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New optical soliton solutions for nonlinear complex fractional Schrödinger equation via new auxiliary equation method and novel (G'/G)-expansion method

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Abstract. In this research, we apply two different techniques on nonlinear complex fractional nonlinear Schrödinger equation which is a very important model in fractional quantum mechanics. Nonlinear Schrödinger equation is one of the basic models in fibre optics and many other branches of science. We use the conformable fractional derivative to transfer the nonlinear real integer-order nonlinear Schrödinger equation to nonlinear complex fractional nonlinear Schrödinger equation. We apply new auxiliary equation method and novel (G'/G)-expansion method on nonlinear complex fractional Schrödinger equation to obtain new optical forms of solitary travelling wave solutions. We find many new optical solitary travelling wave solutions for this model. These solutions are obtained precisely and efficiency of the method can be demonstrated.

Keywords. Nonlinear complex fractional Schrödinger equation; new auxiliary equation method; novel (G'/G)-expansion method; optical solitary travelling wave solutions; kink and antikink.

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1. Introduction

At the beginning of the twentieth century, experimental guides proposed that atomic particles were also wave-like in nature. For example, electrons were established to give diffraction patterns when passed through a double slit just like light waves. Therefore, it was logical to suppose that a wave equation could demonstrate the attitude of atomic particles. Nonlinear complex fractional Schrödinger equation is one of the most fundamental equation in fractional quantum mechanics which was formulated by Nick Laskin in 1999. The properties of the nonlinear complex fractional Schrödinger equation are the same as linearity, real energy eigenstates, space and time derivatives, local conservation of probability, positive energy, analytic continuation to diffusion and regularity. Many properties of it discover new one by getting new form of solitary travelling wave solutions. However, physically interpreting the wave is one of the main philosophical problems of quantum mechanics. This great model appeared to all the world as one of the results obtained by extending the Feynman path integral, form the Brownian-like to Hévy-like quantum mechanical paths. Nick Laskin in 1999 put the formula of nonlinear complex fractional Schrödinger equation as follows:

$$ih\frac{\partial\psi}{\partial t} = D_\alpha(-h^2\Delta)^{\alpha/2}\psi + V\psi, \tag{1.1}$$

where ψ is the Schrödinger wave function which represents the quantum mechanical probability amplitude for the particle, V is the potential energy which is a function of (p, t) such that p is 3D positive vector, t is the time, h is the Planck constant, Δ is the Laplace operator, D_α is a scale constant and α belongs to an open interval (0, 1). Nonlinear complex fractional Schrödinger equation is considered as one of the most powerful models because of its many applications. We just mention a few applications of this model to show how this model has a great influence on science.

- The nuclear charge of the atom when $V(p)$ refers to the energy of the hydrogen-like atom can be defined as the negative value of the mathematical calculation of the product of atomic number (Z) and the square of the electron charge (e) divided by the absolute value of position vector (p).

$$V(p) = -Z\,\frac{e^2}{|p|}.$$

- Infinite potential in one dimension can be considered as the evidence of the discrete energy spectrum and has the following value:

$$V(x) = \begin{cases} \infty \Rightarrow x \;<\; -\alpha, \\ 0 \Rightarrow -\alpha \;<\; x \;<\; \alpha, \\ \infty \Rightarrow x \;>\; -\alpha. \end{cases}$$

- Fractional quantum oscillator which is fractional quantum mechanical with Hamiltonian operator

$$H_{\alpha,\,\beta} = D_\alpha\left(-h^2\,\Delta\right)^{\alpha/2} + Q^2\,|p|^{\beta},$$

where $1 < \alpha \le 2$, $1 < \beta \le 2$ and Q is the interaction constant.
- Fractional quantum mechanics in solid state system.

Many researchers applied several techniques on nonlinear complex fractional Schrödinger equation [1–22]. In ref. [1], McLachlan applied a variational method, in ref. [2], Kaup and Newell applied inverse scattering techniques, in ref. [3], Whitehead proved the existence of a class of exact eigenvalues and eigenfunctions, in ref. [4], Ray applied the methods of Burgan *et al*, in ref. [5], Gendenshtein applied type of hidden symmetry, in ref. [6], Ziolkowski applied the homogeneous method, in ref. [7], Gagnon and Winternitz applied Lie symmetry groups, in ref. [8], Malfliet and Hereman applied the tanh method, in ref. [9], Kanna and Lakshmanan obtained explicit multisoliton solutions (up to four-soliton solutions), in ref. [10], Yan applied the generalised method, in ref. [11], Kruglov *et al* applied ansatz method, in ref. [12], Biazar and Ghazvini applied He's homotopy perturbation method, in ref. [13], Ikhdair and Sever applied the Nikiforov–Uvarov method, in ref. [14], Wazwaz applied the variational iteration method, in ref. [15], Ma and Chen applied symmetry algebra, in ref. [16], Tezcan and Sever applied an appropriate coordinate transformation, in ref. [17], Taghizadeh *et al* applied the first integral method, in ref. [18], Arda and Sever applied Laplace transform approach, in ref. [19], Seadawy applied the function transformation method, in ref. [20], Seadawy applied variational method, in ref. [21], Seadawy applied the amplitude ansatz method, in ref. [22], Yue *et al* applied a class of ordinary differential equations. All of these great researchers applied several methods as we mentioned [23–34]. Over the years, we believe in the importance of this equation and that there are many properties that have not been discovered so far. In this research, we used two methods to find new forms of solutions to this amazing model.

The rest of this paper is organised as follows: In §2, we use new auxiliary equation method [35] and novel $\left(G'/G\right)$-expansion method [36,37] to get the exact and solitary travelling wave solutions of nonlinear complex fractional nonlinear Schrödinger equation. In §3, conclusions are given.

2. Formulation for nonlinear complex fractional Schrödinger equation

Consider the nonlinear complex fractional Schrödinger equation in the form [37,39]

$$\frac{\partial^\alpha Q}{\partial t^\alpha} + i\,\frac{\partial^2 Q}{\partial x^2} + \frac{\partial}{\partial x}(|Q|^2 Q) = 0, \tag{2.1}$$

such that $0 < \alpha < 1$.

Using the travelling wave transformation $Q(x,t) = v(\xi)\mathrm{e}^{i\,\eta}$ and conformable fractional derivative

$$\xi = i\,k\left(x + \frac{2\,\omega\,t^\alpha}{\alpha}\right),$$

$$\eta = \left(\omega\,x + \frac{\epsilon\,t^\alpha}{\alpha}\right)$$

to transfer the nonlinear complex fractional Schrödinger equation to nonlinear integer order Schrödinger equation and for further properties about conformable fractional derivative you can see [35,40–42], we obtain

$$\begin{cases} \dfrac{\partial^\alpha\,Q}{\partial\,t^\alpha} = i\;\left(\epsilon\,v + 2\,\omega\,k\,v'\right)\mathrm{e}^{i\,\eta}, \\ \dfrac{\partial^2\,Q}{\partial\,x^2} = -\;\left(\omega^2\,v + 2\,\omega\,k\,v' + k^2\,v''\right)\mathrm{e}^{i\,\eta}, \\ \dfrac{\partial}{\partial\,x}\,(|Q|^2 Q) = i\;\left(\omega\,v^3 + 3\,k\,v^2\,v''\right)\mathrm{e}^{i\,\eta}. \end{cases} \tag{2.2}$$

Substituting (2.2) into eq. (2.1), we get that the nonlinear complex fractional Schrödinger equation transform into nonlinear ordinary differential equation as follows [43–49]:

$$\left(\epsilon - \omega^2\right)v - k^2\,v'' + \omega\,v^3 + 3\,k\,v^2\,v' = 0. \tag{2.3}$$

Balancing between the highest derivative term and nonlinear term in eq. (2.3), (v'' and $v^2\,v'$) $\Rightarrow$ $(N + 2 = 2\,N + N + 1) \Rightarrow \left(N = \frac{1}{2}\right)$. So, we use another transformation $v(\xi) = u^{1/2}(\xi)$ in eq. (2.3). We obtain

$$4\,\omega\,u^3 + 4\,(\epsilon - \omega^2)\,u^2 + 6\,k\,u^2\,u' + k^2\,u'^2 - 2\,k\,u\,u'' = 0. \tag{2.4}$$

Balancing between the highest derivative term and nonlinear term in eq. (2.3), ($u\,u''$ and $u^2\,u'$) $\Rightarrow$ $(N + N + 2 = 2\,N + N + 1) \Rightarrow (N = 1)$.

2.1 *Exact and solitary wave solution of nonlinear complex fractional Schrödinger equation by using new auxiliary equation method*

Using the default for precision solution using new auxiliary equation method on nonlinear complex fractional Schrödinger equation, we obtain

$$u(\xi) = a_0 + a_1\, a^{f(\xi)}. \tag{2.5}$$

Substituting eq. (2.6) and its derivatives into eq. (2.4) and equating the coefficient of different power of $a^{i\,f(\xi)}$ to zero, we obtain a system of algebraic equations by solving it with any computer program like Maple, Mathematica, Matlab and so on. We get

Case I

$$\alpha = 0, \quad \beta = \frac{-2\,\omega}{3\,k}, \quad \sigma = 0, \quad \epsilon = \frac{10\,\omega^2}{9},$$
$$a_0 = 0, \quad a_1 = a_1,$$

so that, the exact travelling wave solution of nonlinear complex fractional Schrödinger equation (2.4) is in the form

$$u(\xi) = a_1\, a^{f(\xi)}. \tag{2.6}$$

Therefore, the solitary wave solutions:

$$u(\xi) = \frac{a_1\left(-\left(1+\mathrm{e}^{2\,\beta\,\xi}\right) \pm \sqrt{2\left(\mathrm{e}^{4\,\beta\,\xi}+1\right)}\right)}{\mathrm{e}^{2\,\beta\,\xi}-1}, \tag{2.7}$$

$$Q(x,t) = \left[\frac{a_1\left(-\left(1+\mathrm{e}^{2i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}\right) \pm \sqrt{2\left(\mathrm{e}^{4\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}+1\right)}\right)}{\mathrm{e}^{2\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}-1}\right]^{1/2} \mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)} \tag{2.8}$$

or

$$u(\xi) = \frac{a_1\left(-\left(1+\mathrm{e}^{2\,\beta\,\xi}\right) \pm \sqrt{\mathrm{e}^{4\,\beta\,\xi}+6\,\mathrm{e}^{2\,\beta\,\xi}+1}\right)}{2\,\mathrm{e}^{2\,\beta\,\xi}}, \tag{2.9}$$

$$Q(x,t) = \left[\frac{a_1\left(-\left(1+\mathrm{e}^{2\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}\right) \pm \sqrt{\mathrm{e}^{4\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}+6\,\mathrm{e}^{2\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}+1}\right)}{2\,\mathrm{e}^{2\,i\,\beta\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)}}\right]^{1/2} \mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)}. \tag{2.10}$$

Case II

$$\alpha = 0, \beta = 2\,\frac{\omega}{k}, \quad \sigma = 2\,\frac{a_1}{k},$$
$$\epsilon = 2\,\omega^2, \quad a_0 = 0, \quad a_1 = a_1,$$

so that, the exact travelling wave solution of nonlinear complex fractional Schrödinger equation (2.4) is in the form

$$u(\xi) = a_1\, a^{f(\xi)}. \tag{2.11}$$

Therefore, the solitary wave solutions when $\beta^2 - \alpha\,\sigma < 0$ and $\sigma \neq 0$ are

$$u(\xi) = a_1\left[\frac{-\beta}{\sigma} + \frac{\sqrt{-\beta^2}}{\sigma}\tan\left(\frac{\sqrt{-\beta^2}}{2}\,\xi\right)\right], \tag{2.12}$$

$$Q(x,t) = \left[a_1\left[\frac{-\beta}{\sigma} + \frac{\sqrt{-\beta^2}}{\sigma}\tan\left(\frac{\sqrt{\beta^2}}{2}\,k \times\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\right]\right]^{1/2} \mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)} \tag{2.13}$$

or

$$u(\xi) = a_1\left[\frac{-\beta}{\sigma} + \frac{\sqrt{-\beta^2}}{\sigma}\cot\left(\frac{\sqrt{-\beta^2}}{2}\,\xi\right)\right], \tag{2.14}$$

$$Q(x,t)=\left[a_1\left[\frac{-\beta}{\sigma}+\frac{\sqrt{-\beta^2}}{\sigma}\cot\left(\frac{\sqrt{\beta^2}}{2}k\times\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.15}$$

When $\beta^2-\alpha\,\sigma>0$ and $\sigma\neq 0$

$$u(\xi)=a_1\left[\frac{-\beta}{\sigma}-\frac{\beta}{\sigma}\tanh\left(\frac{\beta}{2}\,\xi\right)\right], \tag{2.16}$$

$$Q(x,t)=\left[a_1\left[\frac{-\beta}{\sigma}-\frac{\beta}{\sigma}\times\tanh\left(\frac{i\,k\,\beta}{2}\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.17}$$

or

$$u(\xi)=a_1\left[\frac{-\beta}{\sigma}-\frac{\beta}{\sigma}\coth\left(\frac{\beta}{2}\,\xi\right)\right], \tag{2.18}$$

$$Q(x,t)=\left[a_1\left[\frac{-\beta}{\sigma}-\frac{\beta}{\sigma}\times\coth\left(\frac{i\,k\,\beta}{2}\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.19}$$

When $\beta=k,\ \sigma=2\,k,\ \alpha=0$

$$u(\xi)=\frac{a_1\,\mathrm{e}^{k\,\xi}}{1-\mathrm{e}^{k\,\xi}}, \tag{2.20}$$

$$Q(x,t)=\left[\frac{a_1\,\mathrm{e}^{i\,k^2\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1-\mathrm{e}^{i\,k^2\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.21}$$

When $\alpha=0$

$$u(\xi)=\frac{a_1\,\beta\,\mathrm{e}^{\beta\,\xi}}{1+\frac{\sigma}{2}\,\mathrm{e}^{\beta\,\xi}}, \tag{2.22}$$

$$Q(x,t)=\left[\frac{a_1\,\beta\,\mathrm{e}^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1+\frac{\sigma}{2}\,\mathrm{e}^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.23}$$

Case III

$$\omega=\frac{-3}{2}\,\beta\,k,\quad \sigma=0,\quad \epsilon=\frac{5}{2}\,\beta^2k^2,$$
$$a_0=a_0,\quad a_1=a_1$$

so that, the exact travelling wave solution of nonlinear complex fractional Schrödinger equation (2.4) is in the form

$$u(\xi)=a_0+a_1\,a^{f(\xi)}. \tag{2.24}$$

Therefore, the solitary wave solutions, when $\beta=k,\ \alpha=2\,k,\ \sigma=0$ are

$$u(\xi)=a_0+a_1\left[\mathrm{e}^{k\,\xi}-1\right], \tag{2.25}$$

$$Q(x,t)=\left[a_0+a_1\left[\mathrm{e}^{i\,k^2\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}-1\right]\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.26}$$

When $2\,\beta=\alpha+\sigma$

$$u(\xi)=a_0+a_1\left[1-\alpha\,\mathrm{e}^{\frac{1}{2}(\alpha-\sigma)\xi}\right], \tag{2.27}$$

$$Q(x,t)=\left[a_0+a_1\left[1-\alpha\,\mathrm{e}^{\frac{i\,k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}\right]\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.28}$$

or

$$u(\xi)=a_0-a_1\left[\alpha\,\mathrm{e}^{\frac{1}{2}(\alpha-\sigma)\xi}+1\right], \tag{2.29}$$

$$Q(x,t)=\left[a_0-a_1\left[\alpha\,\mathrm{e}^{\frac{i\,k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+1\right]\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.30}$$

When $\alpha=0$

$$u(\xi)=a_0+\frac{a_1\,\beta\,\mathrm{e}^{\beta\,\xi}}{1+\frac{\sigma}{2}\,\mathrm{e}^{\beta\,\xi}}, \tag{2.31}$$

$$Q(x,t)=\left[a_0+\frac{a_1\,\beta\,\mathrm{e}^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1+\frac{\sigma}{2}\,\mathrm{e}^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]^{1/2}\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.32}$$

Case IV

$$\alpha=\frac{a_0\,(\beta\,k-2\,a_0)}{k\,a_1},$$
$$\omega=\frac{1}{2}\,\beta\,k-2\,a_0,\quad a_0\left(\frac{\beta\,k}{2}-\omega\right),\quad a_1=\frac{k\,\sigma}{2}$$

so that, the exact travelling wave solution of nonlinear complex fractional Schrödinger equation (2.4) is in the form

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2} - \omega\right) + k\,\sigma\,a^{f(\xi)}\right]. \tag{2.33}$$

Therefore, the solitary wave solutions, when $\beta^2 - \alpha\,\sigma < 0$ and $\sigma \neq 0$ are

$$u(\xi) = \frac{1}{2}\left[-\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{-(\beta^2 - \alpha\,\sigma)} \times \tan\left(\frac{\sqrt{-(\beta^2 - \alpha\,\sigma)}}{2}\,\xi\right)\right], \tag{2.34}$$

$$Q(x,t) = \left[\frac{1}{2}\left[-\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 - \alpha\,\sigma)} \times \tan\left(\frac{k\sqrt{(\beta^2 - \alpha\,\sigma)}}{2} \times \left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.35}$$

or

$$u(\xi) = \frac{1}{2}\left[-\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{-(\beta^2 - \alpha\,\sigma)} \times \cot\left(\frac{\sqrt{-(\beta^2 - \alpha\,\sigma)}}{2}\,\xi\right)\right], \tag{2.36}$$

$$Q(x,t) = \left[\frac{1}{2}\left[-\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{-(\beta^2 - \alpha\,\sigma)} \times \cot\left(\frac{k\sqrt{(\beta^2 - \alpha\,\sigma)}}{2}\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.37}$$

When $\beta^2 - \alpha\,\sigma > 0$ and $\sigma \neq 0$

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 - \alpha\,\sigma)} \times \tanh\left(\frac{\sqrt{(\beta^2 - \alpha\,\sigma)}}{2}\,\xi\right)\right], \tag{2.38}$$

$$Q(x,t) = \left[\frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 - \alpha\,\sigma)} \times \tanh\left(\frac{k\sqrt{-(\beta^2 - \alpha\,\sigma)}}{2} \times \left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.39}$$

or

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 - \alpha\,\sigma)} \times \coth\left(\frac{\sqrt{(\beta^2 - \alpha\,\sigma)}}{2}\,\xi\right)\right], \tag{2.40}$$

$$Q(x,t) = \left[\frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 - \alpha\,\sigma)} \times \coth\left(\frac{k\sqrt{-(\beta^2 - \alpha\,\sigma)}}{2} \times \left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.41}$$

When $\beta^2 + \alpha^2 > 0$, $\sigma \neq 0$ and $\sigma = -\alpha$

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 + \alpha^2)} \times \tanh\left(\frac{\sqrt{(\beta^2 + \alpha^2)}}{2}\,\xi\right)\right], \tag{2.42}$$

$$Q(x,t) = \left[\frac{-1}{2}\left[\left(\omega + \frac{\beta k}{2}\right) + k\sqrt{(\beta^2 + \alpha^2)} \times \tanh\left(\frac{k\sqrt{-(\beta^2 + \alpha^2)}}{2}\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.43}$$

or

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{(\beta^2+\alpha^2)} \times \coth\left(\frac{\sqrt{(\beta^2+\alpha^2)}}{2}\,\xi\right)\right], \tag{2.44}$$

$$Q(x,t) = \left[\frac{1}{2}\left[\left(\frac{\beta\, k}{2} - \omega\right) - k\left[\beta + \sqrt{\beta^2+\alpha^2} \times \coth\left(\frac{k\sqrt{-(\beta^2+\alpha^2)}}{2}\left(x + \frac{2\,\omega\, t^{\alpha}}{\alpha}\right)\right)\right]\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.45}$$

When $\beta^2 + \alpha^2 < 0,\ \sigma \neq 0$ and $\sigma = -\alpha$

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2+\alpha^2)} \times \tan\left(\frac{\sqrt{-(\beta^2+\alpha^2)}}{2}\,\xi\right)\right], \tag{2.46}$$

$$Q(x,t) = \left[\frac{-1}{2}\left[\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2+\alpha^2)} \times \tan\left(\frac{k\sqrt{(\beta^2+\alpha^2)}}{2}\left(x + \frac{2\,\omega\, t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.47}$$

or

$$u(\xi) = \frac{-1}{2}\left[\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2+\alpha^2)} \times \cot\left(\frac{\sqrt{-(\beta^2+\alpha^2)}}{2}\,\xi\right)\right], \tag{2.48}$$

$$Q(x,t) = \left[\frac{-1}{2}\left[\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2+\alpha^2)} \times \cot\left(\frac{k\sqrt{(\beta^2+\alpha^2)}}{2}\left(x + \frac{2\,\omega\, t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.49}$$

When $\beta^2 - \alpha^2 < 0$ and $\sigma = \alpha$

$$u(\xi) = \frac{1}{2}\left[-\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2-\alpha^2)} \times \tan\left(\frac{\sqrt{-(\beta^2-\alpha^2)}}{2}\,\xi\right)\right], \tag{2.50}$$

$$Q(x,t) = \left[\frac{1}{2}\left[-\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2-\alpha^2)} \times \tan\left(\frac{k\sqrt{(\beta^2-\alpha^2)}}{2}\left(x + \frac{2\,\omega\, t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.51}$$

or

$$u(\xi) = \frac{1}{2}\left[-\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2-\alpha^2)} \times \cot\left(\frac{\sqrt{-(\beta^2-\alpha^2)}}{2}\,\xi\right)\right], \tag{2.52}$$

$$Q(x,t) = \left[\frac{1}{2}\left[-\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{-(\beta^2-\alpha^2)} \times \cot\left(\frac{k\sqrt{(\beta^2-\alpha^2)}}{2}\left(x + \frac{2\,\omega\, t^{\alpha}}{\alpha}\right)\right)\right]\right]^{1/2} \times e^{i\left(\omega x + \frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.53}$$

When $\beta^2 - \alpha^2 > 0$ and $\sigma = \alpha$

$$u(\xi) = \frac{1}{2}\left[-\left(\omega + \frac{\beta\, k}{2}\right) + k\sqrt{\beta^2-\alpha^2} \times \tanh\left(\frac{\sqrt{\beta^2-\alpha^2}}{2}\,\xi\right)\right], \tag{2.54}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\left(\omega+\frac{\beta k}{2}\right)+k\sqrt{\beta^2-\alpha^2}\times\tanh\left(\frac{k\sqrt{-(\beta^2-\alpha^2)}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^\alpha}{\alpha}\right)} \tag{2.55}$$

or

$$u(\xi)=\frac{1}{2}\left[-\left(\omega+\frac{\beta k}{2}\right)+k\sqrt{\beta^2-\alpha^2}\times\coth\left(\frac{\sqrt{\beta^2-\alpha^2}}{2}\,\xi\right)\right], \tag{2.56}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\left(\omega+\frac{\beta k}{2}\right)+k\sqrt{\beta^2-\alpha^2}\times\coth\left(\frac{k\sqrt{-(\beta^2-\alpha^2)}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^\alpha}{\alpha}\right)}. \tag{2.57}$$

When $\alpha\,\sigma<0,\ \sigma\neq 0$ and $\beta=0$

$$u(\xi)=\frac{1}{2}\left[-\omega+k\sqrt{-\alpha\,\sigma}\,\tanh\left(\frac{\sqrt{-\alpha\,\sigma}}{2}\,\xi\right)\right], \tag{2.58}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\omega+k\sqrt{-\alpha\,\sigma}\times\tanh\left(\frac{k\sqrt{\alpha\,\sigma}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^\alpha}{\alpha}\right)} \tag{2.59}$$

or

$$u(\xi)=\frac{1}{2}\left[-\omega+k\sqrt{-\alpha\,\sigma}\,\coth\left(\frac{\sqrt{-\alpha\,\sigma}}{2}\,\xi\right)\right], \tag{2.60}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\omega+k\sqrt{-\alpha\,\sigma}\times\coth\left(\frac{k\sqrt{\alpha\,\sigma}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^\alpha}{\alpha}\right)}. \tag{2.61}$$

When $\beta=0$ and $\alpha=-\sigma$

$$u(\xi)=\frac{1}{2}\left[-\omega+k\,\sigma\left[\frac{-\left(1+e^{2\alpha\xi}\right)\pm\sqrt{2\left(e^{4\alpha\xi}+1\right)}}{e^{2\alpha\xi}-1}\right]\right], \tag{2.62}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\omega+k\,\sigma\left[\frac{-\left(1+e^{2i\alpha k\left(x+\frac{2\omega t^\alpha}{\alpha}\right)}\right)\pm\sqrt{2\left(e^{4i\alpha k\left(x+\frac{2\omega t^\alpha}{\alpha}\right)}+1\right)}}{e^{2i\alpha k\left(x+\frac{2\omega t^\alpha}{\alpha}\right)}-1}\right]\right]\right]^{1/2} e^{i\left(\omega x+\frac{\epsilon t^\alpha}{\alpha}\right)} \tag{2.63}$$

or

$$u(\xi) = \frac{1}{2}\left[-\omega + k\,\sigma\left[\frac{-\left(1+e^{2\alpha\xi}\right) \pm \sqrt{e^{4\alpha\xi}+6\,e^{2\alpha\xi}+1}}{2\,e^{2\alpha\xi}}\right]\right], \tag{2.64}$$

$$Q(x,t) = \left[\frac{1}{2}\left[-\omega + k\,\sigma\left[\frac{-\left(1+e^{2\,i\,\alpha\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}\right) \pm \sqrt{e^{4\,i\,\alpha\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+6\,e^{2\,i\,\alpha\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+1}}{2\,e^{2\,i\,\alpha\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]\right]\right]^{1/2} \times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.65}$$

When $\beta^2 = \alpha\,\sigma$

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) - \frac{k\,(\beta\,\xi+2)}{\xi}\right], \tag{2.66}$$

$$Q(x,t) = \left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + \frac{i\left(i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)+2\right)}{\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}\right]\right]^{1/2} \times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.67}$$

When $\beta = k,\ \sigma = 2\,k$ and $\alpha = 0$

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + \frac{k\,\sigma\,e^{k\,\xi}}{1-e^{k\,\xi}}\right], \tag{2.68}$$

$$Q(x,t) = \left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + \frac{k\,\sigma\,e^{i\,k^2\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1-e^{i\,k^2\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]\right]^{1/2} \times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.69}$$

When $2\,\beta = \alpha + \sigma$

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + k\,\sigma\left[\frac{1-\alpha\,e^{\frac{i k}{2}(\alpha-\sigma)\xi}}{1-\sigma\,e^{\frac{1}{2}(\alpha-\sigma)\xi}}\right]\right], \tag{2.70}$$

$$Q(x,t) = \left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + k\,\sigma\left[\frac{1-\alpha\,e^{\frac{i k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1-\sigma\,e^{\frac{i k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]\right]\right]^{1/2} \times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)} \tag{2.71}$$

or

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + k\,\sigma\left[\frac{\alpha\,e^{\frac{1}{2}(\alpha-\sigma)\xi}+1}{-\sigma\,e^{\frac{1}{2}(\alpha-\sigma)\xi}-1}\right]\right], \tag{2.72}$$

$$Q(x,t) = \left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + k\,\sigma\left[\frac{\alpha\,e^{\frac{i k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+1}{-\sigma\,e^{\frac{i k}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}-1}\right]\right]\right]^{1/2} \times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.73}$$

When $-2\,\beta = \alpha + \sigma$

$$u(\xi) = \frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right) + k\,\sigma\left[\frac{e^{\frac{1}{2}(\alpha-\sigma)\,\xi}+\alpha}{e^{\frac{1}{2}(\alpha-\sigma)\,\xi}+\sigma}\right]\right], \tag{2.74}$$

$$Q(x,t)=\left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right)+k\,\sigma\left[\frac{e^{\frac{ik}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+\alpha}{e^{\frac{ik}{2}(\alpha-\sigma)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}+\sigma}\right]\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.75}$$

When $\alpha=0$

$$u(\xi)=\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right)+k\,\sigma\left[\frac{\beta\,e^{\beta\,\xi}}{1+\frac{\sigma}{2}\,e^{\beta\,\xi}}\right]\right], \tag{2.76}$$

$$Q(x,t)=\left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right)+k\,\sigma\left[\frac{\beta\,e^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}{1+\frac{\sigma}{2}\,e^{i\,\beta\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}}\right]\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.77}$$

When $\beta=\alpha=\sigma\neq 0$

$$u(\xi)=\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right)-\frac{k\,(\alpha\,\xi+2)}{\xi}\right], \tag{2.78}$$

$$Q(x,t)=\left[\frac{1}{2}\left[\left(\frac{\beta k}{2}-\omega\right)+\frac{i\,k\,\sigma\left(i\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)+2\right)}{\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.79}$$

When $\beta=\alpha=0$

$$u(\xi)=\frac{-1}{2}\left[\omega+\frac{2k}{\xi}\right], \tag{2.80}$$

$$Q(x,t)=\left[\frac{-1}{2}\left[\omega-\frac{2i}{\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)}\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}. \tag{2.81}$$

When $\beta=0$ and $\alpha=\sigma$

$$u(\xi)=\frac{1}{2}\left[-\omega+k\,\sigma\left[\tan\left(\frac{\alpha\,\xi+C}{2}\right)\right]\right], \tag{2.82}$$

$$Q(x,t)=\left[\frac{1}{2}\left[-\omega+k\,\sigma\left[\tan\left(\frac{i\,\alpha\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)+C}{2}\right)\right]\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.83}$$

where C is an arbitrary constant.

2.2 *Exact and solitary wave solutions of nonlinear complex fractional Schrödinger equation by using novel (G'/G)-expansion method*

Using the default for precision solution using novel (G'/G)-expansion method on nonlinear complex fractional Schrödinger equation, we obtain

$$u(\xi)=\frac{a_{-1}}{\left(d+\frac{G'}{G}\right)}+a_0+a_1\left(d+\frac{G'}{G}\right). \tag{2.84}$$

Substituting eq. (2.84) and its derivatives into eq. (2.4) and equating the coefficient of different power of $\left(d+\frac{G'}{G}\right)^{i}$ to zero, we obtain a system of algebraic equations by solving it with any computer program like Maple, Mathematica, Matlab and so on. We get

Case I

$$\lambda=\frac{\mu\,(2\,d\,a_0+a_{-1})}{d\,(d\,a_0+a_{-1})},\quad \omega=\frac{3\,\mu\,a_{-1}k}{2\,d\,(d\,a_0+a_{-1})},$$

$$v=\frac{d^2a_0+d\,a_{-1}+\mu\,a_0}{d\,(d\,a_0+a_{-1})},$$

$$\epsilon=\frac{5\,k^2\mu^2{a_{-1}}^2}{2\,d^2\,(,a_0+a_{-1})^2},\quad a_{-1}=a_{-1},$$

$$a_0=a_0,\quad a_1=0.$$

From the coefficients in Case I, we obtained the exact travelling wave solutions of nonlinear complex fractional Schrödinger equation in the following form:

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{G'}{G}\right)}. \tag{2.85}$$

Therefore, the solitary travelling wave solutions will be in the following forms: When $\Omega=\lambda^2-4\,\lambda\,\mu+4\,\mu>0$ and $\lambda(v-1)\neq 0$ or $\mu(v-1)\neq 0$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\tanh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)\right)\right)}, \tag{2.86}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\tanh\left(\frac{i\,k\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)}\right]^{1/2}\,\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.87}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\coth\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)\right)\right)}, \tag{2.88}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\coth\left(\frac{i\,k\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)}\right]^{1/2}\,\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.89}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}(\lambda+\sqrt{\Omega}\,(\tanh(\sqrt{\Omega}\,\xi)\pm i\,\mathrm{sech}(\sqrt{\Omega}\,\xi)))\right)}, \tag{2.90}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\left(\tanh\left(i\,k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm i\,\mathrm{sech}\left(i\,k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)}\right]^{1/2} \times\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.91}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\left(\coth\left(\sqrt{\Omega}\,\xi\right)\pm\,\mathrm{csch}\left(\sqrt{\Omega}\,\xi\right)\right)\right)\right)}, \tag{2.92}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{\Omega}\,\left(\coth\left(i\,k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm\,\mathrm{csch}\left(i\,k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)}\right]^{1/2} \times\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.93}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{4(v-1)}\left(2\,\lambda+\sqrt{\Omega}\left(\tanh\left(\frac{\sqrt{\Omega}}{4}\,\xi\right)\pm\,\coth\left(\frac{\sqrt{\Omega}}{4}\,\xi\right)\right)\right)\right)}, \tag{2.94}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{1}{4(v-1)}\left(2\,\lambda+\sqrt{\Omega}\left(\tanh\left(\frac{i\,k\sqrt{\Omega}}{4}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm\,\coth\left(\frac{i\,k\sqrt{\Omega}}{4}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)}\right]^{1/2} \times\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.95}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\,(A^2+B^2)}-A\,\sqrt{\Omega}\,\cosh\left(\sqrt{\Omega}\,\xi\right)}{A\,\sinh\left(\sqrt{\Omega}\,\xi\right)+B}\right)\right)}, \tag{2.96}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\,(A^2+B^2)}-A\,\sqrt{\Omega}\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{A\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)}\right]^{1/2}\,\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.97}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\,(A^2+B^2)}+A\,\sqrt{\Omega}\,\cosh\left(\sqrt{\Omega}\,\xi\right)}{A\,\sinh\left(\sqrt{\Omega}\,\xi\right)+B}\right)\right)}, \tag{2.98}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\,(A^2+B^2)}+A\,\sqrt{\Omega}\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{A\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)}\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.99}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)-\lambda\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}\right)}, \tag{2.100}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\cosh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}\right)}\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.101}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}\right)}, \tag{2.102}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\sinh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\cosh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\sinh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}\right)}\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.103}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\cosh\left(\sqrt{\Omega}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(\sqrt{\Omega}\,\xi\right)-\lambda\,\cosh\left(\sqrt{\Omega}\,\xi\right)\pm i\,\sqrt{\Omega}}\right)}, \tag{2.104}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm i\,\sqrt{\Omega}}\right)}\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.105}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\sinh\left(\sqrt{\Omega}\,\xi\right)}{\sqrt{\Omega}\,\cosh\left(\sqrt{\Omega}\,\xi\right)-\lambda\,\sinh\left(\sqrt{\Omega}\,\xi\right)\pm i\,\sqrt{\Omega}}\right)}, \tag{2.106}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm i\,\sqrt{\Omega}}\right)}\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.107}$$

where A, B are arbitrary real constants and $A^2+B^2>0$.

When $\Omega=\lambda^2-4\,\lambda\,\mu+4\,\mu<0$ and $\lambda(v-1)\neq 0$ or $\mu(v-1)\neq 0$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \sqrt{-\Omega}\,\tanh\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)\right)\right)}, \tag{2.108}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \sqrt{-\Omega}\,\tanh\left(\frac{i\,k\,\sqrt{-\Omega}}{2}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)}\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.109}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d - \frac{1}{2(v-1)}\left(\lambda + \sqrt{-\Omega}\,\coth\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)\right)\right)}, \tag{2.110}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d - \frac{1}{2(v-1)}\left(\lambda + \sqrt{-\Omega}\,\coth\left(\frac{i\,k\,\sqrt{-\Omega}}{2}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)}\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.111}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \sqrt{-\Omega}\,\left(\tan\left(\sqrt{-\Omega}\,\xi\right) \pm \sec\left(\sqrt{-\Omega}\,\xi\right)\right)\right)\right)}, \tag{2.112}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \sqrt{-\Omega}\,\left(\tan\left(i\,k\,\sqrt{-\Omega}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right) \pm \sec\left(\sqrt{-\Omega}\,\xi\right)\right)\right)\right)}\right]^{1/2} \times \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.113}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d - \frac{1}{2(v-1)}\left(\lambda + \sqrt{-\Omega}\,\left(\cot\left(\sqrt{-\Omega}\,\xi\right) \pm \csc\left(\sqrt{-\Omega}\,\xi\right)\right)\right)\right)}, \tag{2.114}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d - \frac{1}{2(v-1)}\left(\lambda + \sqrt{-\Omega}\,\left(\cot\left(i\,k\,\sqrt{-\Omega}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right) \pm \csc\left(i\,k\,\sqrt{-\Omega}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)}\right]^{1/2} \times \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.115}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d + \frac{1}{4(v-1)}\left(-2\,\lambda + \sqrt{-\Omega}\,\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\,\xi\right) - \cot\left(\frac{\sqrt{-\Omega}}{4}\,\xi\right)\right)\right)\right)}, \tag{2.116}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d + \frac{1}{4(v-1)}\left(-2\,\lambda + \sqrt{-\Omega}\,\left(\tan\left(\frac{i\,k\,\sqrt{-\Omega}}{4}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right) - \cot\left(\frac{i\,k\,\sqrt{-\Omega}}{4}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)}\right]^{1/2} \times \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.117}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega\,(A^2-B^2)} - A\,\sqrt{-\Omega}\,\cos\left(\sqrt{-\Omega}\,\xi\right)}{A\,\sin\left(\sqrt{-\Omega}\,\xi\right) + B}\right)\right)}, \tag{2.118}$$

$$Q(x,t) = \left[a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega\,(A^2-B^2)} - A\,\sqrt{-\Omega}\,\cos\left(i\,k\,\sqrt{-\Omega}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{A\,\sin\left(i\,k\,\sqrt{-\Omega}\,\left(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right) + B}\right)\right)}\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.119}$$

$$u(\xi) = a_0 + \frac{a_{-1}}{\left(d + \frac{1}{2(v-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega\,(A^2-B^2)} + A\,\sqrt{-\Omega}\,\cos\left(\sqrt{-\Omega}\,\xi\right)}{A\,\sin\left(\sqrt{-\Omega}\,\xi\right) + B}\right)\right)}, \tag{2.120}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{-\Omega\,(A^2-B^2)}+A\sqrt{-\Omega}\cos\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}{A\sin\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)+B}\right)\right)}\right]^{1/2}e^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)},\tag{2.121}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{2\,\mu\cos\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)}{\sqrt{-\Omega}\sin\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)+\lambda\cos\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)}\right)},\tag{2.122}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{2\,\mu\cos\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}{\sqrt{-\Omega}\sin\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)+\lambda\cos\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}\right)}\right]^{1/2}e^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)},\tag{2.123}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\sin\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)}{\sqrt{-\Omega}\cos\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)-\lambda\sin\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)}\right)},\tag{2.124}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\sin\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}{\sqrt{-\Omega}\cos\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)-\lambda\sin\left(\frac{i\,k\sqrt{-\Omega}}{2}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}\right)}\right]^{1/2}e^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)},\tag{2.125}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{2\,\mu\cos(\sqrt{-\Omega}\,\xi)}{\sqrt{-\Omega}\sin(\sqrt{-\Omega}\,\xi)+\lambda\cos(\sqrt{-\Omega}\,\xi)\pm\sqrt{-\Omega}}\right)},\tag{2.126}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{2\,\mu\cos\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}{\sqrt{-\Omega}\sin\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)+\lambda\cos\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\pm\sqrt{-\Omega}}\right)}\right]^{1/2}e^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)},\tag{2.127}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\sin(\sqrt{-\Omega}\,\xi)}{\sqrt{-\Omega}\cos(\sqrt{-\Omega}\,\xi)-\lambda\sin(\sqrt{-\Omega}\,\xi)\pm\sqrt{-\Omega}}\right)},\tag{2.128}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{2\,\mu\sin\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)}{\sqrt{-\Omega}\cos\left(i\,k\sqrt{-\Omega}\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)-\lambda\sin\left(\sqrt{-\Omega}\,i\,k\left(x+\frac{2\,\omega\,t^\alpha}{\alpha}\right)\right)\pm\sqrt{-\Omega}}\right)}\right]^{1/2}e^{i\left(\omega\,x+\frac{\epsilon\,t^\alpha}{\alpha}\right)},\tag{2.129}$$

where A, B are arbitrary real constants and $A^2-B^2>0$.

When $\mu=0$ and $\lambda(v-1)\neq 0$, we have

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{\lambda\,k}{(v-1)\,(k+\cosh(\lambda\,\xi)-\sinh(\lambda\,\xi))}\right)},\tag{2.130}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{\lambda k}{(v-1)\left(k+\cosh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)-\sinh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)}\right)}\right]^{1/2} e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.131}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{\lambda(\cosh(\lambda\xi)+\sinh(\lambda\xi))}{(v-1)(k+\cosh(\lambda\xi)+\sinh(\lambda\xi))}\right)}, \tag{2.132}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d-\frac{\lambda\left(\cosh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+\sinh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)}{(v-1)\left(k+\cosh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+\sinh\left(i\lambda k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)}\right)}\right]^{1/2} e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.133}$$

$$u(\xi)=a_0+\frac{a_{-1}}{\left(d-\frac{1}{(v-1)\xi+C}\right)}, \tag{2.134}$$

$$Q(x,t)=\left[a_0+\frac{a_{-1}}{\left(d+\frac{i}{k(v-1)\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)+C}\right)}\right]^{1/2} e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.135}$$

where C, k are arbitrary constants.

Case II

$$\lambda=\frac{1}{k}\sqrt{\frac{2\epsilon}{5}},\quad \omega=-\frac{3}{2}\sqrt{\frac{2\epsilon}{5}},\quad v=1,\quad a_{-1}=0,$$
$$a_0=a_0,\quad a_1=\frac{-a_0\sqrt{\frac{2\epsilon}{5}}}{d\sqrt{\frac{2\epsilon}{5}}-k\mu}$$

so that, the exact travelling wave solutions of nonlinear complex fractional Schrödinger equation will be in the following form:

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{G'}{G}\right)\right]. \tag{2.136}$$

Therefore, the solitary travelling wave solutions will be in these forms:
When $\Omega=\lambda^2-4\lambda\mu+4\mu>0$ and $\lambda(v-1)\neq 0$ or $\mu(v-1)\neq 0$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}\xi\right)\right)\right)\right], \tag{2.137}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\tanh\left(\frac{ik\sqrt{\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.138}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2}\xi\right)\right)\right)\right], \tag{2.139}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\coth\left(\frac{ik\sqrt{\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.140}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\left(\tanh\left(\sqrt{\Omega}\xi\right)\pm i\,\mathrm{sech}\left(\sqrt{\Omega}\xi\right)\right)\right)\right)\right], \tag{2.141}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\left(\tanh\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\pm i\,\mathrm{sech}\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.142}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\left(\coth\left(\sqrt{\Omega}\,\xi\right)\pm\mathrm{csch}\left(\sqrt{\Omega}\,\xi\right)\right)\right)\right)\right], \tag{2.143}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{2(v-1)}\times\left(\lambda+\sqrt{\Omega}\left(\coth\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\pm\mathrm{csch}\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.144}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{4(v-1)}\times\left(2\lambda+\sqrt{\Omega}\left(\tanh\left(\frac{\sqrt{\Omega}}{4}\xi\right)\pm\coth\left(\frac{\sqrt{\Omega}}{4}\xi\right)\right)\right)\right)\right], \tag{2.145}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{1}{4(v-1)}\times\left(2\lambda+\sqrt{\Omega}\left(\tanh\left(\frac{i\,k\sqrt{\Omega}}{4}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\pm\coth\left(\frac{i\,k\sqrt{\Omega}}{4}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.146}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\left(A^2+B^2\right)}-A\sqrt{\Omega}\cosh\left(\sqrt{\Omega}\,\xi\right)}{A\sinh\left(\sqrt{\Omega}\,\xi\right)+B}\right)\right)\right], \tag{2.147}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\times\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\left(A^2+B^2\right)}-A\sqrt{\Omega}\cosh\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}{A\sinh\left(i\,k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)\right]\right]^{1/2}\times e^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.148}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\left(A^2+B^2\right)}+A\sqrt{\Omega}\cosh\left(\sqrt{\Omega}\,\xi\right)}{A\sinh\left(\sqrt{\Omega}\,\xi\right)+B}\right)\right)\right], \tag{2.149}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\right.\right.$$
$$\left.\left.\times\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{\Omega\,\left(A^2+B^2\right)}+A\,\sqrt{\Omega}\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{A\,\sinh\left(i\,k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)\right]\right]^{1/2}\mathrm{e}^{i(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha})}, \tag{2.150}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{2\,\mu\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)-\lambda\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}\right)\right], \tag{2.151}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\right.\right.$$
$$\left.\left.\times\left(d+\frac{2\,\mu\,\cosh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\cosh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.152}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{2\,\mu\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}{\sqrt{\Omega}\,\cosh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)-\lambda\,\sinh\left(\frac{\sqrt{\Omega}}{2}\,\xi\right)}\right)\right], \tag{2.153}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\right.\right.$$
$$\left.\left.\times\left(d+\frac{2\,\mu\,\sinh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}{\sqrt{\Omega}\,\cosh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\sinh\left(\frac{i\,k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)}\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.154}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{2\,\mu\,\cosh\left(\sqrt{\Omega}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(\sqrt{\Omega}\,\xi\right)-\lambda\,\cosh\left(\sqrt{\Omega}\,\xi\right)\pm i\,\sqrt{\Omega}}\right)\right], \tag{2.155}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\right.\right.$$
$$\left.\left.\times\left(d+\frac{2\,\mu\,\cosh\left(\sqrt{\Omega}\,\xi\right)}{\sqrt{\Omega}\,\sinh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)-\lambda\,\cosh\left(i\,k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm i\,\sqrt{\Omega}}\right)\right]\right]^{1/2}$$
$$\times\,\mathrm{e}^{i\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.156}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{2\,\mu\,\sinh(\sqrt{\Omega}\,\xi)}{\sqrt{\Omega}\,\cosh(\sqrt{\Omega}\,\xi)-\lambda\,\sinh(\sqrt{\Omega}\,\xi)\pm i\,\sqrt{\Omega}}\right)\right], \tag{2.157}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\right.\right.$$
$$\left.\left.\times\left(d+\frac{2\,\mu\,\sinh\Big(\sqrt{\Omega}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\Big)}{\sqrt{\Omega}\,\cosh\Big(\sqrt{\Omega}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\Big)-\lambda\,\sinh\Big(\sqrt{\Omega}\,i\,k\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\Big)\pm i\,\sqrt{\Omega}}\right)\right]\right]^{1/2}\,\mathrm{e}^{i\,\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)},\tag{2.158}$$

where A, B are arbitrary real constants and $A^2+B^2>0$.

When $\Omega=\lambda^2-4\,\lambda\,\mu+4\,\mu<0$ and $\lambda(v-1)\neq 0$ or $\mu(v-1)\neq 0$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{1}{2(v-1)}\left(-\lambda+\sqrt{-\Omega}\,\tanh\left(\frac{\sqrt{-\Omega}}{2}\,i\,k\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right],\tag{2.159}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{1}{2(v-1)}\left(-\lambda+\sqrt{-\Omega}\,\tanh\left(\frac{k\,\sqrt{\Omega}}{2}\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right]\right]^{1/2}\times\mathrm{e}^{i\,\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)},\tag{2.160}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{-\Omega}\,\coth\left(\frac{\sqrt{-\Omega}}{2}\,\xi\right)\right)\right)\right],\tag{2.161}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{-\Omega}\,\coth\left(\frac{k\,\sqrt{\Omega}}{2}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right]\right]^{1/2}\times\mathrm{e}^{i\,\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)},\tag{2.162}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{1}{2(v-1)}\left(-\lambda+\sqrt{-\Omega}\,\left(\tan\Big(\sqrt{-\Omega}\,\xi\Big)\pm\sec\Big(\sqrt{-\Omega}\,\xi\Big)\right)\right)\right)\right],\tag{2.163}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{1}{2(v-1)}\left(-\lambda+\sqrt{-\Omega}\,\left(\tan\left(k\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm\sec\left(k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\,\mathrm{e}^{i\,\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)},\tag{2.164}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{2\,\epsilon/5)}-k\,\mu}\,\left(d-\frac{1}{2(v-1)}\left(\lambda+\sqrt{-\Omega}\,\left(\cot\Big(\sqrt{-\Omega}\,\xi\Big)\pm\csc\Big(\sqrt{-\Omega}\,\xi\Big)\right)\right)\right)\right],\tag{2.165}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d-\frac{1}{2(v-1)}\,\left(\lambda+\sqrt{-\Omega}\,\left(\cot\left(k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\pm\csc\left(k\,\sqrt{\Omega}\,\left(x+\frac{2\,\omega\,t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\,\mathrm{e}^{i\,\left(\omega\,x+\frac{\epsilon\,t^{\alpha}}{\alpha}\right)},\tag{2.166}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\,\epsilon/5}}{d\,\sqrt{(2\,\epsilon/5)}-k\,\mu}\,\left(d+\frac{1}{4(v-1)}\,\left(-2\,\lambda+\sqrt{-\Omega}\,\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\,\xi\right)\,-\cot\left(\frac{\sqrt{-\Omega}}{4}\,\xi\right)\right)\right)\right)\right],\tag{2.167}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{4(v-1)}\left(-2\lambda+\sqrt{-\Omega}\left(\tan\left(\frac{k\sqrt{\Omega}}{4}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\cot\left(\frac{k\sqrt{\Omega}}{4}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.168}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{-\Omega\left(A^2-B^2\right)}-A\sqrt{-\Omega}\cos\left(\sqrt{-\Omega}\,\xi\right)}{A\sin\left(\sqrt{-\Omega}\,\xi\right)+B}\right)\right)\right], \tag{2.169}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\right.\right.\right.$$
$$\left.\left.\left.\times\left(-\lambda+\frac{\pm\sqrt{-\Omega\left(A^2-B^2\right)}-A\sqrt{-\Omega}\cos\left(k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}{A\sin\left(k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.170}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\left(-\lambda+\frac{\pm\sqrt{-\Omega\left(A^2-B^2\right)}+A\sqrt{-\Omega}\cos\left(\sqrt{-\Omega}\,\xi\right)}{A\sin\left(\sqrt{-\Omega}\,\xi\right)+B}\right)\right)\right], \tag{2.171}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{1}{2(v-1)}\right.\right.\right.$$
$$\left.\left.\left.\times\left(-\lambda+\frac{\pm\sqrt{-\Omega\left(A^2-B^2\right)}+A\sqrt{-\Omega}\cos\left(k\sqrt{\Omega}\,i\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}{A\sin\left(k\sqrt{\Omega}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+B}\right)\right)\right]\right]^{1/2}\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.172}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{2\mu\cos\left(\frac{\sqrt{-\Omega}}{2}\xi\right)}{\sqrt{-\Omega}\sin\left(\frac{\sqrt{-\Omega}}{2}\xi\right)+\lambda\cos\left(\frac{\sqrt{-\Omega}}{2}\xi\right)}\right)\right], \tag{2.173}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d-\frac{2\mu\cos\left(\frac{\sqrt{-\Omega}}{2}\,i\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}{\sqrt{-\Omega}\sin\left(\frac{k\sqrt{-\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)+\lambda\cos\left(\frac{k\sqrt{\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}\right)\right]\right]^{1/2}$$
$$\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.174}$$

$$u(\xi)=a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{2\mu\sin\left(\frac{\sqrt{-\Omega}}{2}\xi\right)}{\sqrt{-\Omega}\cos\left(\frac{\sqrt{-\Omega}}{2}\xi\right)-\lambda\sin\left(\frac{\sqrt{-\Omega}}{2}\xi\right)}\right)\right], \tag{2.175}$$

$$Q(x,t)=\left[a_0\left[1-\frac{\sqrt{2\epsilon/5}}{d\sqrt{(2\epsilon/5)}-k\mu}\left(d+\frac{2\mu\sin\left(\frac{\sqrt{-\Omega}}{2}\,i\,k\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}{\sqrt{-\Omega}\cos\left(\frac{k\sqrt{\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)-\lambda\sin\left(\frac{k\sqrt{\Omega}}{2}\left(x+\frac{2\omega t^{\alpha}}{\alpha}\right)\right)}\right)\right]\right]^{1/2}$$
$$\times\mathrm{e}^{i\left(\omega x+\frac{\epsilon t^{\alpha}}{\alpha}\right)}, \tag{2.176}$$

$$u(\xi) = a_0\left[1 - \frac{\sqrt{\frac{2\epsilon}{5}}}{d\sqrt{\frac{2\epsilon}{5}} - k\,\mu}\left(d - \frac{2\,\mu\,\cos\big(\sqrt{-\Omega}\,\xi\big)}{\sqrt{-\Omega}\,\sin\big(\sqrt{-\Omega}\,\xi\big) + \lambda\,\cos\big(\sqrt{-\Omega}\,\xi\big) \pm \sqrt{-\Omega}}\right)\right], \tag{2.177}$$

$$Q(x,t) = \left[a_0\left[1 - \frac{\sqrt{\frac{2\epsilon}{5}}}{d\sqrt{(2\epsilon/5)} - k\,\mu} \times \left(d - \frac{2\,\mu\,\cos\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big)}{\sqrt{-\Omega}\,\sin\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big) + \lambda\,\cos\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big) \pm \sqrt{-\Omega}}\right)\right]\right]^{1/2} \times \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.178}$$

$$u(\xi) = a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu}\left(d + \frac{2\,\mu\,\sin\big(\sqrt{-\Omega}\,\xi\big)}{\sqrt{-\Omega}\,\cos\big(\sqrt{-\Omega}\,\xi\big) - \lambda\,\sin\big(\sqrt{-\Omega}\,\xi\big) \pm \sqrt{-\Omega}}\right)\right], \tag{2.179}$$

$$Q(x,t) = \left[a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu} \times \left(d + \frac{2\,\mu\,\sin\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big)}{\sqrt{-\Omega}\,\cos\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big) - \lambda\,\sin\Big(k\sqrt{\Omega}\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\Big) \pm \sqrt{-\Omega}}\right)\right]\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.180}$$

where A, B are arbitrary real constants and $A^2 - B^2 > 0$.

When $\mu = 0$ and $\lambda(v-1) \neq 0$, we have

$$u(\xi) = a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu}\left(d - \frac{\lambda\,k}{(v-1)\,(k + \cosh(\lambda\,\xi) - \sinh(\lambda\,\xi))}\right)\right], \tag{2.181}$$

$$Q(x,t) = \left[a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu} \times \left(d - \frac{\lambda\,k}{(v-1)\,\Big(k + \cosh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big) - \sinh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big)\Big)}\right)\right]\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.182}$$

$$u(\xi) = a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu}\left(d - \frac{\lambda\,(\cosh(\lambda\,\xi) + \sinh(\lambda\,\xi))}{(v-1)\,(k + \cosh(\lambda\,\xi) + \sinh(\lambda\,\xi))}\right)\right], \tag{2.183}$$

$$Q(x,t) = \left[a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu} \times \left(d - \frac{\lambda\,\Big(\cosh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big) + \sinh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big)\Big)}{(v-1)\,\Big(k + \cosh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big) + \sinh\big(i\,\lambda\,k\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big)\big)\Big)}\right)\right]\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.184}$$

$$u(\xi) = a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu}\left(d - \frac{1}{(v-1)\,\xi + C}\right)\right], \tag{2.185}$$

$$Q(x,t) = \left[a_0\left[1 - \frac{\sqrt{2\,\epsilon/5}}{d\sqrt{(2\,\epsilon/5)} - k\,\mu}\left(d + \frac{i}{k\,(v-1)\,\big(x + \frac{2\,\omega\,t^{\alpha}}{\alpha}\big) + C}\right)\right]\right]^{1/2} \mathrm{e}^{i\left(\omega\,x + \frac{\epsilon\,t^{\alpha}}{\alpha}\right)}, \tag{2.186}$$

where C, k are arbitrary constants.

Note that all the obtained results have been checked with Maple 2017 by putting them back into the original equation and the results are found to be correct.

3. Conclusion

In this research, we succeeded in applying new auxiliary equation method and novel $\left(G'/G\right)$-expansion method on nonlinear complex fractional Schrödinger equation. We obtained new form of solitary travelling wave solutions on this model that can be observed when a comparison is made between our solutions and that obtained in [37–39], that were reported with specific coefficients of the equation as a special case to the results of this paper. This improves the superiority of our methods. Therefore, this paper gives a generalised flavour to nonlinear complex fractional Schrödinger equation in the study of optical solitons. Thus, the paper encourages us to carry out further research, especially with the inclusion of perturbation terms of KE equations. Those results will be soon published.

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