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Heavy flavor baryons in hypercentral model

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Abstract. Heavy flavor baryons containing single and double charm (beauty) quarks with light flavor combinations are studied using the hypercentral description of the three-body problem. The confinement potential is assumed as hypercentral Coulomb plus power potential with power index ν. The ground state masses of the heavy flavor, $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons are computed for different power indices, ν starting from 0.5 to 2.0. The predicted masses are found to attain a saturated value in each case of quark combinations beyond the power index $\nu = 1.0$.

Keywords. Hypercentral constituent quark model; charmed and beauty baryons; hyper-Coulomb plus power potential.

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1. Introduction

Recent experimental observations of a family of doubly charm baryons by SELEX, Fermi Laboratory and most of the other charm baryons discovered by CLEO experiments have generated much interest in the spectroscopy of heavy flavor baryons both experimentally and theoretically [1–9]. Baryons are not only interesting systems to study the quark dynamics and their properties, but also interesting from the point of view of simple systems to study three-body problems. Though there are many theoretical attempts to study the baryons [1–3], many of them do not provide the form factors that reproduce experimental data correctly [1]. For this reason alternate schemes to describe the properties of baryons particularly in the heavy flavor sector are being attempted [1,2]. Here, we employ the hypercentral approach to study the three-body problem, particularly the baryons constituting single- and double-charm (beauty) quarks. The confinement potential is assumed in the hypercentral coordinates of the Coulomb plus power potential form. It should be mentioned that, hypercentral potential contains the effects of the three-body force. As suggested by lattice QCD calculations [10] the three-body forces are important in the study of baryons. For the low-lying resonance states it is a good approximation to simply take the space wave functions of the hyper-Coulomb potential instead of seeking explicit numerical solution with hyperfine interaction.

2. The model

A correct treatment for three-body system is a longstanding problem in physics particularly in atomic and nuclear physics. Other three-body systems of interest are the baryons containing three quarks. Typical interactions among the three quarks are studied using the two-body quark potentials such as the Igsur Karl model, the Capstic and Isgur relativistic model, the chiral model, the harmonic oscillator model etc. The three-body effects are incorporated in such models through two-body and three-body spin-orbit terms. To describe the baryon as a bound state of three constituent quarks, we define the configuration of three particles by two Jacobi vectors $\vec{\rho}$ and $\vec{\lambda}$ as [11]

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2); \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \tag{1}$$

such that

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2}; \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)}. \tag{2}$$

Here m_1, m_2 and m_3 are the constituent quark masses. Further, we introduce the hyperspherical coordinates which are given by the angles

$$\Omega_\rho = (\theta_\rho, \phi_\rho); \quad \Omega_\lambda = (\theta_\lambda, \phi_\lambda) \tag{3}$$

together with the hyper-radius, x and hyperangle ξ respectively defined by

$$x = \sqrt{\rho^2 + \lambda^2}; \quad \xi = \arctan\left(\frac{\rho}{\lambda}\right). \tag{4}$$

As a model Hamiltonian for baryons, we consider

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{P^2}{2m} + V(x). \tag{5}$$

Here the potential V is not purely a two-body interaction but it contains three-body interactions also. If the interaction potential is hypercentral symmetric such that the potential depends on the hyper-radius x only, then the hyper-radial Schrödinger equation corresponds to the Hamiltonian given by eq. (5), and can be written as

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{5}{x}\frac{\mathrm{d}}{\mathrm{d}x} - \gamma(\gamma + 4)\right]\phi_\gamma(x) = -2m[E - V(x)]\,\phi_\gamma(x), \tag{6}$$

where γ is the grand angular quantum number and m is the reduced mass [12] which is defined as

Table 1. Quark model parameters.

Quark masses	$m_u = 338$ MeV
	$m_d = 350$ MeV
	$m_s = 400$ MeV
	$m_c = 1394$ MeV
	$m_b = 4510$ MeV
Model parameter	$b = 13.6$, $\frac{\beta}{m\,\tau} = 1$ (MeV)$^{\nu}$
Spin–spin interaction parameters	$A = 140.7$ MeV
	$\alpha = 850$ MeV

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda} \tag{7}$$

and potential $V(x)$ is taken as [13]

$$V(x) = -\frac{\tau}{x} + \beta x^{\nu} + \kappa + V_{\text{hyp}}(x). \tag{8}$$

Here the hyperfine part of the potential $V_{\text{hyp}}(x)$ is given by [2]

$$V_{\text{hyp}}(x) = A\mathrm{e}^{-\alpha x} \sum_{i \neq j} \sigma_i \cdot \sigma_j, \tag{9}$$

where τ, β, A, κ and α are potential parameters. The energy eigenvalue corresponding to eq. (6), is obtained using virial theorem for different choices of the potential index ν. The trial wave function is taken as the hyper-Coulomb radial wave function given by [2]

$$\psi_{\omega\gamma} = \left[\frac{(\omega-\gamma)!(2g)^6}{(2\omega+5)(\omega+\gamma+4)!}\right]^{1/2} (2gx)^{\gamma} \mathrm{e}^{-gx}. \tag{10}$$

The baryon mass in this hypercentral model is given by

$$M_{\text{B}} = \sum_{i=1}^{3} m_i + \langle H \rangle. \tag{11}$$

The constituent quark mass parameters employed in our calculations are listed in table 1 along with other potential parameters. Here κ is found to be proportional to the reduced mass, the flavor-color degree of freedom ($N_{\text{f}}N_{\text{c}}$) as well as the strong coupling constant α_{s} as

$$\kappa \propto N_{\text{c}} N_{\text{f}} m \alpha_{\text{s}} (1 + O(\alpha_{\text{s}}^2)). \tag{12}$$

It is found that the proportionality constant is equal to 0.41 for the qqQ systems and 0.32 for the QQq systems. The computations are repeated for different choices of ν, from 0.5 to 2.0 and the hyperfine interaction energy is treated perturbatively.

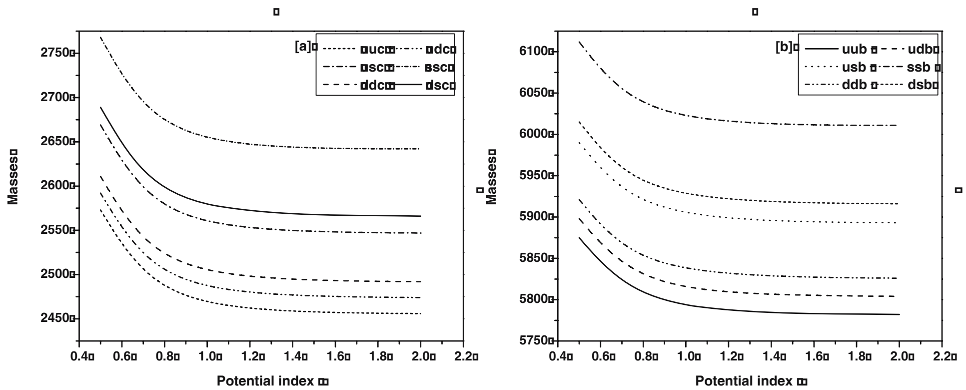

Figure 1. Variation of spin average masses with potential index ν for single heavy baryons. (**a**) Single charm baryons, (**b**) single beauty baryons.

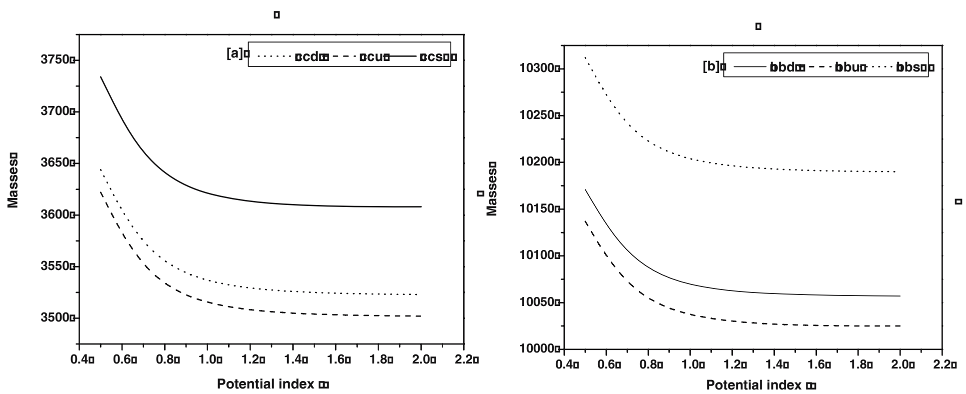

Figure 2. Variation of spin average masses with potential index ν for doubly heavy baryons. (**a**) Doubly charm baryons, (**b**) doubly beauty baryons.

3. Results and discussion

The behavior of the spin average mass of the baryons with the potential index ν in the case of qqQ and qQQ systems are shown in figures 1a,b and 2a,b respectively. It is found that the mass of the baryon decreases as ν increases and attains a saturated value beyond $\nu = 1$. It may be due to the saturation of effective inter-quark interaction within the baryon at potential index $\nu > 1.0$. The computed results for the ground state mass of the single charm, single beauty and double heavy baryons are presented in tables 2, 3 and 4 respectively. We compare our masses at this saturation ($\nu > 1.0$) with other theoretical and existing experimental values.

Our results are found to be in accordance with the known experimental as well as with other theoretical predictions in the case of single heavy baryons at the mass saturation. The variation with the PDG average values are just around 1.0% in the case of single charm baryons and less than 1.0% in the case of single beauty baryons. Consistency has also been found in the case of double heavy systems with the potential index $\nu \geq 1.0$ with other theoretical predictions. Our results at the

Table 2. Single charm baryon masses (masses are in MeV).

Baryon	P.I.(ν)	$J^P = \frac{1}{2}^+$	Others	$J^P = \frac{3}{2}^+$	Others
Σ_c^{++}	0.5	2539	2453 [14]	2607	–
(uuc)	0.7	2463	2454±0.18 [4]	2527	2518±0.6 [4]
	1.0	2432	2460±80 [15]	2495	2440±70 [15]
	1.5	2425		2488	
	2.0	2425		2488	
Σ_c^{+}	0.5	2557	2451 [14]	2627	–
(udc)	0.7	2480	2439 [16]	2546	2518 [16]
	1.0	2449	2453 [17]	2514	2520 [17]
	1.5	2442	2452 [18]	2507	2538 [18]
	2.0	2442	2448 [19]	2507	2505 [19]
			2453 ± 0.4 [4]		2518 ± 2.3 [4]
Σ_c^{0}	0.5	2575	2452 [14]	2647	–
(ddc)	0.7	2497	2454±0.18 [4]	2566	2518±0.5 [4]
	1.0	2466		2533	
	1.5	2460		2526	
	2.0	2460		2526	
Ξ_c^{+}	0.5	2630	2466 [14]	2708	–
(usc)	0.7	2550	2481 [16]	2625	2654 [16]
	1.0	2518	2468 [17]	2591	2650 [17]
	1.5	2512	2473 [18]	2584	2680 [18]
	2.0	2512	2496 [19]	2584	2633 [19]
			2468±0.4 [4]		2647±1.4 [4]
			2410±50 [15]		2550±80 [15]
Ξ_c^{0}	0.5	2648	2472 [14]	2729	–
(dsc)	0.7	2567	2471±0.4 [4]	2645	2646±1.2 [4]
	1.0	2536		2611	
	1.5	2529		2604	
	2.0	2529		2604	
Ω_c^{0}	0.5	2723	2698 [14]	2813	–
(ssc)	0.7	2639	2698 [16]	2726	2768 [16]
	1.0	2607	2710 [17]	2692	2770 [17]
	1.5	2601	2678 [18]	2684	2752 [18]
	2.0	2601	2701 [19]	2684	2759 [19]
			2680±70 [15]		2660±80 [15]
			2698±2.6 [4]		

saturated value of the masses are very close (< 1.0% difference) to the theoretical predictions of Gershtain *et al* [23] and Kiselev *et al* [21]. However, the predictions of Albertus *et al* [20] are found to be nearer to our predicted masses at $\nu = 0.5$. The recent observations of SELEX group [24] on double charmed baryonic state

Table 3. Single beauty baryon masses (masses are in MeV)

Baryon	P.I.(ν)	$J^P = \frac{1}{2}^+$	Others	$J^P = \frac{3}{2}^+$	Others
Σ_b^+	0.5	5862	5820 [14]	5889	
(uub)	0.7	5803	5770±70 [15]	5828	5780±70 [15]
	1.0	5778	$5808^{+02}_{-2.3}$±1.7 [9]	5801	$5829^{+1.6}_{-1.8}$±1.7 [9]
	1.5	5772		5793	
	2.0	5772		5793	
Σ_b^-	0.5	5908	5820 [14]	5937	–
(ddb)	0.7	5849	$5816^{+01}_{-01} \pm 1.7$ [9]	5875	$5837^{+2.1}_{-1.9}$±1.7 [9]
	1.0	5823		5847	
	1.5	5816		5840	
	2.0	5816		5840	
Σ_b^0	0.5	5884	5624 [14]	5912	–
(udb)	0.7	5825	5805 [3]	5851	5834 [3]
	1.0	5800	5820 [17]	5823	5850 [17]
	1.5	5793	5847 [18]	5816	5871 [18]
	2.0	5793	5789 [19]	5816	5844 [19]
Ξ_b^0	0.5	5974	5624 [14]	6007	–
(usb)	0.7	5913	5805 [3]	5943	5963 [3]
	1.0	5887	5820 [17]	5915	5980 [17]
	1.5	5880	5847 [18]	5907	5959 [18]
	2.0	5880	5789 [19]	5907	5967 [19]
			5760±60 [15]		5900±80 [15]
Ξ_b^-	0.5	5997	5800 [14]	6032	–
(dsb)	0.7	5936		5967	
	1.0	5909		5938	
	1.5	5903		5931	
	2.0	5903		5931	
Ω_b^-	0.5	6092	6040 [14]	6132	–
(ssb)	0.7	6028	6065 [3]	6064	6088 [3]
	1.0	6001	6060 [17]	6035	6090 [17]
	1.5	5994	6040 [18]	6028	6060 [18]
	2.0	5994	6037 [19]	6028	6090 [19]
			5990±70 [15]		6000±70 [15]

Ξ_{cc}^+ and Ξ_{cc}^{*+} are found to be very close to our predicted values. Our predicted mass difference $M(\Xi_{cc}^{*+}) - M(\Xi_{cc}^+)$ of 73.3 MeV is extremely close to the lattice QCD prediction of 76.6 MeV [15]. New experimental results are expected to provide the masses of many of the double heavy flavor charm and beauty baryons.

Table 4. Doubly heavy baryon masses (masses are in MeV).

Baryon	P.I.(ν)	$J^P = \frac{1}{2}^+$	Others	$J^P = \frac{3}{2}^+$	Others
Ξ_{cc}^{++}	0.5	3583	3612^{+17} [20]	3660	3706^{+23} [20]
(ccu)	0.7	3505	3620 [16]	3578	3727 [16]
	1.0	3475	3480 [21]	3545	3610 [21]
	1.5	3468	3740 [22]	3537	3860 [22]
	2.0	3468	3478 [23]	3537	3610 [23]
			3541 [24]		
Ξ_{cc}^{+}	0.5	3604	3605±23 [25]	3684	3685±23 [25]
(ccd)	0.7	3525	3620 [16]	3601	3727 [16]
	1.0	3494	3480 [21]	3567	3610 [21]
	1.5	3487	3740 [22]	3560	3860 [22]
	2.0	3487	3478 [23]	3560	3610 [23]
			3443 [24]		3520 [24]
Ω_{cc}^{+}	0.5	3687	3702^{+41} [20]	3782	3783^{+22} [20]
(ccs)	0.7	3604	3778 [16]	3693	3872 [16]
	1.0	3572	3590 [21]	3659	3690 [21]
	1.5	3566	3760 [22]	3651	3900 [22]
	2.0	3566	3590 [23]	3651	3690 [23]
			3733±09 [25]		3801±09 [25]
Ξ_{bb}^{0}	0.5	10105	10197^{+10}_{-17} [20]	10170	10236^{+09}_{-17} [20]
(bbu)	0.7	10032	10202 [16]	10092	10237 [16]
	1.0	10004	10090 [21]	10060	10130 [21]
	1.5	9998	10300 [22]	10053	10340 [22]
	2.0	9998	10093 [23]	10053	10133 [23]
			10314±47 [26]		10333±45 [26]
Ξ_{bb}^{-}	0.5	10137	10197^{+10}_{-17} [20]	10206	10236^{+09}_{-17} [20]
(bbd)	0.7	10063	10202 [16]	10127	10237 [16]
	1.0	10034	10090 [21]	10095	10130 [21]
	1.5	10028	10300 [22]	10087	10340 [22]
	2.0	10028	10314±47 [26]	10087	10333±45 [26]
Ω_{bb}^{-}	0.5	10269	10260^{+14}_{-34} [20]	10355	10297^{+05}_{-28} [20]
(bbs)	0.7	10190	10359 [16]	10270	10389 [16]
	1.0	10160	10180 [21]	10236	10200 [21]
	1.5	10154	10340 [22]	10228	10380 [22]
	2.0	10154	10180 [23]	10228	10200 [23]
			10365±40 [26]		10383±39 [26]

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