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Improving amplitude-modulated signals by re-scaled and twice sampling vibrational resonance methods

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Abstract. We present the re-scaled vibrational resonance (VR) method and the twice sampling VR method to improve the amplitude-modulated signal. Two different kinds of signals are considered. One is the amplitude-modulated harmonic signal. The other is the amplitude-modulated aperiodic binary signal. Both the VR methods have an excellent effect on the signal improvement. For the re-scaled VR method, the scale parameter is the key factor to determine the resonance output. For the twice sampling VR method, the frequency reduced ratio or the minimal random pulse width stretched ratio is the crucial factor. By choosing appropriate key factors, the output can achieve the strongest resonance and lead to the optimal signal improvement.

Keywords. Amplitude-modulated signal; aperiodic signal; vibrational resonance; re-scaled method; twice sampling method.

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1. Introduction

The amplitude-modulated signal is widely used in a variety of engineering and scientific fields [1–7]. It is quite an important topic to improve the weak signal in the amplitude-modulated form. There are different ways to improve a weak signal. Among them, some methods according to the response properties of a nonlinear system have attracted more and more attention. For example, the stochastic resonance (SR) [8] and vibrational resonance (VR) [9] methods are well known. Especially, they can be used to improve the weak amplitude-modulated signal [10–15]. Furthermore, the SR and VR are very important and interesting phenomena of different nonlinear models [16–22].

In the earlier works of SR or VR related to the amplitude-modulated signal excited system, the adiabatic approximation theory should be satisfied. In this precondition, the signal is usually in the low-frequency or slow variable form. However, in the engineering fields, the signal that characterises useful information may be in arbitrary high-frequency form. As a result, we need to improve the signal without the limitation of the frequency value. With the development of SR and VR, there are two important methods to solve this problem. One is the re-scaled method [23–28]. The other is the twice sampling method [29,30]. Based on these two different methods, the high-frequency signal can match the system parameters effectively. Then, strong SR or VR can be achieved. However, to our knowledge, all studies are focussed on the external signal case. There is no work on SR or VR under the excitation of the amplitude-modulated signal especially when the signal frequency is high. Moreover, most of the studies are carried out in the classical bistable system which may not be the optimal system for signal processing. On comparing VR with SR, we found that VR is much easier to control. Considering these problems, we shall further study VR in a general bistable system subjected to an amplitude-modulated high-frequency signal. Two different signal forms will be considered.

The first kind of signal is the harmonic signal in the amplitude-modulated form. Its concrete expression is

$$s(t) = [f + F\cos(\Omega t)]\sin(\omega t), \tag{1}$$

where $s(t)$ is the amplitude-modulated signal that will be improved by the re-scaled and twice sampling VR methods. f and ω are the amplitude and frequency of the character signal, which usually transmit the useful information. f is modulated by the other harmonic signal with amplitude F and frequency Ω. We call $F\cos(\Omega t)$ as the auxiliary signal in this paper. Moreover, the signal satisfies $f \ll 1$ and $\omega \ll \Omega$. In the following analysis, we label $\Omega = n\omega$ with n being a positive real number.

The second kind is the aperiodic binary signal in the amplitude-modulated form. Its concrete expression is

$$s(t) = [f + F\cos(\Omega t)]\sum_{j=-\infty}^{+\infty} R_j\Gamma(t - jT), \tag{2}$$

where $s(t)$ still means the amplitude-modulated signal. The character signal is the aperiodic binary signal with amplitude f, which is modulated by the auxiliary signal with amplitude F and frequency Ω. R_j is a random number generator of $+1$ or -1 with an independent Gaussian distribution. $\Gamma(t)$ is a random pulse with minimal width T. Usually, for VR phenomenon to occur, we have $T \gg 2\pi/\Omega$. In the following analysis, we let $\Omega = n(2\pi/T)$ with n being also a positive real number.

On comparing this work with earlier studies, there are two major differences in the signal form. On the one hand, in [14,15], the frequency ω is low, i.e. $\omega \ll 1$. The signal can be improved by the traditional VR theory. In this work, the frequency ω is an arbitrarily large value. It extends the potential applications of VR. However, the VR phenomenon cannot be realised in this case according to the traditional VR theory. Hence, re-scaled and twice sampling VR methods are proposed to solve the problem. On the other hand, although aperiodic VR is studied in [31], in that work, the signal form is not in the amplitude-modulated form. In engineering fields, the random pulse is usually modulated by another signal [10,12]. As a result, it is important to improve aperiodic binary signal in the amplitude-modulated form. Moreover, in the earlier work [31], the random pulse width cannot be arbitrary because we need to match the system parameters with the minimal random pulse width. To our knowledge, no theoretical framework is proposed to clarify the match condition in detail. We shall solve this problem in the present work.

To obtain some more general results, we use a general bistable system instead of the classical bistable system. Specifically, the governing equation in which the VR phenomenon will be discussed is as follows:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V(x)}{\partial x} + s(t), \tag{3}$$

where $V(x)$ is a bistable potential function with the expression

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{\alpha+1}|x|^{\alpha+1}. \tag{4}$$

$V(x)$ has one unstable equilibrium $x_0 = 0$ and two stable equilibria $x_{1,2} = \pm(a/b)^{1/(\alpha-1)}$. The exponent $\alpha > 1$ can have fractional or integer values. This kind of potential function is widely used in modelling some engineering problems [32–34]. If $\alpha = 3$, the potential function is the classical bistable potential function which has been used in volumes of literatures.

The rest of the paper is organised as follows. In §2, we shall give the re-scaled and twice sampling VR theories to process the amplitude-modulated harmonic signal with an arbitrary frequency. In §3, the re-scaled and twice sampling VR methods are proposed to improve the amplitude-modulated aperiodic signal with an arbitrary minimal random pulse width. Moreover, the effect of the fractional exponent α on the VR will be discussed. In §4, the main results of this work will be presented.

2. The amplitude-modulated harmonic signal case

First, we use the re-scaled and twice sampling VR methods to improve the amplitude-modulated harmonic signal, i.e. the signal described in eq. (1). Under the excitation of this kind of signal, eq. (3) turns to

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)|x(t)|^{\alpha-1} + [f + F\cos(\Omega t)]\sin(\omega t). \tag{5}$$

2.1 *Re-scaled VR*

If $\omega \ll 1$, the parameters a and b are usually in the order of 1. For example, a and b may lie in 0.1 or 1 for several times. If $\omega \gg 1$, the parameters a and b should be large to induce the VR phenomenon.

Let

$$\tau = \beta t, \quad x(t) = z(\tau), \tag{6}$$

where β is the re-scaled parameter and τ is the new time scale. Substituting it into eq. (5), we have

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = \frac{a}{\beta}z(\tau) - \frac{b}{\beta}z(\tau)|z(\tau)|^{\alpha-1} + \frac{1}{\beta}\left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sin\left(\frac{\omega}{\beta}\tau\right). \tag{7}$$

Then, letting

$$a_1 = \frac{a}{\beta}, \quad b_1 = \frac{b}{\beta}, \tag{8}$$

eq. (7) turns to

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = a_1 z(\tau) - b_1 z(\tau)|z(\tau)|^{\alpha-1} + \frac{1}{\beta}\left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sin\left(\frac{\omega}{\beta}\tau\right). \tag{9}$$

By choosing an appropriate re-scaled parameter β, the excitation in eq. (9) can be in the low-frequency form. Furthermore, compare eq. (9) with eq. (5). Although the excitation has the same frequency, the amplitude of the excitation in eq. (9) is $1/\beta$ of the original strength in eq. (5). Equation (9) aims to obtain the parameter matching condition only. Hence, to obtain the equivalent system to eq. (5), we need to recover the amplitude of the excitation. That is to say, the equivalent system to eq. (5) is in the form

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = a_1 z(\tau) - b_1 z(\tau)|z(\tau)|^{\alpha-1} + \left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sin\left(\frac{\omega}{\beta}\tau\right). \tag{10}$$

As a result, the original signal can be improved in the following system, i.e.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)|x(t)|^{\alpha-1} + \beta[f + F\cos(\Omega t)]\sin(\omega t). \tag{11}$$

Furthermore, in eq. (10), the frequency of the signal is low and the system parameters are in the order of 1. The traditional VR can occur in eq. (10). In eq. (11), the frequency of the signal is an arbitrary high value and the system parameters may be large and need to match the signal frequencies automatically. Through the above procedures, the VR can occur in the high-frequency signal excited system. We name VR in eq. (11) as re-scaled VR. In the following, we shall give some numerical examples to verify the re-scaled VR theory.

To quantify the amplification of the harmonic component through the nonlinear system, an index called response amplitude at the frequency ω is defined, specifically

$$Q = \sqrt{B_s^2 + B_c^2}/f, \tag{12}$$

where B_s and B_c are the Fourier coefficients of the time series. Furthermore, B_s and B_c are calculated by

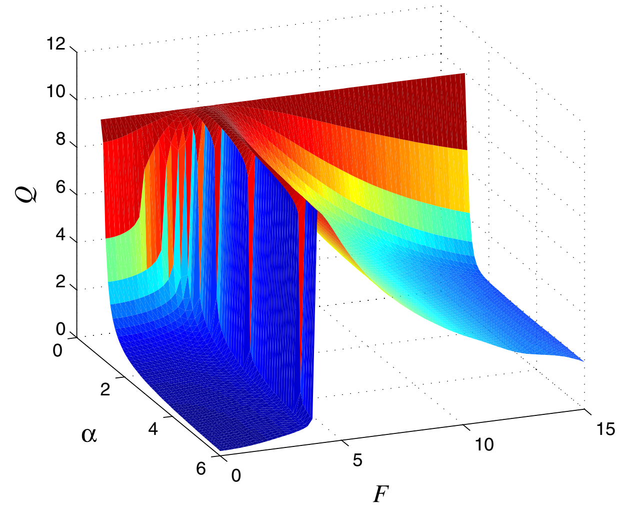

Figure 1. The response amplitude vs. the fractional exponent α and the auxiliary signal amplitude F. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $\omega = 10$, $n = 50$ and $\beta = 100$.

$$B_s = \frac{2}{nT}\int_0^{nT} x(t)\sin(\omega t)\,\mathrm{d}t,$$
$$B_c = \frac{2}{nT}\int_0^{nT} x(t)\cos(\omega t)\,\mathrm{d}t. \tag{13}$$

In fact, the response amplitude Q expresses the amplification of the low-frequency signal by the cooperation of the nonlinear system and the high-frequency signal. When the re-scaled VR occurs, the response amplitude achieves the maximal value.

In figure 1, the response amplitude Q is plotted as a function of α and F. On the one hand, with the increase of α, the response amplitude will decrease. On the other hand, with the increase of F, the resonance peak appears clearly. In other words, re-scaled VR occurs in nonlinear system. Furthermore, for a larger α, the resonance curve is much more apparent.

To express the dependence of Q on the variables F and α better, we give several two-dimensional curves in figure 2. If F is a controllable parameter, as shown in figure 2a, the response amplitude Q has an identical peak value for different values of α. In other words, the fractional exponent α does not influence the peak of the Q–F curve. In figure 2b, the response amplitude Q vs. the fractional exponent α is presented. When $F = 2.0$ and 5.0, Q is nonlinearly decreasing with α. When $F = 3.3$, Q increases first and then decreases with α. There is a weak resonance peak at $\alpha = 1.5$. It indicates that Q is a nonlinear function of α. Moreover, the monotonicity of the Q–α curve depends on the value of F closely. In summary, in figures 1 and 2, the output of the system can be controlled by the parameters F and α effectively. They also show that there is

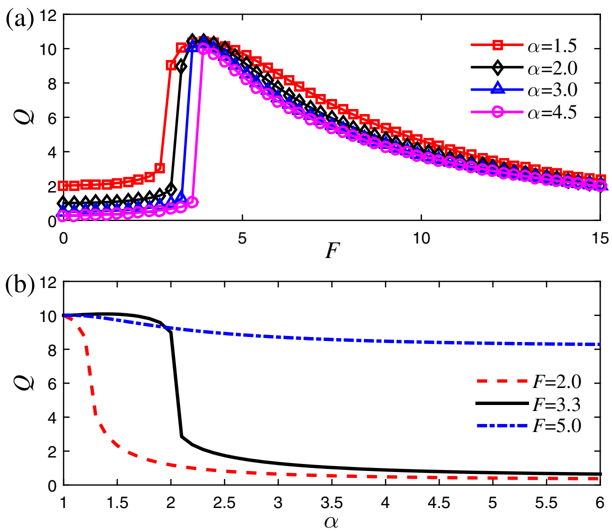

Figure 2. (**a**) The response amplitude Q vs. the auxiliary signal amplitude F and (**b**) the response amplitude Q vs. the fractional exponent α. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $\omega = 10$, $n = 50$ and $\beta = 100$.

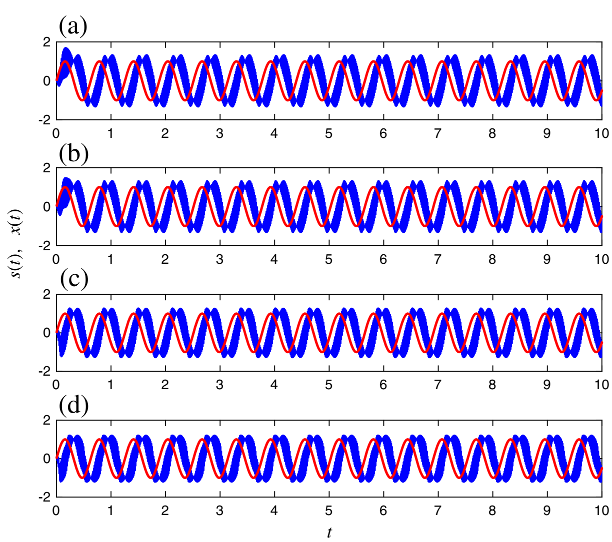

Figure 3. Time series of the output (blue lines) and the signal $10f\,\sin(\omega t)$ (red lines). (**a**) $\alpha = 1.5$, (**b**) $\alpha = 2.0$, (**c**) $\alpha = 3.0$ and (**d**) $\alpha = 4.5$. Other simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $\omega = 10$, $F = 3.9$, $n = 50$ and $\beta = 100$.

a fractional value which can induce optimal resonance. Furthermore, optimal resonance usually does not occur at $\alpha = 3$. In other words, the traditional bistable system may not be the optimal system for the signal processing by the VR method.

In figure 3, we show some output time series corresponding to the resonance peaks in figure 2a. That is to say, the output has achieved the optimum in figure 3. To make a striking contrast, 10 times the strength of the weak signal $f\,\sin(\omega t)$, i.e. $10f\,\sin(\omega t)$,

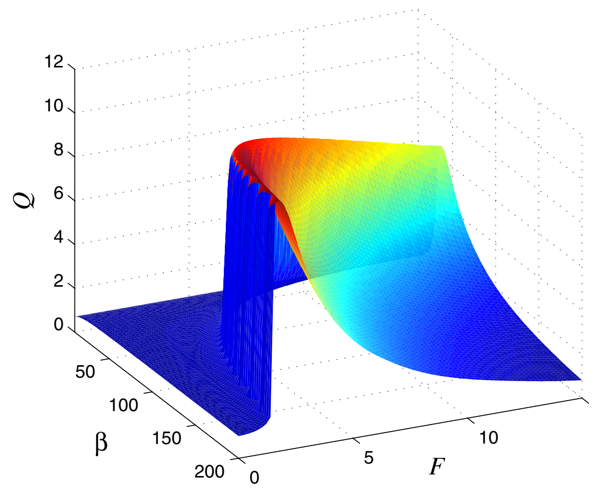

Figure 4. The response amplitude Q vs. the scale parameter β and the auxiliary signal amplitude F. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$, $\omega = 10$ and $n = 50$.

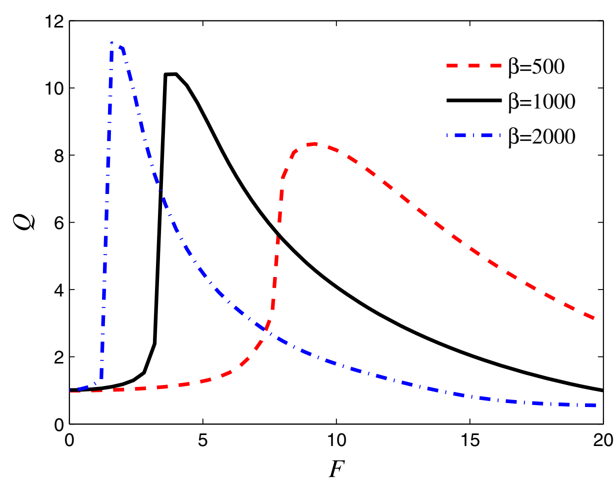

Figure 5. The response amplitude Q vs. the auxiliary signal amplitude F under different values of β. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$, $\omega = 10$ and $n = 50$.

is plotted for comparison. Evidently, in the output, the low-frequency component ω in eq. (1) is improved and the high-frequency Ω is suppressed excellently.

In figure 4, the response amplitude Q vs. the scale parameter β and the auxiliary signal amplitude F is given in a three-dimensional surface. Obviously, the parameter F can induce re-scaled VR in a wide scope of β. Furthermore, the scale parameter β influences the location and height of the response amplitude Q. In other words, β determines the resonance degree and the strength of the auxiliary signal which can induce optimal resonance output. It can also be seen clearly in figure 5 by choosing different values of β and keeping other parameters fixed. We find that with the

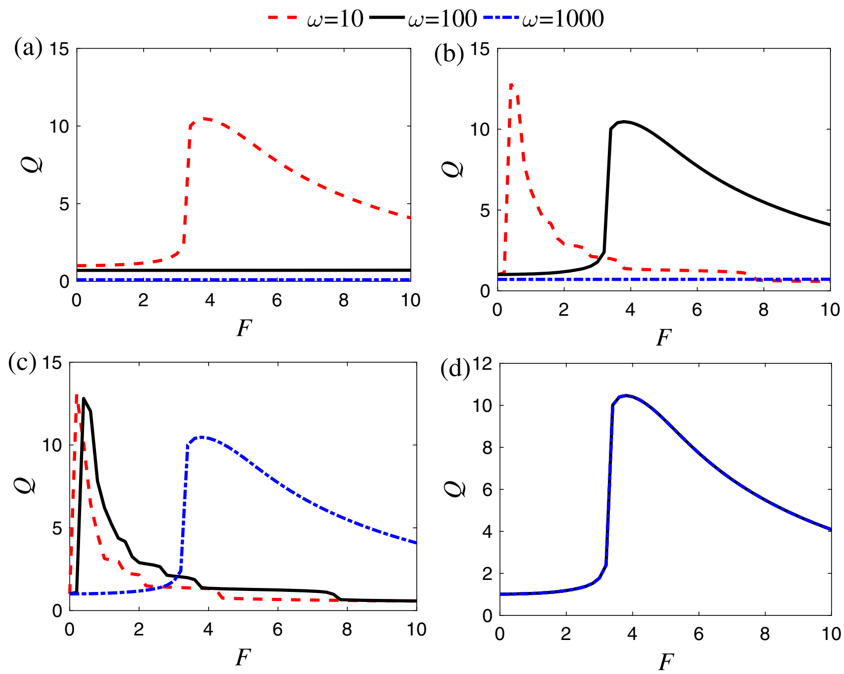

Figure 6. The response amplitude Q vs. the auxiliary signal amplitude F under different values of β and ω. (**a**) $\beta = 100$, (**b**) $\beta = 1000$, (**c**) $\beta = 10{,}000$ and (**d**) $\beta = 10,\ 100$ and 1000 corresponding to $\omega = 100,\ 1000$ and 10000, respectively. Other simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$ and $n = 50$.

increase of β, the maximal value of Q increases and the corresponding F, which is located at the resonance peak, turns smaller. It is because larger β makes the equivalent frequency ω/β in eq. (10) to have a smaller value, which induces a stronger resonance.

The effect of the scale parameter β is further explained in figure 6. In figure 6a, we let $\beta = 100$. Hence, for $\omega = 10$, 100 and 1000, the low-frequency ω/β is 0.1, 1 and 10 in the equivalent system described by eq. (10), respectively. For $\omega = 100$ and 1000, the excitation frequency ω/β is 1 and 10, respectively. They are large values. It leads to the corresponding results, which are very small in the subplot. In figure 6b, the scale parameter is 1000. It changes the low-frequency ω/β to be 0.01, 0.1 and 1. The response amplitude is improved for $\omega = 10$ and 100. For $\omega = 1000$, the response amplitude cannot be improved almost through the VR method. In figure 6c, the low-frequency ω/β in eq. (10) is 0.001, 0.01 and 0.1. The response amplitude curves are presented as the subplot. Interestingly and importantly, in figure 6d, ω and β have different values. However, ω/β has the same value in three lines. It is because we always have $\omega/\beta = 0.1$ for them. Therefore, the response amplitude curves are completely identical for three curves. It indicates that we can obtain the same results by choosing an appropriate scale parameter under different excitation frequencies.

From figures 1–6, the amplitude-modulated signal with an arbitrary frequency is improved by the re-scaled VR method. The results show the validity of the re-scaled VR method.

2.2 *Twice sampling VR*

Besides the re-scaled method to process the high-frequency signal, the twice sampling idea is another important way to process the signal. It has been successfully applied in extracting the weak character information in the strong noisy background [29,30]. Specifically speaking, the procedure of the twice sampling VR is as follows. First, we carry out twice sampling for the original input signal. We use a constant γ as the frequency reduced ratio. The original sampling frequency is f_{s0}. The twice sampling frequency is f_s. Then, the frequency reduced ratio is $\gamma = f_{s0}/f_s$. Through this transformation, the original frequency ω will be reduced to ω/γ. Then, we input the transformed signal to the nonlinear system. Finally, the output is transformed inversely to the original sampling frequency to obtain the new output time series. The new output time series are analysed and the twice sampling VR will be obtained with the aid of an auxiliary signal.

In figure 7, the dependence of the response amplitude Q on the auxiliary signal amplitude F and the frequency reduced ratio γ is shown. The resonance phenomenon is obvious. By choosing appropriate F and γ, the weak amplitude-modulated signal can be improved to a large extent. Moreover, the maximal of the resonance peak depends on the frequency reduced ratio closely.

The effect of the frequency reduced ratio γ is shown in figure 8. In figure 8a, according to the frequency reduced ratio, the original frequency $\omega = 10$, 100 and 1000 is reduced to 0.1, 1 and 10, respectively, by the twice sampling process. Their corresponding response amplitude results are shown as the subplots. In figure 8b, the original frequency is reduced to 0.01, 0.1 and 1 before the signal inputs the nonlinear system. It leads to the response amplitude curves shown in figure 8b. In figure 8c, the original frequency is reduced to 0.001, 0.01 and 0.1. In figure 8d, although the original frequency and the frequency reduced ratio have different values, they have the same frequency after the twice sampling process and before they input the nonlinear system. Hence, the three curves in figure 8d are identical completely. That is to say, the response amplitude mainly depends on the frequency of the twice sampled signal, i.e. the value of γ.

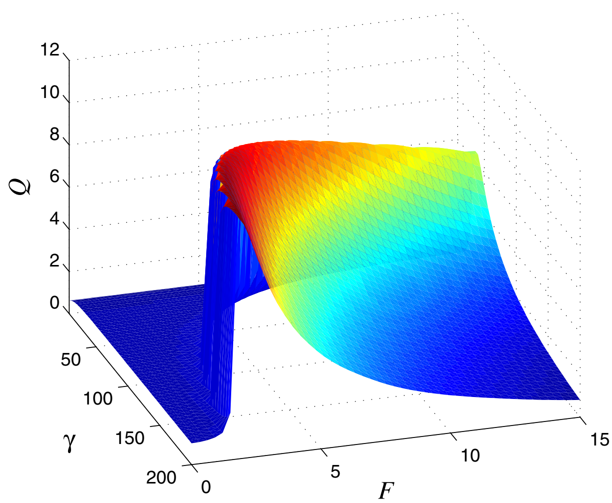

Figure 7. The response amplitude Q vs. the auxiliary signal amplitude F and the frequency reduced ratio γ. The simulation parameters are: $a = 1$, $b = 1$, $\alpha = 2$, $f = 0.1$, $\omega = 10$ and $n = 50$.

From figures 7 and 8, we know that the twice sampling method can induce VR of the amplitude-modulated signal at an arbitrary frequency value. The system can achieve optimal output. Moreover, on comparing figures 7 and 8 with figures 4 and 6, we find that the twice sampling VR has the same effect on improving the weak amplitude-modulated signal at an arbitrary frequency as that of the re-scaled VR method.

3. The amplitude-modulated aperiodic binary signal case

In this section, we study VR in eq. (3) when the signal is in the amplitude-modulated aperiodic form that is described in eq. (2). The governing equation turns to

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)|x(t)|^{\alpha-1} + [f + F\cos(\Omega t)] \sum_{j=-\infty}^{+\infty} R_j \Gamma(t - jT). \quad (14)$$

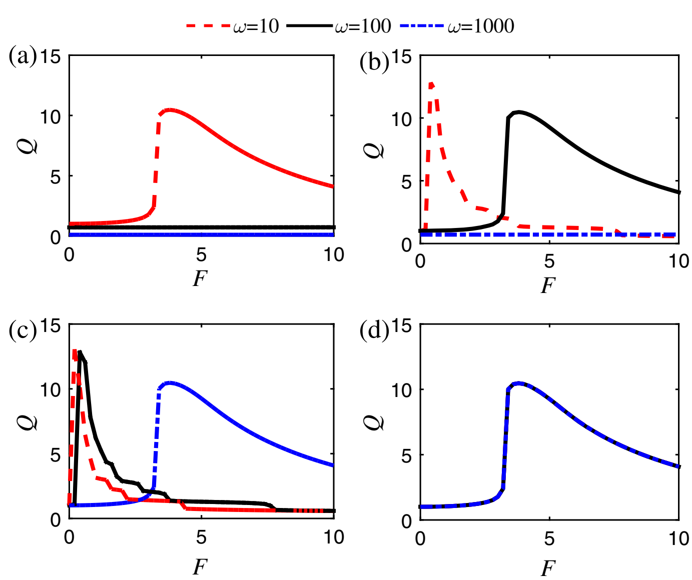

Figure 8. The response amplitude Q vs. the auxiliary signal amplitude F under different values of γ and ω. (**a**) $\gamma = 100$, (**b**) $\gamma = 1000$, (**c**) $\gamma = 10000$ and (**d**) $\gamma = 100$, 1000 and 10,000 corresponding to $\omega = 10$, 100 and 1000, respectively. Other simulation parameters are: $a = 1$, $b = 1$, $\alpha = 2$, $f = 0.1$ and $n = 50$.

3.1 *Re-scaled VR*

We still use the transform formula in eq. (6). Then, substituting it into eq. (14), we obtain

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = \frac{a}{\beta}z(\tau) - \frac{b}{\beta}z(\tau)|z(\tau)|^{\alpha-1} + \frac{1}{\beta}\left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sum_{j=-\infty}^{+\infty} R_j\Gamma\left(\frac{\tau}{\beta} - j\frac{T}{\beta}\right). \tag{15}$$

By eq. (8), we have

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = a_1 z(\tau) - b_1 z(\tau)|z(\tau)|^{\alpha-1} + \frac{1}{\beta}\left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sum_{j=-\infty}^{+\infty} R_j\Gamma\left(\frac{\tau}{\beta} - j\frac{T}{\beta}\right). \tag{16}$$

Recovering the signal to the original strength, then we obtain

$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = a_1 z(\tau) - b_1 z(\tau)|z(\tau)|^{\alpha-1} + \left[f + F\cos\left(\frac{\Omega}{\beta}\tau\right)\right]\sum_{j=-\infty}^{+\infty} R_j\Gamma\left(\frac{\tau}{\beta} - j\frac{T}{\beta}\right). \tag{17}$$

Finally, we use the following equation to induce the VR phenomenon, i.e.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)|x(t)|^{\alpha-1} + \beta[f + F\cos(\Omega t)]\sum_{j=-\infty}^{+\infty} R_j\Gamma(t - jT). \tag{18}$$

Some numerical examples will be given to verify the method. To quantify the degree of the VR phenomenon of an aperiodic signal, the character cross-correlation coefficient is needed to be calculated. Specifically, we use C_{sx} to label the cross-correlation coefficient between the input signal and the system output. When C_{sx} achieves a large enough value, resonance may occur and the weak input aperiodic signal may be enhanced. The cross-correlation coefficient C_{sx} is calculated by

$$C_{sx} = \frac{\sum_{j=1}^{m}[s(j) - \bar{s}][x(j) - \bar{x}]}{\sqrt{\sum_{j=1}^{m}[s(j) - \bar{s}]^2 \sum_{j=1}^{m}[x(j) - \bar{x}]^2}}, \tag{19}$$

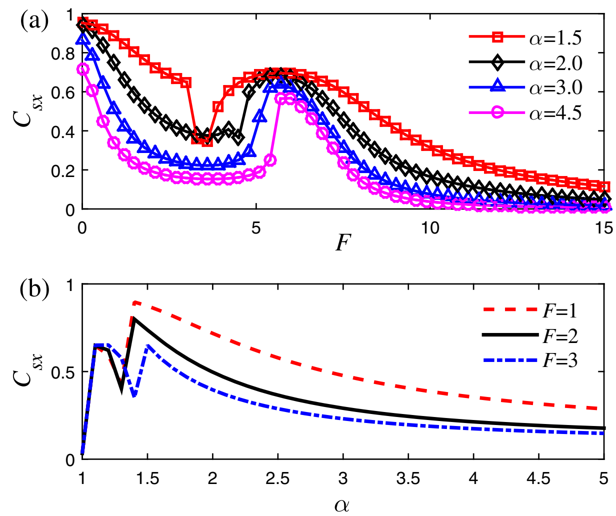

Figure 9. (**a**) The cross-correlation coefficient C_{sx} vs. the auxiliary signal amplitude F and (**b**) the cross-correlation coefficient C_{sx} vs. the fractional exponent α. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $T = 0.4$, $n = 50$ and $\beta = 100$.

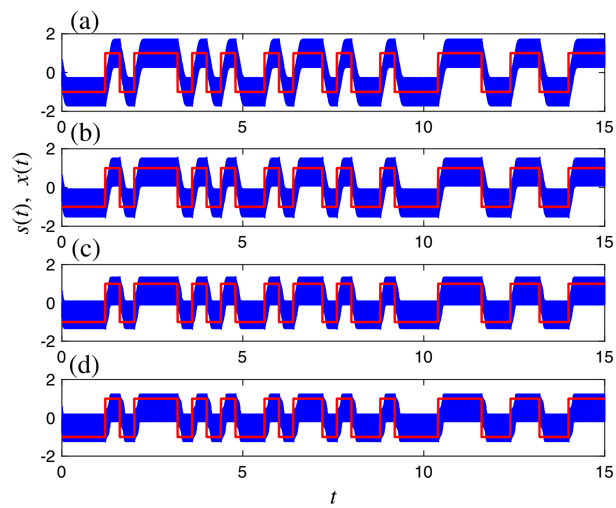

Figure 10. Time series of the output (blue lines) and the signal $10s(t)$ (red lines). (**a**) $\alpha = 1.5$, (**b**) $\alpha = 2.0$, (**c**) $\alpha = 3.0$ and (**d**) $\alpha = 4.5$. Other simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $T = 0.4$, $n = 50$ and $\beta = 100$.

where $\bar{u}$ and $\bar{x}$ are the averages of the input signal and the system output, respectively.

In figure 9a, the curves of C_{sx}–F are plotted. First, the curves present the VR phenomenon with the increase of F under different values of α. It demonstrates the effectiveness of the re-scaled VR method for the amplitude-modulated aperiodic binary signal. Secondly, for the curves in the subplot of figure 9a, we find that the peak value of the curve will decrease with the increase of α. The reason for it will be discussed in figure 10. Moreover, the effect of the fractional exponent α is different from that in figure 2a. In figure 9b, under different values of F, the curves C_{sx} vs. the fractional exponent α are given. Apparently, resonance can be induced by the fractional exponent. In other words, when the excitation amplitude is fixed, the classical bistable system (for $\alpha = 3$) is not the optimal one for the VR system. It is also one of the reasons for choosing the fractional power system.

In figure 10, the optimal output corresponding to the resonance peaks in figure 9a is given. In each subplot of figure 10, the input signal is improved to a great degree by the twice sampling VR method. What is worth mentioning is that the amplitude of the output will decrease with the increase of α. It is consistent with the result in figure 9a in which the resonance peaks decrease with the increase of α.

In figure 11, the three-dimensional curve shows the dependence of the cross-correlation coefficient on the scale parameter β and the auxiliary signal amplitude F. The scale parameter β is a key factor to determine the location and value of the resonance peak. It is the same as its effect in the amplitude-modulated harmonic excitation case shown in figure 4.

To make the re-scaled VR in the amplitude-modulated aperiodic binary signal excited system clear further, we give the resonance curves in figure 12 under different values of T and β. In figure 12a, the equivalent minimal random pulse period in eq. (17) is $T = 40$, 4 and 0.4, under the scale parameter $\beta = 100$. Obviously, the curve corresponding to $T = 0.4$ has the maximal resonance value. In figure 12b, the equivalent minimal random

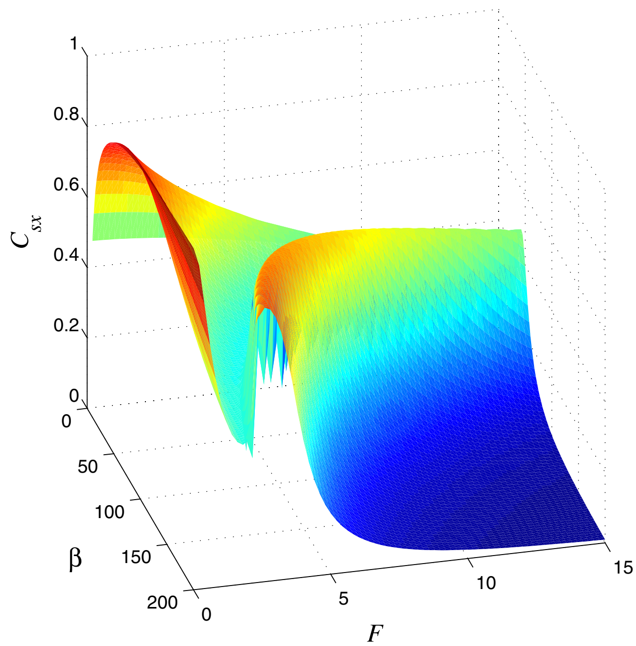

Figure 11. The cross-correlation coefficient C_{sx} vs. the scale parameter β and the auxiliary signal amplitude F. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $f = 0.1$, $T = 0.4$ and $n = 50$.

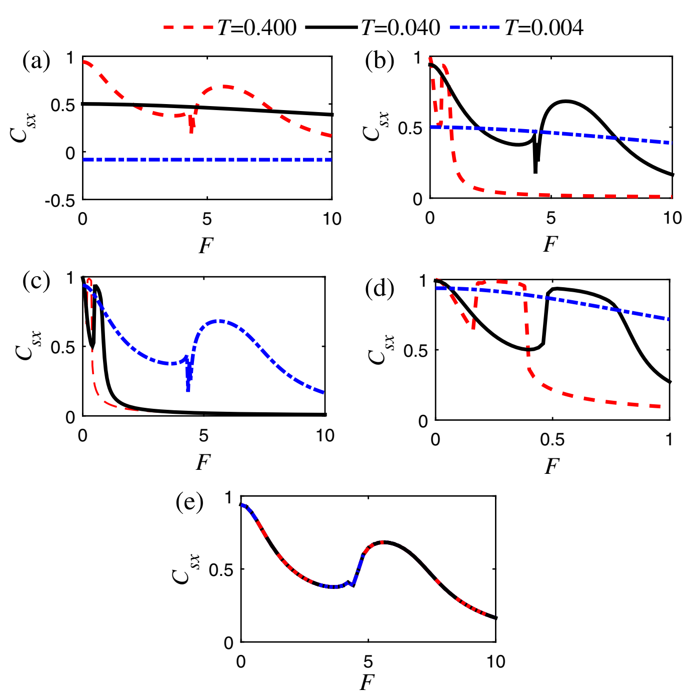

Figure 12. The cross-correlation coefficient C_{sx} vs. the auxiliary signal amplitude F under different values of β and T. (**a**) $\beta = 100$, (**b**) $\beta = 1000$, (**c**) and (**d**) $\beta = 10000$ and (**e**) $\beta = 10$, 100 and 1000 corresponding to $T = 0.4$, 0.04 and 0.004, respectively. Other simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$ and $n = 50$.

pulse period in eq. (17) is $T = 400$, 40 and 4, under the scale parameter $\beta = 1000$. Although the curves are corresponding to $T = 0.4$ and 4.0, both have the resonance peaks. However, the resonance peak corresponding to $T = 0.4$ has a larger value. In figure 12c, the equivalent minimal random pulse period in eq. (17) is $T = 4000$, 400 and 40, under the scale parameter $\beta = 10000$. There are obvious resonance peaks. Figure 12d is the locally enlarged image for figure 12c. In figure 12d, we can observe the structure of the resonance peak much more clearly. In figure 12e, by choosing different values of T and β, we make the equivalent minimal random pulse width in eq. (17) to have the same value. As a result, the curves in figure 12d are identical completely. The results shown in figure 12 are the same as in figure 6.

From figures 9–12, the validity of the re-scaled VR method to process the amplitude-modulated aperiodic binary signal is verified. The results are similar to those in the amplitude-modulated harmonic signal excited system.

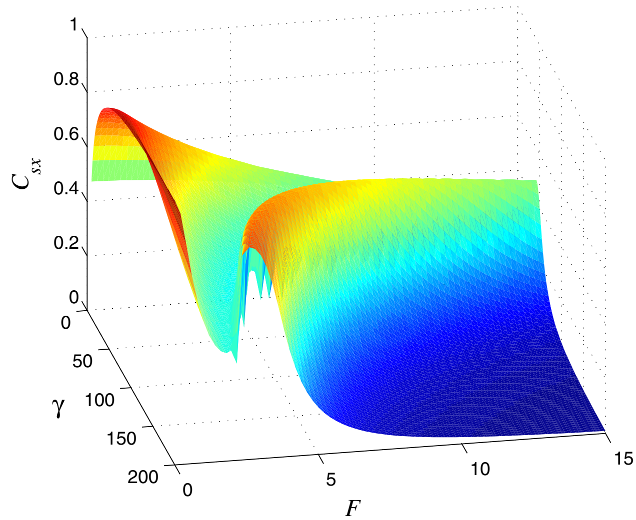

Figure 13. The cross-correlation coefficient C_{sx} vs. the auxiliary signal amplitude F and the minimal random pulse width stretching ratio γ. The simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$, $T = 0.4$ and $n = 50$.

3.2 *Twice sampling VR*

The amplitude-modulated aperiodic binary signal can also be processed by the twice sampling VR method. The procedure is the same as that used in §2.2. In the following, we use some numerical examples to verify it.

In figure 13, the three-dimensional curve of the cross-correlation coefficient vs. the parameters γ and F is given. Here, γ is used to label the stretched ratio of the minimal random pulse width. Specifically, the amplitude-modulated aperiodic binary signal will be extended γ times in the time scale before it inputs the nonlinear system. Certainly, the output will transform to the original time scale to carry out the cross-correlation coefficient analysis. Apparently, the value and location

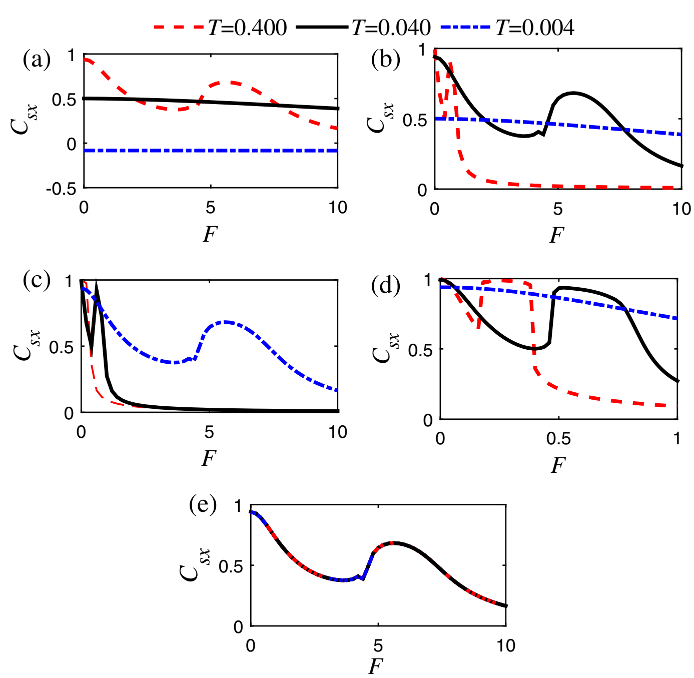

Figure 14. The cross-correlation coefficient C_{sx} vs. the auxiliary signal amplitude F under different values of γ and T. (**a**) $\gamma = 100$, (**b**) $\gamma = 1000$, (**c**) and (**d**) $\gamma = 10000$ and (**e**) $\gamma = 100$, 1000 and 10000 corresponding to $T = 0.4$, 0.04 and 0.004, respectively. Other simulation parameters are: $a_1 = 1$, $b_1 = 1$, $\alpha = 2$, $f = 0.1$ and $n = 50$.

of the resonance peak depend on γ closely. The results in figure 13 are similar to that in figure 7.

In figure 14, under different values of γ and T, the cross-correlation coefficient C_{sx} vs. the auxiliary signal amplitude F is plotted. The results in this figure are similar to that in figures 8 and 12. The equivalent minimal random pulse width γT is the crucial factor to determine the improvement of the character signal.

As to the two proposed methods, although their signal processes are different, they can achieve the same goal. Furthermore, we can construct the experimentally realisable equipment using hardware systems, such as a signal amplifier, a sampling processor, a bistable electrocircuit, etc. In fact, we have described a similar problem in our earlier papers [35,36].

4. Conclusions

The amplitude-modulated signal is an important signal used in many disciplines. It is important to improve an amplitude-modulated signal with an arbitrary frequency or pulse width. In this paper, we present the re-scaled VR method and the twice sampling VR method to process the amplitude-modulated harmonic signal and amplitude-modulated aperiodic binary signal, respectively.

For the amplitude-modulated harmonic signal case, by the re-scaled VR method, the harmonic component of the character signal can be improved excellently. The scale parameter is the crucial factor. By choosing an appropriate scale parameter, we can obtain an ideal resonance output and then achieve great improvement of the character signal. Without this analysis, it is difficult to choose the system parameters to achieve optimal resonance, especially when the considered frequency is high. When the twice sampling VR method is used, we can achieve the optimal output by choosing an appropriate frequency reduced ratio. The effect of the frequency reduced ratio is the same as that of the scale parameter used in the re-scaled VR method.

For the amplitude-modulated aperiodic binary signal case, the re-scaled VR method and the twice sampling VR method are still having good performance in signal improvement. The scale parameter and the minimal random pulse width stretched ratio are key factors when the two methods are applied. By choosing appropriate values of the scale parameter and the minimal random pulse width stretched ratio, the weak amplitude-modulated aperiodic binary signal can be improved greatly.

The results in this paper give two VR methods to improve the amplitude-modulated signal. By our methods, there is no limitation in the frequency or the minimal random pulse width. We can obtain the match condition of the system parameters with the signal parameters quickly and accurately. It provides an effective way to improve the weak amplitude-modulated signal. However, some questions are still not answered in the present paper. For example, if the modulated signal is submerged into noise, especially for the strong noise case, how to use the re-scaled VR method or the twice sampling VR method to extract the weak amplitude-modulated signal with an arbitrary frequency or pulse width? In the engineering field, such as mechanical engineering, the vibration signal collected from a bearing with inner or rolling faults is an amplitude-modulated signal with high frequency. Can we use the proposed VR methods to extract the characteristics successfully? Our future works will try to answer these questions.

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