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Kink degeneracy and rogue potential solution for the (3+1)-dimensional B-type Kadomtsev–Petviashvili equation

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Abstract. In this paper, we obtained the exact breather-type kink soliton and breather-type periodic soliton solutions for the (3+1)-dimensional B-type Kadomtsev–Petviashvili (BKP) equation using the extended homoclinic test technique. Some new nonlinear phenomena, such as kink and periodic degeneracies, are investigated. Using the homoclinic breather limit method, some new rational breather solutions are found as well. Meanwhile, we also obtained the rational potential solution which is found to be just a rogue wave. These results enrich the variety of the dynamics of higher-dimensional nonlinear wave field.

1. Introduction

In recent years, solitary wave solutions of nonlinear evolution equations have begun playing important roles in nonlinear science fields, especially in nonlinear physical science. The solitary wave solution can provide physical information and more insight into the physical aspects of the problem thus leading to further applications [1]. It is well known that there are many methods for finding special solutions of nonlinear partial differential equations, such as the inverse scattering method [1], the homogeneous balance method [2], the Darboux transformation method [3,4], the Hirota's bilinear method [5,6], the improved tanh-method [7], the Lie group method [8], the extended homoclinic test approach [9–11], and so on.

In this work, we consider the (3+1)-dimensional B-type Kadomtsev–Petviashvili (BKP) equation

$$u_{zt} - 3(u_x u_y)_x - u_{xxxy} + 3u_{xx} = 0, \tag{1}$$

where $u: R_x \times R_y \times R_z \times R_t \rightarrow R$. The BKP equation was given this name because it is a B-type KP equation [12–14]. The well-known BKP equation possesses many integrable structures such as Lax formulation and the multiple soliton solutions. Exact solutions of the BKP equation have been studied by means of some effective approaches, such as the complex travelling wave solution [15], periodic solutions, multiple soliton solutions [16], Wronskian solution [17] and the Pfaffian solution [18]. However, to our best knowledge, the berather-type kink and rational breather solutions to the (3+1)-dimensional BKP equation (1) have not yet been studied. Therefore, in this paper, an approach of seeking rational breather-wave solution, called the homoclinic breather limit method [19,20], is proposed and applied. Exact breather kink wave and periodic breather solitary solutions are obtained, kink and periodic degeneracy are investigated, new rational breather solutions and rogue potential solution are constructed by homoclinic breather limit process or by Taylor expansion [21,22].

2. Homoclinic breather limit method

Consider a high-dimensional nonlinear evolution equation of the general form

$$P(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, ...) = 0, \qquad (2)$$

where $u = u(x, y, z, t)$ and P is a polynomial of u and its derivatives. The basic idea of the homoclinic breather limit method can be expressed in the following five steps:

Step 1

By Painlevé analysis [10], a transformation

$$u = T(f), \qquad (3)$$

is made for some new unknown function f.

Step 2

By using the transformation in Step 1, the original equation can be converted into Hirota's bilinear form

$$G(D_t, D_x, D_y, D_z; f, f) = 0, \qquad (4)$$

where the D-operator [23] is defined by

$$Q(D_x, D_y, D_z, D_t, ...)F(x, y, z, t, ...) \cdot G(x, y, z, t, ...)$$
$$= Q(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_z - \partial_{z'}, \partial_t - \partial_{t'}, ...)$$
$$\times F(x, y, z, t, ...)G(x, y, z, t, ...)|_{x'=x, y'=y, z'=z, t'=t, ...}, (5)$$

where Q is a polynomial of $D_x, D_y, D_t, ...$

Step 3

As we know, the breather of integrable PDE is usually in the form of a rational function as the numerator and denominator are the combination of functions of cos, sin, cosh, sinh, and so f can be conjectured as a combination of cos and cosh (or sin and sinh). Then, substitute this trial form to the bilinear equation, eq. (5), to get a set of algebraic equation for some parameters, solve the above set of equation to obtain homoclinic breather wave solution, which was called the extended homoclinic test approach (EHTA) in [24].

Step 4

Let the period of periodic wave go to infinite in homoclinic breather wave solution. We can then obtain a rational breather wave solution.

Step 5

Solving the potential of breather wave solution in Step 3 and letting p tends to zero, we can obtain a rational homoclinic (heteroclinic) wave and this wave is just a rogue wave [25–47].

3. Applications

3.1 *Kink degeneracy and new rational breather solution*

By using Painlevé test, we can assume that

$$u(x, y, z, t) = 2(\ln f)_x, \qquad (6)$$

where $f(x, y, z, t)$ is an unknown real function. Substituting eq. (6) into eq. (1), we obtain the following bilinear form:

$$(D_z D_t - D_x^3 D_y + 3D_x^2)f \cdot f = 0, \qquad (7)$$

where

$$D_z D_t f \cdot f = 2(f f_{zt} - f_z f_t),$$

$$D_x^3 D_y f \cdot f = 2(f_{xxxy} f - f_{xxx} f_y + 3f_{xx} f_{xy} - 3f_x f_{xxy}).$$

With regard to eq. (7), we can seek the solution in the form

$$f = e^{-p_1 \xi} + \delta_1 \cos(p\eta) + \delta_2 e^{p_1 \xi}, \qquad (8)$$

where $\xi = x + a_1 y + b_1 z + c_1 t$, $\eta = x + a_2 y + b_2 z + c_2 t$, $a_1, b_1, c_1, a_2, b_2, c_2, p_1, p, \delta_1, \delta_2$ are real constants to be determined. Substituting eq. (8) into eq. (7) and equating all the coefficients of different powers of $e^{\xi}, e^{-\xi}, \sin(\eta), \cos(\eta)$ and the constant term to zero, we can obtain a set of algebraic equations for $p, p_1, a_i, b_i, c_i, \delta_i (i = 1, 2)$ as follows:

$$\begin{cases} -3a_1 p_1^2 - a_2 p_1^2 + 3a_2 p^2 + c_2 b_1 + a_1 p^2 \\ +c_1 b_2 + 6 = 0 \\ -3a_2 p^2 p_1^2 - 3a_1 p^2 p_1^2 + a_1 p_1^4 - 3p_1^2 + 3p^2 \\ +b_2 c_2 p^2 - c_1 b_1 p^2 + a_2 p^4 = 0 \\ 12\delta_2 p_1^2 - 4a_2 \delta_1^2 p^4 - b_2 c_2 \delta_1^2 p^2 - 3\delta_1^2 p^2 \\ -16a_1 \delta_2 p_1^4 + 4b_1 c_1 \delta_2 p_1^2 = 0. \end{cases} \qquad (9)$$

Solving eq. (9) with the aid of *Maple*, we get the following results:

$$\begin{cases} a_1 = -\dfrac{p^2 \left(a_2\delta_1^2 p^2 \left(p^2 + 2p_1^2\right) \atop +p_1^2 \left(a_2\delta_1^2 p_1^2 + 4\delta_1^2 - 16\delta_2\right)\right)}{4\delta_2 p_1^2 \left(p^2 + p_1^2\right)^2}, \\[2em] c_2 = -\dfrac{p^2 \left(a_2 p^2 + 3 - 2a_2 p_1^2\right) - 3p_1^2 \left(3 + a_2 p_1^2\right)}{b_2 \left(p^2 + p_1^2\right)}, \\[2em] b_1 = -\dfrac{b_2 \left(a_2\delta_1 p^4 \left(p^2 + 2p_1^2\right) + 4p^2 p_1^2 \left(\delta_1^2 - \delta_2^2\right) \atop +p_1^4 \left(a_2\delta_1^2 p^2 + 12\delta_2\right)\right)}{4\delta_2 p_1^2 \left(p^2 + p_1^2\right) \left(a_2 p^2 + a_2 p_1^2 + 3\right)}, \\[2em] c_1 = -\dfrac{\left(3p^2 - p_1^2\right) \left(a_2 p^2 + a_2 p_1^2 + 3\right)}{b_2 \left(p^2 + p_1^2\right)}, \end{cases}$$

$$(10)$$

where $a_2, b_2, \delta_1, \delta_2, p, p_1$ are some free real constants. There are different choices for δ_1, δ_2 and p in (10). Here, we specially take $\delta_i, i = 1, 2$ and p such as $\delta_1 = -2\sqrt{p^2 + 1}, \delta_2 = 2p^2 + 1, p_1 = p$ in eq. (10), so that it is more easy to get the form of $0/0$ as $p \to 0$, in order to obtain rational breather solution. In this case, eq. (10) can be rewritten as

$$a_1 = -\frac{a_2 p^2 + a_2 - 1}{2p^2 + 1},$$

$$b_1 = -\frac{a_2 p^2 (b_2 p^2 + 3p^2 + b_2 + 3) + 8p^2 + 6}{2(2a_2 p^2 + 3)(2p^2 + 1)},$$

$$c_1 = -\frac{2a_2 p^2 + 3}{b_2},$$

$$c_2 = \frac{2a_2 p^2 + 3}{b_2}. \tag{11}$$

Substituting eq. (10) into eq. (8), we have

$$f(x, y, z, t) = 2\sqrt{2p^2 + 1}\cosh(p(x + H_1 y + K_1 z + L_1 t)$$
$$+ \frac{1}{2}\ln(2p^2 + 1)) - 2\sqrt{p^2 + 1}$$
$$\times \cos(p(x + a_2 y + b_2 z - L_1 t)), \tag{12}$$

where

$$H_1 = -\frac{a_2 p^2 + a_2 - 1}{2p^2 + 1},$$

$$K_1 = -\frac{b_2(2a_2 p^4 + 2a_2 p^2 + 4p^2 + 3)}{(2a_2 p^2 + 3)(2p^2 + 1)},$$

$$L_1 = -\frac{2a_2 p^2 + 3}{b_2}.$$

Substituting eq. (11) into eq. (6) yields the solution of the (3+1)-D BKP equation as follows:

$$u(x, y, z, t) = \frac{\begin{array}{c}(2p\sqrt{2p^2 + 1}\sinh(p(x + H_1 y + K_1 z + L_1 t) + \frac{1}{2}\ln(2p^2 + 1)) \\ + 2\sqrt{p^2 + 1}\sin(p(x + a_2 y + b_2 z - L_1 t)))\end{array}}{\begin{array}{c}\sqrt{2p^2 + 1}\cosh(p(x + H_1 y + K_1 z + L_1 t) + \frac{1}{2}\ln(2p^2 + 1)) \\ - 2\sqrt{p^2 + 1}\cos(p(x + a_2 y + b_2 z - L_1 t))\end{array}}. \tag{13}$$

The solution $u(x, y, z, t)$ represented by eq. (13) is a breather-type kink soliton. It is generated by the interaction between the soliton with variable $X = p(x + H_1 y + K_1 z + L_1 t) + \frac{1}{2}\ln(2p^2 + 1)$ and the periodic wave with variable $Y = p(x + a_2 y + b_2 z - L_1 t)$.

If $p \to 0$ in eq. (13), we can get the rational breather solution as follows:

$$u(x, y, z, t) = \frac{4b_2^2(2x + y - z + b_2 z)}{(b_2(x + a_2 y + b_2 z) + 3t)^2 + (b_2(x + y - z) - (a_2 b_2 y + 3t))^2 + 2b_2^2}. \tag{14}$$

The solution $u(x, y, z, t)$ represented by eq. (14) is a new rational breather solution. Note that u tends to zero in eq. (15), when $t \to \pm\infty$, and so it is no longer kinky. Such a surprising feature of weakly dispersive long wave is first obtained. Meanwhile, this shows that kink is degenerated when the period of breather wave tends to infinity in the breather kink wave. Figures 1, 2, 3 and 4 exhibit the evolution breather kink wave and rational breather wave in the (x, t) and (x, y) planes, respectively. This is a new nonlinear phenomenon till now.
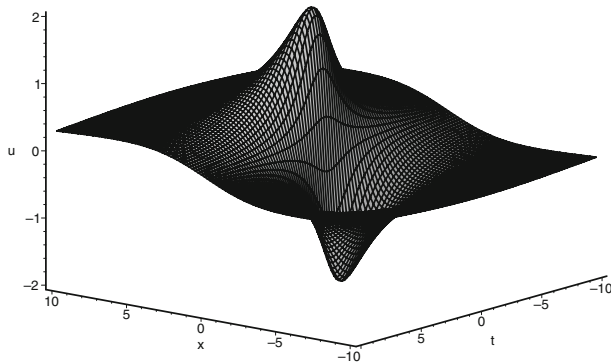
3.2 *Kinky periodic degradation and new rational breather solution*

In this section, we apply the homoclinic breather limit method to the (3+1)-dimensional BKP equation.

By choosing the special test function, we obtained a kinky periodic-wave solution and a new rational breather solution.
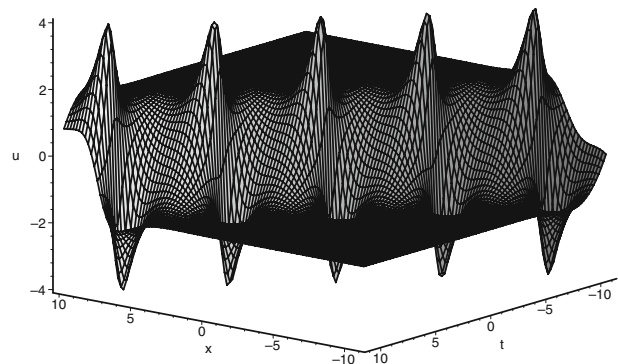
Figure 1. The breather-type kink soliton solution when $a_2 = 1, b_2 = 5, p = 1, y = z = 0$.

Figure 2. The rational breather solution when $a_2 = 1$, $b_2 = 5$, $y = z = 0$.

Suppose that the solution of eq. (7) is

$$f(x, y, z, t) = e^{-p(x+b_1z+d_1)}$$
$$+ \delta_1 \cos(p_1(y + b_2z + ct + d_2))$$
$$+ \delta_2 e^{p(x+b_1z+d_1)}, \quad (15)$$

where $b_1, b_2, c, d_1, d_2, \delta_1, \delta_2, p, p_1$ are free real constants. Substituting eq. (15) into eqs (7), and equating all the coefficients of different powers of $e^{p(x+b_1z+d_1)}$,

$e^{-p(x+b_1z+d_1)}$, $\sin(p_1(y+b_2z+c_2t+d_2))$, $\cos(p_1(y+b_2z+c_2t+d_2))$ and constant term to zero, we can obtain a set of algebraic equations for $c, b_i, \delta_i (i = 1, 2)$. Solving the system with the aid of *Maple*, we get the following results:

$$b_1 = \frac{p^2}{c}, \qquad p_1 = \sqrt{\frac{3}{b_2c}} p, \quad \delta_2 = \frac{1}{4}\delta_1^2. \quad (16)$$

Substituting eq. (16) into eq. (15) and taking $b_2c > 0$, we have

$$f(x, y, z, t) = |\delta_1| \cosh\left(p\left(x + \frac{p^2}{c}z + d_1\right)\right.$$
$$\left. + \ln\left(\frac{1}{2}|\delta_1|\right)\right)$$
$$+ \delta_1 \cos\left(\sqrt{\frac{3}{b_2c}} p(y + b_2z + ct + d_2)\right). \quad (17)$$

Substituting eq. (17) into eq. (6) yields the kinky periodic soliton solution of the (3+1)-D BKP equation as follows:

$$u(x, y, z, t) = \frac{2p|\delta_1| \sinh\left(p\left(x + \frac{p^2}{c}z + d_1\right) + \ln\left(\frac{1}{2}|\delta_1|\right)\right)}{|\delta_1| \cosh\left(p\left(x + \frac{p^2}{c}z + d_1\right) + \ln\left(\frac{1}{2}|\delta_1|\right)\right) + \delta_1 \cos\left(\sqrt{\frac{3}{b_2c}} p(y + b_2z + ct + d_2)\right)}. \quad (18)$$

The solution $u(x, y, z, t)$ represented by eq. (18) can be considered as a kink soliton of the variable

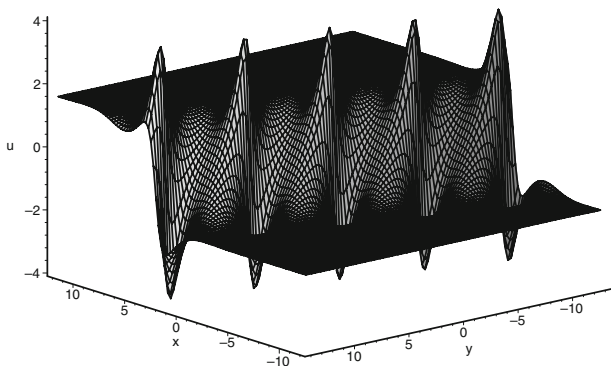$$X = p\left(x + \frac{p^2}{c}z + d_1\right) + \ln\left(\frac{1}{2}|\delta_1|\right)$$

spread along the direction of variable

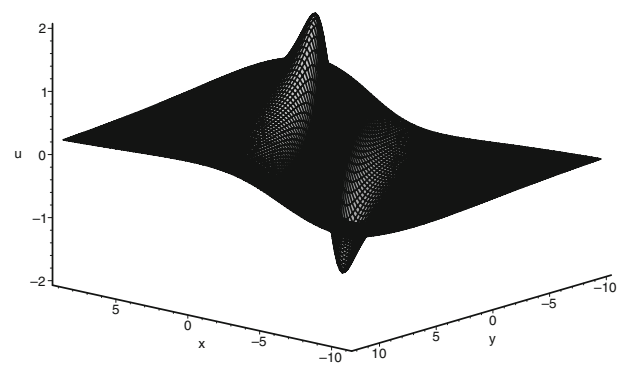$$Y = \sqrt{\frac{3}{b_2c}} p(y + b_2z + ct + d_2)$$

(see figure 5).

Especially, for the same reason as dealing with eq. (10), we choose $\delta_1 = -2$ in eq. (18), while $p \to 0$, we can get the rational breather solution as follows:
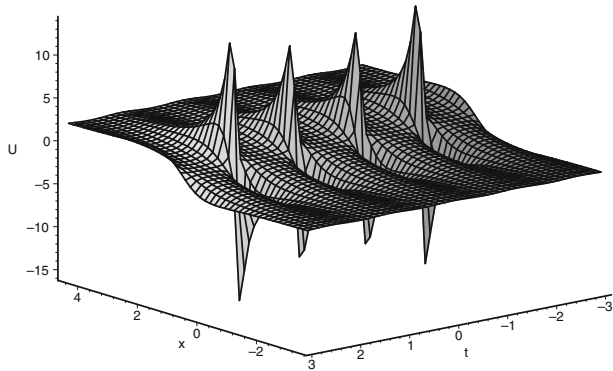
$$u(x, y, z, t) = \frac{4b_2c(x + d_1)}{b_2c(x + d_1)^2 + 3(y + b_2z + ct + d_2)^2}. \quad (19)$$

Figure 3. The breather-type kink soliton solution when $a_2 = 1, b_2 = 5, p = 1, t = z = 0$.

Figure 4. The rational breather solution when $a_2 = 1$, $b_2 = 5, t = z = 0$.

Figure 5. The kinky periodic soliton solution when $b_2 = 1/4, c = 2, \delta_1 = 1, p = 1, d_1 = d_2 = y = z = 0$.

The solution $u(x, y, z, t)$ represented by eq. (19) is a breather wave which no longer has periodic kink feature. Here, periodic kink degeneracy occurs when the period of the periodic wave tends to infinity. It was observed that the periodic kink feature of the solution disappeared when p tends to zero. More importantly, we obtained a new rational breather wave solution (see figure 6).

3.3 *Periodic degeneracy and new rational breather solution*

In this section, we obtained a breather-type periodic soliton solution and a rational breather solution by choosing another special test function. Suppose that the solution of eq. (7) is

$$f(x, y, z, t) = e^{-p(y+b_1z+ct+d_1)}$$
$$+\delta_1 \cos(p_1(x + b_2z + d_2))$$
$$+\delta_2 e^{p(y+b_1z+ct+d_1)}, \qquad (20)$$

where $b_1, b_2, c, \delta_1, \delta_2, p, p_1$ are free real constants.

Figure 6. The rational breather solution when $b_2 = 1/4, c = 2, d_1 = d_2 = y = z = 0$.

Substituting eq. (20) into eqs (7), and equating all the coefficients of different powers of $e^{p(y+b_1z+ct+d_1)}$, $e^{-p(y+b_1z+ct+d_1)}$, $\sin(p_1(x + b_2z + d_2))$, $\cos(p_1(x + b_2z+d_2))$ and constant term to zero, we can obtain a set of algebraic equations for $c, b_i, \delta_i (i = 1, 2)$. Solving the system with the aid of *Maple*, we get the following results:

$$b_2 = -\frac{1}{3}b_1p^2, \quad p_1 = \sqrt{\frac{b_1c}{3}}p, \quad \delta_2 = \frac{1}{4}\delta_1^2. \qquad (21)$$

Substituting eq. (21) into eq. (20) and taking $b_1c > 0$, we have

$$f(x, y, z, t) = |\delta_1| \cosh\left(p(y + b_1z + ct + d_1)\right.$$
$$+ \ln\left(\frac{1}{2}|\delta_1|\right)\right)$$
$$+ \delta_1 \cos\left(\sqrt{\frac{b_1c}{3}}p\left(x - \frac{1}{3}b_1p^2z + d_2\right)\right), \qquad (22)$$

Substituting eq. (22) into eq. (6), we obtain a breather-type periodic soliton solution of BKP equation as follows:

$$u(x, y, z, t) = -\frac{2}{\sqrt{3}} \frac{\sqrt{b_1c}\,p\delta_1 \sin\left(\sqrt{\frac{b_1c}{3}}p\left(x - \frac{1}{3}b_1p^2z + d_2\right)\right)}{|\delta_1| \cosh\left(p(y+b_1z+ct+d_1) + \ln(\frac{1}{2}|\delta_1|)\right) + \delta_1 \cos\left(\sqrt{\frac{b_1c}{3}}p\left(x - \frac{1}{3}b_1p^2z + d_2\right)\right)}. \qquad (23)$$
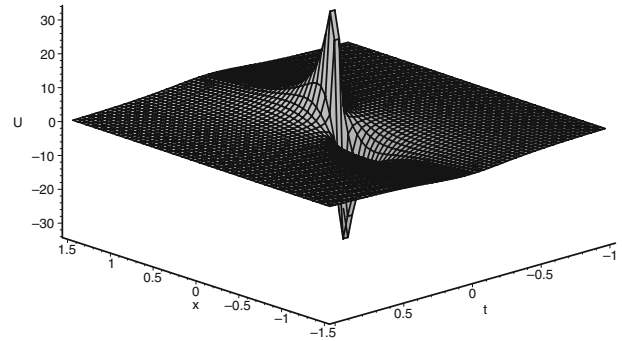
The solution $u(x, y, z, t)$ represented by eq. (23) can be considered as a soliton of variable

$$X = p(y + b_1z + ct + d_1) + \ln\left(\frac{1}{2}|\delta_1|\right)$$

spread along the direction of variable

$$Y = \sqrt{\frac{b_1c}{3}}p\left(x - \frac{1}{3}b_1p^2z + d_2\right)$$

(see figure 7).

Similar to the way we deal with eq. (10), here we choose $\delta_1 = -2$ in eq. (23), when $p \to 0$, and we can get the rational breather solution as follows (figure 8):

$$u(x, y, z, t) = \frac{4b_1c(x + d_1)}{3(y + b_1z + ct + d_2)^2 + b_1c(x + d_1)^2}. \qquad (24)$$

Solution $u(x, y, z, t)$ represented by eq. (24) is a breather wave which no longer has periodic feature. Here, periodic degeneracy occurs when the period of the periodic wave tends infinity. This is a strange and interesting physical phenomenon which causes the evolution of shallow water waves having small amplitudes. It is observed that the periodic feature of the solution disappeared when p tends to zero. More importantly, we obtained a new rational breather wave solution(see figure 10).

3.4 *Rogue potential solution*

In this section, we solve the potential of eq. (13) and let p tend to zero. We then obtain a rational homoclinic (heteroclinic) wave and this wave is just a rogue wave.

Solving the potential of eq. (13), we have

$$\phi = -(u(x, y, z, t))_x$$
$$= \frac{2p^2(-p^2 + 2\sqrt{2p^2 + 1}\sqrt{p^2 + 1}\sinh(p(x + H_1 y + K_1 z + L_1 t) + \frac{1}{2}\ln(2p^2 + 1))\sin(p(x + a_2 y + b_2 z - L_1 t)))}{(2\sqrt{2p^2 + 1}\cosh(p(x + H_1 y + K_1 z + L_1 t) + \frac{1}{2}\ln(2p^2 + 1)) - \sqrt{p^2 + 1}\cos(p(x + a_2 y + b_2 z - L_1 t)))^2} \quad (25)$$

where

$$H_1 = -\frac{a_2 p^2 + a_2 - 1}{2p^2 + 1},$$

$$K_1 = -\frac{b_2(2a_2 p^4 + 2a_2 p^2 + 4p^2 + 3)}{(2a_2 p^2 + 3)(2p^2 + 1)},$$

$$L_1 = -\frac{2a_2 p^2 + 3}{b_2}$$

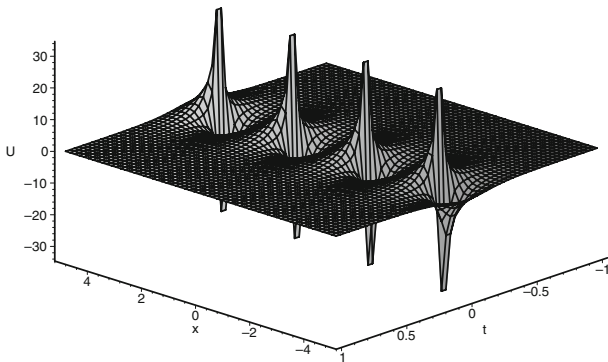and ϕ is a breather-type periodic soliton (see figure 9).

Let $p \to 0$ and $a_2 = 1$ in eq. (25). By computing, we obtain the rational breather wave, and it is just a rogue wave as follows (see figure 10):

$$U_{\text{rogue wave}} = \frac{-8b_2^2(6t(b_2 z + 3t - b_2 x) + 2b_2(x + y + b_2 z)(3t - b_2 x + b_2 z) + b_2^2)}{((b_2(x + y + b_2 z) + 3t)^2 + (b_2(z - x) + 3t)^2 + b_2^2)^2}. \quad (26)$$

U contains two waves with different velocities and directions. It is easy to verify that $U_{\text{rogue wave}}$ is a rational breather-type wave. In fact, $U_{\text{rogue wave}}$ contains two waves with different velocities and directions. From figure 10, we can see that $U_{\text{rogue wave}}$ has one upper dominant peak and two small holes. The spatial structure of the function $U_{\text{rogue wave}}$ is similar to the structure of the rogue waves which has been a point of hot discussion in recent years. In fact, $U \to 0$ for fixed x as y, z and $t \to \pm\infty$. So, $U_{\text{rogue wave}}$ is not only a rational breather wave but also a rogue wave solution, the amplitude of which is three times higher than its surrounding waves and $U_{\text{rogue wave}}$ generally forms in a short time.
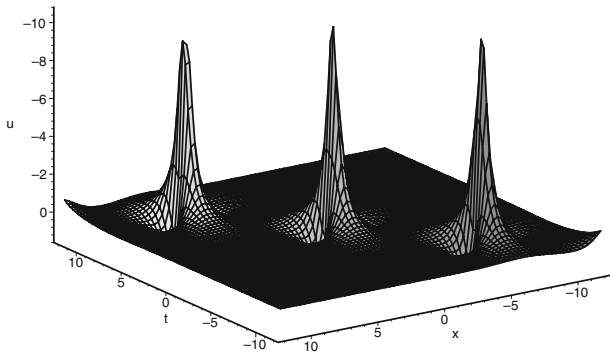
Remark. By using the same methodology as for eq. (13), we can solve the potential of solutions of eqs (18) and
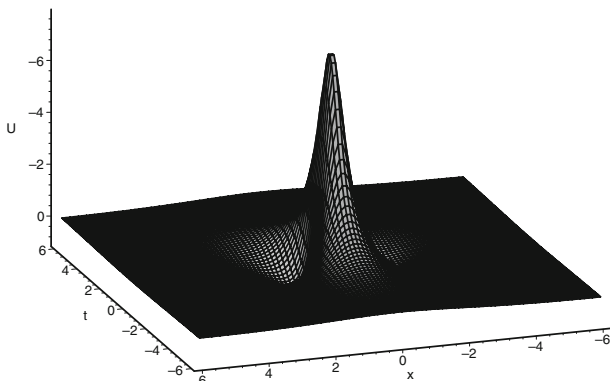

Figure 7. The breather-type periodic soliton solution when $b_1 = 1$, $p = 2$, $c = 4$, $\delta_2 = 2$, $d_1 = d_2 = y = z = 0$.
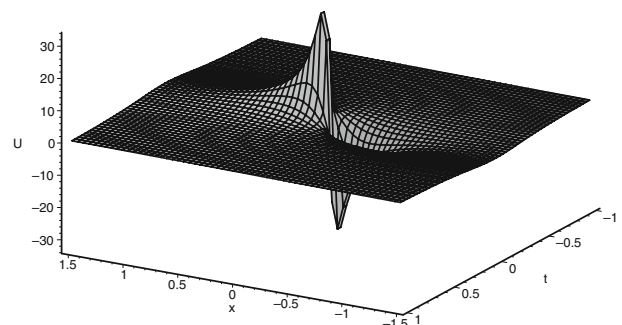
Figure 8. The rational breather solution when $b_1 = 1$, $c = 4$, $d_1 = d_2 = y = z = 0$.

Figure 9. The breather-type periodic soliton ϕ when $a_2 = 1$, $b_2 = 4$, $p = \frac{1}{2}$, $y = z = 0$.

Figure 10. $U_{\text{rogue wave}}$ when $b_2 = 4$, $y = z = 0$.

(23) in §3.2 and 3.3 respectively when $p \to 0$, to get rogue potential solutions.

4. Conclusion

In summary, by successfully applying the extended homoclinic test technique to the (3+1)-dimensional B-type Kadomtsev–Petviashvili equation, we obtained exact kink breather, kinky periodic and periodically breather solitary solutions. By using the homoclinic breather limit method proposed in this work, we obtained some new rational breather solutions. Furthermore, we investigated two new physical phenomena, kink and periodic degeneracy. Our results show different dynamics of high-dimensional systems. Meanwhile, we also obtained the rational potential solution which is just a rogue wave. This method is simple and straightforward. In the future, we shall investigate other types of nonlinear evolution equations and non-integrable systems.

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