PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 79, No. 2
August 2012
pp. 211–222

Relativistic models of a class of compact objects

RUMI DEB[1], BIKASH CHANDRA PAUL[1,2,*] and RAMESH TIKEKAR[3]
[1]IUCAA Reference Centre, Physics Department, North Bengal University, Darjeeling 734 013, India
[2]Physics Department, North Bengal University, Siliguri, Darjeeling 734 013, India
[3]Inter-University Centre for Astronomy and Astrophysics, P.O. Box 4, Ganeshkhind P.O., Pune 411 007, India
*Corresponding author. E-mail: bcpaul@iucaa.ernet.in

MS received 27 December 2011; revised 18 February 2012; accepted 1 March 2012

Abstract. A class of general relativistic solutions in isotropic spherical polar coordinates which describe compact stars in hydrostatic equilibrium are discussed. The stellar models obtained here are characterized by four parameters, namely, λ, k, A and R of geometrical significance related to the inhomogeneity of the matter content of the star. The stellar models obtained using the solutions are physically viable for a wide range of values of the parameters. The physical features of the compact objects taken up here are studied numerically for a number of admissible values of the parameters. Observational stellar mass data are used to construct suitable models of the compact stars.

Keywords. Relativistic star; compact object.

PACS Nos 04.20.Jb; 04.40.Dg; 95.30.Sf

1. Introduction

The discovery of compact stellar objects, such as X-ray pulsars (Her X1, millisecond pulsar SAX J1808.43658) and X-ray sources (4U 1820-30 and 4U 1728-34) which are regarded as the probable strange star candidates, has led to critical studies of relativistic models of such stellar configurations [1–10]. There are several such astrophysical as well as cosmological situations where one needs to consider the equation of state of matter involving matter densities of the order of 10^{15} g cm^{-3} or higher, exceeding the nuclear density. The conventional approach of obtaining models of relativistic stars in equilibrium heavily relies on the availability of definite information about the equation of state of its matter content. Our knowledge about possible equation of state inside a superdense strange star at present is limited. In this context, Vaidya–Tikekar [1] and Tikekar [3] have shown that in the absence of definite information about equation of state of matter content of stellar configuration, the alternative approach of prescribing suitable ansatz geometry for the interior physical three-space of the configuration leads to simple easily tractable models of such stars which are physically viable. Relativistic models of superdense stars based on different solutions of Einstein's field equations obtained by using Vaidya–Tikekar approach of assigning different geometries with physical three-spaces of such objects have been studied by several workers [6,7,9,10]. Pant and Sah [2] obtained a class of relativistic static non-singular analytic solutions in isotropic form describing the space-time of static spherically symmetric distribution of matter. The solution has been found to lead to a physically viable causal model of neutron star with a maximum mass of $4M_{\odot}$.

In this paper we discuss a class of solution of relativistic field equations as obtained in ref. [2] and examine the physical plausibility of several models of a class of neutron stars using numerical procedures to explore the possibility of using it to describe the interior of a compact star. It is possible to estimate the radius of a star when its mass is known. It is also possible to determine the variation of matter density on its boundary surface and that at the centre of a superdense star for the prescribed geometry. The plan of the paper is as follows: in §2 the relevant relativistic field equations have been set up and their solution is discussed. In §3 several features of physical relevance have been reported. In §4, stellar models are discussed with the observational stellar mass data for different values of the parameters λ, k, A and R. Finally in §5, we give a brief discussion.

2. Field equation and solution

The Einstein's field equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{1}$$

where $g_{\mu\nu}$, R, $R_{\mu\nu}$ and $T_{\mu\nu}$ are the metric tensor, Ricci scalar, Ricci tensor and energy–momentum tensor respectively. We use the following form of the space-time metric given by

$$\mathrm{d}s^2 = \mathrm{e}^{\nu(r)}\mathrm{d}t^2 - \mathrm{e}^{\mu(r)}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2) \tag{2}$$

with

$$\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2 \tag{3}$$

using isotropic spherical polar coordinate. In the next section we use systems of units with $8\pi G = 1$, $c = 1$ respectively.

The energy–momentum tensor for a spherical distribution of matter in the form of perfect fluid in equilibrium is given by

$$T_{\mu}^{\mu} = \mathrm{diag}(\rho, -p, -p, -p), \tag{4}$$

where ρ and p are energy density and fluid pressure of matter respectively. Using the space-time metric given by eq. (2), the Einstein's field eq. (1) gives the following equations:

$$\rho = -\mathrm{e}^{-\mu}\left(\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right) \tag{5}$$

$$p = \mathrm{e}^{-\mu}\left(\frac{\mu'^2}{4} + \frac{\mu'}{r} + \frac{\mu'\nu'}{2} + \frac{\nu'}{r}\right) \tag{6}$$

$$p = \mathrm{e}^{-\mu}\left(\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\mu'}{2r} + \frac{\nu'}{2r}\right). \tag{7}$$

Now, pressure isotropy condition from eqs (6) and (7) leads to the following relation between metric variables μ and ν:

$$\nu'' + \mu'' + \frac{\nu'^2}{2} - \frac{\mu'^2}{2} - \mu'\nu' - \frac{1}{r}\left(\nu' + \mu'\right) = 0. \tag{8}$$

It is a second-order differential equation which permits a solution [2] as follows:

$$\mathrm{e}^{\nu/2} = A\left(\frac{1-k\alpha}{1+k\alpha}\right), \qquad \mathrm{e}^{\mu/2} = \frac{(1+k\alpha)^2}{1+(r^2/R^2)}, \tag{9}$$

where R, λ, k and A are arbitrary constants. In the above we denote

$$\alpha(r) = \sqrt{\frac{1+(r^2/R^2)}{1+\lambda(r^2/R^2)}}. \tag{10}$$

We observe that the geometry of the three-space with metric

$$\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)}{1+(r^2/R^2)} \tag{11}$$

is that of a three-sphere immersed in a four-dimensional Euclidean space. Accordingly, the geometry of physical space is given by

$$\mathrm{d}s^2 = A^2\frac{(1-k\alpha)^2}{(1+k\alpha)^2}\mathrm{d}t^2 - \frac{(1+k\alpha)^4}{1+(r^2/R^2)}(\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)), \tag{12}$$

where $\alpha(r)$ is given by eq. (10). Hence the geometry of the three-space obtained at $t =$ constant section of the space-time of metric (12) is a deviation introduced in spherical three-space and the parameter k is a geometrical parameter measuring inhomogeneity of the physical space. However, for $k = 0$, the space-time metric (12) degenerates into that of Einstein's static Universe filled with matter of uniform density. The space-time metric of Pant and Sah [2] is a generalization of the Buchdahl solution, the physical three-space associated with which has the same feature. For $\lambda = 0$, the solution reduces to that obtained by Buchdahl which is an analog of a classical polytrope of index 5. However, for $\lambda > 0$, the solution corresponds to finite boundary models. Pant and Sah [2] obtained a class of non-singular analytic solution of the general relativistic field equations for a static spherically symmetric material distribution which is matched with Schwarzschild's empty space-time. In this paper we study physical properties of compact objects taking different values of R, λ, k and A as permitted by the field equations. Using the solution given by eq. (9) in eqs (5)–(7), one obtains explicit expressions for the energy density and fluid pressure as follows:

$$\rho = \frac{12(1+\lambda k\alpha^5)}{R^2(1+k\alpha)^5}, \tag{13}$$

$$p = \frac{4(\lambda k^2\alpha^6 - 1)}{R^2(1+k\alpha)^5(1-k\alpha)}. \tag{14}$$

The exterior Schwarzschild line element is given by

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r_0}\right)\mathrm{d}t^2 - \left(1 - \frac{2m}{r_0}\right)^{-1}\mathrm{d}r^2 - r_0^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \tag{15}$$

where m represents the mass of the spherical object. This metric can be expressed in an isotropic form [11] as

$$\mathrm{d}s^2 = \left(\frac{1-(m/2r)}{1+(m/2r)}\right)^2 \mathrm{d}t^2 - \left(1+\frac{m}{2r}\right)^4 (\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2) \tag{16}$$

using the transformation $r_0 = r\,(1+(m/2r))^2$ where r_0 is the radius of the compact object. This form of the Schwarzschild metric will be used here to match at the boundary with the interior metric given by eq. (12).

3. Physical properties of a compact star

The solution given by eq. (9) is useful to study physical features of a compact star in a general way which are outlined as follows:

(1) In this model, ρ and p are determined using eqs (13) and (14). We note that ρ is obviously positive for any positive λ and k, while $p \geq 0$ leads to two different cases: (i) $\lambda > 1/k^2\alpha^6$ with $k < 1/\alpha$ and (ii) $\lambda < 1/k^2\alpha^6$ with $k > 1/\alpha$.

(2) At the boundary of the star ($r = b$), the interior solution should be matched with the isotropic form of Schwarzschild exterior solution, i.e.,

$$\mathrm{e}^{\frac{\nu}{2}}|_{r=b} = \left(\frac{1-(m/2b)}{1+(m/2b)}\right), \qquad \mathrm{e}^{\frac{\mu}{2}}|_{r=b} = \left(1+\frac{m}{2b}\right)^2. \tag{17}$$

(3) The physical radius of a star, r_0, is determined knowing the radial distance where the pressure at the boundary vanishes (i.e., $p(r) = 0$ at $r = b$). The physical radius is related to the radial distance ($r = b$) through the relation $r_0 = b\,(1+(m/2b))^2$ [11].

(4) The ratio m/b is determined using eqs (9) and (16), which is given by

$$\frac{m}{b} = 2\left(\frac{1+k\alpha}{\sqrt{1+y^2}} - 1\right). \tag{18}$$

(5) The density inside the star should be positive, i.e., $\rho > 0$.

(6) Inside the star the stellar model should satisfy the condition, $\mathrm{d}p/\mathrm{d}\rho < 1$ for the sound propagation to be causal.

The usual boundary conditions are that the first and second fundamental forms be continuous across the boundary $r = b$. Applying the boundary conditions we determine A which is given by

$$A = \frac{(1-(m/2b))}{(1+(m/2b))}\left(\frac{\sqrt{1+\lambda(b^2/R^2)} + k\sqrt{1+(b^2/R^2)}}{\sqrt{1+\lambda(b^2/R^2)} - k\sqrt{1+(b^2/R^2)}}\right). \tag{19}$$

Equating eqs (9) and (16) at the boundary ($r = b$), we get an eighth-order polynomial equation in y (here b/R is replaced by y):

$$\begin{aligned}[(1+A)^4 &+ k^4(1-A)^4 - 8(1+A)^2 + 16 - 2k^2(1-A^2)^2 - 8k^2(1-A)^2] \\ &+ [2\lambda(1+A)^4 - 16\lambda(1+A)^2 - 8(1+A)^2 + 32(1+\lambda) \\ &- 2k^2(1-A^2)^2(1+\lambda) - 8k^2(2+\lambda)(1-A)^2 + 2k^4(1-A)^4]y^2 \\ &+ [\lambda^2(1+A)^4 - 8\lambda^2(1+A)^2 - 8\lambda(1+A)^2 + (1+4\lambda+\lambda^2) \\ &- 2\lambda k^2(1-A^2)^2 - 8k^2(1-A)^2(1+2\lambda) + k^4(1-A)^4]y^4 \\ &- [8\lambda^2(1+A)^2 - 32(1+\lambda) - 8\lambda k^2(1-A)^2]y^6 + 16\lambda^2 y^8 = 0, \qquad (20)\end{aligned}$$

where λ, k and A are constants. Imposing the condition that pressure at the boundary vanishes in eq. (14), we determine y which is given by

$$y = \sqrt{\frac{1-(\lambda k^2)^{1/3}}{(\lambda k^2)^{1/3} - \lambda}}. \qquad (21)$$

Thus, the size of a star is determined by k and λ. It is evident that a real y is permitted when $k > \lambda$ with $\lambda < 1$, or when $k < \lambda$ with $\lambda > 1$. Using eqs (20) and (21), a polynomial equation in λ, k and A is obtained. Although eq. (20) is a polynomial of degree eight we note that only one realistic solution for y is obtained for different domains of the values of any pair of parameters namely, A, k and λ. Subsequently, the other parameters may be determined. For example, when $A = 2$, we found that λ and k satisfy the inequalities $2.9 \le k \le 5$ and $1.4877 \times 10^{-6} \le \lambda \le 0.04$, and when $A = 4$, the range of permitted values are $1.7 \le k \le 2.3$ and $0.0185 \le \lambda \le 0.0653$. However, for a given λ, e.g., when $\lambda = 0.15$, we note that the permitted values of A lie in the range $3.6 < A < 5.6$ and when $\lambda = 0.1318$, one obtains realistic solution for $3.5 < A < 5.8$.

The square of the acoustic velocity $\mathrm{d}p/\mathrm{d}\rho$ takes the form

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{6\lambda k\alpha^5(1-k\alpha)(1+k\alpha) - 5(1-k\alpha)(\lambda k^2\alpha^6 - 1) + (\lambda k^2\alpha^6 - 1)(1+k\alpha)}{15(1-k\alpha)^2(\lambda\alpha^4(1+k\alpha) - (1+\lambda k\alpha^5))}. \qquad (22)$$

The variation of $\mathrm{d}p/\mathrm{d}\rho$ for $\lambda = 0.1318$ and $k = 2.2268$ displayed in table 1. It is evident that $\mathrm{d}p/\mathrm{d}\rho$ is maximum at the centre and gradually decreases outward. It is also

Table 1. Variation of $\mathrm{d}p/\mathrm{d}\rho$ with radial distance r for $\lambda = 0.1318$ and $k = 2.2268$.

r in units of R km	$\mathrm{d}p/\mathrm{d}\rho$
0	0.521
0.1	0.518
0.2	0.513
0.3	0.504
0.4	0.496
0.41	0.495
0.42	0.495

Table 2. Variation of $dp/d\rho$ with radial distance r for different values of λ and k.

r in units of R km	$dp/d\rho$ for $\lambda = 0.1211$ and $k = 2.2$	$dp/d\rho$ for $\lambda = 0.1318$ and $k = 2.2268$	$dp/d\rho$ for $\lambda = 0.15$ and $k = 2.2681$
0	0.524	0.521	0.520
0.1	0.521	0.518	0.520
0.2	0.514	0.513	0.513
0.3	0.504	0.504	0.508
0.4	0.494	0.496	

found that inside the star the constraint $dp/d\rho < 1$ is always maintained which ensures causality. In table 2, variation of $dp/d\rho$ from the centre to the boundary for different values of λ and k are presented. It is evident that as λ increases $dp/d\rho$ decreases at the centre. The variation of the central density with λ and k are displayed in tables 3 and 4 for $A = 2$ and $A = 4$ respectively. It is evident that the central density (ρ_c) decreases with an increase in λ. Thus stellar models with larger λ accommodate a denser compact object compared to that for lower values of λ and k. The variation of pressure and density with radial distance are drawn employing eqs (13) and (14) which are shown in figures 1–4. As it is not possible to express pressure in terms of density, we study the behaviour of pressure and density inside the curve numerically. In figure 5, the variation of pressure with density is plotted for different model parameters.

4. Physical analysis

In this section we analyse the physical properties of compact objects numerically. For given values of λ and k, the radial coordinate at which the pressure vanishes may be determined from eq. (14). The mass-to-radial distance m/b is estimated from eq. (18), which in turn determines the physical size of the compact star (r_0). For a given set of values of the parameters λ, A and k, the mass (m) and radius of a compact object is obtained in terms of the model parameter R. Thus, for a known mass of a compact star, R is determined which in turn determines the corresponding radius. As the equation to determine the parameters in the model is highly non-linear and intractable in known functional form, we adopt a numerical technique in the next section.

Table 3. Variation of central density for $A = 2$ for different values of λ and k.

λ	k	ρ_c in units of $(1.9 \times 10^{15})/R^2$ kg/m^3
1.4877×10^{-6}	2.9	0.0133
1.3836×10^{-5}	3	0.0117
0.0048	4	0.0039
0.0400	5	0.0019

Table 4. Variation of central density for $A = 4$ for different values of λ and k.

λ	k	ρ_c in units of $(1.9 \times 10^{15})/R^2$ kg/m^3
0.0185	1.7	0.0863
0.0289	1.8	0.0734
0.0432	1.9	0.0633
0.0876	2.1	0.0496
0.1211	2.2	0.0453
0.15	2.268	0.0431

The radial variation of pressure and density of compact stars for different parameters are shown in figures 1–5. It is evident that as λ is increased both the pressure and density at the centre is found to decrease and at the same time it corresponds to a smaller size accommodating more mass.

For a given mass of a compact star [12], it is possible to estimate the corresponding radius in terms of the parameter R. We note that for a given mass of a compact star known from observation, the radius of the star may be estimated from a given R. However, as the radius of a neutron star is $\leq$10 km, it is possible to obtain a class of stellar model taking different R so that the size of the star satisfies the upper bound. In the next sections we consider a few such stars whose masses are known from observations.

We present below four different models using stellar mass data [12–14]:

Model 1: We consider the X-ray pulsar, Her X-1 [12,15,16], which is characterized by mass $M = 1.47 M_\odot$, where $M_\odot =$ the solar mass, and found that it permits a star with radius $r_0 = 4.921$ km, for $R = 0.081$ km. The compactness of the star in this case is $u = M/r_0 = 0.30$. The ratio of the density at the boundary to that at the centre of the star is 0.0003 which is possible for the set of parameters $\lambda = 1.48 \times 10^{-6}$ and $k = 2.9$. Taking different values of R we get different models but a physically realistic model is obtained which accommodates a compact star with radius $\sim$10 km. For example, if

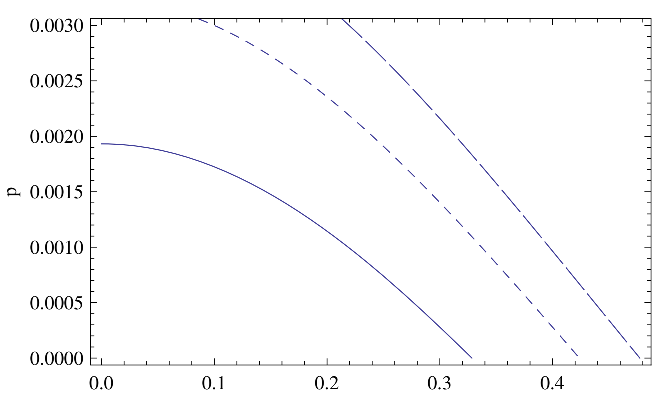

Figure 1. Plots of variation of pressure with radial distance (in units of R) for $\lambda = 0.15$ (solid line), for $\lambda = 0.1318$ (dashed line) and for $\lambda = 0.1211$ (broken line).

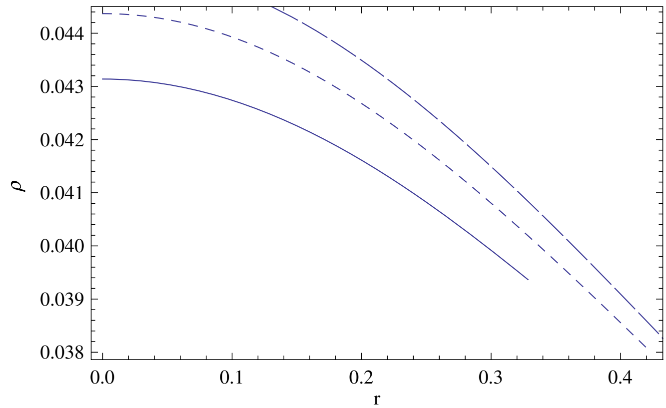

Figure 2. Plots of variations of density with radial distance (in units of R) for $\lambda = 0.15$ (solid line), for $\lambda = 0.1318$ (dashed line) and for $\lambda = 0.1211$ (broken line).

$R = 2.504$ km, one obtains a compact object with radius $r_0 = 7.791$ km. In the later case we note that the ratio of the density at the boundary to that at the centre is very high (0.99). The compactness of the star is 0.189 which is permitted for the set of parameters $\lambda = 0.0393$ and $k = 4.99$ with $A = 2$.

Model 2: We consider the X-ray pulsar, 4U 1700- 37, which is characterized by mass $M = 2.44 M_\odot$ [12]. We note that for $A = 4$, $\lambda = 0.1211$ and $k = 2.2$, the corresponding radius of this star is $r_0 = 8.197$ km with $R = 1.819$ km. The ratio of the density at the boundary to that at the centre of the star in this case is 0.820. However, for $A = 2$, $\lambda = 0.1656$ and $k = 2.3$, a compact object is permitted with radius $r_0 = 8.110$ km when $R = 0.135$ km. The ratio of the density at the boundary to that at the centre of the star in this case is 0.0003. Another stellar model is obtained for $A = 2$, $\lambda = 0.0393$ and $k = 4.99$, where the ratio of the density at the boundary to that at the centre is 0.99. In the later case the values are more compared to the values one obtains taking $A = 4$. However, both the cases permit a star with compactness factor $u = 0.3$.

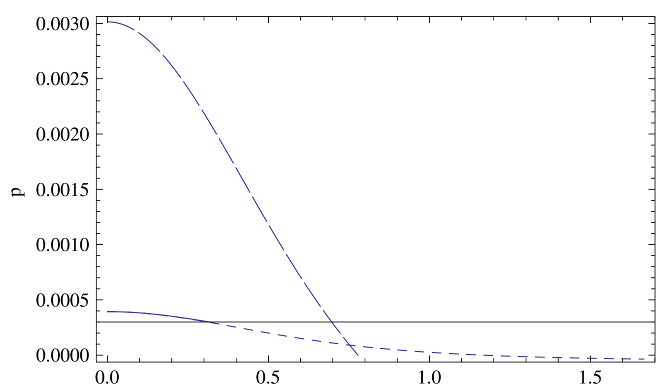

Figure 3. Plots of variations of pressure with radial distance (in units of R) for $A = 4$ (solid line), for $A = 3$ (broken line) and for $A = 2$ (dashed line).

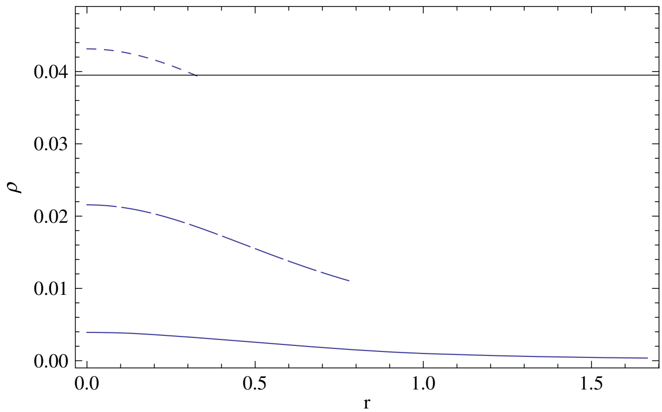

Figure 4. Plots of variations of density with radial distance (in units of R km) for $A = 4$ (solid line), $A = 3$ (broken line) and $A = 2$ (dashed line).

Model 3: We consider the neutron star, J1518+4904, which is characterized by mass $M = 0.72 M_{\odot}$ [12]. For $\lambda = 0.1211$, $k = 2.2$ and $A = 4$, the radius of the star estimated here is $r_0 = 2.419$ km with $R = 0.537$ km. The ratio of the density at the boundary to that at the centre of the star is 0.82. In this case the compactness factor u of the star $= 0.3$. For $A = 2$ we note the following: (i) when $\lambda = 1.48 \times 10^{-6}$ and $k = 2.9$, it admits a star with radius $r_0 = 2.4$ km for $R = 0.04$ km and (ii) when $\lambda = 0.0393$ and $k = 4.99$, it admits a star with radius $r_0 = 3.816$ km for $R = 1.226$ km. The ratio of the density at the boundary to that at the centre of the star in the first case is 0.0003 and that in the later case is 0.988. However, the compactness factor for the former is 0.3 which is higher than that in the second case (0.189).

Model 4: We consider the neutron star, J1748-2021 B, which is characterized by mass $M = 2.74 M_{\odot}$ [12]. For $A = 4$, $\lambda = 0.1318$ and $k = 2.2268$, a star of radius $r_0 = 9.281$ km with $R = 2.247$ km is permitted. The ratio of the density at the boundary to that at the centre of the star is 0.856. The compactness factor $u = 0.3$. In the other case one obtains a star with radius $r_0 = 8.467$ km with $R = 3.406$ km when $\lambda = 0.1656$ and $k = 2.3$. A

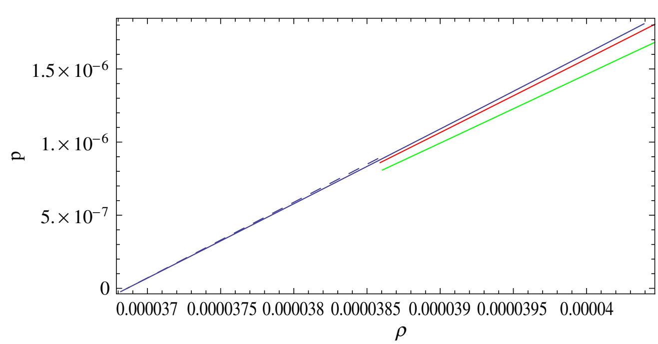

Figure 5. Plots of variations of pressure with density for $\lambda = 0.0876$ (green line), for $\lambda = 0.1318$ (red line), for $\lambda = 0.15$ (solid line) and for $\lambda = 0.165633$ (dashed line).

Table 5. Variation of mass and radius of a compact star for different values of λ and k for $R = 0.2$ km.

$A = 2$	M (mass) ($M_\odot$)	r_0 (radius) (km)
$\lambda = 1.48 \times 10^{-6}$, $k = 2.9$	3.61	12.087
$\lambda = 1.17 \times 10^{-5}$, $k = 2.99$	2.69	9.250
$\lambda = 1.38 \times 10^{-5}$, $k = 3.0$	2.63	9.067
$\lambda = 3.93 \times 10^{-2}$, $k = 4.99$	0.12	0.622

star of smaller size is thus permitted in the later case with compactness factor (0.32) than that of the formal model.

For $A = 2$, stellar model admits a star with radius $r_0 = 13.154$ km for $R = 2.74$ km, $\lambda = 0.138 \times 10^{-5}$ and $k = 3$. However, a smaller star with radius $r_0 = 8.380$ km is permitted here when $R = 0.181$ km with $\lambda = 1.17 \times 10^{-5}$ and $k = 2.99$. The ratio of the density at the boundary to that at the centre in the first case is 0.0017 which is higher than in the later (0.0015). The compactness factor in the former model is 0.20 which is lesser than the later case 0.32.

5. Discussions

In this paper, we present general relativistic solution for a class of compact stars which are in hydrostatic equilibrium considering the isotropic form for a static spherically symmetric matter distribution. The general relativistic solution obtained by Pant and Sah [2] is employed here to study compact objects. We use isotropic form of the exterior Schwarzschild solution to match at the boundary of the compact object. The stellar models discussed here contains four parameters λ, A, k and R. The observed mass of a star determines R for known values of λ, A, k.

We note the following: (i) In figure 1, variation of pressure with radial distance is plotted for different values of λ for same values of A and K. The figure shows that as λ increases pressure decreases inside the star. (ii) In figure 2, radial variation of density is plotted for different λ. We note higher density for lower λ. (iii) The variation of $\mathrm{d}p/\mathrm{d}\rho$ within the star for a given set of values of λ and k are shown in table 1. The causality condition is obeyed inside the star and $\mathrm{d}p/\mathrm{d}\rho$ is maximum at the centre which is however found to decrease monotonically radially outward. For different values of λ

Table 6. Variation of mass and radius of a compact star for different values of λ and k for $R = 2.5$ km.

$A = 4$	M (mass) ($M_\odot$)	b_0 (radius) (km)
$\lambda = 0.1211$, $k = 2.2$	3.35	11.268
$\lambda = 0.1318$, $k = 2.2268$	2.82	10.324
$\lambda = 0.15$, $k = 2.2681$	2.45	8.409
$\lambda = 8.76 \times 10^{-2}$, $k = 2.1$	4.19	13.688
$\lambda = 0.1656$, $k = 2.3$	1.79	6.214

Table 7. Variation of the ratio of the density at the boundary to the density at the centre of a compact star for different values of λ and k.

	$\lambda = 1.48 \times 10^{-6}$, $k = 2.9$	$\lambda = 8.76 \times 10^{-2}$, $k = 2.1$	$\lambda = 0.1211$, $k = 2.2$	$\lambda = 0.1318$, $k = 2.2268$	$\lambda = 0.15$, $k = 2.2681$	$\lambda = 0.1656$, $k = 4.99$
ρ_b/ρ_0	1.89×10^{-5}	0.45	0.54	0.58	0.69	0.81

and k, values of $dp/d\rho$ are also shown in table 2. It is evident that $dp/d\rho$ decreases when λ and k values increase. (iv) Variation of central density for different values of λ and k with $A = 2$ and $A = 4$ are presented in tables 3 and 4 respectively. We note that the central density decreases as the value for the pair (λ and k) increases. From tables 3 and 4 similar tendency for central density is found to exist when A is increased. As the isotropic Schwarzschild metric is singular at $m = 2b$, the model considered here may be useful for representing a strange star with $m \neq 2b$ or $m < 2b$. (v) In tables 5 and 6, the mass of a star with its maximum size is shown for different values of λ and k taking density of the star $\rho_b = 2 \times 10^{15}$ g/cm^3 at the boundary. We obtain here a class of relativistic stars for different values of λ, A, k and R. (vi) The density profile of a given star with different values of λ and k is shown in table 7. As λ increases the ratio of the density at the boundary to that at the centre is found to increase accommodating more compact stars. (vii) In figure 3, variation of pressure with radial distance is plotted for different values of A. It is evident that as A increases pressure decreases. (viii) In figure 4, variation of density with radial distance is plotted for different values of A. We note that as A is increased both the density and the pressure decrease. But the size of a star increases with an increase in A thereby accommodating more compact stars. (ix) In figure 5, variation of pressure with density is plotted for different values of λ. We note that for a given density, pressure is more for higher λ, and this leads to a star with higher central density.

In §4, we present models of the neutron stars that are tested for some known compact objects. As the equation of state is not known we analyse star of known geometry here. The radii of the compact stars, namely, neutron stars, are also estimated here for known mass with a given R. The parameter R permits a class of compact objects, some of which are relevant observationally. By considering the observed masses of the compact objects, namely, X-ray pulsars Her X-1, 4U 1700-37 and neutron stars J1518+4904, J1748-2021 B, we analyse the interior of the star. We obtain a class of compact star models for various R with given values of k, λ and A. The stellar models obtained here can accommodate highly compact objects. However, a detailed study of the stellar composition at high pressure and density will be taken up elsewhere.

Acknowledgements

BCP would like to acknowledge fruitful discussions with Mira Dey and Jisnu Dey while visiting IUCAA, Pune. Authors would like to thank IUCAA, Pune and IRC, Physics Department, North Bengal University (NBU) for providing facilities to complete the work. BCP would like to thank the University Grants Commission, New Delhi for financial support. RT is thankful to UGC for its award of Emeritus Fellowship. The authors would like to thank the referee for constructive criticism.

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