Hunting in Groups

Dynamics of Chase and Escape

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The dynamics of group chase-and-escape have attracted many physicists. In this article, we highlight the biological behaviors and the underlying mathematical rules and physical laws involved during the process of hunting in groups. We discuss some recently proposed stochastic models for such a system, which reveal various universal statistical features and show how slight variation of the behavioral rules or the interaction parameters can lead to very different statistical behaviors. A number of possible challenging questions that might lead to the development of a more life-like model for this novel enthralling dynamical problem are addressed.

1. Introduction

One of the highly fascinating tactical natural instincts in the animal kingdom is *hunting*. Hunting is a consequence of competition for survival – a deadly and most stunning game between prey and predators. Cooperative hunting in groups has a long and fascinating history with a special place in poetry, art and literature. A successful hunt requires a great deal of cooperation and coordination within the group. The Neanderthal man devised several ingenious ways of trapping animals and coordinated groups for hunting [1]. Since then, hunting has dealt a profound impact on human civilization and it was one of the distinctive features of several primitive cultures of our ancestors. The rich variety of hunting skills in the animal kingdom, namely, the amazing art of falconry, the hunting artistry of indigenous foraging people, the clever hunting tactics of chimpanzees,

and the group hunting of hyenas, wild dogs, wolves and lions, have intrigued and motivated evolutionary biologists, computer scientists, physicists and mathematicians for a long time [1–4].

Hunting in groups involves two main actions: *chasing* and *escaping*. The group chasing and escaping process depends crucially on the differences in the number of predators to prey. The simplest possible case among them is the interaction of one chaser and one target [2]. A relatively complicated case is when a target escapes from many chasers, which has been extensively studied in the last few decades [5, 6]. A far more realistic and dynamically perplexing case is when a group of targets escapes from many chasers [7], in which the chasers cooperate among themselves and form packs to trap the flock of targets. In reality, many animal species such as zebra, wildebeest, deer, fish and various insects, aggregate in groups forming flocks, herds, schools and swarms for the benefit of avoiding interaction with grievous predators and searching for food efficiently [8]. On the other hand, most of the predator species that live in groups form packs, prides, pods and clans to successfully capture the prey, particularly the large, agile, or dangerous prey [8, 9]. Recent ecological investigations [10, 11] reveal that group formation strongly stabilizes prey–predator dynamics and therefore, grouping has a profound impact on the underlying stability of many ecosystems. In order to create a life-like animation for such a stunning game, it is necessary to unveil the underlying mathematical rules and physical laws involved in the dynamics of chase-and-escape. To achieve such a goal, joint endeavor of ecologists, mathematicians, physicists and computer scientists is necessary.

Surprisingly, hunting in groups in which a group of predators hunt a group of gregarious prey, has recently attracted many physicists [7, 12, 13]. Physicists are interested in the existence of some universal features possibly

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shared among many different situations. In statistical physics, the term *universality* manifests the fact that quite disparate systems behave in a remarkably similar way irrespective of their details. This universality is characterized by the scale invariance behavior of the systems leading to the power–law relationships among the physically relevant parameters. Although such scale invariance is widely observed in biology [14, 15], the basis for scale-invariant behavior has remained elusive. Statistical physics models for chase-and-escape allow us to model the interaction between prey and predators in very different ways.

Each individual is controlled by certain mathematical rules, physical laws and biological behaviors. The action of chasers in group hunting involves a number of consecutive states – watch, approach, attack-group, attack individual and capture [2, 8]. On the other hand, the targets in a group follow the consecutive states – watch, move away and avoid capture [3, 8]. Several biological factors such as vigilance posture, confusion effect (discussed in Section 4), etc., and the related physical factors such as group size, the relative distance among the individuals, speed of the individuals, sighting range, alignment, the strength of repulsive or attractive force, the space dimension, etc., are involved in the process of hunting and consequently, several natural questions arise when we tackle this problem using statistical mechanics. How the individual group members aggregate, share information, respond to perturbations, collectively move and temporally synchronize are some of the challenging and bewildering issues that have to be addressed in a more realistic dynamic model of hunting in groups.

It was Kamimura and Ohira [7], who for the first time, offered a simple and convincing statistical physics model that demonstrates various inherent dynamic features of group chase-and-escape. Despite its simplicity, this model is able to reproduce rich and complex behavior of the

chasing and escaping process in groups [16, 17]. In this model, although each chaser independently moves in order to catch one of the nearest targets, some group of chasers are simultaneously formed. A comprehensive discussion of this elegant model is provided in the following section.

2. Discrete Models with the Simplest Encounter Strategies

To carry out the numerical simulations, Kamimura and Ohira [7] considered the chase-and-escape process in a two-dimensional square lattice of size 100×100 squares with periodic boundary conditions. Chasers and targets are initially placed randomly on the edges of the lattice. Thus, each site is either empty or occupied by a chaser or a target as depicted in *Figure* 1. Chasers can sense the positions of the targets at an arbitrarily predefined distance and, at each time step, they try to move to one of the nearest neighboring sites (one lattice spacing) in order to decrease the distance from the nearest target. If the positions of the target and chaser are (x_T, y_T) and (x_C, y_C) respectively, then $d = \sqrt{(x_T - x_C)^2 + (y_T - y_C)^2}$ represents the distance of the target from the chaser. As chasers move and approach their targets, the targets try to evade capture by making a distance of one lattice spacing in a direction away from the nearest chaser (i.e., in increasing d) at every

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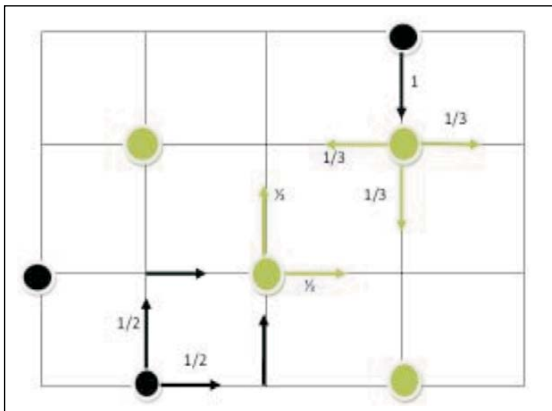
Figure 1. Predator–prey configuration on a square lattice as dictated by the simplest group chase-and-escape model. Predators (black circles) and prey (green circles) are initially placed randomly on the edges of the square lattice. The arrows are the possible nearest-neighbor hopping directions in the next time-step with indicated probabilities.

If there is more than one direction that can increase (or decrease) the distance, the target (or chaser) chooses one of the directions randomly and thereby the notion of stochasticity enters into the model.

time step. If two or more nearest chasers are present, the target chooses one of them randomly and tries to escape from it. If there is more than one direction that can increase (or decrease) the distance, the target (or chaser) chooses one of the directions randomly and thereby the notion of stochasticity enters into the model. In a two-dimensional square lattice as show in *Figure* 1, the chasers or the targets choose one of the two possible sites to move to, with an equal probability of 1/2. However, if chaser and target are on the same x- or y-axis, i.e., $|x_T - x_C|(|y_T - y_C|) = 0$, chasers have one choice, while targets have three choices with equal probability of 1/3. If a chaser is on the nearest-neighboring site to the target, it hops to that site and thereby the prey is caught. After the catch, the chaser pursues the remaining targets in the same manner. However, if a chaser is on the nearest-neighboring site to another chaser, it does not move and stays in the same site.

In this intuitive discrete time-step model, N_C number of chasers and N_T number of targets are initially placed randomly on a 100×100 square lattice. In each time steps the chasers and targets move by one lattice spacing following the above-mentioned simple chase- and-escape rules. Kamimura and Ohira, in their numerical simulations obtained the information about various interesting statistical features, particularly, the lifetime distribution τ (the average time required to catch a target), total catch time T (the time at which the last target is caught), and the predation rate for different numbers of prey-predator. Usually, τ and T are a decreasing function of the number of chasers (N_C) present; such that the more the chasers, the shorter the time needed to complete the catch. However, the above simulation strongly confirmed the existence of two different power-law regimes, namely, $\tau g N_C^{-3/4}$ at high chaser concentration, and $\tau \sim N_C^{-3/4}$ at low chasers concentration as schematically depicted in *Figure* 2. This result indicates
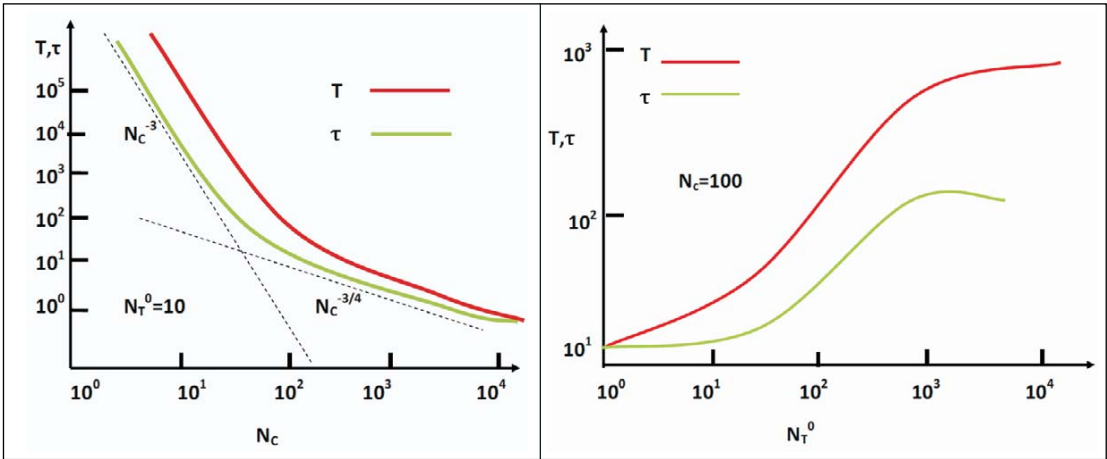
that, when the number of chasers is very large, targets are caught after a few time steps. On the other hand, when there are fewer chasers, targets are able to escape from initial caged configurations and the chase process lasts for a longer time. The crossover is observed when the number of chasers (N_C) is about five times that of the targets (N_T). It has also been found that, for a fixed number of chasers, the total catch time (or, lifetime of final target) T increases monotonically with increasing N_T, whereas the lifetime of typical targets τ peaks around $N_T = 10^3$ and slightly decreases again as schematically shown in *Figure* 3. The explanation of these results was given by Kamimura and Ohira by looking at the time evolution of the system in their numerical simulation. It suggests that when the number of targets N_T is large, they form clusters and thereby evade chasers. Thus, the game lasts for a long time and the group of chasers needs more time (T) to catch the last target. In addition, a group of chasers can efficiently catch targets by surrounding the cluster of targets leading to the peak of the lifetime of a typical target (τ) as shown in *Figure* 3.

Figure 3 (right). Schematic representation of Kamimura and Ohira's result for the lifetime of final (T) and typical (τ) targets for a fixed number of chasers $N_C = 100$.

2.1 *Speed Matters*

In Kamimura and Ohira's model, the speed of chasers and targets are assumed to be equal. Thus, neither of

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the species is superior to the other with respect to speed. In such a situation, the predators that are directly chasing the prey can never catch it as long as the prey is not not caged [6]. Cooperation among the group is therefore very much essential, as it helps a group of chasers to catch a target by surrounding it in such a way that the target cannot escape from them. This is, in fact, one of the main reasons why the predators form groups to enhance the probability of predation [8, 10]. Recently, Iwama and Sato [13] incorporated some fast chasers in their group chase-and-escape model (on a 160×120 square lattice) and investigated through numerical simulations the dependence of the lifetimes τ and T on the number of chasers. Their simulations suggest that due to the existence of fast chasers (which can move two lattice sites at each time step), the total catch time T changes in the region of small N_{C}, where the capture of targets may be completely dominated by the fast chasers. It should be noted that, in this model, the rules for the movement of chasers and targets are slightly different from that of Kamimura and Ohira. In Kamimura and Ohira's model, the positions of chasers and targets are updated simultaneously, while in case of Iwama and Sato's model, the positions are updated asynchronously. This difference in the behavioral rule together with the dominant role of the fast chasers leads to different power law behavior at low chaser concentration (small N_{C}) : $\tau \sim N_{\mathrm{C}}^{-1.5}$ in Iwama and Sato's model and $\tau \sim N_{\mathrm{C}}^{-3}$ in Kamimura and Ohira's model. However, at high chaser concentration, both the models produce the same power law behavior (namely, $\tau \sim N_{\mathrm{C}}^{-3/4}$).

2.2 Searching Ability

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One of the key determinant factors of hunting ability, which in turn plays a crucial role in the chase-and-escape process, is the *search range* of the prey and predators. The search range for a chaser (target) is defined as the range over which it can recognize the existence of a

target(chaser). The above results obtained from discrete models are based on the assumption that the targets and chasers have unlimited search range. In reality, the search range for chasers as well as targets may not be the same. On an average, the sighting range of juvenile individuals can be expected to be larger [20] than that of adults hinting the search range of individuals to be age-specific. Numerical investigations on the survival of a single prey encountered by a group of predators confirmed that when the sighting range of the prey and predators are same (equal to one lattice spacing), the motion of the prey is effectively a diffusive motion; whereas, when prey has a sighting range that is smaller than that of the predators, it undergoes an effective superdiffusive motion during encounter [6]. Thus, we may expect deviations from the above simulation results when one considers, for instance, a group of chasers having sufficiently short-search range with a group of targets having long-search range. In that case, the time evolution of the chase-and-escape process in the Kamimura and Ohira's system has shown that, for an appropriately low number of chasers, targets gather in relatively low-density areas of chasers and momentarily hide from the short-range chasers who cannot recognize their existence. Eventually, a sufficiently long-time is required for such short-range chasers to find out the group of targets and finally catch them [7]. To our knowledge, this model prediction has not been empirically tested. Thus, from an ecological point of view, it would be highly interesting to explore whether there is biological support for this typical simulation result with natural organisms.

To investigate further the influence of search range on the dynamics, Kamimura and Ohira incorporate two different types of chasers, namely, the smart chasers having unlimited search range and random walkers having zero search range. Their simulation results for $N_T = 10$ suggest that if only a small number of smart chasers (5 to

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10) is available, the presence of random walkers significantly contribute to the catching event. However, if the number of smart chasers increases to 30–40, the presence of random walkers does not have any appreciable contribution in the catching event. In other words, if the ratio of the smart chasers to the target is less than 3, the presence of randomly walking chasers enhances the catching event. In contrast, if the ratio of the smart chasers to the target is between 3 and 4, the smart chasers are enough to finish the game most efficiently. This is, in fact, a very interesting result of their model which has not been empirically tested yet. This simplest version of the group chase-and-escape model, although crude, unveiled surprising results, which recently inspired to formulate a number of remarkable mathematical models [12, 13, 17]. The salient features of one such model are highlighted in the following section.

3. A More Realistic Model

The dramatic revolution in the field of computation due to the advent of modern highly efficient computers enables scientists to resolve many challenging problems with potentially far more realistic approaches. By incorporating more complicated distinct set of rules and algorithms in mathematical modelling, it becomes possible to simulate more realistic emergent behavior. It is worthwhile to mention here that the collective motion of many biological organisms, namely, flock of birds, herd of wildebeests, school of fish and swarm of insects, have attracted scientists for a long time [18]. It was T Vicsek and his collaborators, who proposed a very simple simulation model analogous to the model of a ferromagnet and demonstrated that it exhibits rich characteristics of collective behaviors similar to those of natural groups [19]. The simulation was based on the rule that at each time step, a given particle, driven with a constant absolute velocity, has a tendency to move in the average direction of motion of its neighbors while being simultaneously subjected to noise.

On the basis of Vicsek's model for collective motion of self-propelled organisms, recently Angelani [12] studied the phenomena of collective chase-and-escape process. He carried out his simulations for a group of predators chasing a group of prey while incorporating intergroup interactions, attraction (for predators) and repulsion (for preys) from nearest neighbor of the opposite group. The model consists of N_C number of chasers and N_T number of initial targets that decrease over time due to catching events. The predation event is described by the elimination of targets when they enter the capture sphere (of radius r_c). The simulation was performed a box of length L where each individual is described by position vector ($\boldsymbol{r}_i$ for ith individual) and velocity vector ($\boldsymbol{v}_i$ for ith individual) in a two-dimensional space. Each individual moves at constant speed v_0 and their positions and velocities are updated at each time step Δt according to

$$\boldsymbol{r}_i(t + \Delta t) = \boldsymbol{r}_i(t) + \boldsymbol{v}_i(t + \Delta t)\Delta t \,, \qquad (1)$$

$$\boldsymbol{v}_i(t + \Delta t) = v_0 \hat{\boldsymbol{v}}_i^{(\mathrm{int})}(t) \,. \qquad (2)$$

where the unit vector $\boldsymbol{v}^{(\widehat{\mathrm{int}})}$ solely depends on the various intra- and intergroup interacting terms given by

$$\hat{\boldsymbol{v}}^{(\mathrm{int})} = R_\eta \left[\hat{\boldsymbol{v}}_i^{(\mathrm{al})} \right] + \beta \boldsymbol{f}^{(\mathrm{rep})} + \gamma \hat{\boldsymbol{f}}_i^{(\mathrm{CT})} \,. \qquad (3)$$

Here the first term describes the self-propulsion and alignment effect. The direction of motion of any individual i is dictated by the average velocity of all the individuals (including i) belonging to the same group in the spherical neighborhood S_i of radius r_0 centered on i

$$\boldsymbol{v}_i^{(\mathrm{al})} = \sum_{j \in S_i^{(\mathrm{al})}} \boldsymbol{v}_j \,. \qquad (4)$$

The presence of noise, indicated by the operator R_η, rotates the vector by a random angle lying in between

$-\pi$ and $+\pi$. More specifically, it rotates uniformly in the interval $[-\eta\,\pi, \eta\,\pi]$ with the noise $\eta \in [0,1]$. In fact, the noise term η is chosen in such a way that its effect is only to disturb the self-propelling velocity and not the chase or escape force. The second term in (3) represents the repulsive force with relative strength β. The role of this repulsive force is to prevent overlapping among the individuals and can be expressed as

$$\boldsymbol{f}_i^{(\text{rep})} = \sum_{j \in S_i^{(\text{rep})}} \boldsymbol{f}(r_i - r_j) , \qquad (5)$$

where $\boldsymbol{f}$ is the pair-repulsive force and the sum is over particles of the same group within a sphere of radius r_e surrounding particle i. The form of this pair-repulsive force can be chosen in many different ways. One possible choice is

$$\boldsymbol{f}(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}}}{1 + \exp[(r - r_f)/\sigma]} , \qquad (6)$$

where $r = |\boldsymbol{r}|$, r_f is the repulsion length scale, and σ is the steepness. The last term in (3) describes the chase-and-escape force with relative strength γ. Kamimura and Ohira [7] as well as Angelani [12] have chosen the form of this force as

$$\hat{\boldsymbol{f}}_i^{(\text{CT})} = p\hat{\boldsymbol{r}}_{ik_i} , \qquad (7)$$

where $p = -1$ for chasers and $p = +1$ for targets, with k_i as the closest target (or chaser) to any chaser (or target) within the sighting radius r_s of ith individual. The simulation of the above model was performed with different sizes of the square lattice (40×40, 50×50, 100×100) and different noise levels ($\eta = 0$, 0.2, 0.4, 0.6, 0.8, 1). The value of the other parameters were chosen (e.g., $\beta = 5, \gamma = 1.2, \sigma = 0.5, r_0 = 0, r_e = 5, r_f = 1$ and unbounded sighting radius r_s). Here, the units of time and length are r_c, respectively, in such a way that the relative strength of the different terms in (3) ensure the correct relative dominance in the following order:

non-overlapping (dictated by (5)), chase or escape, and self-propulsion. The simulation with these values of the parameters yields two different scaling regimes for total catch time (T) as observed in Kamimura and Ohira's model. However, in contrast to Kamimura and Ohira's results, Angelani observed that $T \sim N_C^{-1}$ for high chaser density and $T \sim N_C^{-2}$ for low density. Although the scaling exponents are distinctly different in both the models, they were able to distinguish the two different physical regimes, namely, a fast catch regime for a relatively large number of chasers ($T \sim N_C^{-1}$), and a slow catch regime for a relatively small number of chasers. At this point, it is worthwhile to contrast these stochastic model simulation results with that of direct observational evidences of animal behavior in several ecological systems.

In the above mathematical models, all the chasers are assumed to be of proficient hunters that abide by the same chasing rules. This gives rise to a power-law decrease in total catch time T with N_C indicating that the hunting success increases with increasing number of chasers. In contrast, ecological investigations on hunting behavior of carnivores, primates, birds and insects [1,20,21,22] suggest that the peak of hunting success is realizable only in small groups of chasers. Empirical studies on the wolves (*Canis lupus*) and hunting elk (*Cervus elaphus*) in Yellowstone National Park [20] suggest that there is a nonlinear relationship between the group size and hunting success, reflecting the transition from cooperative motion to free-riding with increasing group size. Here the term free-riding implies the act of withholding effort by an individual predator who participates only to remain nearby to gain access to the kill. In addition, there is an untested theoretical hypothesis which states that hunting success declines with increasing group size of predators due to overcrowding leading to the increasing number of time-consuming collisions among the proficient and unskillful (often juvenile) predators. This

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interference hypothesis is well-demonstrated in foraging experiments involving homogenous groups of robots [23], where the group task efficiency is reduced in groups with more than four robots due to increase in the number of time-consuming collisions.

All these behavioral studies indicate that the formation of large groups is unrelated to hunting success despite apparent cooperation among the individual hunters. Thus, a more general mathematical model is needed which can capture all these biological behaviors via suitable mathematical and physical rules.

A number of interesting results have emerged from Angelani's study [12]. He investigated the influence of noise in the dynamics of group chase-and-escape by introducing the first term in (3). The presence of noise changes the decision of a target in choosing the proper direction of escape and thereby dooms its destiny. This is reflected in the reduced catch time T in the presence of different levels of noise η in Angelani's numerical simulations. Further, the alignment rule, given by (4), is included in the model to describe organisms moving in a swarm or a flock. However, Angelani found that the inclusion of alignment rule in prey–prey interactions yielded no relevant differences in the catch times and predation rates as compared to the no-alignment case. This indicates that escaping together is not advantageous in this chase-and-escape model. Thus, this numerical simulation provides a hint that the cooperative escape strategies observed in many animal species may rely on some other mechanisms (such as confusion and vigilance effect discussed in Section 4) not included in this model. The following section highlights some of the important distinctive factors that have been established from ecological investigations and/or observational evidences.

A slightly more technical point about the Angelani's model is his investigation of a more efficient escape

strategy of the targets by considering a general form of escape force that depends on the weighted average of predator directions. In addition to (7), Angelani proposed another form of escape force for targets based on the weighted average over close predators within a sphere of radius r_s surrounding the target i, expressed as

$$\boldsymbol{f}_i^T = \sum_{k \in S_i} h(r_{ik}) \hat{\boldsymbol{r}}_{ik}. \tag{8}$$

He considered two different types of weight functions, namely, a power-law weight $h(r) = r^{-w}$ and an exponential-law weight $h(r) = e^{-kr}$, where the weight exponents w and k determine the escape strategy. It has been found that prey are able to survive for a longer time when they escape by choosing the weighted average of predator directions. This effect is more pronounced for the faster targets and the efficient escape takes place when $h(r) = r^{-2}$. This suggests that the survival probability of targets increases as they escape more efficiently by considering the weighted average of neighboring chasers, with a weight exponent $w = 2$. On the other hand, for the exponential weight $h(r) = e^{-kr}$, the total catch time is found to be lower, indicating a less efficient escape than that of the other case. The escape strategy, coming from the numerical simulations, is model-dependent. Finding such escape trajectory is important to model a more realistic situation and thereby enhance our understanding of the problem.

4. The Road Ahead

The results of the two discussed statistical physics models provide us with a glimpse of the underlying physical processes that take place in group chase-and-escape. However, there are several natural considerations which may lead to a more intriguing model close to reality. Here, I would like to point out some biologically relevant features that have not been taken into account in the models mentioned here.

Natural prey fields are heterogeneous in structure with variety of irregular patches, rocky paths, shallow pits and so on.

4.1 *The Structure of the Prey Field*

The two discussed statistical physics models are based on the assumption that the interactions take place on two-dimensional smooth and homogeneous landscapes. However, natural prey fields are heterogeneous in structure with variety of irregular patches, rocky paths, shallow pits and so on. Skillful predators, such as human hunters, take advantage of such a complex heterogeneous landscape, implementing different techniques such as the stampeding of animals towards narrow gorges or cliff edges and the digging of pits or other forms of ambush [1, 3]. Further, the dynamics of chase-and-escape, being highly nonlinear, are grossly different in varying space dimensions. For example, chasing and escaping between a herd of wildebeests and a pack of lions (in two-dimensional prey fields) is distinctly different from that of flocks of birds or schools of fish (three-dimensional prey fields). With increasing space dimensions, there is an increase in the number of degrees-of-freedom, and consequently, more perplexing violent dynamical interaction is conceivable. Realization of the underlying complex dynamics in such a general space dimension requires deeper theoretical thought.

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Final outcomes of the chase-and-escape in the discussed discrete time-step models depend on the size of the lattice as it involves inescapable corners at the boundaries. This type of simplest lattice design is artificial for many group chaser and escape systems as it is difficult to apply in avian and aquatic systems. However, it can work for terrestrial systems where there are landscape features that permit the 'cornering' of prey. For example, African wild dogs and lions trap prey against hard landscape features that prevent their escape [1, 2, 8]. It is also important to note that in these models, the targets and chasers are allowed to move on the sites of the two-dimensional lattice. However, in some previous study on collective motion of interacting entities [19, 24],

particles are allowed to move on off-lattice space. Thus, it would be worthwhile to numerically investigate the group chase-and-escape process on an off-lattice space, which might lead to better physics insight.

4.2 *Collective Escape Strategies*

The discussed statistical mechanics models considered only the simple escape strategy where the targets can escape from the chasers by moving a distance of one lattice spacing in the square lattice at each time step. Such dynamics bear a little resemblance with reality, since many natural prey species acquire skillful escape strategies, such as running in zigzags, overturning, jumping, etc., to enhance their survival probability during a predator encounter. Thus, a Vicsek-type mathematical model [19], in which chasers and targets are allowed to move continuously (off-lattice) on a plane, will be biologically more relevant. Further, Angelani's numerical analysis [12] suggests that the targets have higher survival probability when they escape more efficiently by choosing the weighted average of predator directions.

Several species of grievous predators, on the other hand, employ clever hunting tactics to enhance encounter rates. How should the prey move in order to maximize its chance of not being caught up to time t or, how should the predators forage in order to enhance the encounter rate, depend on several factors. One of the reasons that the prey move in groups is to produce the *'confusion effect'* [25, 26] which reduces the ability of grievous predators to focus on a specific individual and capture it. Due to this confusing perception effect, predators find it hard to track multiple moving targets. This can be seen in the very effective escape manoeuvres displayed by, for example, starling flocks under attack. The high density borders that are often observed may represent a feature that enhances such anti-predatory tactics, creating a 'wall' effect to increase the predator's confusion

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For the foraging prey, the ability to escape from predators depends on how much time a prey remains vigilant (head-up) for predators.

[25–27]. Observational investigations of juvenile wintering redshank birds attacked and killed by Eurasian sparrow hawks [28] suggest that with increasing size of the prey groups, the probability of detection and confusion increases and becomes maxima for a group of nearly 20 redshanks. The confusion effect strongly influences the attack success rate which declines exponentially with group size. However, in the mathematical model of Angelani [12], although an alignment effect was introduced ($r_0 = 5$) to incorporate the collective escape, no distinctive difference was observed with that of the no-alignment case ($r_0 = 0$). A set of rules and algorithms has to be developed to incorporate the confusion effect in a life-like chase and escape model.

As I have already pointed out, in many group hunting instances observed in Nature, the speed of chasers and targets is not really equal. If the chasers are superior compared to the targets with respect to their speed, then the encounter rate increases and, as a result, the total catch time T decreases. This feature has been predicted via numerical simulation of the discrete model with simple encounter rules proposed by Iwama [13]. It has also been suggested that, for slow predators, a strong cooperation among the individuals is necessary to trap the prey [6, 13]. This is also observed in the Angelani's simulation as 'spike-like' events (having very short time intervals) corresponding to cage trapping of a prey group by many chasers converging on it from different directions. Further, in a group of predators or a flock of prey, all individuals are not equally efficient – some are younger than the others and therefore, each individual's effort is not fully countable in such collective group activities. Sometimes, some highly skillful prey safely perform miraculous escape from the relatively weak predators even after they are captured. For the foraging prey, the ability to escape from predators depends on how much time a prey remains vigilant

(head-up) for predators. By joining larger groups, prey share vigilance which enhances the probability of any animal scanning and thus detecting a predator at any one time [29]. Both the probability of detection (vigilance) and confusion exponentially increases with group size, resulting in an exponential decline in attack success with group size [28]. These collective features are not observed in the discussed simple mathematical models. Thus, these models are not capable of introducing the effect of collective escape strategies, particularly, the confusion and vigilance effects, and hence, a far stronger mathematical model is required which can capture the effect of such collective biological behavior.

4.3 *Levy Walks*

When there is an uncertainty or lack of information about the location of the targets, predators adopt different random search strategies lending different chances of finding the targets [30, 31]. Recent investigation of field data for various marine predators [30] strongly suggests that predators have higher encounter rates with prey in heterogeneous natural environments when adopting Levy walks [31, 32]. Levy walk and Levy flight is the natural generalization of Brownian motion [33] to situations of strong fluctuations where more rapid superdiffusive motion takes place. Unlike Brownian motion, Levy walk follows a power-law distribution for the move lengths given by $P(l_i) \sim l_i^{-\gamma}$, where the exponent γ is a number somewhere between 1 and 3. Theoretical investigations [31, 32, 34, 35] suggest that Levy walks and Levy flights across random prey distributions increase new patch encounter probability compared to the simple Brownian motion, with an optimal search having a probability distribution of $P(l_i) \sim l_i^{-2}$. These widespread evidences of Levy flight behavior among various animal species triggers the enormous possibility of incorporating such features into the chase-and-escape model, which in near future, might unveil the large-scale long-time

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features of such an intriguing phenomena observed in Nature.

All the above issues related to biological behaviors, play a crucial role in the dynamics of group chase-and-escape and pose a formidable challenge among mathematicians, computer scientists, ecologists and physicists as well. From the point-of-view of physics, it will be highly interesting to incorporate such features and verify the existence of any universal scaling relation among them. The main theoretical challenge arises from the fact that the governing dynamics for such a system is highly nonequilibrium with so many degrees of freedom. By virtue of being far from equilibrium, such systems are typically nonlinear; i.e., their response to perturbation is often not proportional to the magnitude of the perturbation, as for systems near equilibrium. Although theoretical models and empirical studies have progressively improved our understanding of an animal's collective behavior, the actual dynamical interactions, governed by nonlinear dynamics, are still unclear. However, it can be noted that, using powerful and well-established techniques developed in the fields of condensed matter physics, statistical physics, and computational physics, physicists continued investigations on the emergence of collective motion in multicomponent systems of biological and ecological interest [36, 37].

In recent years, the collective coherent motion of a large number of self-propelled biological organisms has been investigated by formulating the hydrodynamic equations of motion from the underlying symmetry arguments, and then applying the Wilsonian renormalization group approach [38–40]. Simply speaking, in this approach, the dynamical quantities appearing in the equations of motion, acquire corrections that take into account all the dynamical effects from all the (length and time) scales coming into the picture. Various universal statistical features such as scale invariance, long-range order,

spontaneous symmetry breaking, phase transitions, etc. automatically emerge from the renormalization group calculations. The dynamics of group chase-and-escape, however, seems to be far more complex than the dynamics of flocking. Motivated by the success of renormalization group analysis for the collective motion, one can proceed further to formulate the governing hydrodynamic equations of motion for the group chase-and-escape following the Kamimura–Ohira and Angelani's discrete models and, incorporating symmetry-allowed nonlinearities together with appropriate interaction terms, such as those given in (3).

5. Final Remarks

Understanding the ubiquitous complex behavior of natural biological systems is very important as it holds clues to the evolution of sociality, and also for the development of novel technological solutions, particularly, in many defence and security scenarios in the modern threat environment. Most of the widespread natural systems, spanning from the nanoscales to the cosmological scales, are far from thermodynamic equilibrium. Such systems with many degrees of freedom is a topic of great interest in many branches of science such as physics, biology, sociology, economics, chemistry, ecology and so on. The dynamics of group chase-and-escape belongs to the same class of problems. Kamimura and Ohira were able to address a number of physically interesting aspects for group chase-and-escape through their model simulations. Recently, based on the Vicsek's model for collective motion of self-propelled organisms, Angelani proposed a model for collective chase-and-escape process and obtained several interesting universal features via numerical simulations. These versatile and magnificent contributions enhanced our understanding of the underlying dynamics of such a nonequilibrium system.

However, simulation of the real chase-and-escape pro-

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cesses that have been observed in many ecological studies needs more attention. Particularly, the discrete statistical physics models that we have discussed, need to be extended by incorporating more complex animal behavior and strategies such as the different group speeds, different escape and hunting skills, distinct search ranges for prey and predators, the role of alignment rules, geometrically complex landscapes and so on. Inspired by the bold and successful attempt of the above discrete models here, we have discussed various inherent dynamical features of the problem by highlighting their importance in modeling a more realistic situation. We hope this introductory article will ignite intellectual passion in analytical minds to carry out further research along this line to enlighten the deep physical insight of such an increasingly complex natural phenomena.

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