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Modelling of the magnetic and magnetostrictive properties of high permeability Mn–Zn ferrites

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Abstract. This paper presents the results of modelling of both magnetic and magnetostrictive properties of high permeability $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrites. The parameters of energy-based Jiles–Atherton–Sablik (J–A–S) model were calculated for each experimental hysteresis loop on the basis of evolutionary strategies and Hook–Jevis optimization method. Finally, high conformity between experimental and modelling results was achieved. This high conformity indicates that the presented results create new opportunity of modelling of the properties of inductive components based on ferrites as well as quantitative description of magnetization process.

Keywords. Magnetic and magnetostrictive hysteresis modelling; soft ferrites.

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1. Introduction

Despite the growing popularity of soft metallic glasses in the market of magnetic cores for inductive components, high permeability Mn–Zn ferrites are still important materials for technical applications [1]. From the applications point of view, the magnetic, magnetostrictive and magnetoelastic properties of Mn–Zn ferrites should be considered together [2]. Only in such a case, the complete description of properties of high permeability ferrites, required for effective development process, can be provided for both designers and users of mechatronic, inductive components.

Because magnetization process is highly sophisticated, its description requires the physical model. The description of functional properties of the magnetic material can be given by the model's parameters – linked with physical properties of the material. That is why energy-based Jiles–Atherton–Sablik model of magnetic [3] and magnetostrictive hysteresis [4] seems to be the most suitable one. Moreover, this model enables further integration of magnetoelastic properties [5].

2. Experimental and calculation procedures

Magnetic characteristics of the frame-shaped sample made of $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite were measured in quasi-static conditions at room temperature. Measurements were carried out using HBPL hysteresis graph. HBPL hysteresis graph consists of precise function generator, voltage-to-current converter as well as the an ultra-stable integrator. Before each measurement the drift compensation of the integrator as well as sample demagnetization were performed.

For the measurements of magnetostrictive characteristics, semiconductor strain-gauge sensors were applied [6]. The magnetostriction was measured in the direction of the magnetizing field at room temperature.

The volume density d_v of the ferrite sample was 4940 kg/m^3. The frame-shaped sample was 70 mm high, 30 mm wide and 15 mm thick. Magnetizing coil had 20 turns, whereas detecting coil consisted of 50 turns. Until now, the deterministic method of calculation of parameters of Jiles–Atherton–Sablik model was not presented in the literature. Due to the fact that iteration-based method of calculation of the model's parameters is often not effective enough [7], different methods of optimization were used [8]. Generally speaking, this optimization is based on minimization of the target function F where

$$F = \sum_{1}^{n} (B_{mi} - B_{di})^2. \tag{1}$$

B_{mi} is the value of flux density B calculated from the model for the magnetizing field H and B_{di} is the experimental value of flux density B for the same magnetizing field H.

It should be indicated that function F exhibits considerable number of local minima. As a result, all gradient-based methods of optimization of the function F are strongly dependent on initial conditions of the optimization. For this reason, soft-computing methods such as genetic algorithms, evolutionary strategies or simulated annealing should be applied for the minimization of the function F.

From the engineering point of view, application of the genetic algorithms for the determination of Jiles–Atherton–Sablik model parameters (as it was presented in [9]) seems to be reasonable, but not optimal. This is mainly caused by the fact that genetic algorithms are less effective than evolutionary strategies for the minimization of continuous functions [10].

For the above reason the evolutionary strategies were applied with the following procedure. First, Jiles–Atherton–Sablik model's parameters were determined utilizing $(\mu + \lambda)$ evolutionary strategy [10], during the minimization of function F based on the experimental results. Next, typical Hook–Jevis gradient optimization [10] was used with low initial step. Additional application of gradient optimization gives the possibility of final adjustment of the model's parameters.

It should be indicated that in the present investigation, the parameters of the model of magnetic and magnetostrictive hysteresis loops were calculated simultaneously during the optimization process. Such solution increases reliability and coherence of the results.

3. Modelling of the magnetic properties

In Jiles–Atherton–Sablik model, the anhysteric magnetization M_{ah} (necessary to calculate the magnetization M) is calculated from the Langevin equation [5]:

$$M_{\mathrm{ah}} = M_{\mathrm{s}} \left(\coth \frac{H_{\mathrm{e}}}{a} - \frac{a}{H_{\mathrm{e}}} \right), \tag{2}$$

where M_{s} is the saturation magnetization, a determines the shape of the hysteresis loop and $H_{\mathrm{e}} = H + \alpha M$ is the effective magnetic field caused by the magnetizing field H and considering (by parameter α) the magnetic interactions between moments.

In the Jiles–Atherton–Sablik model, the magnetization M is the sum of reversible magnetization M_{rev} and irreversible magnetization M_{irr}:

$$M = M_{\mathrm{rev}} + M_{\mathrm{irr}}. \tag{3}$$

Reversible magnetization $M_{\mathrm{rev}} = c(M_{\mathrm{ah}} - M_{\mathrm{irr}})$ takes into account the c parameter, which is the reversibility coefficient. The irreversible magnetization M_{irr} is given by the differential equation [5]

$$\frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}H} = \frac{M_{\mathrm{ah}} - M_{\mathrm{irr}}}{k\delta - (\alpha M_{\mathrm{ah}} - M_{\mathrm{irr}} \frac{\mathrm{d}M_{\mathrm{irr}}}{\mathrm{d}M})}, \tag{4}$$

where k is the average energy required for breaking the pinning sites and δ is equal to $+1$ for increasing magnetizing field H, and -1 for decreasing magnetic field.

Table 1 presents the above parameters calculated for experimentally measured hysteresis loop (H was increasing linearly up to 80 A/m) of $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite sample. Calculated value of k confirms, that for high permeability materials, k is close to the coercive force H_{c} [5] of the material.

In figure 1 both experimental and modelled hysteresis loops are presented. It should be highlighted that high correspondence of experimental and modelled characteristics were achieved. This high correspondence of experimental and modelled characteristics is confirmed by the fact that Pearson r^2 indicator is surpassing 99%.

4. Modelling of the magnetostrictive properties

In a simplified model, the magnetostriction λ can be calculated from quadratic equation as a function of magnetization M [11]:

Table 1. Model parameters calculated for the magnetic hysteresis loop of $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite.

Parameter	Value	Unit
a	9.42	A/m
k	7.45	A/m
c	0.83	–
M_{s}	$3.41 \cdot 10^5$	A/m
α	$2.08 \cdot 10^5$	–

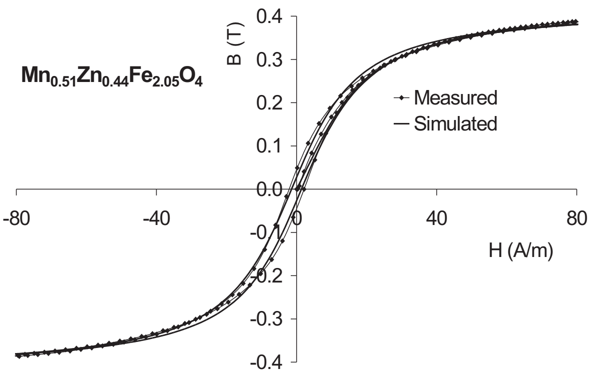

Figure 1. Magnetic hysteresis loop $B(H)$ of Mn–Zn ferrite measured and calculated from the model.

$$\lambda = \frac{3\lambda_s}{2M_s^2}M^2, \tag{5}$$

where λ_s is the saturation magnetostriction. However, this quadratic model does not give possibility of modelling of the hysteresis in $\lambda(B)$ characteristics or 'lift-off' phenomenon, as is presented in figures 2a and 3a. Because hysteresis in $\lambda(B)$ and 'lift-off' are confirmed by experiments [6], for physical modelling of the magnetostriction, different approaches should be considered.

In Jiles–Atherton–Sablik model, the calculation of the magnetostriction λ requires calculation of the magnetic energy density ϕ_{mag} [5]:

$$\phi_{mag}(M) = \frac{1}{2}\mu_0(\alpha M^2 + \alpha''(M_{ah} - M)^2 - 2\alpha'(M_{ah} - M)H), \tag{6}$$

where α' describes hysteresis in $\lambda(B)$ relation and α'' is connected with 'lift-off' phenomenon. Finally, the magnetostriction λ can be calculated from eqs (7)–(9):

$$\lambda(M) = t_1\left(\sqrt{1 + t_2 \cdot \phi_{mag}(M_s)} - \left(1 + t_2 \cdot \sqrt{\phi_{mag}(M_s) - \phi_{mag}(M)}\right)\right), \tag{7}$$

$$t_1 = \frac{4b(1+\nu)}{9Y}, \tag{8}$$

$$t_2 = \frac{9Y}{2b^2(1+\nu)^2}, \tag{9}$$

where Y is the Young's modulus of the material, ν is its Poisson ratio and b is the isotropic magnetoelastic coupling constant [5]. Magnetostrictive parameters calculated from the experimental hysteresis $\lambda(H)$ loop of $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite are presented in table 2.

Table 2. Model parameters calculated for magnetostrictive hysteresis loop of $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite.

Parameter	Value	Unit
b	$4.16 \cdot 10^3$	N/m^2
Y	$1.49 \cdot 10^{11}$	N/m^2
ν	0.35	–
α'	$3.01 \cdot 10^{-5}$	–
α''	$2.88 \cdot 10^{-3}$	–

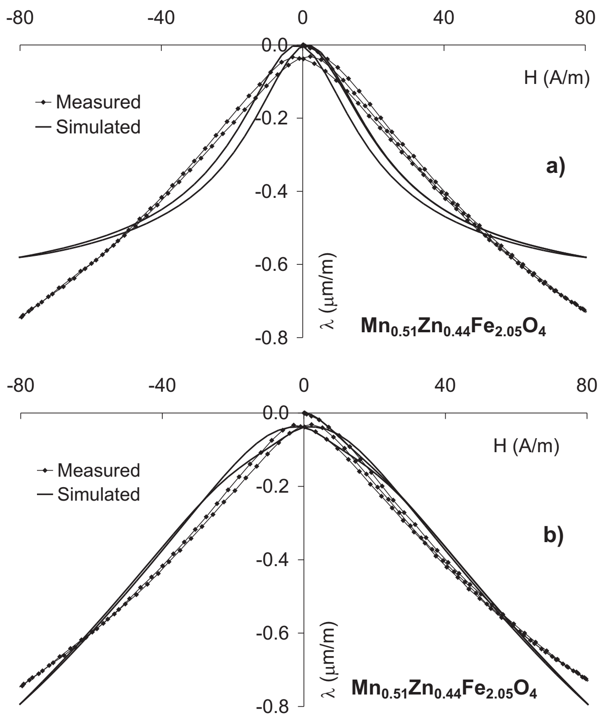

Figure 2. Magnetostrictive hysteresis loop $\lambda(H)$ of Mn–Zn ferrite measured and calculated from the model. (**a**) Quadratic dependence, (**b**) J–A–S model.

Figures 2b and 3b present respectively the $\lambda(H)$ and $\lambda(B)$ hysteresis loops calculated according to Jiles–Atherton–Sablik model.

It should be indicated that in saturation, the magnetization M and anhysteric magnetization $M_{\rm ah}$ are equal to $M_{\rm s}$. In such a case, saturation magnetostriction $\lambda_{\rm s}$ can be calculated from eqs (6)–(9) as

$$\lambda(M_{\rm s}) = \lambda_{\rm s} = t_1 \left(\sqrt{1 + \tfrac{1}{2} t_2 \mu_0 \alpha M_{\rm s}^2} - 1 \right). \tag{10}$$

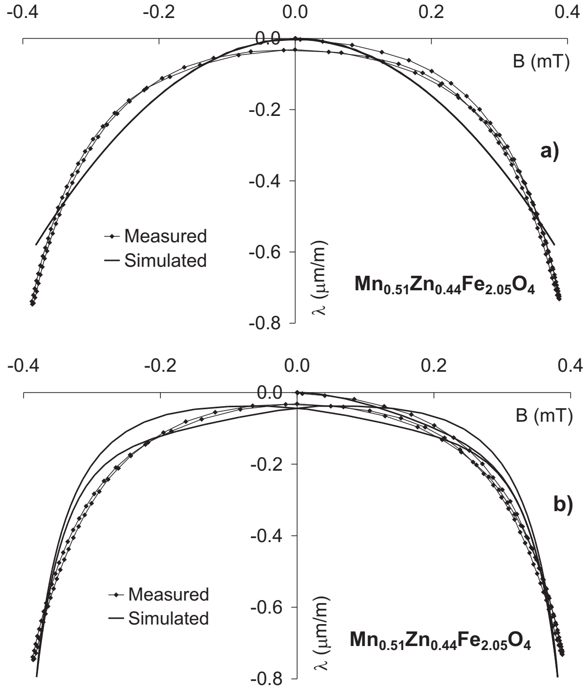

Figure 3. Magnetostrictive hysteresis loop $\lambda(B)$ of Mn–Zn ferrite measured and calculated from the model. (**a**) Quadratic dependence, (**b**) J–A–S model.

For parameters given in table 2, value of saturation magnetostriction λ_s, calculated from eq. (10), is equal to 1.3 μm/m. This result corresponds appropriately to the results of experimental measurements of saturation magnetostriction λ_s of other high permeability Mn–Zn ferrites [12].

It should be indicated that modelling of the magnetostrictive hysteresis loops $\lambda(H)$ and $\lambda(B)$ gives worse correspondence between experimental and calculated characteristics than modelling of $B(H)$ characteristics. This suggests that the presented model of magnetostrictive hysteresis still requires some improvements.

On the other hand, the Pearson r^2 indicator for experimental and modelled $\lambda(H)$ characteristics is about 98.

5. Conclusion

Jiles–Atherton–Sablik model is a good option for modelling the magnetic hysteresis loop $B(H)$ of high permeability $Mn_{0.51}Zn_{0.44}Fe_{2.05}O_4$ ferrite. In this case the Pearson r^2 indicator, between experimental and calculated data, is exciding 99%.

In the case of the modelling of $\lambda(H)$ and $\lambda(B)$ hysteresis loops, the correspondence achieved (r^2 about 98%) was much worse. In this case, however both 'lift-off' and hysteresis in $\lambda(B)$ characteristic were observed. This fact proves that Jiles–Atherton–Sablik model guarantees better results, than quadratic model.

Acceptable correspondence between experimental and modelled data confirms that evolutionary strategies $(\mu+\lambda)$ together with Hook–Jevis gradient optimization can be used for the determination of Jiles–Atherton–Sablik model's parameters. Moreover, the achieved results indicate that such a model may be sufficient for a great majority of technical applications, where the prediction of magnetic and magnetostrictive properties of soft ferrites is required.

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