$$-2n\tan^{-1}\sqrt{\left(\frac{h^2}{xM}-1\right)}+\log\frac{1-\sqrt{(1-M/x)}}{1+\sqrt{(1-M/x)}}=C, \tag{1}$$

where C is the constant of integration, and M is given by the quadratic equation

$$M(x^2+y^2+h^2)=(M^2+h^2)x. \tag{2}$$

But Asutosh remarked [3, p.115], "this form of the equation is so complicated that it would be a hopeless task to have to trace the curve from it; indeed, it is so unsymmetrical and inelegant that Professor Forsyth in his splendid work on Differential Equations (p.131) does not at all give the answer."

In paper [iv], Asutosh gave an elegant solution by which the trajectory was "represented by a pair of remarkably simple equations which admit of an interesting geometrical interpretation" [3, p.115]. Professor Forsyth quoted Asutosh's solution of Mainardi's problem in subsequent editions of his book on differential equations.

Asutosh's simplification of Mainardi's solution is as follows:

Asutosh's Solution: Assume

$$Mx=h^2\cos^2\phi, \quad C=2n\lambda\,, \tag{3}$$

where λ is a new constant. Substitute in (1) to get

$$\log\left[\left\{1-\sqrt{\left(1-\tfrac{M}{x}\right)}\right\}/\left\{1+\sqrt{\left(1-\tfrac{M}{x}\right)}\right\}\right]=2n\lambda+2n\phi$$

[6] Here, a 'trajectory' refers to a curve which intersects every member of the given system of confocal ellipses at a constant angle. If the (constant) angle of intersection is a right angle, then the curve is called an orthogonal trajectory; if the angle of intersection is *not* a right angle, then the curve is called an 'oblique trajectory'.

"It was remarkable that a real result was obtained by consideration of an imaginary point".
– Arthur Cayley on a paper of Asutosh Mukopadhyay

$$\implies \quad \left\{1-\sqrt{\left(1-\tfrac{M}{x}\right)}\right\} / \left\{1+\sqrt{\left(1-\tfrac{M}{x}\right)}\right\} = e^{2n(\lambda+\phi)}$$

$$\implies \quad 1/\sqrt{\left(1-\tfrac{M}{x}\right)} = \left\{1+e^{2n(\lambda+\phi)}\right\} / \left\{1-e^{2n(\lambda+\phi)}\right\}$$

$$\implies \quad 1-\tfrac{M}{x} = \left(\tfrac{1-e^{2n(\lambda+\phi)}}{1+e^{2n(\lambda+\phi)}}\right)^2$$

$$\implies \quad \tfrac{M}{x} = \tfrac{4\,e^{2n(\lambda+\phi)}}{(1+e^{2n(\lambda+\phi)})^2}.$$

But $M = (h^2/x)\cos^2\phi$. Substituting this value of M and taking the positive square root

$$\frac{h}{x}\cos\phi = \frac{2\,e^{n(\lambda+\phi)}}{1+e^{2n(\lambda+\phi)}}.$$

So,

$$x = h\cos\phi\cosh n(\lambda+\phi).$$

Again substituting the value of M in (2) and simplifying, we get

$$\frac{x^2}{h^2\cos^2\phi} - \frac{y^2}{h^2\sin^2\phi} = 1,$$

or,

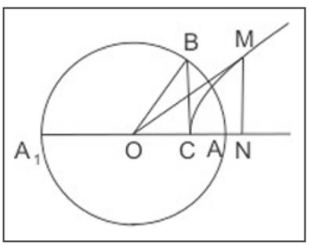
$$y = h\,\sin\phi\,\sinh n(\lambda+\phi).$$

Therefore, the coordinates of any point on the trajectory may be expressed in a very neat and symmetrical form, in terms of a parameter ϕ, viz., we have

$$x = h\,\cos\phi\cosh n(\lambda+\phi), \quad y = h\,\sin\phi\sinh n(\lambda+\phi). \tag{4}$$

Asutosh also gave a brilliant and intricate geometrical interpretation of the equations represented by (4). The interpretation is as follows *(Figure 2)*:

Figure 2.



Let A, A_1, be the foci of confocal ellipses, such that OA= h and O is the mid-point of AA_1. A circle with center O and radius h is drawn. Any point B on the circle is taken such that $\angle AOB = \phi$. Let C be the foot of the perpendicular from B on OA. Then,

$$OC = h\cos\phi, \quad BC = h\sin\phi.$$

A hyperbola with centre O and transverse and conjugate axes as OC and BC respectively is drawn. Let M be a point on the hyperbola such that the area of the hyperbolic sector OCM equals $n(\lambda + \phi)$ times the area of the triangle OCB. Asutosh proved that M is a point on the required trajectory, i.e., a point having the coordinates displayed in (4). We sketch his argument here (the reader can verify the details).

Let N be the foot of the perpendicular from M on the line OA; let $\alpha = h\cos\phi$ and $\beta = h\sin\phi$. Thus, the point M on the hyperbola is of the form $(\alpha\cosh\psi, \beta\sinh\psi)$ and the area of the triangle OMN is $\frac{1}{2}\alpha\beta\cosh\psi\sinh\psi$. Using integration, the area of the portion CMN can be seen to be $\frac{1}{2}\alpha\beta\cosh\psi\sinh\psi - \frac{1}{2}\alpha\beta\psi$, showing that the area of OCM is $\frac{1}{2}\alpha\beta\psi$. Hence, by hypothesis, $\psi = n(\lambda + \phi)$. Thus, the coordinates of M satisfy (iv).

The paper [vii] on a general theorem on the differential equations of trajectories is a development of the earlier paper [iv]. "Asutosh did not rest satisfied with giving an elegant solution of Mainardi's problem and a geometric interpretation." [4, p.62] Asutosh firmly believed that, "every simple mathematical result could be established by a correspondingly simple process" [3, p.119] and re-examined the whole question to see if the artificial process of Mainardi could be simplified. The investigation led him to a very general theorem which shows (roughly) that:

> *Whenever the coordinates of any point on a curve can be expressed by means of a single variable parameter, the coordinates of the corresponding point on the trajectory may be similarly expressed.* [3, p.119]

The precise statement of the general theorem is given below.

Theorem. *Let the coordinates of any point on a plane* *curve be represented by means of a variable parameter* θ, *by the equations*

$$x = f_1(\theta, a), \quad y = f_2(\theta, b), \tag{5}$$

where a, b *are arbitrary constants. We consider the system of curves* (5) *obtained by varying* a *and* b *subject to a condition which can be analytically represented by the relations*

$$a = F_1(\psi, h), \quad b = F_2(\psi, h), \tag{6}$$

where ψ *is a parameter and* h *a given constant. Then, the coordinates of the corresponding point on the oblique trajectory, which intersects any such curve at an angle* α, *is given by a similar expression*

$$X = f_1(\theta, F_1(\psi, h)), \quad Y = f_2(\theta, F_2(\psi, h)), \tag{7}$$

where ψ *is a function of* θ *detemined by the differential equation*

$$\frac{\mathrm{d}\psi}{\mathrm{d}\theta} = \frac{nL}{N - nM}, \tag{8}$$

where

$$n = \tan\alpha, \tag{9}$$

$$L = \left(\frac{\mathrm{d}f_1}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}f_2}{\mathrm{d}\theta}\right)^2, \tag{10}$$

$$M = \left(\frac{\mathrm{d}f_1}{\mathrm{d}\theta}\right)\left(\frac{\mathrm{d}f_1}{\mathrm{d}\psi}\right) + \left(\frac{\mathrm{d}f_2}{\mathrm{d}\theta}\right)\left(\frac{\mathrm{d}f_2}{\mathrm{d}\psi}\right), \tag{11}$$

$$N = \left(\frac{\mathrm{d}f_2}{\mathrm{d}\theta}\right)\left(\frac{\mathrm{d}f_1}{\mathrm{d}\psi}\right) - \left(\frac{\mathrm{d}f_1}{\mathrm{d}\theta}\right)\left(\frac{\mathrm{d}f_2}{\mathrm{d}\psi}\right). \tag{12}$$

Merely stating and proving the general theorem did not satisfy Asutosh; he also applied it to determine the oblique trajectories of: (i) a system of confocal ellipses (Mainardi's problem), (ii) a system of confocal hyperbolas, (iii) a system of parabolas having a common principal axis and which touch each other at their common vertex, (iv) a pencil of coplanar rays radiating from a point (by two methods), (v) a system of circles which touch a given straight line at a given point, (vi) a system of parabolas which have a common focus and principal axis, vii) the system of curves obtained by varying b in the equation $e^x \sin y = ab$, and other problems. He also connected the theorem with the theory of conjugate functions.

William Booth, a "*brilliant but somewhat eccentric Irish Mathematician*" [4, p.63], was a Professor of mathematics of Asutosh at Presidency College. Asutosh was informed by Booth that, during his lectures on differential equations at the University of Dublin, Michael Roberts used to present a solution of Mainardi's problem by the help of elliptic coordinates. Asutosh, who had no opportunity of examining the unpublished solution arrived at by Roberts, worked out his own solution by means of elliptic coordinates but remarked that the result obtained is not in an appreciably simpler form and the method is not suited for general application; the methods and theorems of Asutosh (in papers [iv, vii], of *Box* 3) effectively removes these disadvantages [3, pp. 149–151]. A C Bose observed [4, p.63], "No wonder Asutosh from being a beloved pupil, in subsequent years, became William Booth's valued friend!".

We now discuss some of the other papers of Asutosh Mukhopadhyay listed in *Box* 3.

(a) His first published paper gave an elegant alternative proof of Proposition 25 of Euclid's *Book I*. The proposition states [6, p.55]: "If in any two triangles two sides are equal and if the third side of the first is greater than that of the second, then the angle opposite the third side of the first is greater than the angle opposite to the corresponding side of the second". This new proof is done by the method of superposition and it is shorter than the lengthy proof given in Euclid's book. This proof was discovered by him when he was in school.

(b) Asutosh's second paper titled 'Extension of a theorem of Salmon's' was also published in *Messenger of*

Mathematics, Cambridge. Salmon's theorem states: "*If* A, B, C, D *be any four points on a circle and* O *is the fifth point taken arbitrarily, and if we denote the area of the triangle* BCD *by* [BCD], *etc. then we have*

$$OA^2.[BCD]+OC^2.[ABD] = OB^2.[ACD]+OD^2.[ABC]\text{''}.$$

In his paper, Asutosh has given an extension of the above theorem to $2n$ points in a circle (instead of just 4 points). To arrive at his result he considers initially $n = 3$, that is, the case of a hexagon in the circle.

Asutosh also obtained a three-dimensional analogue of Salmon's theorem. The paper was written when Asutosh was still an undergraduate student.

(c) The third paper [iii in *Box* 3] is perhaps influenced by J L Lagrange's books, *Theorie des function analytic* (1797) and *Lecon Sur le Calculus des function* (1804), and A M Legendre's book, *Traite des functions elliptiques*. In this outstanding paper titled 'A note on elliptic functions', Asutosh established an addition theorem in the theory of elliptic functions by a novel process based on the properties of the ellipse. Regarding the paper, Arthur Cayley (1821–1895) observed (1887):

> "It was remarkable that a real result was obtained by consideration of an imaginary point."

This paper has been referred to in Enneper's *Elliptische Funktionen* [6, p.51]. The paper was written when Asutosh was an MA student.

It was remarkable that a real result was obtained by consideration of an imaginary point.
– Arthur Cayley on a paper of Asutosh Mukopadhyay

(d) The paper 'A memoir on plane analytic geometry', [vi] is the longest of all the papers of Asutosh. It contains 62 pages and is divided into 32 sections, which are practically independent of each other. Before the publication of the paper, Asutosh presented the entire material in a lecture at IACS, Calcutta. Regarding the object and scope of this memoir, Asutosh says [3, p.25], "It is my object in the present paper to bring together a number of theorems in plane analytic geometry which have accumulated in my hands during my study of the subject. . . . I believe that either the theorems themselves, or the methods of establishing them are original.".

Indeed, as A C Bose writes [4, p.67]: "The originality and breadth of treatment of fundamental topics of analytic geometry in this paper, disclose a master hand." The numerous topics "receive a freshness and elegance of treatment from this young mathematician of only 23 years of age, which is simply astonishing!"

We quote one theorem from this paper [3, p.42].

Theorem. *If from any point* P, *tangents are drawn to the conic*

$$S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0,$$

and P *is constrained to move on any curve*

$$F(x, y) = 0,$$

then, the locus of the middle point of the polar chord of P *with regard to S is*

$$F(\frac{x}{1+S}, \frac{y}{1+S}) = 0.$$

(e) Now, a few words about the papers [v, ix, x and xi] in *Box* 3. The paper [v] titled: 'On Monge's differential equation to all conics' is of outstanding merit. "In it, he published a challenge to mathematicians as regards the true geometric interpretation of this famous equation – a challenge which led to a controversy out of which the Indian mathematician emerged with flying colours." [4, pp.63–64].

The French mathematician Gaspard Monge (1746–1818) was the first to find the differential equation of all conics in 1810 (that is, to curves that are represented by the general equation of the second degree in x, y). In the first edition of his book, *A Treatise on Differential Equations* (1859), George Boole (1815–1864), quoting the result (i.e., the differential equation) of Monge made the remark [3, p.158]: "But, here our powers of geometrical interpretation fail, and results such as this can scarcely be otherwise useful than as a registry of integrable forms."

Subsequently, two attempts were made to interpret geometrically the Mongian differential equation. The first attempt was made by Alan Joseph Cunningham (1842–1928) in 1877 and the second one by J J Sylvester (1814–1897) in 1886.

In paper [v], Asutosh discussed in detail the questions of derivation of the Mongian differential equation from the integral equation of the conic and the geometric interpretation of the Mongian differential equation with reference to Boole's remark and Sylvester's interpretation. He showed that the geometric interpretation of Sylvester was not the true interpretation. In a letter written in 1887 to the 23-year old Asutosh, Cayley remarked about his criticism of Sylvester's interpretation that [4, p.66], [6, p.52] "It is of course, all perfectly right".

In paper [ix], Asutosh presented a geometrical interpretation of the differential equation of all parabolas. The investigations led to his paper [x] in which Asutosh laid down two fundamental tests which should be applied if one wishes "to examine whether a proposed interpretation of a given differential equation is relevant or not" [3, p.256].

The paper [x] was in response to a letter written by Cunningham to the Asiatic Society of Bengal referring to the paper [v] of Asutosh and drawing attention to the fact that he had already given the geometric interpretation of the Mongian equation. In the paper, Asutosh critically examined Cunningham's interpretation and rejected it on the ground that it does not satisfy any of the two tests for a true geometrical interpretation of a differential equation. In fact, he showed by actual calculation that Cunningham's interpretation was not the interpretation of the Mongian but of one of its five independent first integrals. Sylvester's attempted interpretation had satisfied one of the two tests, but not the other.

Finally, in paper [xi], Asutosh gave his own interpretation of the Mongian differential equation. Asutosh's geometric interpretation of the Mongian equation was accepted by all. This interpretation has been quoted by Edwards in his book on differential calculus. Cunningham acknowledged [*Nature*, Vol.38, pp.318–319]:

> "*Professor Asutosh Mukhopadhyay has proposed a really excellent mode of geometric interpretation of differential equations in general, viz., writing the equation in the form* $F = 0$*, the geometric meaning of the symbol* F *considered as a magnitude* (*angle, length, area, etc.*) *in any curve whatever (wherein* F *is of course not zero) is, if possible, to be formed; then the geometric meaning of that equation obviously is that the quantity* F *vanishes right round every curve of the family represented by the differential equation. This is the most direct geometric interpretation yet proposed.*".

"Professor Asutosh Mukhopadhyay has proposed a really excellent mode of geometric interpretation of differential equations in general, ..."
– A J Cunningham on a paper of Asutosh Mukopadhyay

Asutosh did his research all by himself. No one was there to guide him. Ganesh Prasad rightly remarked, "Sir Asutosh's contributions to mathematical knowledge were due to his unaided efforts while he was only a college student. After Bhaskara, he was the first Indian to enter into the field of mathematical research[7] as distinguished from astronomical research, and did much which was truly original" [5, p.54]. Ganesh Prasad also said

[7] In making this statement, it appears that Ganesh Prasad was unaware of the Kerala School of Mathematics founded by Madhava (c.1340-1425) and the works of other medieval mathematicians like Narayana Pandita.

"Sir Asutosh's boldness of vision and independence of thought – qualities absolutely necessary for success as a mathematical investigator – showed themselves very early when, as a student of the first year class of Presidency College, he wrote his first paper in 1880".
– *Ganesh Prasad*

[5, p.53]: "Sir Asutosh's boldness of vision and independence of thought – qualities absolutely necessary for success as a mathematical investigator – showed themselves very early when, as a student of the first year class of Presidency College, he wrote his first paper in 1880.".

Asutosh Mukhopadyay's work for enriching mathematics was manifold. Aware of the importance of good textbooks in mathematics, he himself wrote a textbook titled, *An Elementary Treatise on the Geometry of Conics*, which was published in 1893 (London, Macmillan and Company). In the preface of the book, written on 19th April, 1893, one finds, "This work contains elementary proofs of the principal properties of conics, and is intended for students who proceed to the study of the subject after finishing the first six books of Euclid." This 184-page textbook contains chapters on parabola, ellipse, hyperbola with diagrams and illustrations. There are slightly more than 100 propositions (25 on parabola, 35 on ellipse, 42 on hyperbola) and more than 800 exercises. It remained most preferred as the textbook for many years owing to its superiority over the then prevailing books by others. He also wrote a book (with coauthor Shyama Charan Bose) titled, *Arithmetic for Schools* (1901).

4. Promotion of Study of Mathematics and Research

It is true that after the textbook, we have not found any mathematical writing of Asutosh. But, he continued his efforts to update himself with development of mathematical knowledge of his time, to patronise, encourage research and to inspire young talents in study and research. The convocation address delivered on December 26, 1913, shows how well acquainted Asutosh was with the developments in mathematics of that time. In one portion of his address, he says [11, p.114]: "The theory of functions of a real variable – took its rise, partly in the speculation of George Cantor on the nature of the concept of mathematical infinity, and partly in the persistent endeavours of isolated purists to give a more rigid form to proofs of well-known theorems and a greater precision to the conditions under which certain processes are allowable" [11, p.113]. In that address, while informing about the appointment of William Henry Young (1863–1942) as the first occupant of the Chair of mathematics named after Lord Hardinge, he asserted that advanced students would be "stimulated by his teaching and example to undertake fruitful work in the most promising regions of the ever-widening domain of mathematical research."

Asutosh played an important role in the early development of mathematics in India by successfully establishing the Calcutta Mathematical Society (CMS) on September 6, 1908 and starting a research journal, *Bulletin of Calcutta Mathematical Society* in 1909. The foundation of the two mathematical societies in India (the Indian Mathematical Society in 1910, initially established as Analytic Club in 1907 by V Ramaswami Iyer, and the Calcutta Mathematical Society by Asutosh Mukhopadhyay in 1908) constitutes an important milestone in the history of development of mathematics and mathematical research in modern India.

CMS was started with the main objectives of encouraging and promoting mathematical study and research, improving mathematical studies, writing, compiling, publishing journals, books and reports of mathematical sciences, etc. Largely because of Asutosh, CMS could gradually witness the emergence of mathematical activists seriously bent on contributing to the subject through research. He would regularly visit the mathematical society and ask for the progress made in the researches of the young lecturers M N Saha, S N Bose, N R Sen, N N Sen and others whom he had recruited for the mathematics and physics departments of Calcutta University. It is noteworthy that Ganesh Prasad (in 1909), AR Forsyth (in 1913) and W H Young (in 1915) delivered addresses before the Society. What is most notable is that Asutosh was present and presided over each of the 83 meetings the Society held during his lifetime. The early issues of the *Bulletin of CMS* show considerable contact with continental mathematics of that time. The bulletin found a galaxy of mathematicians and scientists like C E Cullis, Shyamadas Mukhopadhyay, W H Young, B N Seal, Ganesh Prasad, C V Raman, D N Mallik, M N Saha, A C Bose, B B Datta, S N Bose, N R Sen, S K Mitra and others, to contribute to the journal.

It is remarkable that under Asutosh's personal initiative, encouragement and inspiration, research workers – some of them very brilliant – began to gather around him. His continuing queries to young researchers of those days about the progress of their research work were a source of inspiration to them. CE Cullis rightly remarked [2, p.15]:

> "*To every band of men engaged in the quest after truth and light, his (Asutosh's) help and encouragement were greatly and unselfishly given, and in learned societies and gatherings he was a dominant figure, his giving appreciation where it was due and advice where it was needed*".

To every band of men engaged in the quest after truth and light, his help and encouragement were greatly and unselfishly given, and in learned societies and gatherings he was a dominant figure, his giving appreciation where it was due and advice where it was needed.
– C E Cullis

5. Epilogue

A lot of information about Asutosh's love for mathematics, his likes and dislikes, pattern of study and living may be deciphered from his diary [2], though it was written only for a very short period. His diary and collection of books and journals bear testimony to his closeness to mathematics and love for creating a specialised culture in the mathematical arena. See *Box* 4 for his inspiration to students.

Box 4. Excerpts from Asutosh Mukhopadhyay's Convocation Addresses [A]

1. "Neglect not in the glare of Western light, the priceless treasures which are your inheritance. In your just admiration for all that is best in the culture of the West, do not, under any circumstances, denationalize yourselves. Do not hesitate to own at all times that you are genuine Indians, and do not fail to rise above the petty vanities of dress and taste. Above all, sedulously cultivate your vernaculars, ..."

2. "Follow the path of virtue, which knows no distinction of country or colour; be remarkable for your integrity as for your learning, and let the world see that there are amongst you:

   'Souls tempered with fire,
   Fervent, heroic and good –
   Helpers and Friends of mankind*.' "

3. "People of Bengal, you have at your doors, the foundations already laid of a great University, a University devoted to the advancement of literature, science and art, to the promotion of Letters as the record of the achievements of the human spirit, to the promotion of Science as the revealer of the laws and the conqueror of the forces of Nature, to the promotion of art as the sunshine and gilding of life, but more than all this, to the investigation of the glorious past of India and the fundamental unity, amidst apparent diversity, of the varied aspects of Indian civilisation which is so deeply calculated to rouse and purify true national instinct and national pride."

4. "I call upon you to take this as your motto, and to join with me in a fervent prayer for the well-being of our motherland in the words of the message of our great national poet, Rabindranath Tagore**:

   'Where the mind is without fear and the head is held high;
   Where knowledge is free;
   Where the world has not been broken up into fragments by narrow domestic walls;
   Where words come out from the depth of truth;
   Where tireless striving stretches its arms towards perfection;
   Where the clear stream of reason has not lost its way into the dreary desert sand of dead habit;
   Where the mind is led forward by Thee into ever-widening thought and action —
   Into that heaven of freedom, my Father, let my country awake!' "

5. "Fellow graduates, you speak of this University as your alma mater. Do you always realise the nobility of this commonplace expression? What a singular endearment it voices – our fostering mother – what fine relation is that for a great institution of learning to bear to all those who throughout the years have learned wisdom at her feet and have gone out into the world, sustained by her strength and inspired by her lofty example."

Box 4. Continued...

Box 4. Continued...

6. "I call upon you, fellow graduates, to join with me, in the words of the warrior poet***, in a solemn pledge of eternal devotion to the spirit of our motherland, the protecting divinity of our alma mater.

   'I vow to thee, my country — all earthly things above —
   Entire and whole and perfect, the service of my love —
   The love that asks no question; the love that stands the test,
   That lays upon the altar the dearest and the best;
   The love that never falters, the love that pays the price,
   The love that makes undaunted the final sacrifice.' "

*From '*Rugby Chapel*' by Matthew Arnold

** The poem '*Prarthana*' from *Naibedya*, included in the English version of *Gitanjali*.

*** The first stanza of the British patriotic song "I vow to thee, my country" by Sir Cecil Spring Rice. The poem, written in 1908, was set to music by Gustav Holst in 1921.

[A] Sir Asutosh Mookerjee: A Tribute (On 150th Birth Anniversary of Sir Asutosh Mookerjee), University of Calcutta and Asutosh Mookerjee Memorial Institute, 2013.

To have a glimpse of his greatness, one can cite dozens of quotes about Asutosh, but I conclude here by giving just three:

> "*He (Asutosh) was mighty in battle. He could have ruled an empire. But he gave the best of his powers to education because he believed that in education rightly interpreted lies the secret of human welfare and the key to every empire's moral strength*" [Sir Michael Sadler, 11, p.195].

> "*Men are always rare in all countries through whom the aspiration of their people can hope to find its fulfillment, who have the thundering voices to say that what is needed shall be done; and Asutosh had that magic voice of assurance. He had the courage to dream because he had the power to fight and the confidence to win, his will itself was the path to the goal*" [Rabindranath Tagore, 11, p.193].

"*Surely the gratitude and support of every Indian, who truly loves his country, is due to the man who has done so much for Indian learning. That man is himself an Indian among Indians, Sir Asutosh Mookerjee*" [Lord Ronaldshay, 2, p.15, 19, p.124].

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Grothendieck's theorem painted by himself on the egg given to John Tate during Easter 1964 – photograph courtesy of Dinesh Thakur. *Resonance* received this just too late to catch the special issue (June 2015).