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Bianchi Type-V model with a perfect fluid and Λ-term

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Abstract. A self-consistent system of gravitational field with a binary mixture of perfect fluid and dark energy given by a cosmological constant has been considered in Bianchi Type-V universe. The perfect fluid is chosen to be obeying either the equation of state $p = \gamma\rho$ with $\gamma \in [0, 1]$ or a van der Waals equation of state. The role of Λ-term in the evolution of the Bianchi Type-V universe has been studied.

Keywords. Bianchi-type; perfect fluid; lambda term.

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1. Introduction

In view of its importance in explaining the observational cosmology, many workers have considered cosmological models with dark energy. In a recent paper, Kremer [1] has modelled the universe as a binary mixture whose constituents are described by a van der Waals fluid and dark energy. Zlatev *et al* [2] showed that 'tracker field', a form of quintessence, may explain the coincidence, adding a new motivation for the quintessence scenario. The fate of density perturbation in a universe dominated by the Chaplygin gas, which exhibits negative pressure was studied by Fabris *et al* [3]. Models with Chaplygin gas were also studied by Bento *et al* [4] and Dev *et al* [5]. These authors restricted their study to a spatially flat, homogeneous and isotropic universe described by a FRW metric. Since the theoretical arguments and recent experimental data support the existence of an anisotropic phase, it makes sense to consider the models of the universe with anisotropic background in the presence of dark energy. Saha [6,7] has studied the role of Λ-term in the evolution of Bianchi Type-I universe in the presence of spinor and/or scalar field with a perfect fluid satisfying equation of state $p = \gamma\rho$. Saha [7,8] has studied the evolution of an anisotropic universe given by a Bianchi Type-I space–time in the presence of a perfect fluid obeying not only $p = \gamma\rho$, but also the van der Waals equation of state. In the present work we have studied the evolution of Bianchi Type-V universe in the presence of a perfect fluid with equation of state $p = \gamma\rho$ or van der Waals fluid [1] and dark energy given by a cosmological constant. We have followed the method due to Saha [7–10] and Kremer [1].

2. Basic equation

The Einstein field equations are in the form

$$R_i^j - \frac{1}{2}\delta_i^j R = kT_i^j + \delta_i^j \Lambda. \tag{2.1}$$

Here R_i^j is the Ricci tensor, R is the Ricci scalar, k is the Einstein gravitational constant and Λ is the cosmological constant. A positive Λ corresponds to the universal repulsion force, while a negative one gives an attractive force. Note that a positive Λ is often taken to be a form of dark energy. We study the gravitational field given by Bianchi Type-V cosmological model and choose it in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\,\mathrm{d}x^2 - a_2^2 \mathrm{e}^{-2mx}\,\mathrm{d}y^2 - a_3^2 \mathrm{e}^{-2mx}\,\mathrm{d}z^2 \tag{2.2}$$

with the metric functions a_1, a_2, a_3 being functions of t only and m is a constant.

The Einstein field equations (2.1) for the Bianchi Type-V space–time, in the presence of the Λ term, can be written in the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1^2} = kT_1^1 + \Lambda. \tag{2.3a}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2} = kT_2^2 + \Lambda. \tag{2.3b}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = kT_3^3 + \Lambda. \tag{2.3c}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1} - \frac{3m^2}{a_1^2} = kT_0^0 + \Lambda. \tag{2.3d}$$

$$\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = \frac{2\dot{a}_1}{a_1}. \tag{2.3e}$$

From (2.3e) we have $a_2 a_3 = a_1^2$.

Here, overhead dot denotes differentiation with respect to t. The energy–momentum tensor of the source is given by

$$T_i^j = (\rho + p)u_i u^j - p\delta_i^j, \tag{2.4}$$

where u^i is the flow vector satisfying

$$g_{ij}u^iu^j = 1. \tag{2.5}$$

Here ρ is the total energy density of a perfect fluid and/or dark energy, while p is the corresponding pressure. p and ρ are related by an equation of state.

In a co-moving system of coordinates, from eq. (2.4) one finds

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = -p. \tag{2.6}$$

Now using eqs (2.3a)–(2.3e) and eq. (2.6) we obtain

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m^2}{a_1^2} = -kp + \Lambda. \tag{2.7a}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{m^2}{a_1^2} = -kp + \Lambda. \tag{2.7b}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{m^2}{a_1^2} = -kp + \Lambda. \tag{2.7c}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{3m^2}{a_1^2} = k\rho + \Lambda. \tag{2.7d}$$

$$a_2a_3 = a_1^2. \tag{2.7e}$$

We follow the method used by Saha [7] to solve eqs (2.7a)–(2.7d) and use $a_2a_3 = a_1^2$. Subtracting eq. (2.7b) from eq. (2.7a), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \tag{2.8}$$

Let V be a function of t defined by

$$V = a_1a_2a_3. \tag{2.9}$$

Then from eqs (2.8) and (2.9) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} = 0. \tag{2.10}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2} = d_1 \exp\left(x_1 \int \frac{\mathrm{d}t}{V}\right), \quad d_1 = \text{constant}, \; x_1 = \text{constant}. \tag{2.11}$$

By subtracting eq. (2.7c) from (2.7a) and eq. (2.7a) from (2.7b), we get

$$\frac{a_1}{a_3} = d_2 \exp\left(x_2 \int \frac{\mathrm{d}t}{V}\right), \quad d_2 = \text{constant}, \; x_2 = \text{constant}, \tag{2.12a}$$

$$\frac{a_2}{a_3} = d_3 \exp\left(x_3 \int \frac{\mathrm{d}t}{V}\right), \quad d_3 = \text{constant}, \; x_3 = \text{constant}, \tag{2.12b}$$

where d_2, d_3, x_2, x_3 are integration constants.

In view of the relations $V = a_1a_2a_3$ we find the following relation between the constants $d_1, d_2, d_3, x_1, x_2, x_3$.

$$d_2 = d_1 d_3, \quad x_2 = x_1 + x_3.$$

Finally from eqs (2.11) and (2.12), we write $a_1(t)$, $a_2(t)$, and $a_3(t)$ in the explicit form.

$$a_1(t) = D_1 V^{1/3} \exp\left(X_1 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.13a}$$

$$a_2(t) = D_2 V^{1/3} \exp\left(X_2 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.13b}$$

$$a_3(t) = D_3 V^{1/3} \exp\left(X_3 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.13c}$$

where D_i $(i = 1,2,3)$ and X_i $(i = 1,2,3)$ satisfy the relation $D_1D_2D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eq. (2.7e) we get

$$X_1 = 0, \quad X_2 = -X_3 = X, \quad D_1 = 1, \quad D_2 = D_3^{-1} = D. \tag{2.14}$$

Then eq. (2.13) can be written as

$$a_1(t) = V^{1/3}, \tag{2.15a}$$

$$a_2(t) = DV^{1/3} \exp\left(X \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.15b}$$

$$a_3(t) = D^{-1} V^{1/3} \exp\left(-X \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.15c}$$

where X and D are constants.

Now, by adding eqs (2.7a), (2.7b), (2.7c) and three times eq. (2.7d), we get

$$\begin{aligned}&\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) - \frac{6m^2}{a_1^2}\\ &\quad = \frac{3k(\rho - p)}{2} + 3\Lambda.\end{aligned} \tag{2.16}$$

From eq. (2.9) we have

$$\frac{\ddot{V}}{V} = \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right). \tag{2.17}$$

From eqs (2.16), (2.17) and (2.15a) we obtain

$$\frac{\ddot{V}}{V} - \frac{6m^2}{V^{2/3}} = \frac{3k(\rho - p)}{2} + 3\Lambda. \tag{2.18}$$

On the other hand, the conservational law for the energy–momentum tensor gives

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + p). \tag{2.19}$$

From (2.18) and (2.19) we have

$$\dot{V}^2 = 3(2k\rho + \Lambda)V^2 + 9m^2V^{4/3} + C_1 \tag{2.20}$$

with C_1 being an integration constant. Let us define the Hubble constant as

$$\frac{\dot{V}}{V} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = 3H. \tag{2.21}$$

From eqs (2.20) and (2.21) we have

$$k\rho = \frac{3}{2}H^2 - \frac{3m^2}{2V^{2/3}} - \frac{\Lambda}{2} - \frac{C_1}{6V^2}. \tag{2.22}$$

It should be noted that the energy density of the universe is a positive quantity. It is believed that at the early stages of evolution when the volume scale V was close to zero, the energy density of the universe was infinitely large. On the other hand, with the expansion of the universe, i.e., with the increase of V, the energy density ρ decreases and an infinitely large V corresponds to a ρ close to zero. In that case, from eq. (2.22), it follows that

$$3H^2 - \Lambda \longrightarrow 0. \tag{2.23}$$

As seen from eq. (2.23), in this case Λ is essentially non-negative. We can also conclude from (2.23) that in the absence of a Λ term, beginning from some value of V the evolution of the universe becomes standstill, i.e., V becomes constant, since H becomes zero, whereas in the case of a positive Λ the process of evolution of the universe never comes to halt. Moreover, it is believed that the presence of the dark energy results in the accelerated expansion of the universe. As far as negative Λ is concerned, its presence imposes some restriction on ρ, namely, ρ can never be small enough to be ignored. It means in that case there exists some upper limit for V as well.

From eqs (2.21), (2.22), and (2.18), we obtain

$$\dot{H} = -\frac{k}{2}(\rho + p) - \frac{2m^2}{V^{2/3}} + \frac{\Lambda}{2} - \frac{C_1}{6V^2}. \tag{2.24}$$

Let us now go back to eq. (2.20). It is in fact the first integral of eq. (2.18) and can be written as

$$\dot{V} = \pm\sqrt{C_1 + 3(2k\rho + \Lambda)V^2 + 9m^2V^{4/3}}. \tag{2.25}$$

On the other hand, rewriting (2.19) in the form

$$\frac{\dot{\rho}}{\rho + p} = -\frac{\dot{V}}{V} \tag{2.26}$$

and taking into account the pressure and the energy density obeying an equation of state of type $p = f(\rho)$, we conclude that ρ and p, hence the right-hand side of eq. (2.18) is a function of V only.

$$\ddot{V} = \frac{3k}{2}(\rho - p)V + 3\Lambda V + 6m^2V^{1/3} \equiv F(V). \tag{2.27}$$

From the mechanical point of view, eq. (2.27) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then the following first integral exists:

$$\dot{V} = \sqrt{2[\varepsilon - U(V)]}. \tag{2.28}$$

Here ε can be viewed as energy and $U(V)$ is the potential of the force F. Comparing eqs (2.25) and (2.28) we find $\varepsilon = C_1/2$ and

$$U(V) = -\left[\frac{3}{2}(k\rho + \Lambda)V^2 + \frac{9}{2}m^2V^{4/3}\right]. \tag{2.29}$$

Finally, we write the solution to eq. (2.25) in quadrature form

$$\int \frac{\mathrm{d}V}{\sqrt{C_1 + 3(k\rho + \Lambda)V^2 + \frac{9}{2}m^2V^{4/3}}} = t + t_0, \tag{2.30}$$

where the integration constant t_0 can be taken to be zero, since it only gives a shift in time.

Essentially we have followed the method due to Saha [7].

3. Universe filled with perfect fluid

In this section we consider the case when the source field is given by a perfect fluid. Here we study two possibilities: (i) The energy density and the pressure of the perfect fluid are connected by a linear equation of state and (ii) the equation of state is a nonlinear (van der Waals) one.

3.1 *Universe as a perfect fluid with* $p_{\rm PF} = \gamma\rho_{\rm PF}$

In this subsection we consider the case when the source field is given by a perfect fluid obeying the equation of state

$$p_{\rm PF} = \gamma\rho_{\rm PF}. \tag{3.1}$$

Here γ is a constant and lies in the interval $\gamma \in [0,1]$. Depending on its numerical value γ describes the following types of universe.

$$\gamma = 0 \quad \text{(dust universe)} \tag{3.2a}$$

$$\gamma = 1/3 \quad \text{(radiation universe)} \tag{3.2b}$$

$$\gamma \in (1/3, 1) \quad \text{(hard universe)} \tag{3.2c}$$

$$\gamma = 1 \quad \text{(Zeldovich universe or stiff matter).} \tag{3.2d}$$

In view of eq. (3.1), from eq. (2.19) for the energy density and pressure one obtains

$$\rho_{\rm PF} = \frac{\rho_0}{V^{1+\gamma}}, \quad p_{\rm PF} = \frac{\gamma\rho_0}{V^{1+\gamma}}, \tag{3.3}$$

where ρ_0 is a constant of integration. For V from eq. (2.30) one find

$$\int \frac{{\rm d}V}{\sqrt{C_1 + 3(k\rho_0 V^{1-\gamma} + \Lambda V^2) + 9m^2 V^{4/3}}} = t. \tag{3.4}$$

In the absence of the Λ term one immediately finds

$$\int \frac{{\rm d}V}{\sqrt{C_1 + 3k\rho_0 V^{1-\gamma} + 9m^2 V^{4/3}}} = t. \tag{3.5}$$

3.2 *Universe as a van der Waals fluid*

Here we consider the case when the source field is given by a perfect fluid with a van der Waals equation of state in the absence of dissipative process. The pressure of the van der Waals fluid $p_{\rm w}$ is related to its energy density $\rho_{\rm w}$ [1] by

$$p_{\rm w} = \frac{8W\rho_{\rm w}}{3-\rho_{\rm w}} - 3\rho_{\rm w}^2. \tag{3.6}$$

In (3.6) the pressure and the energy density are written in terms of dimensionless reduced variables and W is a parameter connected with a reduced temperature.

Inserting eq. (3.6) into (2.24), on account of eq. (2.22) we find

$$\dot{H} = -\frac{\begin{array}{r}(\frac{3}{2}H^2 - \frac{3m^2}{2V^{2/3}} - \frac{\Lambda}{2} - \frac{C_1}{6V^2})[(8W+3)k - 10(\frac{3}{2}H^2 - \frac{3m^2}{2V^{2/3}} - \frac{\Lambda}{2} - \frac{C_1}{6V^2}) \\ + \frac{3}{k}(\frac{3}{2}H^2 - \frac{3m^2}{2V^{2/3}} - \frac{\Lambda}{2} - \frac{C_1}{6V^2})]\end{array}}{2[3k - (\frac{3}{2}H^2 - \frac{3m^2}{2V^{2/3}} - \frac{\Lambda}{2} - \frac{C_1}{6V^2})]}$$
$$-\frac{2m^2}{V^{2/3}} + \frac{\Lambda}{2} - \frac{C_1}{6V^2}. \tag{3.7}$$

It can be easily verified that eq. (3.7) in the absence of Λ term and $C_1 = 0$ and $k = 3$, reduces to

$$\dot{H} = -\frac{3}{2}\left[\frac{1}{2}\left(H^2 - \frac{m^2}{V^{2/3}}\right) + \frac{8W\left(\frac{1}{2}\left(H^2 - \frac{m^2}{V^{2/3}}\right)\right)}{3 - \frac{1}{2}\left(H^2 - \frac{m^2}{V^{2/3}}\right)}\right.$$
$$\left. -3\left(\frac{1}{2}\left(H^2 - \frac{m^2}{V^{2/3}}\right)\right)^2\right] - \frac{2m^2}{V^{2/3}}. \tag{3.8}$$

4. Some particular cases

Case I. $\gamma = 1/3$ (disordered radiation)
For $C_1 = 0$, eq. (3.4) reduces to

$$\int \frac{\mathrm{d}V}{\sqrt{3k\rho_0 V^{2/3} + 3\Lambda V^2 + 9m^2 V^{4/3}}} = t \tag{4.1}$$

which gives

$$V = \left[\mathrm{e}^{m^2}\sqrt{\frac{3}{k\rho_0}}t - \frac{2k\rho_0}{3m^2}\right]^{3/2}, \quad \text{when } \Lambda = \frac{9m^4}{4k\rho_0}, \tag{4.2a}$$

$$V = \left[\left(\frac{k\rho_0}{\Lambda} - \frac{9m^4}{4\Lambda^2}\right)^{1/2}\sinh\left(2\sqrt{\frac{\Lambda}{3}}t\right) - \frac{3m^2}{2\Lambda}\right]^{3/2}, \quad \text{when } \Lambda > \frac{9m^4}{4k\rho_0}, \tag{4.2b}$$

$$V = \left[\left(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda}\right)^{1/2}\cosh\left(2\sqrt{\frac{\Lambda}{3}}t\right) - \frac{3m^2}{2\Lambda}\right]^{3/2}, \quad \text{when } \Lambda < \frac{9m^4}{4k\rho_0}. \tag{4.2c}$$

We consider these subcases separately.

Case I(a). $\Lambda = 9m^4/4k\rho_0$

Then from eqs (2.15) and (4.2a), we obtain

$$a_1(t) = (\mathrm{e}^{C_2 t} - C_3)^{1/2}, \tag{4.3a}$$

$$a_2(t) = D(\mathrm{e}^{C_2 t} - C_3)^{1/2} \times \exp\left[\frac{2X}{C_2 C_3}\left(\frac{1}{\sqrt{C_3}}\tan^{-1}\sqrt{\frac{C_3}{(\mathrm{e}^{C_2 t} - C_3)}} - \frac{1}{\sqrt{(\mathrm{e}^{C_2 t} - C_3)}}\right)\right], \tag{4.3b}$$

$$a_3(t) = D^{-1}(\mathrm{e}^{C_2 t} - C_3)^{1/2} \times \exp\left[-\frac{2X}{C_2 C_3}\left(\frac{1}{\sqrt{C_3}}\tan^{-1}\sqrt{\frac{C_3}{(\mathrm{e}^{C_2 t} - C_3)}} - \frac{1}{\sqrt{(\mathrm{e}^{C_2 t} - C_3)}}\right)\right], \tag{4.3c}$$

where

$$C_2 = m^2\sqrt{\frac{3}{k\rho_0}} \quad \text{and} \quad C_3 = \frac{2k\rho_0}{3m^2}.$$

From eqs (3.3) and (4.2a), we have

$$\rho = \rho_0\left[\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t} - \frac{2k\rho_0}{3m^2}\right]^{-2} \tag{4.4a}$$

and

$$p = \frac{\rho_0}{3}\left[\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t} - \frac{2k\rho_0}{3m^2}\right]^{-2}. \tag{4.4b}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2 and the deceleration parameter q. They are defined as

$$\theta = 3H. \tag{4.5}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{4.6}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}AH^2. \tag{4.7}$$

$$q = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1. \tag{4.8}$$

In this case these quantities are

$$\theta = \frac{3m^2}{2}\sqrt{\frac{3}{k\rho_0}}\frac{\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t}}{\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t} - \frac{2k\rho_0}{3m^2}} \tag{4.9}$$

$$A = \frac{8X^2}{3a^2}\frac{\left[\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t} - \frac{2k\rho_0}{3m^2}\right]^3}{\mathrm{e}^{2m^2\sqrt{\frac{3}{k\rho_0}}t}} \tag{4.10}$$

$$\sigma^2 = X^2\left[\mathrm{e}^{m^2\sqrt{\frac{3}{k\rho_0}}t} - \frac{2k\rho_0}{3m^2}\right] \tag{4.11}$$

$$q = -1 + \frac{4k\rho_0}{3m^2}\mathrm{e}^{-m^2\sqrt{\frac{3}{k\rho_0}}t}. \tag{4.12}$$

For a finite value of t, pressure and density tend to infinity. Therefore, the model has a future singularity in finite time.

Case I(b). $\Lambda > \frac{9m^4}{4k\rho_0}$

Then for small t (i.e. near singularity $t = 0$),

$$\sinh\left(2\sqrt{\frac{\Lambda}{3}}t\right) \approx 2\sqrt{\frac{\Lambda}{3}}t. \tag{4.13}$$

Then eq. (4.2b) reduces to

$$V = \left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right]^{3/2}. \tag{4.14}$$

From eqs (2.15) and (4.14), we obtain

$$a_1(t) = (C_4 t - C_5)^{1/2}, \tag{4.15a}$$

$$a_2(t) = D(C_4 t - C_5)^{1/2}\ \exp\left[-\frac{2X}{C_4\sqrt{C_4 t - C_5}}\right], \tag{4.15b}$$

$$a_3(t) = D^{-1}(C_4 t - C_5)^{1/2}\ \exp\left[\frac{2X}{C_4\sqrt{C_4 t - C_5}}\right], \tag{4.15c}$$

where

$$C_4 = \frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} \quad \text{and} \quad C_5 = \frac{3m^2}{2\Lambda}.$$

From eqs (3.3) and (4.14), we have

$$\rho = \rho_0 \left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right]^{-2} \tag{4.16a}$$

and

$$p = \frac{\rho_0}{3} \left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right]^{-2}. \tag{4.16b}$$

With the use of eqs (4.5)–(4.8) we can express the physical quantities as

$$\theta = \frac{\sqrt{3}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2}}{\left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right]} \tag{4.17}$$

$$A = \frac{8X^2}{3a^2} \left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right]^{3} \tag{4.18}$$

$$\sigma^2 = X^2 \left[\frac{2}{\sqrt{3}}\left(k\rho_0 - \frac{9m^4}{4\Lambda}\right)^{1/2} t - \frac{3m^2}{2\Lambda}\right] \tag{4.19}$$

$$q = 1. \tag{4.20}$$

For a finite value of t, pressure and density become infinite. Therefore, the model has a future singularity in finite time.

Case I(c). $\Lambda < \frac{9m^4}{4k\rho_0}$

Then for small t (i.e. near singularity $t = 0$),

$$\cosh\left(2\sqrt{\frac{\Lambda}{3}}t\right) \approx 1 + \frac{\Lambda}{3}t^2. \tag{4.21}$$

Then eq. (4.2c) reduces to

$$V = \left[\left(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9}\right)^{1/2} t^2 + \left(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda}\right)^{1/2} - \frac{3m^2}{2\Lambda}\right]^{3/2}. \tag{4.22}$$

From eqs (2.15) and (4.22), we obtain

$$a_1(t) = (C_6 t - C_7)^{1/2}, \tag{4.23a}$$

$$a_2(t) = D(C_6 t - C_7)^{1/2} \ \exp\left[\frac{X}{C_7\sqrt{C_6}}\left(\frac{C_6 t^2}{C_6 t^2 + C_7}\right)^{1/2}\right], \tag{4.23b}$$

$$a_3(t) = D^{-1}(C_6 t - C_7)^{1/2} \ \exp\left[-\frac{X}{C_7\sqrt{C_6}}\left(\frac{C_6 t^2}{C_6 t^2 + C_7}\right)^{1/2}\right], \tag{4.23c}$$

where

$$C_6 = \left(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9}\right)^{1/2} \quad \text{and} \quad C_7 = \left(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda}\right)^{1/2} - \frac{3m^2}{2\Lambda}.$$

From eqs (3.3) and (4.22), we have

$$\rho = \rho_0 \left[\left(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9}\right)^{1/2} t^2 + \left(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda}\right)^{1/2} - \frac{3m^2}{2\Lambda}\right]^{-2} \tag{4.24a}$$

and

$$p = \frac{\rho_0}{3} \left[\left(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9}\right)^{1/2} t^2 + \left(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda}\right)^{1/2} - \frac{3m^2}{2\Lambda}\right]^{-2}. \tag{4.24b}$$

With the use of eqs (4.5)–(4.8) we can express the physical quantities as

$$\theta = \frac{3(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9})^{1/2} t}{[(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9})^{1/2} t^2 + (\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda})^{1/2} - \frac{3m^2}{2\Lambda}]} \tag{4.25}$$

$$A = \frac{2X^2}{3(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9}) t^2 [(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9})^{1/2} t^2 + (\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda})^{1/2} - \frac{3m^2}{2\Lambda}]} \tag{4.26}$$

$$\sigma^2 = \frac{X^2}{[(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9})^{1/2} t^2 + (\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda})^{1/2} - \frac{3m^2}{2\Lambda}]^3} \tag{4.27}$$

$$q = -\frac{[(\frac{9m^4}{4\Lambda^2} - \frac{k\rho_0}{\Lambda})^{1/2} - \frac{3m^2}{2\Lambda}]}{(\frac{m^4}{4} - \frac{k\rho_0\Lambda}{9})^{1/2}} \frac{1}{t^2}. \tag{4.28}$$

This model has no singularity.

Case II. $\gamma = -1$

For $C_1 = 0$, eq. (3.4) reduces to

$$\int \frac{\mathrm{d}V}{\sqrt{3(k\rho_0+\Lambda)V^2+9m^2V^{1/3}}} = t \tag{4.29}$$

which gives

$$V = \left[\frac{3m^2}{2(k\rho_0+\Lambda)}\right]^{3/2}\left[\cosh\left(2\sqrt{\frac{k\rho_0+\Lambda}{3}}t\right)-1\right]^{3/2}. \tag{4.30}$$

For small t (i.e. near singularity $t=0$)

$$\cosh\left(2\sqrt{\frac{k\rho_0+\Lambda}{3}}t\right) \approx 1+\left(\frac{k\rho_0+\Lambda}{3}\right)t^2. \tag{4.31}$$

Then eq. (4.30) reduces to

$$V = \frac{m^3}{2\sqrt{2}}t^3. \tag{4.32}$$

From eqs (2.15) and (4.32), we obtain

$$a_1(t) = \frac{mt}{\sqrt{2}}, \tag{4.33a}$$

$$a_2(t) = D\frac{mt}{\sqrt{2}}\ \exp\left(-\frac{\sqrt{2}X}{m^3}\frac{1}{t^2}\right), \tag{4.33b}$$

$$a_3(t) = D^{-1}\frac{mt}{\sqrt{2}}\ \exp\left(\frac{\sqrt{2}X}{m^3}\frac{1}{t^2}\right). \tag{4.33c}$$

From eqs (3.3) and (4.32), we obtain

$$\rho = \rho_0 \tag{4.34a}$$

and

$$p = -\rho_0. \tag{4.34b}$$

With the use of eqs (4.5)–(4.8) we can express the physical quantities as

$$\theta = \frac{3}{t}, \tag{4.35}$$

$$A = \frac{16X^2}{m^6t^6}, \tag{4.36}$$

$$\sigma^2 = \frac{8X^2}{m^6t^4}, \tag{4.37}$$

$$q = 0. \tag{4.38}$$

This model has no singularity. The anisotropy and shear die out as $t\to\infty$.

5. Conclusion

The Bianchi Type-V universe has been considered for a mixture of a perfect fuid and dark energy given by cosmological constant. The solution has been obtained in quadrature form. The particular cases of disordered radiation and inflation have been studied in detail. Their singularities have also been studied.

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