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Relativistic star solutions in higher-dimensional pseudospheroidal space-time

P K CHATTOPADHYAY[1] and B C PAUL[2,*]
[1]Department of Physics, Alipurduar College, P.O. Alipurduar Court, Dist. Jalpaiguri 736 122, India
[2]Physics Department, North Bengal University, Dist. Darjeeling 734 013, India
*Corresponding author. E-mail: bcpaul@iucaa.ernet.in

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Abstract. We obtain relativistic solutions of a class of compact stars in hydrostatic equilibrium in higher dimensions by assuming a pseudospheroidal geometry for the space-time. The space-time geometry is assumed to be $(D-1)$ pseudospheroid immersed in a D-dimensional Euclidean space. The spheroidicity parameter (λ) plays an important role in determining the equation of state of the matter content and the maximum radius of such stars. It is found that the core density of compact objects is approximately proportional to the square of the space-time dimensions (D), i.e., core of the star is denser in higher dimensions than that in conventional four dimensions. The central density of a compact star is also found to depend on the parameter λ. One obtains a physically interesting solution satisfying the acoustic condition when λ lies in the range $\lambda > (D+1)/(D-3)$ for the space-time dimensions ranging from $D = 4$ to 8 and $(D+1)/(D-3) < \lambda < (D^2-4D+3)/(D^2-8D-1)$ for space-time dimensions ≥ 9. The non-negativity of the energy density (ρ) constrains the parameter with a lower limit $(\lambda > 1)$. We note that in the case of a superdense compact object the number of space-time dimensions cannot be taken infinitely large, which is a different result from the braneworld model.

Keywords. Relativistic star; compact star; higher dimensions.

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1. Introduction

The framework of higher dimensions in understanding the issues in both cosmology and astrophysics draw lot of interest in the last couple of decades. Kaluza and Klein [1] independently initiated the study to unify gravity with electromagnetic interaction, by introducing an extra dimension, which is essentially an extension of Einstein's GR in five dimensions, which is of much interest in particle physics and cosmology. In the last couple of decades, various attempts were made to address some of the issues of stellar objects in higher-dimensional framework. The success of string theories in higher dimensions led to a spurt in activities to generalize the known four-dimensional astrophysical and cosmological results in the framework of higher-dimensional geometry. It is interesting to explore if there is any effect due to incorporation of extra dimensions in the theory. Attempts have been made to build cosmological models [2] in higher dimensions which may undergo a spontaneous compactification leading to a product space $M^4 \times M^d$, with M^d describing the compact inner space, which may describe the present Universe satisfactorily. In the context of localized source, higher-dimensional versions of the spherically symmetric Schwarzschild and Reissner–Nordstrom black holes [3,4], Kerr black holes [5,6], black holes in compactified space-time [7], no-hair theorem [8], Hawking radiation [5] and Vaidya solution [9] have been generalized. Shen and Tan [10] obtained a global regular solution of the higher-dimensional Schwarzschild space-time. The equation of state (EOS) and the behaviour of the matter in the central core region of highly compact objects like neutron stars, strange stars etc. are not well known. The relativistic models of the superdense stars in equilibrium may be studied in conventional approach by prescribing the EOS for the fluid forming the interior of the star or solving the Einstein's field equations connecting the interior space-time geometry with the dynamical variables of their physical content. To simplify the complexity of the field equations of such compact objects, the equations are solved by assuming a simple spatial geometry. Vaidya–Tikekar [11] and Tikekar [12] in this context proposed specific ansatz characterized by two geometrical parameters namely, λ and R prescribing specific 3-spheroidal geometries for the 3-space of the interior space-time of the star and discussed the various features and suitability of the two-parameter class of models that are obtained. An alternative to Vaidya–Tikekar ansatz for the physical space of the interior of a compact relativistic stars assuming a pseudospheroidal geometry was proposed by Tikekar and Thomas [13] and the core–envelope models of such stars are also investigated in the literature [14,15]. The space-time geometry is also considered to obtain a singularity-free cosmological model [16] in the presence of non-adiabatic dissipative heat flow. It may be interesting to note that the nature of superdense stars based on Vaidya–Tikekar ansatz have been extensively studied in spheroidal geometry by Maharaj and Leach [17], Mukherjee *et al* [18], Gupta and Jassim [19] and Paul [20,21] on the basis of the general solutions of the Einstein's equations.

Recently, a class of relativistic solution of compact star assuming a pseudo-spheroidal geometry in four dimensions was obtained by Tikekar and Jotania [22]. The motivation of the paper is to look for the effects of embedding the compact star in a higher-dimensional space-time. By assuming an isotropic pressure here, we obtain non-singular solution inside the D-dimensional sphere with finite values of the density and pressure at the centre of the star which obeys all the necessary energy conditions.

The paper is organized as follows: In §2, we give higher-dimensional Einstein field equation and its solution. In §3, the physical properties of a compact star is presented and finally in §4 a brief discussion is given.

2. Einstein's field equation and its solution

The Einstein's field equation in higher dimensions is given by

$$R_{AB} - \frac{1}{2} g_{AB} R = 8\pi G_D T_{AB}, \tag{1}$$

where D is the total number of space-time dimensions and the alphabets A, B correspond to $(0, 1, 2, \ldots, D-1)$, $G_D = GV_{D-4}$ is the gravitational constant in D dimensions (G denotes the four-dimensional Newton's constant, V_{D-4} is the volume of the extra dimension). R_{AB} is the Ricci tensor and T_{AB} is the energy–momentum tensor. We consider a higher-dimensional spherically symmetric static space-time in the form

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu(r)}\,\mathrm{d}t^2 + \mathrm{e}^{2\mu(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_n^2, \tag{2}$$

where $n = D - 2$ and $\mathrm{d}\Omega_n^2 = \mathrm{d}\theta_1^2 + \sin^2\theta_1 \mathrm{d}\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 \mathrm{d}\theta_3^2 + \cdots + (\sin^2\theta_1 \sin^2\theta_2 \cdots \sin^2\theta_{n-1} \mathrm{d}\theta_n^2)$, represents the metric on the n-sphere in polar co-ordinates.

The matter content of the star will be assumed to be a perfect fluid in equilibrium with energy–momentum tensor

$$T_{\mu\nu} = (\rho + p) u_{AB} - p g_{AB}, \tag{3}$$

where ρ and p represent the matter density and fluid pressure respectively. For a static star the unit D-velocity field u^A of the fluid has the expression

$$u^A = (\mathrm{e}^{-\nu}, 0, \ldots, 0). \tag{4}$$

The field equation (1) then reduces to the following set of three equations:

$$\frac{n\mu' \mathrm{e}^{-2\mu}}{r} + \frac{n(n-1)}{2} \frac{(1 - \mathrm{e}^{-2\mu})}{r^2} = 8\pi G_D \rho, \tag{5}$$

$$\frac{n\nu' \mathrm{e}^{-2\mu}}{r} - \frac{n(n-1)}{2} \frac{(1 - \mathrm{e}^{-2\mu})}{r^2} = 8\pi G_D; p, \tag{6}$$

$$\mathrm{e}^{-2\mu} \left[\nu'' + \nu'^2 - \mu'\nu' - \frac{(n-1)(\mu' - \nu')}{r} \right] - \frac{(n-1)(n-2)}{2} \frac{(1 - \mathrm{e}^{-2\mu})}{r^2} = 8\pi G_D; \rho, \tag{7}$$

where overhead dash denotes the derivative with respect to r. Using pressure isotropy condition, we get from eqs (6) and (7), the following equation:

$$\nu'' + \nu'^2 - \mu'\nu' - \frac{\nu'}{r} - \frac{(n-1)\mu'}{r} - \frac{(n-1)(1 - \mathrm{e}^{2\mu})}{r^2} = 0. \tag{8}$$

As μ and ν are functions of r, it is evident from eq. (8) that if μ is known then ν can be determined in terms of r and vice-versa.

To solve the system of eqs (5), (6) and (8) we use the ansatz in pseudospheroidal space-time as

$$\mathrm{e}^{2\mu} = \frac{1 + \lambda r^2/R^2}{1 + r^2/R^2}, \tag{9}$$

where R is an arbitrary constant which measures the size of the star and λ is a constant parameter, which represents the spheroidicity and leads to a regular space for $\lambda > 1$, flat when $\lambda = 1$ and open hyperboloid when $\lambda = 0$. Substituting eq. (9) in eq. (8) we get

$$(1 + \lambda - \lambda x^2)\Psi_{xx} + \lambda x \Psi_x + (n-1)(1-\lambda)\Psi = 0, \tag{10}$$

where $\Psi = \mathrm{e}^{\nu(r)}$ and the subscript represents the differentiation with respect to x.

Using the transformation $z = \sqrt{\lambda/(\lambda-1)}x$, where $x^2 = 1 + (r^2/R^2)$, the above equation reduces to

$$(1 - z^2)\Psi_{zz} + z\Psi_z + (n-1)(1-\lambda)\Psi = 0. \tag{11}$$

It is a second-order linear differential equation. The general solutions are given below in the known forms for three different cases:

Case i. $1 < \lambda < 2$

In this case the values of $n = D - 2$ and λ will be such that $\beta (= \sqrt{n - (n-1)\lambda}\,)$ is positive and the corresponding solution becomes

$$\begin{aligned}\Psi = {} & E[\beta\sqrt{z^2-1}\cosh(\beta\eta) - z\sinh(\beta\eta)] \\ & + F[\beta\sqrt{z^2-1}\sinh(\beta\eta) - z\cosh(\beta\eta)].\end{aligned} \tag{12}$$

Case ii. $\lambda = 2$

In this case $\beta = \sqrt{2-n}$ which is positive definite for $n < 2$ and the solution is given by eq. (12). However, for $n = 2$ one gets $\beta = 0$. In the latter case one recovers the solution obtained by Tikekar and Thomas [13].

Case iii. $\lambda > 2$

In this case $\beta = \sqrt{(n-1)\lambda - n}$ and is positive for any value of n, and the corresponding solution is

$$\begin{aligned}\Psi = {} & E[\beta\sqrt{z^2-1}\cos(\beta\eta) - z\sin(\beta\eta)] \\ & + F[\beta\sqrt{z^2-1}\sin(\beta\eta) - z\cos(\beta\eta)],\end{aligned} \tag{13}$$

where E and F are constants of integration, and $z = \cosh(\eta)$. It reduces to the solutions obtained by Tikekar and Jotania [22] for $D = 4$.

3. Physical properties of a compact star

From eqs (5) and (6) we determine the variation of energy density and pressure inside a compact stellar object qualitatively. The energy density and pressure are given by

$$\rho = \frac{n}{16\pi G_D R^2 (z^2-1)}\left[n - 1 + \frac{2}{(\lambda-1)(z^2-1)}\right], \tag{14}$$

$$p_r = -\frac{1}{8\pi G_D R^2 (z^2-1)} \left[\frac{n(n-1)}{2} - \frac{nz}{\lambda - 1} \frac{\Psi_z}{\Psi} \right]. \tag{15}$$

In the above it is useful to consider $\lambda > 1$ and $z > 1$. The parameter R which is related to the size of the compact object can be evaluated from the knowledge of the central density (ρ_0) of the star, which is given by

$$\rho_0 = \frac{n(n+1)(\lambda - 1)}{16\pi G_D R^2}. \tag{16}$$

For a given space-time dimension (D), the energy density and pressure at a point inside the compact object is determined in terms of the parameter R and λ and the two constants E and F. The energy density is positive definite inside the compact star when

$$\lambda > 1 - \frac{2}{(n-1)(z^2-1)}. \tag{17}$$

From the above inequality it is clear that the energy density at the centre will be positive definite if $\lambda > 1$ for a given space-time dimension. The constants E and F can be determined from the boundary conditions, i.e. by matching the solution with the exterior higher-dimensional Schwarschild solution at the boundary of a compact star, say $r = b$ and the idea that pressure should vanish at the surface of the star, i.e. $p(b) = 0$, where b represents the radius of the star. Now the other two parameters λ and n ($=D-2$) are yet to be determined. The stellar model is obtained for various values of λ in different space-time dimensions. It appears from eq. (15) that the pressure may have negative value inside the star, which is undesirable. To keep the pressure positive inside the star we obtain

$$\frac{\Psi_z}{\Psi} \geq \frac{(n-1)(\lambda - 1)}{2z}. \tag{18}$$

From the above inequality it is evident that the parameter λ plays a very important role in the determination of the equation of state of matter inside the compact object. Let us now determine the variation of pressure with respect to the energy density in order to determine the causality conditions. Using eqs (14) and (15) one obtains

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{z(z^2-1)^2(\Psi_z/\Psi)^2 + (z^2-1)(\Psi_z/\Psi)}{z(n-1)(z^2-1)(\lambda-1) + 4z}. \tag{19}$$

Here the acoustic condition ($(\mathrm{d}p/\mathrm{d}\rho) < 1$) is satisfied when

$$\frac{1}{z^2-1}\left(-\frac{1}{2z} - \Omega\right) \leq \frac{\Psi_z}{\Psi} \leq \frac{1}{z^2-1}\left(-\frac{1}{2z} + \Omega\right), \tag{20}$$

where

$$\Omega = \sqrt{4 + (\lambda-1)(n-1)(z^2-1) + \frac{1}{4z^2}}.$$

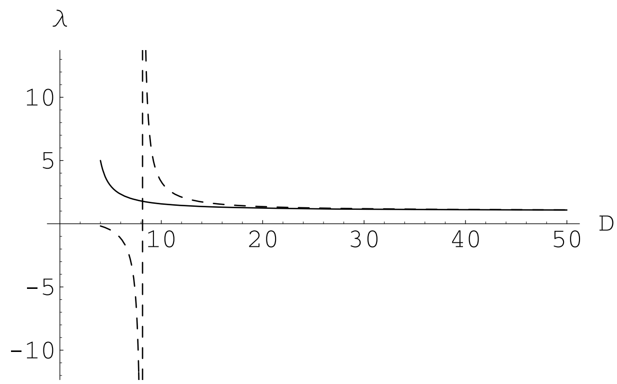

Figure 1. The variation of the parameter $\lambda_1 < (D^2-4D+3)/(D^2-8D-1)$ with the dimension D (dotted line) and that $\lambda_2 > (D+1)/(D-3)$ with the dimension D (solid line).

A realistic solution is permitted when Ψ_z/Ψ is real, which is attained when λ satisfies the following inequalities:

$$\text{(i)} \qquad \lambda = \lambda_1 < \frac{D^2-4D+3}{D^2-8D-1}, \tag{21}$$

$$\text{(ii)} \qquad \lambda = \lambda_2 > \frac{D+1}{D-3}. \tag{22}$$

In four dimensions the corresponding values are $\lambda < -3/17$ and $\lambda > 5$. To determine the allowed values of λ we study the variation of λ with space-time dimensions D. We plot a graph showing $\lambda < (D^2-4D+3)/(D^2-8D-1)$ with D (dotted line) and $\lambda > (D+1)/(D-3)$ with D (solid line) in figure 1. It is evident from figure 1 that for space-time dimension $D < 9$, λ can pick up values greater than $(D+1)/(D-3)$. In four dimensions one obtains a lower bound $\lambda > 5$ and in eight dimensions it is comparatively lower which is $\lambda > 1.8$. But for $D \geq 9$, the value of λ will lie in the limit $(D+1)/(D-3) < \lambda < (D^2-4D+3)/(D^2-8D-1)$. For example, in nine and ten dimensions λ satisfies different limiting values given by $1.667 < \lambda < 6$ and $1.572 < \lambda < 3.32$ respectively. It is also evident from the graph that for a large number of space-time dimensions λ tends to unity. Using inequality relations (18) and (20) we get an upper bound on the possible physically relevant square of the reduced radius (denoted by $Y = (b/R)^2$) which is given by

$$Y \leq \frac{[\sqrt{17}(D-\lambda) - [(D-2)^2(1-\lambda) + (D-2)(1+3\lambda) + (6-2\lambda)]}{\lambda(D-3)[(1-\lambda)(D-3)+4]}. \tag{23}$$

The upper limit on Y for various D and λ are given in table 1. It is evident from table 1 that the upper limit on the reduced radius decreases as λ increases for a given space-time dimension and the tendency is displayed in rows. Also the reduced

Table 1. The variation of $Y = (b/R)^2$ with λ for a given value of D.

			$(b/R)^2$			
$D = n + 2$	$\lambda = 2$	$\lambda = 3$	$\lambda = 5$	$\lambda = 7$	$\lambda = 10$	$\lambda = 100$
4					0.8753	0.0436
5			0.8928	0.4286	0.2379	0.0162
6			0.3333	0.1790	0.1038	0.0073
7		0.5773	0.1517	0.0809	0.0459	0.0029
8		0.2648	0.0621	0.0286	0.0136	0.0003
9	0.9041	0.1111	0.0088			
10	0.4211	0.0200				
11	0.1830					
12	0.0418					
13						

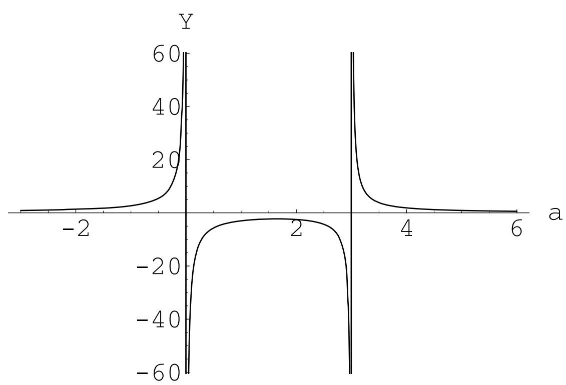

Figure 2. Variation of Y $(= (b/R)^2)$ with λ for $D = 5$.

radius decreases with space-time dimensions for a given λ which are presented in columns. The gap in the table is because of the fact that Y becomes negative for a given space-time dimension D with the particular value of λ. It is evident that λ plays an important role in the theory to accommodate a pseudospheroidal higher-dimensional space-time. In the case of spheroidal geometry it was shown [20] that for a given value of λ there is a limiting value for the space-time dimensions so that a physically realistic solution exists. However, in the case of pseudospheroidal space-time geometry, a large value of D for a given λ is also permitted. For a large number of dimensions $Y \to -(1/\lambda)$. In figures 2 and 3, the upper bound of compact stars with both positive and negative values of λ for $D = 5$ and 10 are shown respectively. It is evident that a star in $D = 5$ or in ten-dimensional space-time requires to be specified by $\lambda > 3$ and $\lambda > 1.6$ respectively. Thus it is evident that for a given space-time dimension, there is a lower bound on λ to make Y a positive quantity. Otherwise Y becomes negative and the corresponding reduced radius becomes an imaginary quantity which is unphysical.

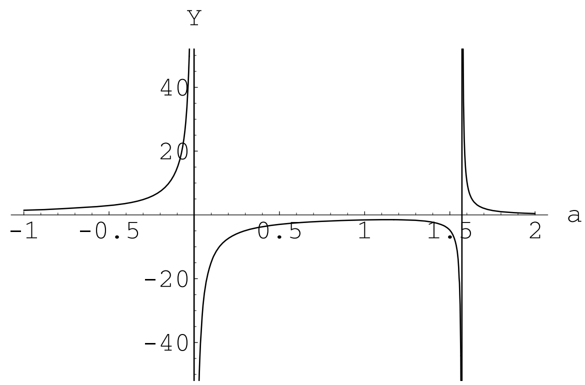

Figure 3. Variation of $Y\ (=(b/R)^2)$ with λ for $D = 10$.

Table 2. The variation of $(\mathrm{d}p/\mathrm{d}\rho)$ with r (in units of R) for a given value of n $(=D-2)$ and λ.

	$(\mathrm{d}p/\mathrm{d}\rho)$			
Distance (r)	$n = 2, \lambda = 6$	$n = 3, \lambda = 4$	$n = 7, \lambda = 2$	$n = 8, \lambda = 2$
0	0.1854	0.2674	0.6000	0.7159
0.1	0.1896	0.2654	0.5869	0.6967
0.2	0.1988	0.2570	0.5490	0.6423
0.3	0.2043	0.2369	0.4900	0.5606
0.4	0.1975	0.2019	0.4158	0.4621
0.5	0.1740	0.1531	0.3330	0.3572
0.6	0.1354	0.0963	0.2481	0.2545

However, for a given space-time dimension D, there are gaps in the table for some λ because it leads to unphysical solution corresponding to $Y > 1$. Thus in pseudospheroidal space-time we determine the allowed values of the parameter λ for a given space-time dimension for a realistic compact star solution.

The variation of $\mathrm{d}p/\mathrm{d}\rho$ with the distance (in the scale of parameter R) from the core of the star (HER-X1) for a given λ and space-time dimension ($n = D - 2$) are shown in table 2.

Table 2 shows that inside the compact star of a size given by eq. (23), the acoustic condition is maintained satisfactorily. Now, to understand the variation of energy density around the centre of a compact star (HER-1) with pressure we consider two examples with $D = 5$ and 10 and the corresponding variations are shown in figures 4 and 5 respectively for $\lambda = 3.2$. It is evident that as the space-time dimensions increase the core of the star becomes denser than that of a compact star of lower dimensions.

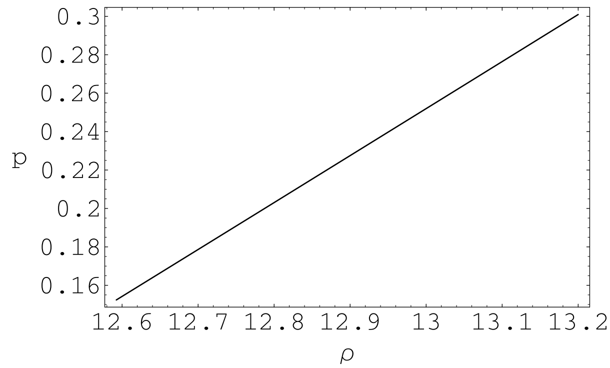

Figure 4. Energy density ρ along x-axis and pressure p (in the unit of $8\pi G_D R^2$) in y-axis for $D = 5$ with $E = 0.417$ and $F = -0.196$.

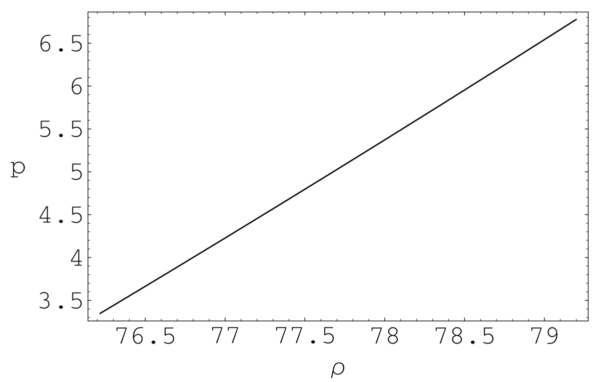

Figure 5. Energy density ρ along x-axis and pressure p (in units of $8\pi G_D R^2$) in y-axis for $D = 10$ with $E = -0.346$ and $F = -0.14$.

The size of the star is defined by $r = b$ and perfect fluid distribution of matter is extended up to the radius, outside which ($r > b$) the fluid distribution is described by the higher-dimensional analog of the Schwarzschild metric [10], which is

$$\mathrm{d}s^2 = -\left(1 - \frac{C}{r^{n-1}}\right)\mathrm{d}t^2 + \left(1 - \frac{C}{r^{n-1}}\right)^{-1}\mathrm{d}r^2 + \mathrm{d}r^2\mathrm{d}\Omega_n^2, \tag{24}$$

where C is a constant related to the mass of the star and $n = D - 2$. The mass of a higher-dimensional compact star is given by

$$M = \frac{nA_nC}{16\pi G_D}, \tag{25}$$

where $A_n = 2\pi^{n+1/2}/\Gamma((n+1)/2)$. The ratio of mass to radius is given by

$$\frac{M}{b} = \frac{(\lambda+1)}{2}\frac{b^{D-2}}{R^2(1+\lambda\frac{b^2}{R^2})}. \tag{26}$$

The mass contained within a star of radius r is given by

$$m(r) = 4\pi A_n \int_0^r r'^{D-2} \rho(r') \mathrm{d}r', \tag{27}$$

where $\rho(r')$ represents energy density at $r = r'$. The actual mass of a star can be obtained using eq. (22) and integrating it up to $r = b$, the maximum size of the compact object. The mass of a compact star is thus a function of both the parameter λ and space-time dimensions.

4. Discussion

In this paper we present a higher-dimensional relativistic solution of a spherically symmetric compact star in hydrostatic equilibrium. We obtain a class of solutions for a relativistic star assuming a pseudospheroid geometry in the framework of higher dimensions. The effects of higher dimensions on the mass, size and density of the star is presented. The compact star solution obtained by Tikekar and Jotania [22] is recovered for $D = 4$. It may be pointed out here that for $\lambda = 2$ in the usual four-dimensional space-time ($n = 2$), the solution obtained by Tikekar and Thomas [13] is relevant. Considering a compact object embedded in a higher-dimensional space-time and geometry characterized by the parameter λ of a $(D-1)$ pseudospheroidal space, the solutions are obtained here. The positivity condition of energy density constrains the parameter so that $\lambda > 1$. In order to satisfy acoustic condition ($\frac{\mathrm{d}p}{\mathrm{d}\rho} < 1$), the parameter λ must satisfy a lower bound $\lambda > (D+1)/(D-3)$ for space-time dimensions $D < 9$ and within the limit $(D+1)/(D-3) < \lambda < (D^2-4D+3)/(D^2-8D-1)$ for space-time dimensions $D \geq 9$. Instead of considering an equation of state $p = p(\rho)$ for the fluid inside the compact star we took an ansatz given by eq. (9). The ansatz considered here is important to determine the energy density and the pressure inside the compact object in terms of the parameter λ and we analysed the model qualitatively. It is found that the acoustic condition is obeyed inside the star. The maximum size of a star is determined by both λ and space-time dimensions. It is found that higher dimensions increase the central density of the star which increases as the square of the space-time dimensions. Consequently, for a given radius of a star the central density is comparatively more for a star if it is embedded in higher dimensions than the usual four-dimensional space-time. From eq. (16) we note that when λ tends to unity, central density of the star goes to zero which is not acceptable. Thus we note that a pseudospheroidal geometry does not allow a very large number of space-time dimensions. The result is not in support of what one obtains in brane-world cosmological models [23,24]. Also, for a given dimension of the space-time and radius we note that the higher-dimensional space-time accommodates a comparatively dense star if λ is increased. It is evident that on increasing the space-time dimensions one accommodates a more massive compact star for a given radius as compared to four dimensions. The stability of the solution due to radial perturbation and its applications will be discussed elsewhere.

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