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Nonlinear propagation of ion-acoustic waves in a degenerate dense plasma

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Abstract. Nonlinear propagation of ion-acoustic (IA) waves in a degenerate dense plasma (with all the constituents being degenerate, for both the non-relativistic or ultrarelativistic cases) have been investigated by the reductive perturbation method. The linear dispersion relation and Korteweg–de Vries (KdV) equation have been derived, and the numerical solutions of KdV equation have been analysed to identify the basic features of electrostatic solitary structures that may form in such a degenerate dense plasma. The implications of our results in compact astrophysical objects, particularly, in white dwarfs and neutron stars, have been briefly discussed.

Keywords. Solitary wave; degenerate plasma; dense plasma; KdV equation.

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1. Introduction

Presently, the main theoretical concern is to understand the environment of the compact objects having their interiors supporting themselves via degenerate pressure. The degenerate pressure, which arises due to the combined effect of Pauli's exclusion principle (Wolfgang Ernst Pauli, 1925) and Heisenberg's uncertainty principle (Werner Heisenberg, 1927), depends only on the fermion number density, not on its temperature. This degenerate pressure has a vital role for studying the electrostatic perturbation in matters existing in extreme conditions [1–6]. The extreme conditions of matter are caused by the significant compression of the interstellar medium. High density of degenerate matter in these compact objects (which are, in fact, 'relics of stars') is one of these extreme conditions. These interstellar compact objects, having ceased burning thermonuclear fuel, thereby no longer generating thermal pressure, are contracted significantly, and as a result, the density of their interiors becomes extremely high for providing non-thermal pressure through degenerate pressure of their constituent particles and particle–particle interaction. The

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observational evidence and theoretical analysis imply that these compact objects support themselves against gravitational collapse by degenerate pressure.

The degenerate electron number density in such a compact object is so high (e.g. in white dwarfs it can be of the order of 10^{30} cm^{-3}, or even more [7]) that the electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in vacuum. The equation of state for degenerate electrons in such interstellar compact objects are mathematically explained by Chandrasekhar [3] for two limits, namely non-relativistic and ultrarelativistic limits. The interstellar compact objects provide us cosmic laboratories for studying the properties of the medium (matter), as well as waves and instabilities [8–16] in such a medium at extremely high densities (degenerate state) for which quantum as well as relativistic effects become important [8,15]. The quantum effects on linear [11,13,16] and nonlinear [12,14] propagation of electrostatic and electromagnetic waves have been investigated using the quantum hydrodynamic (QHD) model [8,16], which is an extension of classical fluid model in a plasma, and by using the quantum magnetohydrodynamic (QMHD) model [11–14], which involves spin-$\frac{1}{2}$ and one-fluid MHD equations.

Recently, a number of theoretical investigations have also been done of the nonlinear propagation [17–20] of electrostatic waves in degenerate quantum plasma by a number of authors, e.g. Hass [21], Misra and Samanta [22], Misra *et al* [23] etc. However, these investigations are based on the electron equation of state which is valid for the non-relativistic limit. In 2011, Misra and Shukla [24] considered the nonlinear propagation of electrostatic wave packets in an ultrarelativistic (UR) degenerate dense electron–ion plasma. One year later, they discussed [25] the nonlinear propagation of electrostatic wave packets in a collisional plasma composed of strongly coupled ions and relativistically degenerate electrons. Some investigations have been done of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state which is valid for ultrarelativistic limit [26]. To the best of our knowledge, no theoretical investigation has been done to study the extreme condition of matter for both non-relativistic and ultrarelativistic limits. Therefore, in this paper, we consider a degenerate dense plasma containing non-relativistic degenerate cold ion fluid and both non-relativistic and ultrarelativistic degenerate electrons for studying the basic features of solitary waves in such a degenerate dense plasma. The model is relevant for compact interstellar objects (e.g., white dwarf, neutron star, etc.).

The paper is organized as follows. The governing equations are given in §2. The numerical analysis is given in §3 and a brief summary is provided in §4.

2. Governing equations

We consider the propagation of electrostatic perturbation in a degenerate dense plasma containing non-relativistic degenerate cold ion and degenerate electron fluids. Thus, at equilibrium we have $n_{\mathrm{i}0} = n_{\mathrm{e}0} = n_0$, where $n_{\mathrm{i}0}$ ($n_{\mathrm{e}0}$) is the ion (electron) number density at equilibrium. The nonlinear dynamics of the electrostatic waves propagating in such a degenerate plasma is governed by

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{i}} u_{\mathrm{i}}) = 0, \tag{1}$$

$$\frac{\partial u_{\rm i}}{\partial t} + u_{\rm i}\frac{\partial u_{\rm i}}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_{\rm i}}\frac{\partial n_{\rm i}^{\alpha}}{\partial x} = 0, \tag{2}$$

$$n_{\rm e}\frac{\partial \phi}{\partial x} - K_2\frac{\partial n_{\rm e}^{\gamma}}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\rm e} - n_{\rm i}, \tag{4}$$

where $n_{\rm i}$ ($n_{\rm e}$) is the ion (electron) number density normalized by its equilibrium value $n_{\rm i0}$ ($n_{\rm e0}$), $u_{\rm i}$ is the ion fluid speed normalized by $C_{\rm i} = (m_{\rm e}c^2/m_{\rm i})^{1/2}$ with $m_{\rm e}$ ($m_{\rm i}$) being the electron (ion) rest mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_{\rm e}c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{\rm pi} = (4\pi n_0 e^2/m_{\rm i})^{1/2}$, and the space variable (x) is normalized by $\lambda_{\rm s} = (m_{\rm e}c^2/4\pi n_0 e^2)^{1/2}$. The constants $K_1 = n_0^{\alpha-1}K_{\rm i}/m_{\rm i}^2 C_{\rm i}^2$ and $K_2 = n_0^{\gamma-1}K_{\rm e}/m_{\rm i}{C_{\rm i}}^2$. The equations of state used here are given by

$$P_{\rm i} = K_{\rm i}n_{\rm i}^{\alpha}, \tag{5}$$

where

$$\alpha = \frac{5}{3}, \quad K_{\rm i} = \frac{3}{5}\left(\frac{\pi}{3}\right)^{1/3}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_{\rm c}\hbar c, \tag{6}$$

for the non-relativistic limit (where $\Lambda_{\rm c} = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm and $\hbar$ is the Planck constant divided by 2π). For the electron fluid,

$$P_{\rm e} = K_{\rm e}n_{\rm e}^{\gamma}, \tag{7}$$

where

$$\gamma = \alpha, \quad K_{\rm e} = K_{\rm i} \quad \text{for non-relativistic limit} \tag{8}$$

and

$$\gamma = \frac{4}{3}, \quad K_{\rm e} = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{1/3}\hbar c \simeq \frac{3}{4}\hbar c, \tag{9}$$

in the ultrarelativistic limit [1–3,7].

3. Numerical analysis

To examine the electrostatic perturbations propagating in the ultrarelativistic degenerate dense plasma by analysing the outgoing solutions of (1)–(4), we first introduce the stretched coordinates [27]

$$\zeta = -\epsilon^{1/2}(x + V_{\rm p}t), \tag{10}$$

$$\tau = \epsilon^{3/2}t, \tag{11}$$

where $V_{\rm p}$ is the wave phase speed (ω/k where ω is the angular frequency and k is the wave number of the perturbation mode) and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand $n_{\rm i}$, $n_{\rm e}$, $u_{\rm i}$ and ϕ, in power series of ϵ:

$$n_{\rm i} = 1 + \epsilon n_{\rm i}^{(1)} + \epsilon^2 n_{\rm i}^{(2)} + \cdots, \tag{12}$$

$$n_{\rm e} = 1 + \epsilon n_{\rm e}^{(1)} + \epsilon^2 n_{\rm e}^{(2)} + \cdots, \tag{13}$$

$$u_{\rm i} = \epsilon u_{\rm i}^{(1)} + \epsilon^2 u_{\rm i}^{(2)} + \cdots, \tag{14}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{15}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, eqs (1)–(15) give $u_{\rm i}^{(1)} = -3V_{\rm p}\phi^{(1)}/(3V_{\rm p}^2 - 2K_1')$, $n_{\rm i}^{(1)} = 3\phi^{(1)}/(3V_{\rm p}^2 - 2K_1')$, $n_{\rm e}^{(1)} = \phi^{(1)}/\gamma K_2'$ and $V_{\rm p} = \sqrt{\gamma K_2' + (2/3)K_1'}$, where $K_1' = \alpha/m_{\rm i}(\alpha - 1)$. The relation $V_{\rm p} = \sqrt{\gamma K_2' + (2/3)K_1'}$ represents the linear dispersion relation for the ion-acoustic type electrostatic waves in the degenerate plasma under consideration [the new constants introduced here are given by $K_1' = \alpha K_1/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$]. We can express the linear dispersion relation (taking the equilibrium and first-order perturbation only) in one-dimensional form as

$$\frac{\omega^2}{k^2} = \frac{C_{\rm i}^2}{(m_{\rm e}c^2/n_0 K_2') + k^2\lambda_{\rm s}^2} + K_1'. \tag{16}$$

We now examine the dispersion properties of these waves for an interstellar object like white dwarfs (helium, carbon, oxygen-dominated models [1,2]), where mass density [4–6] ρ_0 can vary from $\sim 10^6$ g cm^{-3} to $\sim 10^8$ g cm^{-3}.

We are interested in studying the nonlinear propagation of these dispersive ion-acoustic type electrostatic waves in a degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_{\rm i}^{(1)}}{\partial \tau} - V_{\rm p}\frac{\partial n_{\rm i}^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}\left[u_{\rm i}^{(2)} + n_{\rm i}^{(1)}u_{\rm i}^{(1)}\right] = 0, \tag{17}$$

$$\frac{\partial u_{\rm i}^{(1)}}{\partial \tau} - V_{\rm p}\frac{\partial u_{\rm i}^{(2)}}{\partial \zeta} - u_{\rm i}^{(1)}\frac{\partial u_{\rm i}^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} - K_1'\frac{\partial}{\partial \zeta}\left[\frac{2n_{\rm i}^{(2)}}{3} - \frac{\left(n_{\rm i}^{(1)}\right)^2}{9}\right] = 0, \tag{18}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - \gamma K_2'\frac{\partial}{\partial \zeta}\left[n_{\rm e}^{(2)} + \frac{(\gamma - 2)}{2}\left(n_{\rm e}^{(1)}\right)^2\right] = 0, \tag{19}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = n_{\rm e}^{(2)} - n_{\rm i}^{(2)}. \tag{20}$$

Now, combining eqs (17)–(20) we deduce a modified Korteweg–de Vries equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \zeta} + B\frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \tag{21}$$

where

$$A = \frac{\left(3V_{\mathrm{p}}^2 - 2K_1'\right)^2}{18V_{\mathrm{p}}} \left(\frac{1}{\gamma^2 K_2'^2} + \frac{3\left(27V_{\mathrm{p}}^2 - 2K_1'\right)}{\left(3V_{\mathrm{p}}^2 - 2K_1'\right)^3} \right), \tag{22}$$

$$B = \frac{(3V_{\mathrm{p}}^2 - 2K_1')^2}{18V_{\mathrm{p}}}. \tag{23}$$

The stationary solitary wave solution of (21) is

$$\phi^{(1)} = \phi_m^{(1)} \operatorname{sech}^2\left(\frac{\xi}{\Delta}\right), \tag{24}$$

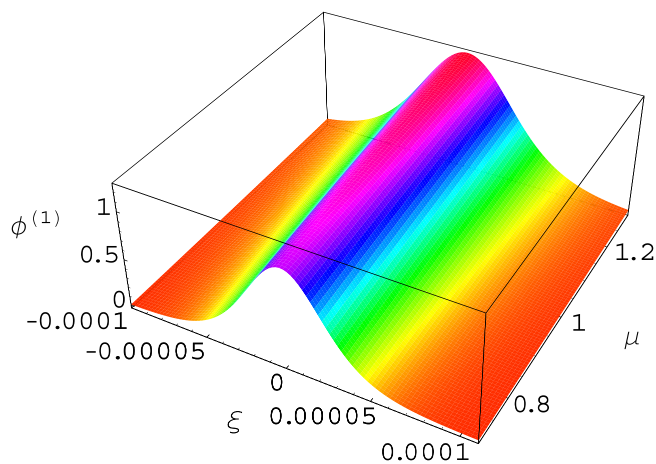

Figure 1. The solitary profiles represented by eq. (24) for $u_0 = 1$ and $n_{i0} = 3 \times 10^{31}$, where both the constituent particles are non-relativistic.

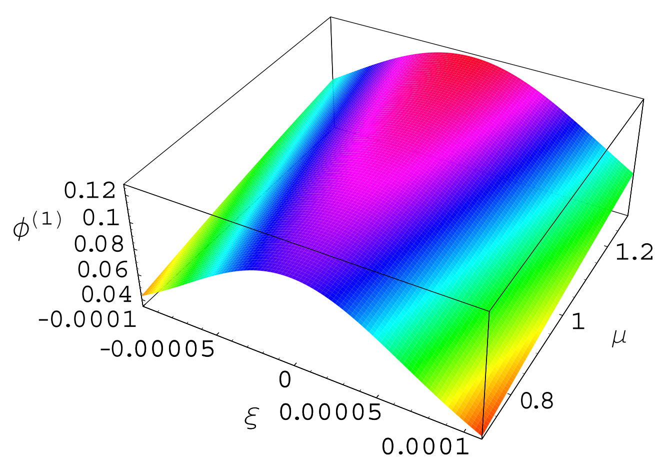

Figure 2. The solitary profiles represented by eq. (24) for $u_0 = 0.1$ and $n_{i0} = 3 \times 10^{31}$, where both the constituent particles are non-relativistic.

where $\xi = \zeta - u_0\tau$, $\phi_m^{(1)} = 3u_0/A$ and $\Delta = (4B/u_0)^{1/2}$. It is obvious from eqs (22) and (24) that the degenerate plasma under consideration supports compressive electrostatic solitary waves which are associated with a positive potential. It is obvious from eqs (22)–(24) that the amplitude $[\phi_m^{(1)}]$ of these solitary structures depends on the density parameter μ, i.e., the ratio of electron to ion number density. It is also seen that the amplitude increases (decreases) with the increase (decrease) of speed u_0. The electrostatic solitary profiles are shown in figures 1–4 where μ is the density ratio of the constituent particles (electron to ion number density).

We now turn to eq. (21) with the term $\phi^{(1)}$ which changes proportionally with the parameter μ. We have numerically solved eq. (21), and have studied the effects of μ on electrostatic solitary structures in both non-relativistic and ultrarelativistic degenerate

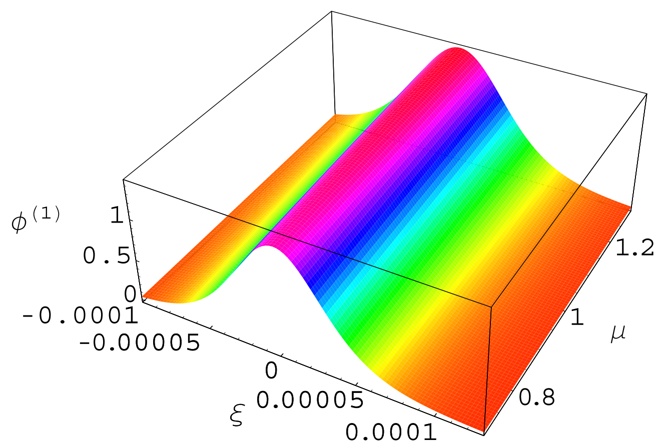

Figure 3. The effect of variation in density ratio of the constituents ($\mu = n_{e0}/n_{i0}$) in solitary profiles for ultrarelativistic electron with $u_0 = 1$.

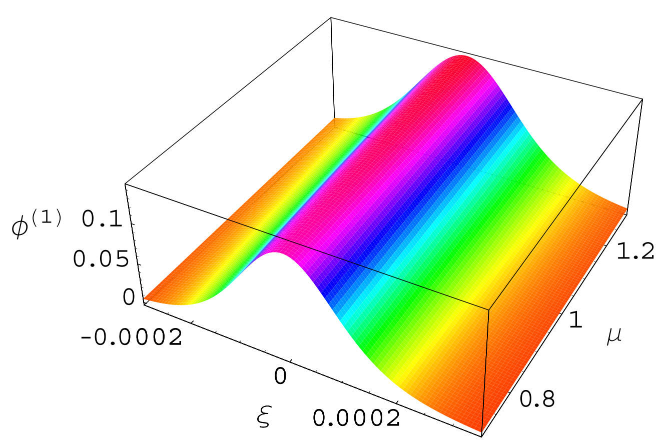

Figure 4. The effect of variation in density ratio of the constituents ($\mu = n_{e0}/n_{i0}$) in solitary profiles for ultrarelativistic electron with $u_0 = 0.1$.

electrons (ion always being non-relativistic degenerate). The results of the first case are depicted in figures 1 and 2. It is observed that the solitary potential increases with the increase of μ, or, in other words, the maximum electron number density results in the maximum wave potential. It also holds good for the ultrarelativistic degenerate electrons (shown in figures 3 and 4).

4. Summary

To summarize, we have investigated electrostatic solitary waves in a degenerate dense plasma, which is relevant to interstellar compact objects [5,28–34]. The degenerate dense plasma is found to support solitary structures [35–38] whose basic features (amplitude, width, speed, etc.) depend only on the plasma number density. It has been shown here that the amplitude, width and speed increase with the increase of the plasma number density, particularly, the maximum number of light particles (electrons). This work is very much effective and quite different from others and is more general than the relevant previous works [7,26]. We hope that our present investigation will be helpful for understanding the basic features of the localized electrostatic disturbances in compact astrophysical objects (e.g., white dwarfs, neutron stars, etc.).

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