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Hybrid synchronization of hyperchaotic Lu system

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Abstract. In this paper, we study the hybrid synchronization between two identical hyperchaotic Lu systems. Hybrid synchronization of hyperchaotic Lu system is achieved through synchronization of two pairs of states and anti-synchronization of the other two pairs of states. Active controls are designed to achieve hybrid synchronization between drive and response systems using the sum and difference of relevant variables of the chaotic systems. Numerical simulations are presented to evaluate the analysis and effectiveness of the controllers.

Keywords. Hybrid synchronization; hyperchaotic Lu system; Lyapunov stability theory; active control.

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1. Introduction

Chaos synchronization, an important topic in nonlinear science, has been developed and studied extensively in the last few years. Since Pecora and Carrol [1] introduced a method to synchronize two identical systems with different initial conditions, a variety of approaches has been proposed for the synchronization of chaotic systems which include complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], intermittent lag synchronization [5], time-scale synchronization [6], intermittent generalized synchronization [7], projective synchronization [8], modified projective synchronization [9,10] and function projective synchronization [11–13].

Complete synchronization is characterized by the equality of state variables while evolving in time. Anti-synchronization is characterized by the disappearance of the sum of relevant variables. Projective synchronization is characterized by the fact that the drive and response system could be synchronized upto a scaling factor whereas in modified projective synchronization (MPS) the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. Projective synchronization has attracted a lot of interest because of its proportionality between synchronized dynamical states. In the application of secure communications,

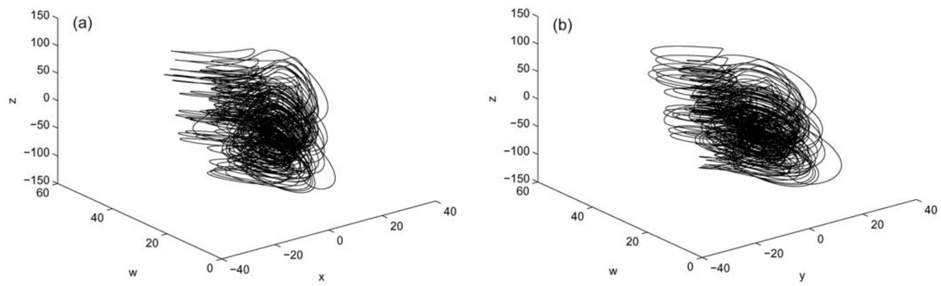

Figure 1(a) and **(b)**. The chaotic attractor for Lu hyperchaotic system.

this feature can be used for achieving fast communication. It is obvious that the complete synchronization (CS) and anti-phase synchronization (AS) are the special cases of the above synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

Recently, Li [14] studied hybrid synchronization behaviour in chaotic systems. In hybrid synchronization scheme, one part of the system is anti-synchronized and the other completely synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the system. The co-existence of CS and AS enhances security in communication and chaotic encryptation schemes.

It is believed that the chaotic systems with higher-dimensional attractor like hyperchaotic systems have much wider applications. In fact, the presence of more than one positive Lyapunov exponents clearly improves the security by generating more complex dynamics. Thus hyperchaos synchronization has become a new subject of active research.

In this work we study the hybrid synchronization behaviour in hyperchotic Lu system [15] using a nonlinear active control method which is simple, efficient and easy to implement in practical applications. We design active controllers so that two pairs of states are synchronized and the other two pairs are anti-synchronized. Numerical results verify that both synchronization and anti-synchronization can co-exist in the hyperchaotic Lu system.

2. System description

The hyperchaotic Lu system [15] is given by

$$\begin{aligned} &\dot{x} = a(y - x) + w, \quad \dot{y} = -xz + cy, \\ &\dot{z} = xy - bz, \quad \dot{w} = xz + rw, \end{aligned} \tag{1}$$

where x, y, z and w are the state variables and a, b, c and r are the real constants.

When $a = 36, b = 3, c = 20, -0.35 \le r \le 1.3$, system (1) is hyperchaotic. The chaotic attractor for the Lu hyperchaotic system is shown in figures 1a and 1b.

3. Hybrid synchronization of Lu hyperchaotic system

In order to observe the hybrid synchronization behaviour in the Lu hyperchaotic system, we have two Lu hyperchaotic systems where the drive system with the four state variables denoted by the subscript 1 drives the response system having identical equations denoted by the subscript 2. However, the initial condition on the drive system is different from that of the response system. The two Lu systems are described by the following equations:

$$\begin{aligned}&\dot{x}_1 = a(y_1 - x_1) + w_1, \quad \dot{y}_1 = -x_1 z_1 + c y_1,\\ &\dot{z}_1 = x_1 y_1 - b z_1, \quad \dot{w}_1 = x_1 z_1 + r w_1.\end{aligned} \tag{2}$$

and

$$\begin{aligned}&\dot{x}_2 = a(y_2 - x_2) + w_2 + u_1, \quad \dot{y}_2 = -x_2 z_2 + c y_2 + u_2,\\ &\dot{z}_2 = x_2 y_2 - b z_2 + u_3, \quad \dot{w}_2 = x_2 z_2 + r w_2 + u_4.\end{aligned} \tag{3}$$

We have introduced four active control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$. These control functions are to be determined for the purpose of hybrid synchronization of the two systems with the same parameters and different initial conditions.

For the hybrid synchronization, we define the state errors between the response system that is to be controlled and the controlling drive system as

$$e_1 = x_2 - x_1, \quad e_2 = y_2 + y_1, \quad e_3 = z_2 - z_1, \quad e_4 = w_2 + w_1. \tag{4}$$

Then the error system is given by

$$\begin{aligned}\dot{e}_1 &= -a e_1 + a e_2 + e_4 - 2a y_1 - 2 w_1 + u_1\\ \dot{e}_2 &= c e_2 - x_2 z_2 - x_1 z_1 + u_2\\ \dot{e}_3 &= -b e_3 + x_2 y_2 - x_1 y_1 + u_3\\ \dot{e}_4 &= r e_4 + x_2 z_2 + x_1 z_1 + u_4.\end{aligned} \tag{5}$$

We re-define the active control function $u = [u_1(t), u_2(t), u_3(t), u_4(t)]^T$ as

$$\begin{aligned}u_1(t) &= 2a y_1 + 2 w_1 + v_1(t)\\ u_2(t) &= x_2 z_2 + x_1 z_1 + v_2(t)\\ u_3(t) &= -x_2 y_2 + x_1 y_1 + v_3(t)\\ u_4(t) &= -x_2 z_2 - x_1 z_1 + v_4(t).\end{aligned} \tag{6}$$

Substituting (6) in (5) gives

$$\begin{aligned}&\dot{e}_1 = -a e_1 + a e_2 + e_4 + v_1(t), \quad \dot{e}_2 = c e_2 + v_2(t),\\ &\dot{e}_3 = -b e_3 + v_3(t), \quad \dot{e}_4 = r e_4 + v_4(t).\end{aligned} \tag{7}$$

Thus, the system (7) to be controlled is a linear system with the control input function $v = [v_1(t), v_2(t), v_3(t), v_4(t)]^T$ as functions of the error states e_1, e_2, e_3 and e_4. When (7) is stabilized by the feedback v, the error will converge to zero as

$t \to \infty$ which implies that the systems (2) and (3) are globally synchronized. To achieve this goal, we choose v such that

$$[v_1(t), v_2(t), v_3(t), v_4(t)]^T = A[e_1(t), e_2(t), e_3(t), e_4(t)], \tag{8}$$

where A is a 4×4 matrix. For (8) to be asymptotically stable, the feedback system of the element of the matrix A must have all of its eigenvalues with negative real parts. Various choices of A are possible. A good choice is

$$A = \begin{bmatrix} -1 & -a & 0 & -1 \\ -d & -2c & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2r \end{bmatrix} \tag{9}$$

with eigenvalues $\lambda_1 = -1, \lambda_2 = -2c, \lambda_3 = -1$ and $\lambda_4 = -2r$.

Thus yielding feedback functions as

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = \begin{bmatrix} -1 & -a & 0 & -1 \\ -d & -2c & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2r \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}. \tag{10}$$

Hence the error system becomes

$$\begin{aligned} \dot{e}_1 &= -(a+1)e_1, \quad \dot{e}_2 = -ce_2, \\ \dot{e}_3 &= -(b+1)e_3, \quad \dot{e}_4 = -re_4. \end{aligned} \tag{11}$$

Theorem. *Systems* (2) *and* (3) *can be exponentially and globally hybrid synchronized for any initial condition with the active nonlinear controllers* (6).

Proof. We choose the Lyapunov function as follows:

$$V(t) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{12}$$

By using the control law, the time derivative of $\dot{V}$ is obtained as

$$\begin{aligned} \dot{V}(t) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\ &= e_1(-(a+1)e_1) + e_2(-ce_2) + e_3(-(b+1)e_3) + e_4(-re_4) \\ &= -(a+1)e_1^2 - ce_2^2 - (b+1)e_3^2 - re_4^2 \\ &\le 0 \end{aligned}$$

which implies that $e_i = 0 (i = 1, 2, 3, 4)$ as $t \to \infty$ and guarantees the exponentially asymptotic stability of the error system (5). Therefore, systems (2) and (3) can achieve exponentially asymptotic hybrid synchronization for any initial condition with active nonlinear controllers (6).

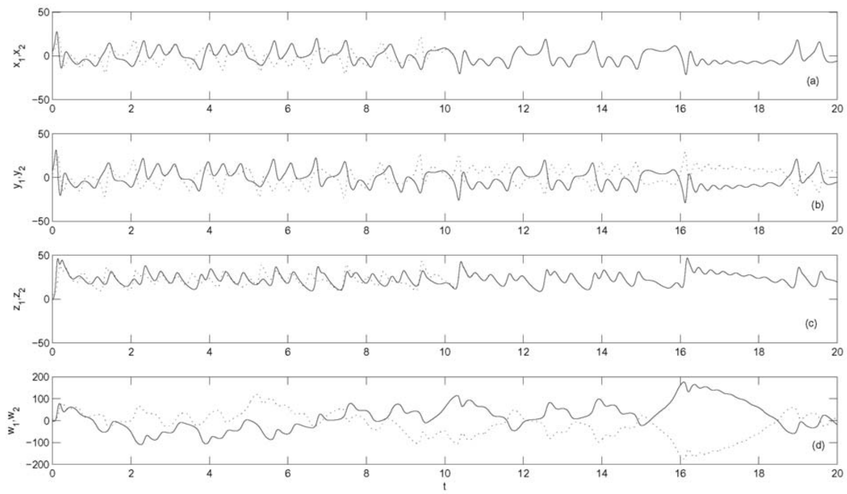

Figure 2. Time response of states for the drive system (x_1, y_1, z_1, w_1) and response system (x_2, y_2, z_2, w_2) with active control law activated after 10 s. (**a**) Signals x_1 and x_2, (**b**) signals y_1 and y_2, (**c**) signals z_1 and z_2 and (**d**) signals w_1 and w_2.

4. Simulation results

In the simulation, the fourth-order Runge–Kutta integration method is used to solve the two systems of differential equations (2) and (3) with time step size equal to 0.01. We select the parameters of the Lu hyperchaotic system as $a = 36, b = 3, c = 20, r = 1.3$ so that the Lu system exhibits hyperchaotic behaviour. The initial values for the drive and response systems are $x_1(0) = 5, y_1(0) = 8, z_1(0) = -1, w_1(0) = -3$ and $x_2(0) = 3, y_2(0) = 4, z_2(0) = 5, w_2(0) = 5$ respectively, while the initial values of error systems (4) are $(e_1(0), e_2(0), e_3(0), e_4(0)) = (-2, 12, 6, 2)$. Figures 2a–2d show the time response of states x_1, y_1, z_1 and w_1 for the drive system (2) and the states x_2, y_2, z_2 and w_2 for the response system (3) when active control is applied after 10 s. Figures 3a–3d display the time response of the error system (4).

Obviously, the synchronization errors converge to zero with exponentially asymptotical speed and hybrid synchronization of the two systems with different initial values are achieved very quickly.

5. Conclusion

This work demonstrates that hybrid synchronization between two identical Lu hyperchaotic systems can be achieved through active control method. Numerical simulations are used to verify the effectiveness of the proposed control techniques.

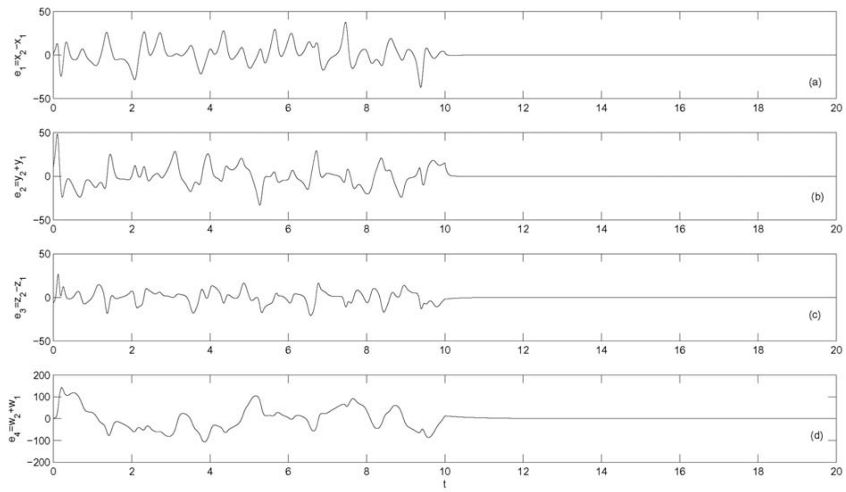

Figure 3. Time response of error states e_1, e_2, e_3 and e_4 with active control law activated after 10 s. (**a**) Error signal $e_1 = x_2 - x_1$, (**b**) error signal $e_2 = y_2 + y_1$, (**c**) error signal $e_3 = z_2 - z_1$ and (**d**) error signal $e_4 = w_2 + w_1$.

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