Pramana – J. Phys. (2022) 96:105
https://doi.org/10.1007/s12043-022-02352-9

Electromagnetic field and spherically symmetric dissipative fluid models

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MS received 28 October 2021; revised 12 January 2022; accepted 23 January 2022

Abstract. This paper studies a few properties of Lemaître–Tolman–Bondi (LTB) space–time to the dissipative cases that may lead to its extension in Maxwell-$f(R, T)$ gravity, where R is the Ricci scalar and T is the trace of energy–momentum tensor. Using Misner and Sharp mass formalism, we have first established a relationship between the Weyl tensor and other physical variables. The role of electric charge in the development of the Bianchi identities was also investigated. The physical importance of the effective form of structure scalars was then analysed in view of some realistic backgrounds. We also discussed the generalisations of LTB and the extension of LTB based on structure scalars and a few symmetry properties under constant curvature conditions.

Keywords. Self-gravitating systems; hydrodynamics; electromagnetic field; structure scalars.

PACS Nos **04.20.Cv; 04.40.Nr**

1. Introduction

The general theory of relativity (GR) is a profound comprehension of the magnificent world, not just a theory. This hypothesis begins with the first man on this planet pondering the enigmatic sky, progressing from the idea that the Sun revolves around the Earth to the proof that the Earth revolves around the Sun, from the Big Bang to black holes, and from real matter to dark matter, GR is always there and no one was aware of it. However, Einstein raised the theory of special relativity, which eventually led to GR. Even after hundreds of years, this idea is still young and obscure. Several studies [1–3] of high-redshift supernovae type Ia, as well as cosmic microwave background oscillations, appear to show that the Universe is expanding at a faster rate than before, confirming the presence of dark energy. As a result, the acceleration of the cosmos has offered some of the most complex challenges in theoretical physics. Such studies provide reasons to think about how gravitational dynamics change at cosmic scales. Many astrophysicists [4,5] have made numerous attempts to characterise the accelerating cosmologies at various epochs. Friedman and Lemaître discovered interstellar expansion, which Hubble confirmed by calculating the red-shift of distant galaxies. Dark matter and dark energy are the driving forces behind the Universe's accelerated expansion. Recent gravitational wave discoveries by LIGO and VIRGO [6,7] have shown a new perspective on the Universe. These gravitational waves will act as standard sirens for calculating the rate of expansion of the Universe.

In order to deal with these mysterious dark cosmic sectors, researchers have shown a strong desire to work on alternative approaches of GR and it has become a new trend in the modern scientific period. To achieve the desired results, the Einstein–Hilbert action has been modified in different ways. $f(G)$ (G is the Gauss–Bonnet term), $f(R, T)$ (R is the Ricci scalar and $T \equiv R_{\mu\nu}T^{\mu\nu}$), $f(R, T, R_{\gamma\delta}T^{\gamma\delta})$, $f(R, \Box R, T)$ ($\Box \equiv \nabla_\mu \nabla^\mu$) and $f(G, T)$ are some modified theories [8–11]. Sotiriou and Faraoni [12] extended GR to $f(R)$ gravity in a straightforward way, where $f(R)$ is a general function of the Ricci scalar R, and $f(R)$ gravity is essentially a family of hypotheses, each defined by a separate function, f of the Ricci scalar R. The role of dark energy in the accelerated expansion of the Universe has been studied using this kind of hypothesis [13]. Capozziello and Laurentis [14] addressed the dark matter problem from the perspective of $f(R)$ gravity. Qadir *et al* [15] also suggested that the modfication of GR could lead to understand a few cosmic puzzles

Published online: 25 May 2022

including quantum gravity effects, etc. Harko *et al* [16] suggested the $f(R,T)$ gravity, based on the extensions of $f(R)$ theory in which the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of stress–energy momentum tensor. The $f(R,T)$ gravity was introduced by extending the idea of $f(R)$ theory. Such an extension is motivated to study fluid-curvature coupling (non-minimal) by the quantum effects or exotic imperfect matter distributions. Such types of assumptions in the gravitational analysis would lead us to see the non-conservative behaviour of stress–energy tensor. This gravity model along with electromagnetic field could lead to very important outcomes as they are expected to provide a fundamental mathematical viewpoint of a broad theoretical detail for the late-time cosmic acceleration, without bothering about the postulates of dark energy. This theory could provide us a bridge between quantum and classical windows.

An anisotropic fluid would be one in which fluid's features are based on the flow direction. The modern Universe, which is governed by dark energy, can be explained by using the notion of anisotropy. At different levels, the anisotropic fluid space–time that occurs spontaneously, causes inhomogeneities. Several researchers [17–20] have recently investigated compact star models with anisotropic fluid arrangements. The presence of anisotropy in wormholes [21], gravastars [22,23] and cavity evolution [24–26], which are known as alternative solutions of the field equations, may also be investigated. Ryblewski and Florkowski [27] developed a framework for highly anisotropic and strongly dissipating hydrodynamical systems. Cavity evolution in relativistic self-gravitating fluids that could lead to understand anisotropy in matter content has been investigated by Herrera and his collaborates [28,29] and afterwards by Yousaf *et al* [30,31].

The direct coupling among Maxwell and gravitational equations, which can be regarded as the electromagnetic waves scattering because of space–time curvature, has a long history of electromagnetic studies in curved space–time. When electromagnetic influences are incorporated in the Lagrangian, Moffat [32] obtained static stellar models in the background of spherical structures. Dehghani introduced [33] a new class of black brane modified gravity models by taking the negative choices of cosmological constant along with an electric charge. Tsagas [34] used a covariant method in GR to study the role of Maxwell force for the warped manifold which is assumed to be coupled with a perfect fluid. Herrera *et al* [35] investigated the importance of charge in the study of structure scalar (SS) that was found from the splitting of the curvature tensor. In the study of astrophysics, these SSs are connected with many physical variables, particularly when the electromagnetic field is present [36,37]. Herrera and Barreto [38] discovered that electromagnetic radiations use rotation in the system and, as a result, are responsible for frame dragging. The role of electric charge on the modelling of different stellar models received key importance in literature [39,40,42,43].

Rej and Bhar [41] investigated the charged strange stars model in $f(R,T)$ modified theory. Using $f(R,T)$ gravity, Houndjo [44] explored the consequences of matter-influenced and accelerated phases in the Universe's expansion. In the context of $f(R,T,Q)$ hypothesis, Bhatti *et al* discussed the gravitational collapse of spherical compact structures [45]. The $f(R,T)$ gravity theory with electromagnetic influences has been popularly utilised in a number of subjects, namely wormholes, the Universe's accelerated acceleration, dark matter, massive pulsars and super-Chandrasekhar meteorites [46–50].

It is well-known that the Einstein field equation solutions for describing the inhomogeneities in a non-dissipative spherical symmetric fluid are regarded as Lemaître–Tolman–Bondi (LTB). It was found, respectively, by Lemaître and Tolman, and studied by Bondi [51]. A finite or infinite spherical dust cloud expands or contracts due to gravity in this solution. Another expression for the LTB metric is the Tolman metric. Humphreys *et al* [52] used LTB as a powerful mathematical model to understand the gravitational implosion and the subject of cosmic censorship. It is now well-accepted that the influence of measurable inhomogeneities in the Universe cannot be ignored when developing efficient cosmological theories. As a result of the new predictions of type Ia supernovas that show that the Universe is expanding, there seems to be a growing interest in LTB [2,53,54]. There are multiple works in the fields of astrophysics and cosmology that have been undertaken in the sense of charged $f(R,T)$ gravity. Electromagnetic influences on the evolution of LTB geometry in modified gravity were studied by Yousaf *et al* [55]. The coordinate transformation for non-static situations to comoving coordinates was shown by Lasky and Lun [56], with the metric being a straightforward generalisation of the LTB space–time to incorporate pressures. Bhatti *et al* [57] studied gravitational collapse and the dark Universe in the electromagnetic field using LTB geometry. Recently, Herrera *et al* [58] described some very important characteristics of LTB space–time. They also generalised this formulation for dissipative cases in GR through SS and symmetry properties.

Sussman derived thermodynamically viable LTB models which were assumed to be coupled with a viscous matter content [59]. Zibin [60] implemented the linear scalar perturbations on the LTB dust cloud to understand the relation between large cosmological voids and luminosity measurements of the supernova with some specific backgrounds. Duffy and Nolan [61] considered some specific configurations of LTB metrics and after applying the perturbation technique, they found finite regular data points at the Cauchy horizon. Firouzjaee and Mansouri [62] explored the creation of Hawking radiation with respect to the apparent horizon for the LTB black hole compact objects. Sussman [63] studied the association of 'look-alike' LTB scalars with an FLRW metric through a weighed scalar averaging procedure. Sussman and Larena [64] provided the relation between entropy and density growth for LTB space–time. After a few years, Sussman and Jaime [65] calculated a few non-static irregular dust LTB models. Paliathanasis [66] described irregular analytical models in a specific gravity theory by considering the modified version of Szekeres space–times. Herrera *et al* [67] noticed a striking resemblance between LTB cosmological models and hyperbolically symmetric matter configurations. They found the non-radiating nature of all non-complex LTB models. In this regard, GLTB can be quite useful for dissipating fluxes that are arbitrarily minimal. From the preceding analysis, it is evident that there is a solid argument for generalising LTB spaces to enable for dissipative fluxes using $f(R, T)$ theory including electromagnetic effects.

In this article, we shall focus on various strategies for determining specific solutions for geodesic radiating fluids having the same features as LTB. The lay-out of this paper is as follows: Section 2 deals with the fundamental idea of $f(R, T)$ gravity with an electromagnetic field and spherical symmetric dissipative viscous matter compositions. A link is also established between the Weyl tensor and inhomogeneous energy density. We shall also take a look at Bianchi identities and how constraint equations are emerged. In §3, we derive expressions for the SS that arise from the orthogonal decomposition of the Riemann tensor, as well as the implications of the electric field. Section 4 investigates the LTB space–time. We generalise the LTB to the dissipative case in §5. In §6 and 7, we investigate the LTB extensions based on symmetry characteristics and SS, respectively. Finally, in the concluding part, we have summarised our findings.

2. Fundamental equations, fluid distribution and kinematic variables

We use the anisotropic fluid structure having the energy–momentum tensor, $T_{\eta\varsigma}$, in the analysis. We assume that the comoving frame under which we define four velocity as $u_\alpha = (-1, 0, 0, 0)$. Presume that the fluid within the relativistic interior is locally anisotropic in existence, having an expression

$$T_{\eta\zeta} = \mu V_\eta V_\zeta + q_\eta V_\zeta + V_\eta q_\zeta + \epsilon l_\eta l_\zeta, \tag{1}$$

where ϵ is the radiation density, q_η indicates the heat flux, l^η is a null four-vector and V_η is the four-velocity of the fluid. The associated line element is provided by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + G^2\,\mathrm{d}r^2 + M^2\,\mathrm{d}\Omega^2, \tag{2}$$

where the metric coefficients M and G depend on t and r only. Both of these function are positive. Also G is dimensionless and M has the same dimensions as r. The vectors satisfy

$$V^\eta V_\eta = -1, \quad V^\eta q_\eta = 0, \quad l^\eta V_\eta = -1, \quad l^\eta l_\eta = 0 \tag{3}$$

and in a comoving coordinate system, we can write

$$V^\eta = \delta^\eta_0, \quad q^\eta = qG^{-1}\delta^\eta_1, \quad l^\eta = \delta^\eta_0 + G^{-1}\delta^\eta_1, \tag{4}$$

where $q^\eta = q\chi^\eta$ and χ^η is a unit four-vector in radial direction, which satisfies

$$\chi^\eta\chi_\eta = 1, \quad \chi^\eta V_\eta = 0, \quad \chi^\eta = G^{-1}\delta^\eta_1. \tag{5}$$

Equation (1) can be written alternatively as

$$T_{\eta\zeta} = \tilde{\mu} V_\eta V_\zeta + \tilde{q}(V_\eta\chi_\zeta + \chi_\eta V_\zeta) + \epsilon\chi_\eta\chi_\zeta, \tag{6}$$

where tilde describes $\tilde{z} = z + \epsilon$.

2.1 *Maxwell f(R, T) field equations*

The action integral for the modified gravity theories can be written as

$$S = \frac{1}{16\pi}\int f(R, T)\sqrt{-g}\,\mathrm{d}^4x + S_m + S_e, \tag{7}$$

where $R \equiv g^{\gamma\zeta}R_{\gamma\zeta}$, $T \equiv g^{\gamma\zeta}T_{\gamma\zeta}$, S_m and S_e are the matter and electromagnetic actions (for details, please see [68]). The stress–energy tensor takes the form

$$T_{\eta\zeta} = -\frac{2\delta(L_m\sqrt{-g})}{\delta g^{\eta\zeta}\sqrt{-g}}, \tag{8}$$

where $T = T_{\eta\zeta}g^{\eta\zeta}$ is the trace of stress–energy momentum tensor. In the electromagnetic field, the energy–momentum tensor is given as

$$E_{\eta\zeta} = \frac{1}{4\pi}\left(F^\eta_\zeta F_{\eta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\eta\zeta}\right), \tag{9}$$

where ϕ_η is the four-potential and $F_{\eta\zeta} = \phi_{\zeta,\eta} - \phi_{\eta,\zeta}$ is the Maxwell field tensor. The Maxwell field equations can be written as

$$F^{\eta\zeta}_{\zeta} = \mu_0 J^\eta, \quad F_{[\eta\zeta;\gamma]} = 0. \tag{10}$$

Take $\mu_0 = 4\pi$ as the magnetic permutability and J^η is the four velocity vector. We can have in comoving coordinate as

$$\phi_\eta = \phi\delta_0^\eta, \quad J^\eta = \sigma V^\eta, \tag{11}$$

where the scalar potential and charge density are described by ϕ and σ, respectively. The charge conservation equation, $J^\eta_\eta = 0$, is a mathematical formula that describes how charges are conserved, and generates

$$s(r) = 4\pi \int \sigma G M^2 \,\mathrm{d}r. \tag{12}$$

Using eq. (10) with $\eta = 0$ and $\phi'(t,r) = 0$ we have

$$\phi' = \frac{sG}{M^2}. \tag{13}$$

Finally, the required Maxwell-$f(R,T)$ field equations after calculations takes the form through Einstein tensor as $(G_{\eta\zeta})$ as

$$G_{\eta\zeta} = 8\pi T^{\mathrm{eff}}_{\eta\zeta} + 8\pi Q_{\eta\zeta}, \tag{14}$$

$$Q_{\eta\zeta} = \frac{1}{f_R} E_{\eta\zeta} \tag{15}$$

and

$$T^{\mathrm{eff}}_{\eta\zeta} = \frac{1}{f_R}\Bigg[T_{\eta\zeta}(1+f_T) + f_T \rho g_{\eta\zeta} + \frac{R}{2} g_{\eta\zeta} \times \left(\frac{f(R,T)}{R} - f_R\right) + (\nabla_\eta \nabla_\zeta - f_R g_{\eta\zeta}\Box)\Bigg]. \tag{16}$$

The charged $f(R,T)$ field equations for our model are

$$\left(\frac{2\dot{G}}{G} + \frac{\dot{M}}{M}\right)\frac{\dot{M}}{M} - \left(\frac{1}{G^2}\right)\left[\frac{2M''}{M} + \left(\frac{M'}{M}\right)^2 - \frac{2G'M'}{GM} - \frac{G^2}{M^2}\right] = 8\pi\tilde{\mu}_{\mathrm{eff}} + Q_0$$
$$= 8\pi\frac{L\tilde{\mu} + \chi_{00}}{f_r} + Q_0 \tag{17}$$

$$-2\left(\frac{\dot{M}'}{M} - \frac{\dot{G}M'}{GM}\right) = -8\pi\tilde{q}_{\mathrm{eff}} G = \frac{-L\tilde{q}G - \chi_{01}}{f_R}8\pi \tag{18}$$

$$-G^2\left(\frac{2\ddot{M}}{M} + \left(\frac{\dot{M}}{M}\right)^2\right) + \left(\frac{M'}{M}\right)^2 - \left(\frac{G}{M}\right)^2$$
$$= 8\pi G^2\epsilon_{\mathrm{eff}} + Q_2 = \frac{L\epsilon G^2 + \chi_{11}}{f_R}8\pi + Q_2 \tag{19}$$

$$-M^2\left(\frac{\ddot{G}}{G} + \frac{\ddot{M}}{M} + \frac{\dot{G}\dot{M}}{GM}\right) + \left(\frac{M}{G}\right)^2\left(\frac{M''}{M} - \frac{G'M'}{GM}\right)$$
$$= 8\pi\chi_{33} + Q_3 = \frac{\chi_{22}}{f_R} + Q_3, \tag{20}$$

where

$$Q_0 = \frac{1}{f_R}\frac{s^2}{M^4}, \quad Q_2 = -\frac{1}{f_R}\frac{s^2G^2}{M^4}$$

and

$$Q_3 = \frac{1}{f_R}\frac{s^2}{M^2}. \tag{21}$$

The primes as well as dots depict the derivatives of r and t. The terms χ_{jj} are listed in the Appendix, and Q_0, Q_2, Q_3 contain charge terms.

In an adiabatic system, Misner–Sharp mass is found to be the energy. In charged $f(R,T)$ theory, Misner and Sharp mass [69] can be written as follows:

$$m = \frac{(M^3)}{2}R^{23}_{23} = \frac{M}{2}\left[\dot{M}^2 - \left(\frac{M'}{G}\right)^2 + 1\right] + \frac{s^2}{2}. \tag{22}$$

The velocity U of the collapsing fluid can be defined in terms of areal velocity with respect to time as

$$U = \dot{M}. \tag{23}$$

Then we can write eq. (22) in the following form:

$$E = \frac{M'}{G} = \left[1 + U^2 - \frac{2m(t,r)}{M} + \frac{s^2}{2}\right]^{1/2}. \tag{24}$$

We have the following relation for mass from eqs (18), (19) and (23) as

$$\dot{m} = -4\pi(\epsilon_{\mathrm{eff}}U + \tilde{q}_{\mathrm{eff}}E)M^2 + \left(\frac{s^2}{2M}\right)^{\cdot} - Q_2\frac{\dot{M}M^2}{2G^2} \tag{25}$$

and

$$m' = 4\pi\left(\tilde{\mu}_{\mathrm{eff}} + \tilde{q}_{\mathrm{eff}}\frac{U}{E}\right)G^2M' + \left(\frac{s^2}{2M}\right)' + M^2\acute{M}Q_0. \tag{26}$$

After integration of eq. (26) we get

$$m = \int_0^r 4\pi M^2\left(\tilde{\mu}_{\mathrm{eff}} + \tilde{q}_{\mathrm{eff}}\frac{U}{E}\right)M'\,\mathrm{d}r + \int_0^r\left[\left(\frac{s^2}{2M}\right)' + M^2M'Q_0\right]\mathrm{d}r. \tag{27}$$

The effects of modified terms can be seen easily from the above expression. We partially integrate eq. (22) to get

$$\frac{3m}{M^3} = 4\pi\tilde{\mu}_{\text{eff}} - \frac{4\pi}{M^3}\int_0^r M^3\left(\tilde{\mu}'_{\text{eff}} - 3\tilde{q}_{\text{eff}}\frac{M'U}{ME}\right)\text{d}r + \frac{3}{M^3}\int_0^r\left[\left(\frac{s^2}{2M}\right)' + M^2M'Q_0\right]\text{d}r. \tag{28}$$

It establishes a link between the matter quantity and physical properties including electric charge, energy density and heat flux.

2.2 *Matching conditions*

We have examined outside $\sum$ and also Vadiya space–time in this instance of limited arrangements; in the dissipationless case, the Schwarzchild space–time is represented by

$$\text{d}s^2 = -2\text{d}\nu\,\text{d}\rho - \left[1 - \frac{2C(\nu)}{\rho}\right]\text{d}\nu^2 + \rho^2\text{d}\Omega^2, \tag{29}$$

where m is the mass of the system. On the matching of interior and exterior manifolds, we get

$$\text{d}t \overset{\Sigma}{=} \text{d}\nu\left(1 - \frac{2C(\nu)}{\rho}\right), \tag{30}$$

$$R \overset{\Sigma}{=} \rho(\nu), \tag{31}$$

$$\left(\frac{\text{d}\nu}{\text{d}t}\right)^{-} 2 \overset{\Sigma}{=} \left(1 - \frac{2C(\nu)}{\rho} + 2\frac{\text{d}\rho}{\text{d}\nu}\right). \tag{32}$$

Through the continuity of second differential condition of Darmois conditions, we get

$$m(t, r) \overset{\Sigma}{=} C(\nu), \tag{33}$$

$$2\left(\frac{\dot{M}'}{M}\right) \overset{\Sigma}{=} -G\left[2\frac{\ddot{M}}{M} + \left(\frac{\dot{M}}{M}\right)^2\right] + \frac{1}{G}\left[\left(\frac{M'}{M}\right)^2 - \left(\frac{G}{M}\right)^2\right]. \tag{34}$$

It is worthy to stress that the matching of eqs (2) and (29) on Σ implies

$$\epsilon \overset{\Sigma}{=} \frac{L}{4\pi\rho^2}. \tag{35}$$

The luminance of the sphere over its exterior is calculated as $L_{\sum}$.

$$L \overset{\Sigma}{=} L_\infty\left(1 - \frac{2m}{\rho} + 2\frac{\text{d}\rho}{\text{d}\nu}\right)^{-1} \tag{36}$$

and

$$L_\infty = \frac{\text{d}C}{\text{d}\nu} \overset{\Sigma}{=} -\left[\frac{\text{d}m}{\text{d}t}\left(\frac{\text{d}\nu}{\text{d}t}\right)^{-1}\right], \tag{37}$$

where L_∞ is the total luminance observed at infinity by an investigator at stationary position.

$$\frac{\text{d}\nu}{\text{d}t} \overset{\Sigma}{=} 1 + z, \tag{38}$$

with

$$\frac{\text{d}\nu}{\text{d}t} \overset{\Sigma}{=} \left(\frac{M'}{G} + \dot{M}\right)^{-1}. \tag{39}$$

As a result, the time for the creation of the black hole is calculated as

$$\left(\frac{M'}{G} + \dot{M}\right) \overset{\Sigma}{=} E + U \overset{\Sigma}{=} 0. \tag{40}$$

Also observe that

$$L \overset{\Sigma}{=} \frac{L_\infty}{(E+U)^2} \tag{41}$$

and from eqs (23), (24), (32) and (39) we get

$$\frac{\text{d}\rho}{\text{d}\nu} \overset{\Sigma}{=} U(U+E). \tag{42}$$

2.3 *The Weyl tensor*

One can define the Weyl tensor as follows:

$$C^{\rho}_{\eta\zeta\mu} = R^{\rho}_{\eta\zeta\mu} - \frac{1}{2}R^{\rho}_{\zeta}g_{\eta\mu} + \frac{1}{2}R_{\eta\zeta}\delta^{\rho}_{\mu} - \frac{1}{2}R_{\eta\mu}\delta^{\rho}_{\zeta} + \frac{1}{2}R^{\rho}_{\mu}g_{\eta\zeta} + \frac{1}{6}R(\delta^{\rho}_{\zeta}g_{\eta\mu} - g_{\eta\zeta}\delta^{\rho}_{\mu}), \tag{43}$$

whose electric part

$$E_{\eta\zeta} = C_{\eta\mu\zeta\nu}V^{\mu}V_{\nu}, \tag{44}$$

having non-vanishing components for our spherical system are found as follows:

$$E_{11} = \frac{2}{3}G^2\varepsilon, \quad E_{22} = -\frac{1}{3}M^2\varepsilon, \quad E_{33} = E_{22}\sin^2\theta, \tag{45}$$

where

$$\varepsilon = \frac{1}{2}\left[\frac{\ddot{M}}{M} - \frac{\ddot{G}}{G} - \left(\frac{\dot{M}}{M} - \frac{\dot{G}}{G}\right)\frac{\dot{M}}{M}\right] + \frac{1}{2G^2}\left[-\frac{M''}{M} + \left(\frac{G'}{G} + \frac{M'}{M}\right)\frac{M'}{M}\right] - \frac{1}{2M^2}. \tag{46}$$

Now, using eqs (17), (19) and (20) in eqs (22) and (46), we have

$$4\pi(\tilde{\mu}_{\text{eff}} - \epsilon_{\text{eff}}) - \varepsilon = \frac{3m}{M^3} - 8\pi\frac{\chi_{33}}{M^2} - Q_0 - \frac{Q_2}{G^2} - \frac{Q_3}{M^2} - \frac{s^2}{2M}. \tag{47}$$

This demonstrates that the Weyl tensor is affected by energy density inhomogeneity, local anisotropy in the presence of effective and usual charge terms with $f(R,T)$ extra degrees of freedom.

2.4 *The Bianchi identities, Weyl tensor and an Ellis equation*

For our spherical relativistic system, the two constrained Bianchi identities are obtained as follows:

$$\dot{\tilde{\mu}}_{\rm eff}+(\tilde{\mu}_{\rm eff}+\epsilon_{\rm eff})\frac{\dot{G}}{G}+\frac{2\dot{M}}{M}\tilde{\mu}_{\rm eff}+\frac{\tilde{q}'_{\rm eff}}{G}+2\frac{M'}{MG}\tilde{q}_{\rm eff}+\frac{1}{8\pi f_R}\left(\frac{s^2}{M^4}\right)^{\cdot}-\frac{1}{8\pi f_R}\times\frac{\dot{G}s^2}{GM^4}+\left(\frac{\dot{G}}{G}+\frac{2\dot{M}}{M}\right)\frac{1}{f_R}\frac{s^2}{8\pi M^4}-Z_0=0=0, \tag{48}$$

$$\dot{\tilde{q}}_{\rm eff}+2\left(\frac{\dot{G}}{G}+\frac{\dot{M}}{M}\right)\tilde{q}_{\rm eff}+\frac{\epsilon'_{\rm eff}}{G}+\frac{2M'}{MG}\epsilon_{\rm eff}-\frac{1}{8\pi f_R}\left(\frac{s^2}{G^2M^4}\right)'-\frac{1}{8\pi f_R}\left(2\frac{G'}{G}+\frac{M'}{M}\right)\frac{s^2}{G^2M^4}-2\frac{M'}{G^2}\frac{1}{8\pi f_R}\frac{s^2}{M^5}-Z_1=0, \tag{49}$$

where Z_0 and Z_1 are due to the non-conserved nature of this theory. A constrain equation for the Weyl tensor that is widely called Ellis equation is found as follows:

$$[\varepsilon-4\pi(\tilde{\mu}_{\rm eff}-\epsilon_{\rm eff})]'+(\varepsilon+4\pi\epsilon_{\rm eff})\frac{3M'}{M}=-12\pi G\frac{\dot{M}}{M}\tilde{q}_{\rm eff}+\frac{4\pi M'}{M^3}\chi_{33}+\frac{4\pi}{M^2}\chi'_{33}+\frac{3M'Q_2}{G^2M}+\left(\frac{Q_2}{2G^2}\right)'-\left(\frac{Q_0}{2}\right)'. \tag{50}$$

The causes of inhomogeneous energy density in Maxwell-$f(R,T)$ gravity can be studied using the above-mentioned equation. Yousaf and his collaborators [70] studied the emergence of irregular energy density in the presence of electric charge with different realistic matter configurations.

3. SS in a charged field

We shall use a collection of SS in our investigation that would be obtained by splitting the Riemann tensor orthogonally. We take the tensors $Y_{\eta\zeta}$ and $X_{\eta\zeta}$ [71] as

$$Y^{\rm eff}_{\eta\zeta}=R_{\eta\gamma\zeta\delta}V^\gamma V^\delta, \tag{51}$$

$$X^{\rm eff}_{\eta\zeta}={}^*R^*_{\eta\gamma\zeta\delta}V^\gamma V^\delta=\frac{1}{2}\eta^{\epsilon\rho}_{\eta\gamma}R^*_{\epsilon\rho\zeta\delta}V^\gamma V^\delta, \tag{52}$$

where $R^*_{\eta\zeta\gamma\delta}=\frac{1}{2}\eta_{\epsilon\gamma\rho\delta}R^{\epsilon\rho}_{\eta\zeta}$ and $\eta_{\epsilon\rho\gamma\delta}$ represents the Levi–Civita tensor. By following the procedure of [58], we found the modified SS as follows:

$$Y^{\rm eff}_T=4\pi(\tilde{\mu}_{\rm eff}+\epsilon_{\rm eff})+\chi_{5B}+Q_7,$$
$$Y^{\rm eff}_{TF}=\varepsilon-4\pi\epsilon_{\rm eff}+\chi_{7B}+Q_8, \tag{53}$$
$$X^{\rm eff}_T=8\pi\tilde{\mu}_{\rm eff}+\chi_{9B}+Q_9,$$
$$X_{TF}=-\varepsilon-4\pi\epsilon_{\rm eff}+\chi_{10B}+Q_{10}. \tag{54}$$

These results are obtained from eqs (51) and (52) after using eqs (6), (14) and (43). Here the values of χ_{5B}, χ_{9B}, χ_{10B}, χ_{7B} are given in Appendix containing the dark source terms. The ordinary SS can be explicitly obtained from the above equations by swapping the effective variables with ordinary one. Also, utilising eqs (28) and (47) with eq. (53) we may get

$$Y^{\rm eff}_{TF}=-8\pi\epsilon_{\rm eff}+\frac{4\pi}{M^3}\int_0^r M^3\left(\tilde{\mu}'_{\rm eff}-3\tilde{q}_{\rm eff}\frac{M'U}{ME}\right)-\frac{3}{M^3}\int_0^r\left(\left(\frac{s^2}{2M}\right)'+M^2M'Q_0\right){\rm d}r+\frac{8\pi\chi_{33}}{G^2}+\chi_{7B}+\left(-Q_0-\frac{Q_2}{G^2}-\frac{Q_3}{M^2}-\frac{s^2}{2M}\right). \tag{55}$$

In the above equation, we configured the relation between scalar null fluid, density inhomogeneity, dissipative variables and Weyl tensor using $Y^{\rm eff}_{TF}$. It is also important to recall that Herrera *et al* [71] used this SS to express Tolman mass. One can use this SS, $Y^{\rm eff}_{TF}$, to analyse the stability of the shear-free condition in the geodesic background. The SS expressed in eqs (53) and (54) in terms of structural variables of the line element (2) can be written as

$$Y^{\rm eff}_T=-2\frac{\ddot{M}}{M}-\frac{\ddot{G}}{G}+\chi_{5B}+Q_7-\frac{Q_0}{2}-\frac{Q_2}{2G^2},$$
$$Y^{\rm eff}_{TF}=-\frac{\ddot{G}}{G}+\frac{\ddot{M}}{M}+\chi_{7B}+Q_8+\frac{Q_2}{2G^2}, \tag{56}$$
$$X^{\rm eff}_T=\left(2\frac{\dot{G}}{G}+\frac{\dot{M}}{M}\right)\frac{\dot{M}}{M}-\frac{1}{G^2}\left[2\frac{M''}{M}+\frac{M'^2}{M^2}-2\frac{G'M'}{GM}\right]+\frac{1}{M^2}+\chi_{9B}+Q_9-Q_0, \tag{57}$$

$$X_{TF}^{\rm eff} = \left(\frac{\dot{M}}{M} - \frac{\dot{G}}{G}\right)\frac{\dot{M}}{M} + \frac{1}{G^2}\left[\frac{M''}{M} - \left(\frac{G'}{G} + \frac{M'}{M}\right)\frac{M'}{M}\right] + \frac{1}{M^2} + \chi_{10B} + Q_{10} + \frac{Q_2}{2G^2}. \quad (58)$$

In term of these SS, the Weyl tensor and energy density inhomogeneity differential equation can be described as

$$(X_{TF}^{\rm eff} + 4\pi\tilde{\mu}_{\rm eff})' = -3\frac{M'}{M}X_{TF}^{\rm eff} + 4\pi\tilde{q}_{\rm eff}(\Theta - \sigma)G + \frac{3M'}{M}\chi_{10B} + \chi'_{10B} + \frac{4\pi M'}{M^3}\chi_{33} + \frac{4\pi}{M^2}\chi'_{33} + Q_4, \quad (59)$$

where Θ and σ are expansion and shear scalars, respectively. One can analyse X_{TF} to be the inhomogeneity factor, after switching the effective energy density with the ordinary one. This result applies to both the charged and non-charged spherically symmetric cases. This means that effective variables due to Maxwell-$f(R, T)$ gravity are trying to reduce the effects in making this SS to be an inhomogeneity factor. For non-dissipative case, eq. (59) reduces to the following relation:

$$(X_{TF}^{\rm eff} + 4\pi\tilde{\mu}_{\rm eff})' = -3\frac{M'}{M}X_{TF}^{\rm eff} + \frac{3M'}{M}\chi_{10B} + \chi'_{10B} + \frac{4\pi M'}{M^3}\chi_{33} + \frac{4\pi}{M^2}\chi'_{33} + Q_4. \quad (60)$$

Now, if we apply constant curvature uncharged condition on the above equation, we notice that $X_{TF} = 0, \Leftrightarrow \mu'_{\rm eff} = 0$, denoting X_{TF} as the factor of inhomogeneity. On the other hand, if $\tilde{\mu}'_{\rm eff} = 0$ then we get

$$(X_{TF}^{\rm eff})' = -3\frac{M'}{M}X_{TF}^{\rm eff} + \frac{3M'}{M}\chi_{10B} + \chi'_{10B} + \frac{4\pi M'}{M^3}\chi_{33} + \frac{4\pi}{M^2}\chi'_{33} + Q_4 \quad (61)$$

giving

$$\ln X_{TF}^{\rm eff} = \ln\frac{f(t)}{C^3},$$

where

$$\ln f(t) = \ln g(r) + \int\left(\frac{1}{X_{TF}^{\rm eff}}\frac{3M'}{M}\chi_{10B} + \chi'_{10B} + \frac{4\pi M'}{M^3}\chi_{33} + \frac{4\pi}{M^2}\chi'_{33} + Q_4\right){\rm d}r$$

$$X_{TF}^{\rm eff} = \frac{f(t)}{M^3}. \quad (62)$$

One can observe that $f(t) = 0$ necessarily implies $X_{TF}^{\rm eff} = 0$, thereby indicating it as an inhomogeneity/regularity factor. It has been proved in GR that the shear-free case implies the existence of zero complex system. The extra degrees of freedom of Maxwell-$f(R, T)$ gravity are trying to reduce the impact of X_{TF} as there are extra terms (χ_{7B} and Q_2) whose disappearance are also necessary to obtain the regular configuration of the non-static LTB geometry.

4. LTB space–time

Now, we consider a geodesic, dissipation-less fluid to obtain the general mathematical formulation of LTB. On integrating eq. (18) with $q_{\rm eff} = \epsilon_{\rm eff} = 0$, we get

$$G(t, r) = \frac{M'}{(1 + \kappa(r))^{1/2}} \quad (63)$$

with κ as an arbitrary function of r. By using the above equation with eq. (2), LTB model becomes

$${\rm d}s^2 = -{\rm d}t^2 + \frac{M'^2}{1 + \kappa(r)}{\rm d}r^2 + M^2({\rm d}\theta^2 + \sin^2\theta\,{\rm d}\phi^2). \quad (64)$$

This metric is usually compatible with an irregular distribution of non-interacting cloud. Equations (22) and (63) provide

$$\dot{M}^2 = \frac{2m}{M} + \kappa(r) - \frac{s^2}{M} \quad (65)$$

and from eqs (25) and (26) we see that

$$m = m(r), \quad (66)$$

$$m' = 4\pi\tilde{\mu}_{\rm eff}M^2M' + \left(\frac{s^2}{2M}\right)' + M^2M'Q_0. \quad (67)$$

In this scenario, one can write [58]

$$\dot{\mu}_{\rm eff} = \mu_{\rm eff}\left(\frac{\dot{G}}{G} + 2\frac{\dot{M}}{M}\right) + Q_A, \quad (68)$$

resulting in

$$\mu_{\rm eff} = \frac{h(r)}{GM^2} + \int Q_A\,{\rm d}r, \quad (69)$$

where $h(r)$ is an integration function.

$$\mu_{\rm eff} = \frac{3h(r)(1 + \kappa(r))^{1/2}}{(M^3)'} + \int Q_A\,{\rm d}r. \quad (70)$$

It shows that energy density inhomogeneity is affected by an electric charge and one can obtain the $f(R, T)$ results in the absence of electric field. Equation (65) can also be written as

$$\dot{M}^2 = \frac{2m}{M^*} + \kappa(r), \quad (71)$$

where $M^* = m - \frac{s^2}{2}$ describes the mass of the charged system. In the background of eq. (64), the well-known SS Y_T^{eff}, Y_{TF}^{eff}, X_{TF}^{eff} and X_T^{eff} take the form

$$Y_{TF}^{\text{eff}} = \frac{\ddot{M}}{M} - \frac{\ddot{M}'}{M'} + \chi_{5B} + Q_8 + \frac{Q_2}{2G^2},$$
$$Y_T^{\text{eff}} = -2\frac{\ddot{M}}{M} - \frac{\ddot{M}'}{M'} + \chi_{7B} + Q_7 - \frac{Q_0}{2} - \frac{Q_2}{2G^2}, \tag{72}$$

$$X_T^{\text{eff}} = \frac{2\dot{M}\dot{M}'}{MM'} + \frac{\dot{M}^2}{M^2} - \frac{\kappa}{M^2} - \frac{\kappa'}{MM'} + \chi_{9B} + Q_9 - Q_0, \tag{73}$$

$$X_{TF}^{\text{eff}} = \frac{\dot{M}^2}{M^2} - \frac{\dot{M}\dot{M}'}{MM'} - \frac{\kappa}{M^2} + \frac{\kappa'}{2M'M} + \chi_{10B} + Q_{10} + \frac{Q_2}{2G^2}. \tag{74}$$

The inclusion of an electromagnetic $f(R,T)$ correction has greatly modified the above versions of SS in the dissipation-less relativistic fluid. One can observe that the expression of Y_{TF}^{eff} is independent of κ, thus indicating that this complexity of the system does not depend on κ even in charged case.

5. Application of LTB to the dissipative case

The LTB space–time clearly excludes dissipative fluxes, as seen in the preceding section. By having dissipative fluxes, we can now solve the issue in the extension of a general LTB metric. We shall make it a condition that all GLTBs become LTBs under the condition of vanishing dissipative fluxes. As we are looking for space–times that are as 'close' to LTB as possible, we shall consider the geodesics motion of the shearing dust cloud along with $\tilde{q}_{\text{eff}} \neq 0$. It is important to note that the consideration of pure dust (i.e., no dispersion) necessarily corresponds to a geodesic fluid. However, this result does not hold if one considers the case of dissipative dust. Therefore, eq. (18) gives

$$G(t,r) = \frac{M'}{(1+B(t,r))^{1/2}}, \tag{75}$$

with

$$1 + B(t,r) = \left[\int 4\pi \tilde{q}_{\text{eff}} M \, \mathrm{d}t + c(r)\right]^2. \tag{76}$$

Since in the non-dissipative case eq. (75) should become eq. (63), it follows that

$$c(r) = (1+\kappa(r))^{1/2}. \tag{77}$$

The line element takes the form using eq. (75) in eq. (2), and we get

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{(M')^2}{\left[\int 4\pi \tilde{q}_{\text{eff}} M \, \mathrm{d}t + c(r)\right]^2}\mathrm{d}r^2 + M^2(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2) \tag{78}$$

or from eq. (77) it becomes

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{(M')^2}{\left[\int 4\pi \tilde{q}_{\text{eff}} M \, \mathrm{d}t + (1+\kappa(r))^{1/2}\right]^2}\mathrm{d}r^2 + M^2(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2). \tag{79}$$

The SS, Y_T^{eff}, Y_{TF}^{eff}, X_T^{eff} and X_{TF}^{eff} of Maxwell-$f(R,T)$ gravity take the form

$$Y_T^{\text{eff}} = -2\frac{\ddot{M}}{M} - \frac{\ddot{M}'}{M'} + \frac{\dot{B}}{1+B}\left(\frac{\dot{M}'}{M'} - \frac{3\dot{B}}{4(1+B)}\right) + \frac{\ddot{B}}{2(1+B)} + \chi_{5B} + Q_7 - \frac{Q_0}{2} - \frac{Q_2}{2G^2}, \tag{80}$$

$$Y_{TF}^{\text{eff}} = \frac{\ddot{M}}{M} - \frac{\ddot{M}'}{M'} + \frac{\dot{B}}{1+B}\left(\frac{\dot{M}'}{M'} - \frac{3\dot{B}}{4(1+B)}\right) + \frac{\ddot{B}}{2(1+B)} + \chi_{7B} + Q_8 + \frac{Q_2}{2G^2}, \tag{81}$$

$$X_T^{\text{eff}} = \frac{2\dot{M}\dot{M}'}{MM'} + \frac{\dot{M}^2}{M^2} - \frac{B}{M^2} - \frac{B'}{MM'} - \frac{\dot{M}\dot{B}}{M(1+B)} + \chi_{9B} + Q_9 - Q_0, \tag{82}$$

$$X_{TF}^{\text{eff}} = \frac{\dot{M}^2}{M^2} - \frac{\dot{M}\dot{M}'}{MM'} - \frac{B}{M^2} + \frac{B'}{2MM'} + \frac{\dot{M}\dot{B}}{2M(1+B)} + \chi_{10B} + Q_{10} + \frac{Q_2}{2G^2}. \tag{83}$$

In the dissipative case, we get SS with electromagnetic effects. If we want to go forward to reach a certain set of straightforward solutions, more criteria will be required. As previously demonstrated, the criterion for selecting such circumstances will be defined by the need that the findings provided, represent the 'closest' scenario to LTB space–time when dissipative fluxes are included. In addition, the final criteria are a bit hazy as well, leaving a lot of space for interpretation.

6. Extension of LTB depending on SS

After analysing eqs (72)–(74) and (80)–(83), we observed that (a) the SS Y_{TF} and Y_T are independent of κ in LTB and (b) there are two different SS, i.e., Y_T and Y_{TF} for the GLTB. In this context, eqs (72) and (80) provide

$$\frac{\dot{B}}{1+B}\left[\frac{\dot{M}'}{M'}-\frac{3\dot{B}}{4(1+B)}\right]+\frac{\ddot{B}}{2(1+B)}=0. \tag{84}$$

On integrating the above equation, we get

$$\frac{M'\dot{B}^{1/2}}{(1+B)^{3/4}}=C_1(r). \tag{85}$$

By using eqs (76) and (77), we obtain

$$C_1(r)=\frac{(8\pi\tilde{q}_{\rm eff}M)^{1/2}M'}{(1+\kappa(r))^{1/2}+\int 4\pi\tilde{q}_{\rm eff}M\,{\rm d}t}, \tag{86}$$

where $C_1(r)$ is an integration function. Equation (85) gives

$$B+1=\frac{4}{\left[C_1(r)^2\int\frac{{\rm d}t}{M'^2}+C_2(r)\right]^2}, \tag{87}$$

where $C_2(r)$ is another integration function, which might have some connections to $M(r)$ as follows. Equation (87) yields

$$\dot{B}=-\frac{8C_1^2(r)}{(M')^2\left[C_1^2(r)\int\frac{{\rm d}t}{M'^2}+C_2(r)\right]^3}. \tag{88}$$

Then the combination of eqs (76) and (88) gives

$$2\pi\tilde{q}_{\rm eff}=\frac{C_1^2(r)}{M(C')^2\left[C_1^2(r)\int\frac{{\rm d}t}{(M')^2}+C_2(r)\right]^2}. \tag{89}$$

One can see that the above equation is independent of charge, rather it depends on $f(R,T)$ terms that are fabricated with the fluid dissipation. Finally, the comparison of eq. (86) with eq. (89) give

$$C_2(r)=\frac{2(1-4\pi\tilde{q}_{\rm eff}MM'^2)}{C(r)+\int 4\pi\tilde{q}_{\rm eff}M\,{\rm d}t}. \tag{90}$$

Thus, we have obtained the GLTB because of modified SS in the presence of electric charge. For the given choices of $C_1,\ C_2$ and R, the role of effective form of dissipative variable $q^{\rm eff}$ can be studied. When the dissipative fluxes disappear from the system, the GLTB becomes LTB.

7. Extension of LTB based on properties of symmetry

In this section, we shall take an approach that depends on the symmetry characterisation to reach GLTB spacetimes. Here we suppose that our metric obeys proper matter collineation. We obtain three equations for the dissipative dust cloud as

$$\xi^0\dot{\tilde{\mu}}_{\rm eff}+\xi^1\tilde{\mu}'_{\rm eff}+2\tilde{\mu}_{\rm eff}\xi^0_{,0}-2\tilde{q}_{\rm eff}G\xi^1_{,0}+2Q_{00}\xi^0_0$$
$$+\dot{Q_{00}}\xi^0+\acute{Q_{00}}\xi^1=0, \tag{91}$$

$$-\xi^0\dot{\tilde{q}}_{\rm eff}-\xi^1\tilde{q}'_{\rm eff}+\tilde{\mu}_{\rm eff}\frac{\xi^0_{,1}}{G}$$
$$-\tilde{q}_{\rm eff}\left(\xi^0_{,0}+\xi^0\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)$$
$$+\epsilon_{\rm eff}G\xi^1_{,0}+\frac{Q_{00}\xi^0_1}{G}+\frac{Q_{11}\xi^1_0}{G}=0, \tag{92}$$

$$\xi^0\dot{\epsilon_{\rm eff}}+\xi^1\epsilon'_{\rm eff}-2\tilde{q}_{\rm eff}\frac{\xi^0_{,1}}{G}$$
$$+2\epsilon_{\rm eff}\left(\xi^0\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1,1\right)$$
$$+\frac{Q_{11}\xi^1_1}{G^2}+\frac{\dot{Q_{00}}\xi^0}{G^2}+\frac{\acute{Q_{11}}\xi^0}{G^2}=0. \tag{93}$$

Next, we shall proceed our analysis by taking the following two subcases:

7.1 *Diffusion approximation*, $\epsilon_{\rm eff}=0,\ q_{\rm eff}\neq 0$

In this scenario, eqs (18) and (75) provide

$$8\pi q_{\rm eff}G=\frac{\dot{B}}{1+B}\frac{M'}{M}. \tag{94}$$

Next, we obtain $\xi^0=Z(t)$ and it follows from eqs (91) and (92) that

$$Z(t)\dot{\mu}_{\rm eff}+\xi^1\mu'_{\rm eff}+2\mu_{\rm eff}\dot{Z}(t)-2q_{\rm eff}G\xi^1_{,0}$$
$$+2Q_{00}\dot{Z(t)}+\dot{Q_{00}}Z(t)+Q'_{00}\xi^1=0, \tag{95}$$

$$Z(t)\dot{q}_{\rm eff}+\xi^1q'_{\rm eff}+q_{\rm eff}\left(\dot{Z}(t)+Z(t)\frac{\dot{G}}{G}\right.$$
$$\left.+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)+\frac{Q_{00}Z'(t)}{G}+\frac{Q_{11}\xi^1_0}{G}=0, \tag{96}$$

which may be simplified as follows:

$$\xi^1[\ln(q_{\rm eff}G\xi^1)]'+Z(t)[\ln(q_{\rm eff}GZ(t))]^{\cdot}$$
$$+\frac{Q_{00}Z'(t)}{G}+\frac{Q_{11}\xi^1_0}{G}=0. \tag{97}$$

Multiplying the above equation by $q_{\rm eff}G$, it becomes

$$(q_{\rm eff}G\xi^1)'+(q_{\rm eff}ZG)^{\cdot}+\frac{Q_{00}Z'(t)}{G}+\frac{Q_{11}\xi^1_{,0}}{G}=0. \tag{98}$$

Additionally, a partial solution may be expressed as

$$q_{\rm eff}G\xi^1=-\dot{\psi}(t,r)+\dot{\chi}(t,r), \tag{99}$$

$$q_{\rm eff}GZ(t)=\psi'(t,r)+\chi'(t,r). \tag{100}$$

From eqs (94) and (100), we have

$$\psi'(t,r)=\frac{Z(t)}{8\pi}\frac{\dot{B}}{1+B}\frac{M'}{M}-\chi'(t,r). \tag{101}$$

Thus, GLTB has been obtained.

7.2 *Streaming-out approximation, $\epsilon \neq 0,\ q=0$*

In this background, we have $q_{\rm eff}=0$ and $\epsilon_{\rm eff}\neq 0$. Then [58]

$$\dot{\mu}_{\rm eff}+\mu_{\rm eff}\left(\frac{\dot{G}}{G}+2\frac{\dot{M}}{M}+\right)+Q_A=0, \tag{102}$$

which after integration gives

$$\mu_{\rm eff}=\frac{j(r)}{GM^2}+\int\frac{Q_A}{\mu_{\rm eff}}{\rm d}t, \tag{103}$$

where $j(r)$ is an integration function. Following that, in this approach, eqs (91)–(93) will take the form

$$\xi^0(\mu_{\rm eff}+\epsilon_{\rm eff})^{\cdot}+\xi^1(\mu_{\rm eff}+\epsilon'_{\rm eff}+2(\mu_{\rm eff}+\epsilon_{\rm eff}\xi^0_{,0}-2\epsilon_{\rm eff}G\xi^1_{,0}+2Q_{00}\xi^0_0+\dot{Q}_{00}\xi^0+Q'_{00}\xi^1=0, \tag{104}$$

$$\xi^0\dot{\epsilon_{\rm eff}}+\xi^1\epsilon'_{\rm eff}-(\mu_{\rm eff}+\epsilon_{\rm eff})\frac{\xi^0_{,1}}{G}+\epsilon_{\rm eff}\left(\xi^0_{,0}+\xi^0\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)-\epsilon_{\rm eff}G\xi^1_{,0}+\frac{Q_{00}\xi^0_1}{G}+\frac{Q_{11}\xi^1_0}{G}=0, \tag{105}$$

$$\xi^0\dot{\epsilon_{\rm eff}}+\xi^1\epsilon'_{\rm eff}-2\epsilon_{\rm eff}\frac{\xi^0_{,1}}{G}+2\epsilon_{\rm eff}\left(\xi^0\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)+\frac{Q_{11}\xi^1_1}{G^2}+\frac{\dot{Q}_{00}\xi^0}{G^2}+\frac{Q'_{11}\xi^0}{G^2}=0. \tag{106}$$

Let us consider $\xi^0=Z(t)$. Then, from eqs (104)–(106) we have

$$Z(t)(\mu_{\rm eff}+\epsilon_{\rm eff})^{\cdot}+\xi^1(\mu_{\rm eff}+\epsilon_{\rm eff})'+2(\mu_{\rm eff}+\epsilon_{\rm eff})\times\dot{Z}(t)-2\epsilon_{\rm eff}G\xi^1_{,0}+2Q_{00}\dot{Z}(t)+\dot{Q}_{00}Z(t)+Q'_{00}\xi^1=0, \tag{107}$$

$$Z(t)\dot{\epsilon_{\rm eff}}+\xi^1\epsilon'_{\rm eff}+\epsilon_{\rm eff}\left(\dot{Z}(t)+Z(t)\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)-\epsilon_{\rm eff}G\xi^1_{,0}+\frac{Q_{00}\xi^0_1}{G}+\frac{Q_{11}\xi^1_0}{G}=0, \tag{108}$$

$$Z(t)\dot{\epsilon_{\rm eff}}+\xi^1\epsilon'_{\rm eff}+2\epsilon_{\rm eff}\left(Z(t)\frac{\dot{G}}{G}+\xi^1\frac{G'}{G}+\xi^1_{,1}\right)=0. \tag{109}$$

We can also write the above expression as

$$Z(t)[\ln(\epsilon_{\rm eff}G^2)]^{\cdot}+\xi^1[\ln(\epsilon_{\rm eff}(G\xi^1)^2)]'+\frac{Q_{11}\xi^1_1}{G^2}+\frac{\dot{Q}_{00}\xi^0}{G^2}+\frac{Q'_{11}\xi^0}{G^2}=0, \tag{110}$$

or alternatively as

$$Z(t)(\epsilon_{\rm eff}G^2)^{\cdot}+\frac{1}{\xi^1}(\epsilon_{\rm eff}(G\xi^1)^2)'+\frac{Q_{11}\xi^1_1}{G^2}+\frac{\dot{Q}_{00}\xi^0}{G^2}+\frac{Q'_{11}\xi^0}{G^2}=0. \tag{111}$$

In this case, we can write Bianchi identity (eq. (49)) in the following form:

$$[\ln(\epsilon_{\rm eff}(GM)^2)]^{\cdot}+\frac{1}{G}[\ln(\epsilon_{\rm eff}M^2)]'+Q_D=0. \tag{112}$$

It can also be written as

$$[\epsilon_{\rm eff}(GM)^2]^{\cdot}+G(\epsilon_{\rm eff}M^2)'+Q_D=0. \tag{113}$$

Thus, one can get the GLTB for a given LTB from the above equation. The details for the procedure of plugging the known values in the above equation can be seen in [58].

8. Conclusion

Using the $f(R,T)$ theory with charge field, several methods for achieving spherically symmetric, geodesic, dissipative dust solutions to Einstein equations were formulated. Both alternatives are aimed to create solutions that are as similar as possible to the non-dissipative situation (LTB). To do so, we looked at LTB space–time dependent on some symmetry properties of LTB and then analysed them despite the presence of structure scalars. In this study, we examined the impact of $f(R,T)$ modifications with charge on some dynamical effects of changing stellar bodies. For this research, we used a spherically symmetric geometry that is thought to be coupled with the anisotropic radiating matter.

We assumed that the relativistic fluid distribution in a charge field has a shearing viscous property and emits radiations in the free streaming and diffusion approximations. There is no scattering as the dissipation occurs. Charged $f(R,T)$ gravity was then believed to mediate the extra degrees of freedom. After that, the associated field equations and dynamical equations were found. We also provided a connection between the Weyl tensor and other physical quantities using Misner–Sharp mass formalism. This relationship could be significant in stellar system modelling. The representation of the Riemann curvature tensor was then broken down using the orthogonal decomposition technique. For our stellar model, we used this strategy with modified $f(R,T)$ gravity. The trace and trace-free sections of tensorial entities were analysed, which are particularly important in the study of gravitational collapse, stellar evolution and other phenomena. These trace and trace-less components are known as $f(R,T)$ structure scalars.

The energy density of the fluid with the input of electric charge and X_T^{eff}, has some correspondence according to our findings. X_{TF}^{eff} controls energy density inhomogeneity with the passage of time in the absence of dissipation. Indeed, Y_T^{eff} is the mass density, while Y_{TF}^{eff} combines both energy density inhomogeneity and local anisotropy. The extra degrees of freedom of Maxwell-$f(R,T)$ gravity is trying to reduce the impact of X_{TF}, thereby making it difficult for the regular compact object to enter in the inhomogeneous window. To obtain the regular configuration of the non-static LTB geometry, the extra terms of $f(R,T)$ theory need to vanish. All our results reduce to GR [58] under the limit $f(R,T)=R$.

Acknowledgements

This work was supported by University of the Punjab Research Project for the fiscal year 2021–2022.

Appendix A

The parts of the equations are given below:

$$\chi_{00}=\left[\mu f_T-\left(f-\frac{R}{2}f_R\right)+\frac{f_R''}{G^2}-\dot{f}_R\left(\frac{\dot{G}}{G}+2\frac{\dot{M}}{M}\right)-f_R'\left(\frac{G'}{G^3}-\frac{2M'}{G^2M}\right)\right], \tag{A.1}$$

$$\chi_{01}=\left[\dot{f}_R'-\frac{\dot{G}}{G}\left(\dot{f}_R+f_R'\right)\right], \tag{A.2}$$

$$\chi_{11}=\left[-\mu G^2f_T+\left(f-\frac{R}{2}f_R\right)G^2-G\dot{G}\dot{f}_R-\frac{G'}{G}f_R'+G^2\ddot{f}_R+\dot{f}_RG^2\left(\frac{\dot{G}}{G}+2\frac{\dot{M}}{M}\right)+f_R'G^2\left(\frac{G'}{G^3}-\frac{2M'}{G^2M}\right)\right], \tag{A.3}$$

$$\chi_{22}=\left[-\mu M^2f_T+\left(f-\frac{R}{2}f_R\right)M^2+\frac{MM'}{G^2}f_R'-M\dot{M}\dot{f}_R+M^2\ddot{f}_R-\frac{M^2}{G^2}f_R''+M^2\left(\frac{\dot{G}}{G}+2\frac{\dot{M}}{M}\right)\dot{f}_R+f_R'\left(\frac{G'}{G^3}-\frac{2M'}{G^2M}\right)M^2\right]. \tag{A.4}$$

Some more terms are given below:

$$\chi_{5B}=\frac{4\pi(1+f_T)}{f_R}(\tilde{q})+\frac{4\pi}{f_R}\left[-\chi_{00}+\frac{\chi_{11}}{G^2}\right]+4\pi\left[3\left(\mu f_T+\Box f_R-f+\frac{R}{2}f_R\right)-v^\zeta v^\gamma\nabla_\gamma\nabla_\zeta f_R-v^\eta v^\delta\nabla_\eta\nabla_\delta f_R+v^\gamma v^\delta\nabla_\gamma\nabla_\delta f_R\right]+8\pi f_R\left[\Box f_R-4\left(\rho f_T+\Box f_R-f+\frac{R}{2}f_R\right)\right], \tag{A.5}$$

$$\chi_{7B}=\frac{4\pi\chi_{11}}{G^2f_R}+\frac{1}{\chi_\eta\chi_\zeta-\frac{h_{\eta\zeta}}{3}}\times\left[4\pi\left(\left(\mu f_T+\Box f_R-f+\frac{R}{2}f_R\right)\times(2v_\eta v_\zeta+g_{\eta\zeta}+\delta_{\eta\zeta})-v_\eta v^\gamma\nabla_\gamma\nabla_\zeta f_R-v_\zeta v^\delta\nabla_\eta\nabla_\delta f_R+v^\gamma v^\delta\nabla_\gamma\nabla_\delta f_R\delta_{\eta\zeta}\right)+\frac{8\pi}{3}h_{\eta\zeta}f_R\left[\Box f_R-4\left(\rho f_T+\Box f_R-f+\frac{R}{2}f_R\right)\right]-\frac{1}{3}h_{\eta\zeta}\left[4\pi(\tilde{\mu}_{\text{eff}}+\epsilon_{\text{eff}})+\chi_{5B}\right]\right], \tag{A.6}$$

$$\chi_{9B}=-\frac{8\pi}{f_R}\left(\chi_{00}+\frac{1+f_T}{f_R}\epsilon\right)+12\pi\epsilon\frac{1+f_T}{f_R}+\frac{8\pi}{f_R}\left(4\left(\mu f_T-f+\frac{R}{2}f_R\right)+\Box f_R\right)+\Box f_R-g^{\eta\zeta}\frac{\pi}{f_R}[\epsilon_\eta^{\epsilon\rho}\epsilon_{\pi\rho\zeta}g^{\mu\pi}\nabla_\mu\nabla_\epsilon f_R-\epsilon_\eta^{\epsilon\rho}\epsilon_{\pi\epsilon\zeta}g^{\mu\pi}\nabla_\mu\nabla_\rho f_R-\epsilon_\eta^{\epsilon\rho}\epsilon_{\rho\theta\zeta}g^{\theta\mu}\nabla_\mu\nabla_\epsilon f_R\,\epsilon_{\epsilon\theta\zeta}g^{\mu\theta}\nabla_\mu\nabla_\rho f_R], \tag{A.7}$$

$$\chi_{10B}=-\frac{4\pi\chi_{11}}{G^2f_R}+\frac{1}{\chi_\eta\chi_\zeta-\frac{h_{\eta\zeta}}{3}}\times\left[\left(4\epsilon g_{\eta\zeta}\frac{1+f_T}{f_R}-4\epsilon h_{\eta\zeta}\frac{1+f_T}{f_R}-\tilde{\mu}h_{\eta\zeta}\frac{8\pi}{3}\right)-\frac{8\pi}{3f_R}\left(3\nabla_\eta\nabla_\zeta f_R-4\left(\rho f_T+\Box f_R-f+\frac{R}{2}f_R\right)h_{\eta\zeta}\right)-\frac{1}{3}h_{\eta\zeta}(8\pi\tilde{\mu}_{\text{eff}}+\chi_{9D})-\frac{\pi}{f_R}\times\left[\epsilon_\eta^{\epsilon\rho}\epsilon_{\pi\rho\zeta}g^{\mu\pi}\nabla_\mu\nabla_\epsilon f_R-\epsilon_\eta^{\epsilon\rho}\epsilon_{\pi\epsilon\zeta}g^{\mu\pi}\nabla_\mu\nabla_\rho f_R\right.\right.$$

$$-\epsilon_{\eta}^{\epsilon\rho}\epsilon_{\rho\theta\zeta}g^{\theta\mu}\nabla_{\mu}\nabla_{\epsilon}f_R$$
$$+\epsilon_{\eta}^{\epsilon\rho}\epsilon_{\epsilon\theta\zeta}g^{\mu\theta}\nabla_{\mu}\nabla_{\rho}f_R\Big]\Big]. \tag{A.8}$$

Appendix B

The charge terms are given as follows:

$$Q^{00} = \frac{1}{f_r}\frac{s^2}{8\pi M^4}, \quad Q^{11} = -\frac{1}{f_r}\frac{s^2}{8\pi G^2 M^4},$$
$$Q^{22} = \frac{1}{R^4}\frac{1}{f_r}\frac{s^2}{8\pi M^2}, \tag{B.1}$$
$$Q^{33} = \frac{1}{R^4\sin^2\theta}\left[\left(\frac{1+f_T}{f_r}\right)\frac{s^2}{8\pi M^2} - f_T\frac{s^2}{2M^4}M^2\right],$$
$$Q_{00} = \left[\left(\frac{1+f_T}{f_r}\right)\frac{s^2}{8\pi M^4} - f_T\frac{s^2}{2M^4}\right], \tag{B.2}$$
$$Q_{11} = -\left(\frac{1+f_T}{f_r}\right)\frac{s^2 G}{8\pi G M^4} + f_T G^2\frac{M^2}{2M^4},$$
$$Q_{22} = \left[\left(\frac{1+f_T}{f_r}\right)\frac{s^2}{8\pi M^2} - f_T\frac{s^2}{2M^4}M^2\right], \tag{B.3}$$
$$Q_{33} = \left[\left(\frac{1+f_T}{f_r}\right)\frac{s^2}{8\pi M^2}\sin^2\theta - f_T\frac{s^2}{2M^4}M^2\sin^2\theta\right], \tag{B.4}$$

$$Q_4 = Q_{10}' + \left(\frac{Q_2}{G^2}\right)' + \frac{3M'}{M}Q_{10} + \frac{3M'}{MG^2}Q_2 - \left(\frac{Q_0}{2}\right)',$$
$$Q_7 = 4\pi(Q_{\gamma\zeta}V^{\zeta}V^{\gamma} + Q_{\gamma\delta}V^{\gamma}V^{\delta}), \tag{B.5}$$
$$Q_8 = \frac{1}{\chi_\eta\chi_\zeta - \frac{h_{\eta\zeta}}{3}}\left(4\pi\left(Q_{\eta\zeta} + Q_{\gamma\zeta}V_\eta V^\gamma + Q_{\gamma\delta}\delta_{\eta\zeta}V^\gamma V^\delta\right) - \frac{1}{3}h_{\eta\zeta}Q_7\right), \tag{B.6}$$
$$Q_9 = -\pi\epsilon^{\rho\epsilon\zeta}\left(\epsilon_{\pi\rho\zeta}Q_{\epsilon}^{\pi} + \epsilon_{\pi\epsilon\zeta}Q_{\rho}^{\pi} + \epsilon_{\rho\theta\zeta}Q_{\epsilon}^{\theta} + \epsilon_{\epsilon\theta\zeta}Q_{\rho}^{\theta}\right), \tag{B.7}$$
$$Q_{10} = \frac{1}{\chi_\eta\chi_\zeta - \frac{h_{\eta\zeta}}{3}}\left(-\pi\epsilon_{\eta}^{\rho\epsilon}(\epsilon_{\pi\rho\zeta}Q_{\epsilon}^{\pi} + \epsilon_{\pi\epsilon\zeta}Q_{\rho}^{\pi} + \epsilon_{\rho\theta\zeta}Q_{\epsilon}^{\theta} + \epsilon_{\epsilon\theta\zeta}Q_{\rho}^{\theta}) - \frac{1}{3}Q_9 h_{\eta\zeta}\right), \tag{B.8}$$

$$Q_A = \left(\frac{1}{f_r}\frac{s^2}{8\pi M^4}\right) - G\dot{G}\left(\frac{1}{f_r}\frac{s^2}{8\pi G^2M^4}\right) + \left(\frac{\dot{G}}{G} + \frac{2\dot{M}}{M}\right)\frac{1}{f_r}\frac{s^2}{8\pi M^4}, \tag{B.9}$$
$$Q_B = -\frac{1}{8\pi f_R}\left(\frac{s^2}{G^2M^4}\right)' - \frac{1}{8\pi f_R}\left(2\frac{G'}{G} + \frac{M'}{M}\right)\frac{s^2}{G^2M^4} - 2\frac{M'M}{G^2}\frac{1}{8\pi f_R}\frac{s^2}{M^6}, \tag{B.10}$$
$$Q_C = \frac{3M'Q_2}{G^2M} + \left(\frac{Q_2}{2G^2}\right)' - \left(\frac{Q_0}{2}\right)', \tag{B.11}$$
$$Q_D = \frac{1}{8\pi f_R}\left(\frac{s^2}{G^2M^4}\right)' - \frac{1}{8\pi f_R}\left(2\frac{G'}{G} + \frac{M'}{M}\right)\frac{s^2}{G^2M^4} - 2\frac{M'}{G^2}\frac{1}{8\pi f_R}\frac{s^2}{M^5}. \tag{B.12}$$

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