

Forward modelling: Magnetic anomalies of arbitrarily magnetized 2D fault sources with analytically defined fault planes

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Based on Poisson's relation, a generalized equation to realize forward modelling of magnetic anomalies due to arbitrarily magnetized 2D listric fault sources in any component is derived in the space domain. The non-planar fault plane of a listric fault structure is described with a generalized polynomial equation. The estimated coefficients of a prescribed polynomial are used to construct the fault plane analytically. The validity of the presented formula is established against the theoretical anomalies that are realized by an analytic equation over a vertical fault structure. It is demonstrated with a synthetic example that the magnetic anomalies in any component produced by a typical listric fault source always have lesser magnitude when compared to the corresponding anomalous field produced by the same structure with a planar fault plane assumption.

Keywords. Listric fault morphology; arbitrary magnetization; magnetic anomaly; forward modelling.

1. Introduction

Faults are planar or gently curved fractures in lithosphere rocks that are caused by tectonic and other related disturbances/movements. Large scale faults formed during rifting, drifting, and evolution of passive continental margins are normally associated with the basin development process. More often than not, fault planes of marginal faults associated with thick sedimentary basins are nonplanar because the primary detachment fracture more often follows a curved path rather than planar (McKenzie 1978; Smith and Bruhn 1984; Jackson 1987; Goussav *et al.* 2006; Chakravarthi 2011).

The geometry and kinematics of listric faults have gained paramount importance in understanding the large-scale extensional processes and exploring commercially viable mineralized targets. Torizin *et al.* (2009) argue that the study of the nature of fault dips with increasing depth could improve the precision of seismic source characterization. Due to the non-planar nature of fault planes, it is indeed difficult to accurately estimate the major extension and throw of faults from surface geologic observations alone (McKenzie 1978; Chakravarthi 2011). In such cases, the existence of magnetization contrast(s) between the displaced/ detached rock masses on either side of fault planes could generate measurable magnetic anomalies, which can be mapped and parameterized to quantify the listric fault morphologies.

A few techniques are available to estimate the parameters of fault structures from observed magnetic anomalies. The use of characteristic curves in the interpretation of magnetic anomalies had been proposed by Moo (1965), Grant and West (1965), Rao and Murthy (1978), and Rao *et al.* (1980). Based on the application of Fourier integral, Sengupta (1974) had proposed a method to analyse the magnetic anomalies caused by vertical fault structures. Stanley (1977) demonstrated that the horizontal gradient of total magnetic anomaly over a vertical contact is the same as the total magnetic anomaly over a thin dyke and that specific points on the gradient are related to the fault parameters. Qureshy and Nalaye (1978) proposed a technique based on decomposing magnetic anomaly into symmetric and anti-symmetric parts. Rao and Babu (1983) presented standard curves for magnetic anomaly interpretation treating the angle of fault plane as 90° and that the magnetization is caused purely by induction. Murthy (1985) had developed an efficient method of interpreting magnetic anomalies of arbitrarily magnetized fault structures, wherein the anomalies across the structure are scaled at two different elevations followed by identifying the maximum and minimum anomalies and their mid points, which in turn were used to estimate the source parameters.

Mushayandebvu *et al.* (2001) had developed a method using extended Euler deconvolution to analyse the anomalies produced by fault structures, while Murthy *et al.* (2001) used Marguardt's (1963) algorithm to analyse the magnetic anomalies. The inversion scheme proposed by Murthy et al. (2001) is noteworthy because their algorithm presumes arbitrary magnetization for the fault structures and analyses the anomalies in any component for the source parameters. Using analytic signal and Euler deconvolution, Doo et al. (2007) had devised a technique to estimate the source parameters of a 2D magnetic contact. Subrahmanyam and Rao (2009) have suggested a simple method that uses a few characteristic positions on the magnetic anomaly to find the parameters of a fault structure. Interpretation techniques based on constrained optimization theory (Asfahani and Tlas 2004) and stochastic algorithms (Asfahani and Tlas 2007) are also available currently to analyse the magnetic anomalies of fault structures. However, the practical utility of all the above techniques is limited to analyse the magnetic anomalies caused by large normal faults having non-planar fault planes.

Chakravarthi (2010) had developed a forward modelling technique to compute the gravity anomalies of listric fault sources, among which the density contrast differs continuously with depth. An automatic inversion scheme to simultaneously estimate the fault parameters (listric) and regional gravity background from a set of observed gravity anomalies was also proposed (Chakravarthi 2011). To the best of authors' knowledge, no algorithm is reported/available explicitly to analyse the magnetic anomalies generated by listric fault sources. Therefore, a need exists to develop suitable techniques to analyse magnetic anomalies produced by fault structures presuming (i) arbitrary magnetization for the source and (ii) non-planar surfaces for fault planes.

In this paper, we derive a generalized equation for computing the magnetic anomaly due to a listric fault morphology in any component (i.e., horizontal, vertical, and total field). A computer code in JAVA is developed to realize forward modelling in an interactive mode. The advantage of this code is that it is platform-independent and works on any GUI-based operating system with at least the jdk 1.6 version installed.

2. Forward modelling: Magnetic anomaly of an arbitrary magnetized 2D listric fault source

In the Cartesian co-ordinate system, let the z-axis is positive vertically downwards and x-axis traverse to the strike of a listric fault source whose geometry is shown in figure 1. The 2D fault structure (infinite strike length) is confined between the depth limits z_T and z_B ($z_B > z_T$) along the z-axis. Here, we treat the sediments within the hanging wall as magnetically transparent, while the magnetic interface (fault plane) is only responsible for generating the anomalies. Further, the structure is bounded on the left by a non-planar surface defined by $f(z) = \sum_{i=0}^{n} f_i z^i$, and towards the right, it extends to infinity. Here, f_i represents a set of coefficients, and n stands for the degree of the polynomial. Because both induced and remanent magnetizations are responsible for generating magnetic anomalies over a geologic structure, we presume that the structure is magnetized in an unknown direction along the resultant of both induced and remanent magnetic vectors. For a 2D source, the resultant magnetic field vector, J, can be resolved spatially into three mutually orthogonal components namely, $J\sin\theta$, $J\cos\theta\cos\delta$, and $J\cos\theta\sin\delta$ along the vertical, parallel to the strike of the source, and perpendicular to the strike of the source in the horizontal plane, respectively. Here,



Figure 1. Schematic diagram showing a conceptual geometry of a typical listric fault source. The x-axis is transverse to the strike of the structure, z-axis is vertically positive downwards, and the fault plane is described by a polynomial of specific degree. The source is striking infinitely along the y-axis.

 θ is the resultant magnetic dip, and δ denotes the resultant magnetic field vector's declination from the source's strike. Because the component resolved along the strike of the body fails to generate magnetic anomalies, the effective magnetization which is responsible for producing the anomalies can be expressed (Murthy 1998) as:

$$J_{ef} = J\sqrt{(1 - \cos^2\theta \cos^2\delta)}.$$
 (1)

The dip of the effective magnetization vector is given by

$$\theta_{ef} = \tan^{-1}(\tan\theta\csc\delta). \tag{2}$$

The effective magnetization always lies in a vertical plane perpendicular to the strike of the source.

Now considering d_c as the density contrast of the structure, the magnetic potential, W, at any point P(0,0) outside the source region can be expressed using the Poisson's relation as (Murthy 1998)

$$W = -\frac{J_{ef}}{Gd_c} \left\{ \frac{\partial U}{\partial x} \cos \theta_{ef} + \frac{\partial U}{\partial z} \sin \theta_{ef} \right\}, \quad (3)$$

where G is universal gravitational constant, and U represents the gravity potential.

The vertical magnetic anomaly ΔV outside the source region can be expressed as:

$$\Delta V = \frac{J_{ef}}{Gd_c} \left[\frac{\partial^2 U}{\partial x \partial z} \cos \theta_{ef} + \frac{\partial^2 U}{\partial z^2} \sin \theta_{ef} \right].$$
(4)

The gravity potential U due to the 2D source at the point P(0,0) is given by

$$U = -Gd_c \int_s \ln(x^2 + z^2) ds, \qquad (5)$$

where s is the cross-sectional area of the structure and (x, z) stands for the source coordinates of an element within the structure. Substituting the expressions for partial derivatives of U from equation (5) in equation (4), we obtain

$$\Delta V = 2J_{ef} \int_{s} \frac{(z^2 - x^2)\sin\theta_{ef} + 2xz\cos\theta_{ef}}{(x^2 + z^2)^2} \, ds. \quad (6)$$

Applying Stokes' theorem, equation (6) can be rewritten as

$$\Delta V = 2J_{ef} \int_{z=z_T}^{z_B} \left[\int_{x=f(z)}^{\infty} \frac{(z^2 - x^2)\sin\theta_{ef} + 2xz\cos\theta_{ef}}{(x^2 + z^2)^2} \, dx \right] dz.$$
(7)

Upon simplification equation (7) takes the form

$$\Delta V = 2J_{ef} \int_{z=z_T}^{z_B} \frac{z \cos \theta_{ef} - f(z) \sin \theta_{ef}}{f^2(z) + z^2} \, dz.$$
(8)

The vertical magnetic anomaly due to the structure at any point $P'(x_j, z_j)$ on the topography along the principal profile can be obtained as:

$$\Delta V(x_j, z_j) = 2J_{ef} \int_{z=z_T}^{z_B} \frac{(z-z_j)\cos\theta_{ef} - (f(z) - x_j)\sin\theta_{ef}}{(f(z) - x_j)^2 + (z-z_j)^2} dz.$$
(9)

Further, the anomaly in horizontal component ΔH at the point P(0,0) can be obtained from equation (3) as:

$$\Delta H = \frac{J_{ef} \sin \vartheta}{Gd_c} \left[\frac{\partial^2 U}{\partial x \partial z} \cos\left(\theta_{ef} - \frac{\pi}{2}\right) + \frac{\partial^2 U}{\partial z^2} \sin\left(\theta_{ef} - \frac{\pi}{2}\right) \right],\tag{10}$$

where ϑ is strike of the body. The horizontal magnetic anomaly due to the structure at any



Figure 2. (a) Comparison of vertical magnetic anomalies obtained from the present method and the analytic equation (Murthy *et al.* 2001), (b) geometry of vertical fault, and (c) differences between the anomalies from the two methods.

point $P'(x_j, z_j)$ on the principal profile outside the source region can be expressed using equation (5) as:

$$\Delta H(x_j, z_j) = 2J_{ef} \sin \vartheta \\ \times \int_{z=z_T}^{z_B} \frac{(z-z_j)\cos(\theta_{ef} - \frac{\pi}{2}) - (f(z) - x_j)\sin(\theta_{ef} - \frac{\pi}{2})}{(f(z) - x_j)^2 + (z-z_j)^2} dz.$$
(11)

From equations (9 and 11), one can realize that the vertical magnetic anomaly produced by a 2D listric fault source is similar to the horizontal magnetic anomaly produced by the same structure but with a different amplitude and phase.

The generalized equation for the magnetic anomaly in any component due to a listric fault source at an observer location at $P'(x_j, z_j)$ can be finally expressed as:

$$\Delta T(x_j, z_j) = 2J' \int_{z=z_T}^{z_B} \frac{(z-z_j)\cos\theta' - (f(z)-x_j)\sin\theta'}{(f(z)-x_j)^2 + (z-z_j)^2} dz$$
(12)

where $J' = J_{ef}\sqrt{(1 - \cos^2 \vartheta \cos^2 \alpha)}$ and $\theta' = \theta_{ef} - \tan^{-1}(\sin \vartheta / \tan \alpha)$.

Equation (12) is a standard form to calculate the magnetic anomaly of a listric fault source in any specific component. For example, by setting α to 90°, 0°, and *i* in equation (12) the vertical, horizontal, and total magnetic anomalies can be realized, respectively. Further, it is more appropriate to solve equation (12) by a numerical approach rather than analytical because of the simple fact that the polynomial, f(z), in the integrand may assume any degree (Chakravarthi 2010, 2011).

We demonstrate the validity of equation (12) by comparing the anomalies (in each component) obtained from the present method against the ones realized from an analytical equation (Murthy *et al.* 2001) over a vertical fault structure (figure 2b). In this case, the assumed parameters of the source are $z_T = 0 \text{ km}, z_B = 4.0 \text{ km}, \theta_{ef} = 30^\circ, J_{ef} = 100 \text{ nT}$ and $\vartheta = 40^\circ$. The anomalies in each component are calculated along a profile in the interval $x_j \epsilon (0, 40 \text{ km})$ on the observational plane $z_j = 0$ and



Figure 3. (a) Comparison of horizontal magnetic anomalies obtained from the present method and the analytic equation (Murthy *et al.* 2001), (b) differences between the horizontal anomalies obtained from the two methods, (c) total magnetic anomalies from the present method and the analytic equation (Murthy *et al.* 2001), and (d) differences between the total anomalies from the two methods.



Figure 4. Structural relationship between Model, View and Controller.

shown in figures 2(a) and 3(a, c), respectively. The differences between the anomalies obtained from the present method and the analytic equation in each component are shown in figures 2(c) and 3(b, d). In the case of vertical anomaly, a maximum difference of -6E-04 nT is observed at the 19th km on the profile (figure 2c), whereas in horizontal

and total components, the observed maximum differences are -2E-04 nT and -4E-04 nT, respectively (figure 3b and d). These insignificant differences between the anomalies in all the components demonstrate the accuracy of the proposed method of forward modelling.

3. Computer code

Based on the algorithm described in the text, a GUI-based software, FRMGLSTRK, coded in JAVA, is developed to compute the magnetic anomalies of a 2D listric fault source in any component.

The code is built on the Model-View-Controller (MVC) architecture according to the structural relationship shown in figure 4. The module 'Model' estimates the coefficients of a prescribed polynomial to construct the geometry of a fault plane and computes the magnetic anomalies of the structure



Figure 5. View module of FRMGLSTRK.



Figure 6. Selection of control points by mouse clicks in the structure panel. The appearance of number of control points in red warrants the selection of addition points.

in the required component. The 'View' module reads the input data and displays the output in both graphical and ASCII forms. The 'Controller' executes the task of passing the required actions to Model and View modules whenever they called for.

Once the batch file of the software is invoked, the view module appears on the monitor, as shown in figure 5. The view module is arranged into the input, graphical, and ASCII layouts (figure 5). The input layout consists of nine input fields and eight action buttons. The graphical layout is divided into the anomaly panel on top and the structural panel at the bottom. The ASCII layout towards the right displays the output in ASCII format.

The input parameters to the code are: profile and ID, number of observations, distance to each observation as measured with reference to the first station (any units), depth to the basement (any units), degree of the polynomial, the strike of the source with reference to magnetic north (degrees), intensity of magnetization (nT), direction of magnetization (degrees), and code number (1 for vertical, 2 for horizontal, and 3 for total magnetic anomaly). The code allows the user to specify the input in two ways: (1) the data can be entered in a notepad and loaded to the code by the 'load file' action button, or (2) data can be directly entered in respective fields of the input layout (figure 5). The action buttons of the input layout are: specify fault coordinates, draw/edit fault plane, forward modelling, save and print, clear, save file, load file, and exit.

After specifying the input parameters and invoking the action button 'specify fault



Figure 7. The number of control points (six) appear in blue indicates the selection of sufficient control points in the structure panel.



Figure 8. Analytically defined fault plane by a 5th degree polynomial. The footwall is represented with solid red and the hanging wall in yellow. The estimated coefficients of the polynomial and the coordinates of six control points are displayed in the lower panel of ASCII layout.

coordinates', the user selects a few control points in the structure panel by means of mouse clicks to construct a fault plane. The code automatically assigns the coordinates (x, z) to each such selected point in the structure panel, and the same is displayed on the right-hand side of the structure panel. The number of selected control points are also displayed in the graphical layout, as shown in figure 6. In addition, the code guides the user to select the optimum number of control points in the structure panel to construct the fault plane. For example, if the number of control points to describe the fault plane is insufficient (depending upon the degree of chosen polynomial), then the number of selected control points is displayed in red (as shown in figure 6). In such a case, the user needs to select a few more points in the structure panel till the font colour turns to blue (figure 7).



Figure 9. Vertical magnetic anomaly over a listric fault morphology. The non-planar fault plane is described with a 5th degree polynomial. The magnitude of anomaly at each observation is displayed in the top panel of ASCII layout.



Figure 10. Output of forward modelling in html format with a print dialogue box attached to it.

Upon invoking the 'draw/edit fault plane' action button in the input layout, the code solves the polynomial coefficients, f_k , by fitting the prescribed polynomial to the coordinates of control points. These estimated coefficients are then used to construct an analytically defined fault plane, as shown



Figure 11. (a) Vertical, horizontal, and total magnetic anomalies obtained from the present method over a listric fault source (fault plane is described with a 5th degree polynomial) whose geometry is shown in (d). The magnetic anomalies in the three components calculated over the same structure by an analytic equation (Murthy *et al.* 2001) presuming a planar fault plane are also shown for comparison.

Table1. Coefficientsof5thdegreepolynomial.

Coefficient	Magnitude
a_0	20.014
a_1	-0.1479
a_2	0.4836
a_3	0.0711
a_4	-0.0023
a_5	0.00039

in figure 8. If the user opts for modifying the fault plane, then the control points in the structure panel are edited by simple drag and drop mouse operations. Accordingly, the coefficients of the polynomial get updated, and the fault plane is reconstructed and displayed in real-time. The user also has the option to change the degree of the polynomial, if required. Once the model space is constructed, the anomalous field in any specific component (by specifying code number 1 for vertical, 2 for horizontal, and 3 for total) can be realized by invoking the 'forward modeling' operator (figure 9). The business logic computes the magnetic anomalies in the required component and displays the response in the anomaly panel, as shown in figure 9. The computed anomalies are also displayed in a tabular form in the ASCII layout (figure 9). The user saves the output by invoking the 'save and print' action button (figure 10).

4. Example

The applicability of method and code is demonstrated on a synthetic listric fault model, whose geometry is shown in figure 11(d). The assumed model space remains the same as in figure 2(b), but in this case, a 5th degree polynomial is used to describe the listric (non-planar) fault plane, as shown in figure 11(d). The coefficients of the chosen polynomial (5th degree) are given in table 1. For such a structure, the magnetic anomalies in vertical, horizontal, and total components are calculated from the present method and compared to the anomalies obtained from the analytic equation (Murthy *et al.* 2001), which presumes the planar surface for the fault plane. In both cases, the observer is on the top of the topography at $z_i = 0$ km. The theoretical anomalies obtained from both methods at 41 equi-spaced observations on a profile in the interval $x_i \epsilon (0, 40 \,\mathrm{km})$ are shown in figure 11(a-c). It is clearly seen from figure 11(a-c)that the magnetic anomalies produced by the listric fault structure differ in magnitude from the corresponding anomalies realized from the analytic equation. Therefore, the assumption of a planar surface for a fault plane should be accepted with caution, particularly when interpreting the magnetic anomalies caused by large normal faults.

5. Conclusions

A generalized equation that combines both analytic and numeric approaches to compute the magnetic anomalies due to an arbitrarily magnetized 2D listric fault source in any component is presented. It is demonstrated with a synthetic example that the magnitude of anomalies produced by a listric fault source, in any component, portray lesser magnitude than the anomalies produced by the same structure with a planar fault surface. Therefore, the application of routine modelling and inversion algorithms that consider the fault planes as planar surfaces to analyse the magnetic anomalies of large normal faults is discouraged.

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Author statement

Ani Nibisha has written the manuscript. Ramamma and Rajeswara Sastry have programmed the algorithm. Chakravarthi has developed the algorithm.

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