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On- and off-shell Jost functions and their integral representations

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Abstract. By judicious exploitation of the transpose operator relation in conjunction with the differential equations of special functions of mathematical physics, integral representations of the on- and off-shell Jost functions are derived from the particular integrals of the inhomogeneous Schrödinger equation. Using the particular integral of the inhomogeneous Schrödinger equation, exact analytical expressions for the Coulomb and Coulomb plus Yamaguchi off-shell Jost solutions are constructed in the maximal reduced form. As a case study, the limiting behaviours and the on-shell discontinuities of the Coulomb plus Yamaguchi Jost solutions are verified numerically.

Keywords. Jost functions; integral representations; Coulomb–Yamaguchi off-shell Jost solutions and functions.

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1. Introduction

The Jost function $f_\ell(k)$ [1] plays an important role in examining the analytic properties of partial wave scattering amplitude which is determined by the behaviour of irregular solution $f_\ell(k,r)$ of the radial Schrödinger equation near the origin. The Jost function has two integral representations [2]; one in terms of the irregular solution $f_\ell(k,r)$ and the other in terms of the regular solution $\varphi_\ell(k,r)$. The integral representation related to the irregular solution $f_\ell(k,r)$ follows directly from the integral equation for on-shell Jost solution $f_\ell(k,r)$, while the other integral representation is derived with particular emphasis on the asymptotic behaviour of the regular solution $\varphi_\ell(k,r)$. The off-shell Jost function [3] is also determined from the off-shell Jost solution $f_\ell(k,q,r)$ in the same way as $f_\ell(k)$ is obtained from $f_\ell(k,r)$. Based on the differential equation approach [4] to the T-matrix, Fuda and Whiting [3] have introduced the concept of off-shell Jost function $f_\ell(k,q)$. It is a function of wave number k and off-shell momentum q. The off-shell Jost function $f_\ell(k,q)$ also has two integral representations; one in terms of off-shell Jost solution $f_\ell(k,q,r)$ and the other involves a free-particle off-shell solution and the on-shell regular solution $\varphi_\ell(k,r)$. The latter was obtained by Fuda [3,5] by using momentum space formalism of the off-shell Jost function and Kowalski's generalization of Sasakawa method [6], while the former one follows directly from the integral representation for $f_\ell(k,q,r)$. One of us derived these integral representations (i) by exploiting the off-shell Jost solution $f_\ell(k,q,r)$ together with the boundary conditions [7] on on-shell regular and irregular solutions and (ii) by making judicious use of the transpose operator relation [8] on the particular solution of inhomogeneous Schrödinger equation for $f_\ell(k,q,r)$ [9] for the partial wave $\ell = 0$ only.

The objective of the present work is to reconstruct these integral representations for both on- and off-shell Jost functions in all partial waves by exploiting the particular solution of the inhomogeneous differential equation for $f_\ell(k,q,r)$ together with transpose operator relation and interacting Green's functions. Section 2 is devoted to develop basic formalism for the integral representations of Jost functions in the representation space approach. Finally, in §3 we construct analytical expressions of the off-shell Jost solutions for Coulomb and Coulomb plus Yamaguchi potentials and study their limiting behaviours with particular emphasis on their on-shell discontinuity both analytically as well as numerically and present concluding remarks.

2. Basic formalism for the integral representations of Jost functions

The off-shell Jost solution $f_\ell(k,q,r)$ for spherically symmetric potential $V(r)$ satisfies the inhomogeneous differential equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - V(r)\right] f_\ell(k,q,r) = (k^2 - q^2)\hat{h}_\ell^{(+)}(qr)\,\mathrm{e}^{i\ell\pi/2}. \tag{1}$$

The off-shell Jost function is obtained as

$$f_\ell(k,q) = \lim_{r\to 0}(qr)^\ell \frac{\mathrm{e}^{-i\ell\pi/2}}{(2\ell+1)!!} f_\ell(k,q,r). \tag{2}$$

The off-shell Jost solution satisfies the asymptotic boundary condition

$$f_\ell(k,q,r) \underset{r\to\infty}{\longrightarrow} \mathrm{e}^{iqr}. \tag{3}$$

It is well known that Riccatti–Hankel function $\hat{h}_\ell^{(+)}(qr)$ satisfies the differential equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + q^2 - \frac{\ell(\ell+1)}{r^2}\right]\hat{h}_\ell^{(+)}(qr) = 0. \tag{4}$$

Equation (4) may be expressed as

$$\mathrm{e}^{i\ell\pi/2}(k^2 - q^2)\hat{h}_\ell^{(+)}(qr) = \left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right]\hat{h}_\ell^{(+)}(qr)\,\mathrm{e}^{i\ell\pi/2}. \tag{5}$$

Combination of eqs (1) and (5) leads to

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right]\left(f_\ell(k,q,r) - \hat{h}_\ell^{(+)}(qr)\,\mathrm{e}^{i\ell\pi/2}\right) = V(r) f_\ell(k,q,r). \tag{6}$$

One can also rewrite eq. (6) in the following form:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - V(r)\right](f_\ell(k,q,r) - \hat{h}_\ell^{(+)}(qr)\,\mathrm{e}^{i\ell\pi/2}) \\ = \mathrm{e}^{i\ell\pi/2}\,V(r)\,\hat{h}_\ell^{(+)}(qr). \tag{7}$$

The particular integrals of eqs (1), (6) and (7) are written as

$$f_\ell(k,q,r) = \mathrm{e}^{i\ell\pi/2}(k^2-q^2)\int_r^\infty \mathrm{d}r'\hat{h}_\ell^{(+)}(qr')G_\ell^{(I)}(r,r'), \tag{8}$$

$$f_\ell(k,q,r) = \mathrm{e}^{i\ell\pi/2}\hat{h}_\ell^{(+)}(qr) + \int_r^\infty \mathrm{d}r'\,V(r')G_\ell^{0(I)}(r,r')f_\ell(k,q,r') \tag{9}$$

and

$$f_\ell(k,q,r) = \mathrm{e}^{i\ell\pi/2}\hat{h}_\ell^{(+)}(qr) + \mathrm{e}^{i\ell\pi/2}\int_r^\infty \mathrm{d}r'\,V(r')\hat{h}_\ell^{(+)}(qr')G_\ell^{(I)}(r,r'). \tag{10}$$

The quantity $G_\ell^{(I)}(r,r')$ is the irregular Green's function [2] for motion in the potential $V(r)$ and is defined as

$$G_\ell^{(I)}(r,r') = -\frac{1}{\Im_\ell(k)}\left[\varphi_\ell(k,r)f_\ell(k,r') - \varphi_\ell(k,r')f_\ell(k,r)\right] \tag{11}$$

with

$$\Im_\ell(k) = \frac{(2\ell+1)!!}{k^\ell}\mathrm{e}^{i\ell\pi/2}f_\ell(k). \tag{12}$$

Here $G_\ell^{0(I)}(r,r')$ is the free-particle Green's function and the quantities $\varphi_\ell(k,r)$ and $f_\ell(k,r)$ stand for the regular and irregular solutions of the Schrödinger equation with potential $V(r)$. Equation (10) will follow automatically from eq. (8). This is as follows.

In conjunction with eq. (5), eq. (8) may be rewritten as

$$f_\ell(k,q,r) = \mathrm{e}^{i\ell\pi/2}\int_r^\infty \mathrm{d}r'G_\ell^{(I)}(r,r')\left[\frac{\mathrm{d}^2}{\mathrm{d}r'^2} + k^2 - \frac{\ell(\ell+1)}{r'^2}\right]\hat{h}_\ell^{(+)}(qr'). \tag{13}$$

By the judicious use of transpose operator relation [8] $\langle\varphi|\,\hat{O}\,|\psi\rangle = \langle\psi|\,\tilde{O}\,|\varphi\rangle$, $\tilde{O} = \hat{O}$ together with the well-known differential equation for the Green's function

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - V(r)\right]G_\ell^{(I)}(r,r') = \delta(r-r') \tag{14}$$

eq. (13) leads to eq. (10). From eqs (2) and (8)–(11), the integral representation of the off- and on-shell Jost functions are obtained as

$$f_\ell(k,q) = \frac{(k^2-q^2)\,q^\ell}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\;\hat{h}_\ell^{(+)}(qr)\varphi_\ell(k,r), \tag{15}$$

$$f_\ell(k,q) = 1 + \frac{q^\ell\,\mathrm{e}^{-i\ell\pi/2}}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\,V(r)\,\varphi_\ell^0(k,r)\,f_\ell(k,q,r) \tag{16}$$

and

$$f_\ell(k,q) = 1 + \frac{q^\ell}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\, V(r)\,\hat{h}_\ell^{(+)}(qr)\varphi_\ell(k,r). \tag{17}$$

Equations (15) and (17) are equivalent. Equation (15) is the most suitable form for deriving analytical expression for off-shell Jost function as it does not involve the potential explicitly. However, one can arrive at eq. (15) from eq. (17) by adopting the following two methods.

The Schrödinger equation for the regular solution $\varphi_\ell(k,r)$ for motion in a potential $V(r)$ may be expressed as

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right]\varphi_\ell(k,r) = V(r)\varphi_\ell(k,r). \tag{18}$$

Substitution of eq. (18) in eq. (17) leads to

$$f_\ell(k,q) = 1 + \frac{q^\ell}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\,\hat{h}_\ell^{(+)}(qr)\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right]\varphi_\ell(k,r). \tag{19}$$

Integrating the above equation twice by parts along with the limiting behaviours $\lim_{r\to 0}\varphi_\ell(k,r) = r^{\ell+1}$ and $\lim_{x\to 0}\hat{h}_\ell^{(+)}(x) = (2\ell-1)!!x^{-\ell}$, one arrives at eq. (15). On the other hand, one can easily obtain eq. (15) by exploiting the transpose operator relation [8] in eq. (19) along with eq. (5).

The corresponding on-shell versions of the Jost function are

$$f_\ell(k) = 1 + \frac{k^\ell\,\mathrm{e}^{-i\ell\pi/2}}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\, V(r)\varphi_\ell^0(k,r)f_\ell(k,r) \tag{20}$$

and

$$f_\ell(k) = 1 + \frac{k^\ell}{(2\ell+1)!!}\int_0^\infty \mathrm{d}r\, V(r)\,\hat{h}_\ell^{(+)}(kr)\varphi_\ell(k,r). \tag{21}$$

Equations (15)–(17) hold good for both short-range and Coulomb potentials.

3. Results and conclusion

For scattering on a short-range potential $f_\ell(k,q)$ is a continuous function of the off-shell momentum. This is, however, not true for Coulomb and Coulomb plus short-range potentials [10] and they exhibit discontinuity at the energy shell. The results for the off-shell Jost function for motion in Coulomb and Coulomb-nuclear potentials have been reported earlier in a number of publications [10–21] but the results for the off-shell Jost solutions for Coulomb and Coulomb-like interactions are relatively new [15–22]. Knowledge of off-shell Jost solutions and Jost functions are required for calculating transition matrices [17,18,20,21,23]. The off-shell Jost function can also be obtained directly from off-shell Jost solution. By using any one of the representations cited in eqs (8)–(10), one can construct off-shell Jost solution for motion in a potential $V(r)$. In the following, we shall construct the expression for off-shell Jost solution for motion in Coulomb plus Yamaguchi potential.

From eq. (8), the off-shell Jost solution for Coulomb–Yamaguchi potential is written as

$$f^{\mathrm{CY}}(k,q,r) = (k^2-q^2)\int_r^\infty \mathrm{d}r'\mathrm{e}^{iqr'}\,G^{(I)}(r,r'), \tag{22}$$

where $G^{(I)}(r,r')$ stands for the irregular Green's function for Coulomb plus Yamaguchi interaction. The above equation involves certain tedious indefinite integrals. To circumvent these difficulties in analytical calculations, the irregular Green's function for Coulomb–Yamaguchi potential is expressed in terms of pure Coulomb irregular Green's function and their integral transforms as

$$G^{(I)}(r,r') = G^{C(I)}(r,r') + \frac{\lambda}{D(k)}\,I(k,\beta,r)\int_r^\infty \mathrm{d}r' g(r')G^{C(I)}(r,r') \tag{23}$$

with

$$I(k,\beta,r) = \int_0^\infty \mathrm{d}r' g(r')\,G^{C(I)}(r,r') \tag{24}$$

and

$$\begin{aligned} G^{C(I)}(r,r') = {} & -2ikrr'\mathrm{e}^{ik(r+r')}\big[\Phi\big(1+i\eta,2;-2ikr'\big)\bar{\Phi}(1+i\eta,2;-2ikr) \\ & - \Phi(1+i\eta,2;-2ikr)\bar{\Phi}\big(1+i\eta,2;-2ikr'\big)\big]. \end{aligned} \tag{25}$$

Here, $g(r) = \mathrm{e}^{-\beta r}$ is the form factor of the Yamaguchi potential and the Fredholm determinant associated with regular or irregular boundary condition

$$\begin{aligned} D(k) = 1 - {} & \frac{\lambda}{(1+i\eta)(\beta-ik)}\left[\frac{1}{(\beta^2+k^2)}\left(\frac{(\beta-ik)}{(\beta+ik)}\right)^{i\eta}\right. \\ & \times {}_2F_1\left(1,\ i\eta;\ 2+i\eta;\ \frac{(\beta+ik)}{(\beta-ik)}\right) \\ & \left. -\frac{1}{2\beta(\beta-ik)}\ {}_2F_1\left(1, i\eta;\ 2+i\eta;\ \left(\frac{(\beta+ik)}{(\beta-ik)}\right)^2\right)\right]. \end{aligned} \tag{26}$$

Combining eqs (8) and (23) in conjunction with eqs (24) and (25), the expression for Coulomb–Yamaguchi off-shell Jost solution [17–22] is obtained as

$$\begin{aligned} f^{\mathrm{CY}}(k,q,r) = {} & f^{\mathrm{C}}(k,q,r) + \lambda\frac{r\mathrm{e}^{ikr}}{D(k)(1+i\eta)(\beta-ik)} \\ & \times\left[\left(\frac{q+k}{q-k}\right)^{i\eta}\ {}_2F_1\left(1,\ i\eta;\ 2+i\eta;\ \frac{\beta+ik}{\beta-ik}\right)\right. \\ & \left.+\frac{\mathrm{e}^{i\pi/2}(q-k)}{(\beta-iq)}\ {}_2F_1\left(1,\ i\eta;\ 2+i\eta;\ \frac{(q-k)(\beta+ik)}{(q+k)(\beta-ik)}\right)\right] \\ & \times\left[\frac{1}{(1+i\eta)(\beta-ik)}\ {}_2F_1\left(1,\ i\eta;\ 2+i\eta;\ \frac{\beta+ik}{\beta-ik}\right)\right. \\ & \times\Phi(1+i\eta,\ 2;\ -2ikr) \\ & +\frac{2ik\,\Gamma(1+i\eta)}{(\beta^2+k^2)}\left(\frac{\beta-ik}{\beta+ik}\right)^{i\eta}\Psi(1+i\eta,\ 2;\ -2ikr) \\ & \left.+\frac{1}{2ik}\sum_{n=0}^{\infty}\frac{\rho^n}{n!}\theta_{n+1}(1+i\eta,\ 2;\ -2ikr)\right] \end{aligned} \tag{27}$$

with the Coulomb off-shell Jost solution

$$f^{\mathrm{C}}(k,q,r) = r\mathrm{e}^{ikr}\left\{\left[\frac{\mathrm{e}^{i\pi/2}(q-k)}{(1+i\eta)}\right]{}_2F_1\left(1,i\eta;2+i\eta;\frac{(q-k)}{(q+k)}\right)\Phi(1+i\eta,2;\right.$$
$$-2ikr) - 2ik\Gamma(1+i\eta)f^{C}(k,q)\Psi(1+i\eta,\ 2;\ -2ikr)$$
$$\left.-\frac{(k^2-q^2)}{2ik}\sum_{n=0}^{\infty}\frac{\rho^n}{n!}\theta_{n+1}(1+i\eta,\ 2;\ -2ikr)\right\} \tag{28}$$

and

$$\rho = (\beta+ik)/2ik. \tag{29}$$

In deriving the above results, we have used the following standard integrals, relation and integral representation [24–26]:

$$\int_0^\infty \mathrm{e}^{-\lambda z}z^{\nu}\Phi(a,c;\,pz) = \frac{\Gamma(\nu+1)}{\lambda^{\nu+1}}\,{}_2F_1\left(a,\nu+1;\,c;\,p/\lambda\right), \tag{30}$$

$$F\left(b,S;1+S+b-d;1-\frac{\mu}{a}\right) = \frac{a^S\Gamma(1+b+S-d)}{\Gamma(1+S-d)\Gamma(S)} \times \int_0^\infty \mathrm{e}^{-ax}x^{S-1}\Psi(b,d;\,\mu x)\mathrm{d}x$$
$$\mathrm{Re}\,S>0,\ 1+\mathrm{Re}\,S>\mathrm{Re}\,d \tag{31}$$

$${}_2F_1\left(a,b;c;z\right) = (1-z)^{c-a-b}\,{}_2F_1\left(c-a,c-b;c;z\right) \tag{32}$$

and

$$\theta_\sigma(a,c;z) = \frac{1}{(c-1)}\left[\Phi(a,c;z)\int_0^z \mathrm{e}^{-z'}z'^{\sigma+c-2}\bar{\Phi}(a,c;z')\mathrm{d}z'\right.$$
$$\left.-\bar{\Phi}(a,c;z)\int_0^z e^{-z'}z'^{\sigma+c-2}\Phi(a,c;z')\mathrm{d}z'\right]. \tag{33}$$

Relatively recently, we have constructed the expressions for off-shell Jost solutions for Coulomb and Coulomb plus Graz-I separable potentials [20] using different approaches to the problem in the representation space. For S-waves the form factors of the Graz-I separable potential coincide with those of Yamaguchi [27]. Our constructed expressions for off-shell Jost solutions for motion in Coulomb and Coulomb plus Yamaguchi potentials produce their correct limiting behaviours and on-shell discontinuity. For example, when $\lambda = 0$, $f^{\mathrm{CY}}(k,q,r) \to f^{\mathrm{C}}(k,q,r)$ and secondly, when $\eta = 0$, pure Yamaguchi Jost solution is obtained [28]. When both λ and η tend to zero $f^{\mathrm{CY}}(k,q,r) = \mathrm{e}^{ikr}$. The on-shell limiting behaviours of $f^{\mathrm{C}}(k,q,r)$ and $f^{\mathrm{C}}(k,q)$ or $f^{\mathrm{CY}}(k,q,r)$ and $f^{\mathrm{CY}}(k,q)$ are given by the singular factor $(q-k)^{-i\eta}$. The corresponding on-shell functions are obtained by using the relation [10,14]

$$f^t(k,r) = \lim_{q\to k}\omega f^t(k,q,r), \qquad k>0 \tag{34}$$

and

$$f^t(k) = \lim_{q\to k}\omega f^t(k,q), \qquad k>0 \tag{35}$$

with

$$\omega = \frac{e^{\pi\eta/2}}{\Gamma(1+i\eta)} \left(\frac{q-k}{q+k}\right)^{i\eta}. \tag{36}$$

Here t stands for either C or CY. We have verified that our expressions in eqs (27) and (28) are at par with the above relations along with $f^{\mathrm{CY}}(k,q,r) \underset{r\to 0}{\to} f^{\mathrm{CY}}(k,q)$.

The Yamaguchi potential with $\lambda = -2.405$ fm^{-3} and $\beta = 1.1$ fm^{-1} provides a reasonable fit to $(p-p)$ scattering data [18]. We have computed $f^{\mathrm{CY}}(k, q, r)$ and $f^{\mathrm{Y}}(k, q, r)$ for various values of $r = 0.1$, 0.01 and 0.0 fm with real positive k and q by taking the limit Im $q \to 0^+$ and our results are portrayed in figures 1–4 as a function of off-shell momentum q for laboratory energies of 10 and 30 MeV respectively. The values of the off-shell Jost solutions for pure Yamaguchi potential have been obtained by turning off the Coulomb interaction in the numerical routine for $f^{\mathrm{CY}}(k, q, r)$. Therefore, the two sets of numbers, namely those for $f^{\mathrm{CY}}(k, q, r)$ and $f^{\mathrm{Y}}(k, q, r)$ are expected to provide a basis for looking into the role of Coulomb interaction in the $(p-p)$ off-shell scattering [18,21]. As expected, $f^{\mathrm{Y}}(k, q, r)$ is a continuous function of the off-shell momentum q. In contrast to this, $f^{\mathrm{CY}}(k, q, r)$ exhibits a discontinuity at the on-shell point $q = k$. It is observed that Re $f^{\mathrm{Y}}(k, q, r)$ and Re $f^{\mathrm{CY}}(k, q, r)$ with $r = 0.1$, 0.01 and 0.0 fm have positive values over the entire range of q under consideration while Im $f^{\mathrm{Y}}(k, q, r)$ and Im $f^{\mathrm{CY}}(k, q, r)$ possess negative values over the entire range for $r = 0.01$ and 0.0 fm and change sign at $q = 2.75$ and 2.50 fm^{-1} with $r = 0.1$ fm for 10 and 30 MeV respectively. It is well known that as $r \to 0$, $f(k, q, r) \to f(k, q)$ and looking closely into our figures, we see that our numerical results support it.

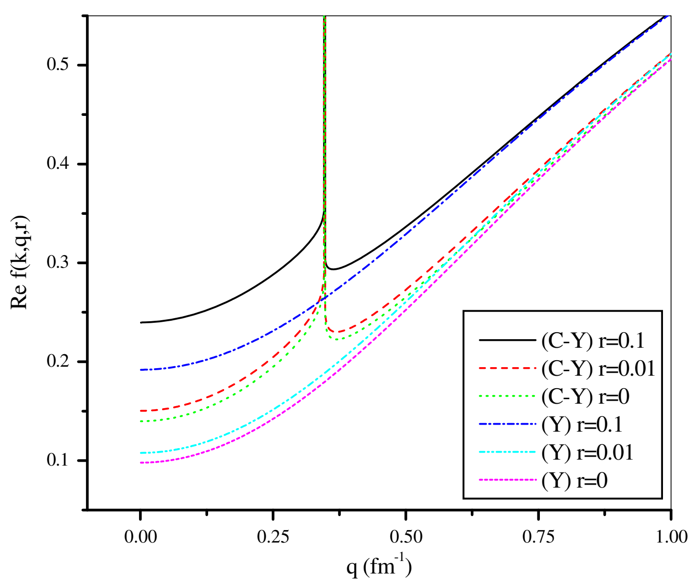

Figure 1. Real part of $f(k, q, r)$ as a function of off-shell momentum q for $E_{\mathrm{Lab}} = 10$ MeV.

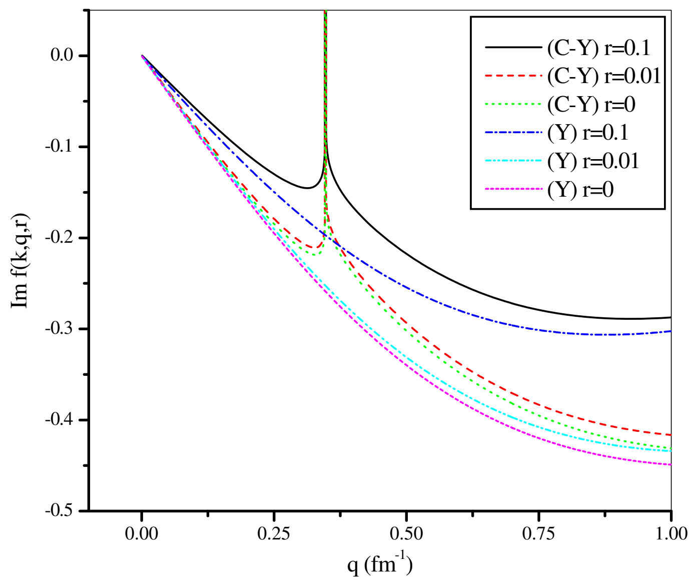

Figure 2. Imaginary part of $f(k, q, r)$ as a function of off-shell momentum q for $E_{\mathrm{Lab}} = 10$ MeV.

In view of the importance of the experiments which involve charged hadrons, the interest in studying potentials consisting of the sum of a short-range finite rank separable potential and Coulomb potential is increased. Calculations in physics involving such

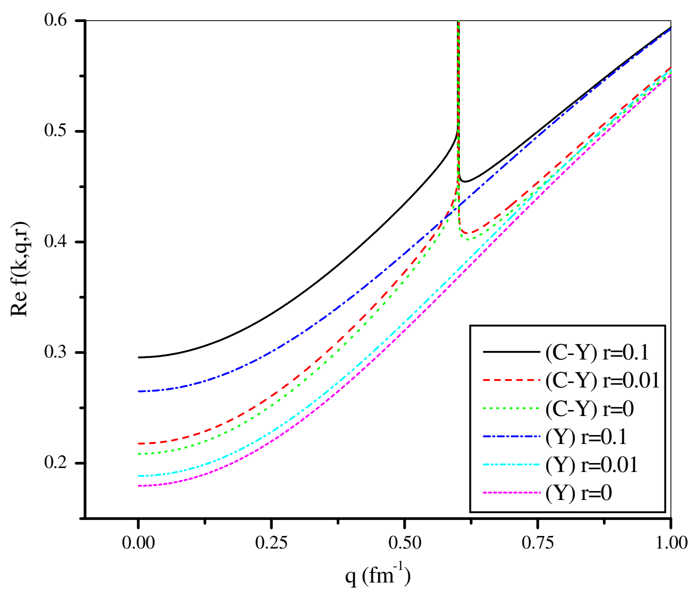

Figure 3. Real part of $f(k, q, r)$ as a function of off-shell momentum q for $E_{\mathrm{Lab}} =$ 30 MeV.

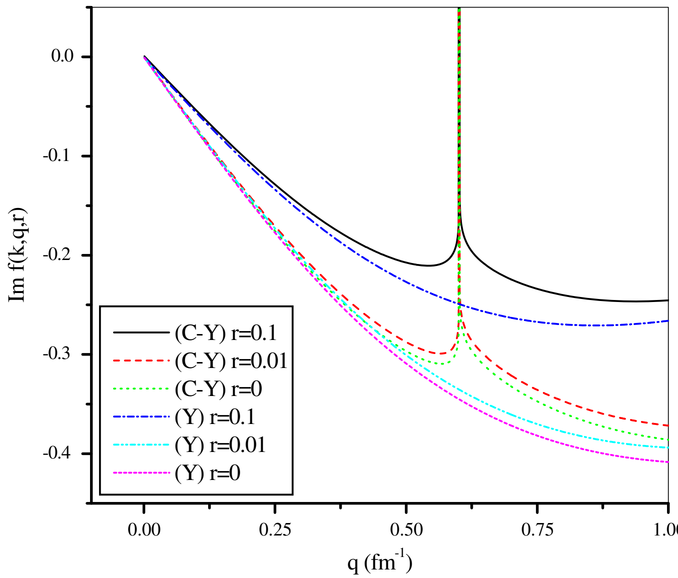

Figure 4. Imaginary part of $f(k, q, r)$ as a function of off-shell momentum q for $E_{\text{Lab}} = 30$ MeV.

interactions generally use off-energy shell elements of the two-particle transition matrices. The fully off- and half-off-shell T-matrix elements are directly related to off-shell Jost solutions and Jost functions. The T-matrix has established importance in nuclear physics with respect to its close relation to experiment. The on-shell T-matrix elements are related to scattering phase shifts and the half-shell element of the T-matrix $T(k, q, k^2)$ defines the transition probability per unit time for change in a system from an initial state to all accessible final states. Here, the time for the two-nucleon scattering process is restricted on one side only [14,29–31]. In this case, the half-off-shell T-matrix is applicable. Also the phase of the half-off-shell T-matrix is the scattering phase shift [3]. In $(p\text{-}2p)$ reaction, the momentum transfer distribution is closely co-related with the distribution of momenta which the nuclear proton had before it was knocked out. In such a case, a single proton is knocked out of the nucleus and the momentum transfer distribution measured [32]. In the three-body problem, the off-shell T-matrix is a direct and transparent link between experimental two-nucleon data and three-nucleon observables. The T-matrix is therefore closely related to the experiment. Any nuclear process depends on these elements in one way or another. Thus, off-shell Jost solutions and Jost functions are immensely popular tools for dynamical calculations in nuclear physics. In contrast to Fuda [3,5] and Newton [2], the present approach is much more simpler and provides a common basis for deriving all the integral representations for the Jost functions.

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