



# Shannon entropy transfer between solar magnetic modes

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**Abstract.** Based on Shannon entropy transfer estimation technique and Stanford's solar global magnetic field harmonic coefficients, several new facts about solar magnetic modes have been found. The entropy transferring between most of the modes has been subjected to a steady modulation with a period near 72 solar rotations (5.38 years). As a rule, the amplitudes of entropy transferring modulation were less or equal to 0.1 bit/solar rotation. These modulations had no relations with intensity or configuration of the solar magnetic fields. Besides, Shannon entropy exchanges have been found between some modes—the sign of dependencies of them. Such modes had sector nature. The influence of a magnetic mode on another is strongest with the delay time between the two modes being twenty to forty solar rotations. Special properties have been revealed for zonal modes also.

**Keywords.** Magnetic fields—solar cycle—Shannon entropy.

## 1. Introduction

The detection of change in magnitude of directional (causal) coupling between non-linear time series of solar activity values is a common subject of interest in solar physics. Understanding causal dependencies may help to simplify descriptions of complex physical processes because it constrains the coupling functions between the dynamical variables. It is particularly necessary to find such organization of observational data-sets which correspond to causal relationships in order to reveal variables that drive the dynamics.

In our investigation we have tried to reveal dependencies between the solar magnetic modes. Such modes have been characterized by their energy density, which can be derived as a sum of squares of global field harmonic coefficients,  $g$ 's and  $h$ 's, which described the mode in spherical presentation of solar magnetic field distribution in current-free approximation (Mikhailutsa 1994):

$$E_{lm} = \frac{l+1}{4l+2}(g_{lm}^2 + h_{lm}^2). \quad (1.1)$$

The indexes  $l$  and  $m$  correspondingly are the 'order' of mode (marks the number of dipoles which construct the mode) and the 'degree' of mode (the number of mode's bipolarity sectors which would be on the solar sphere).

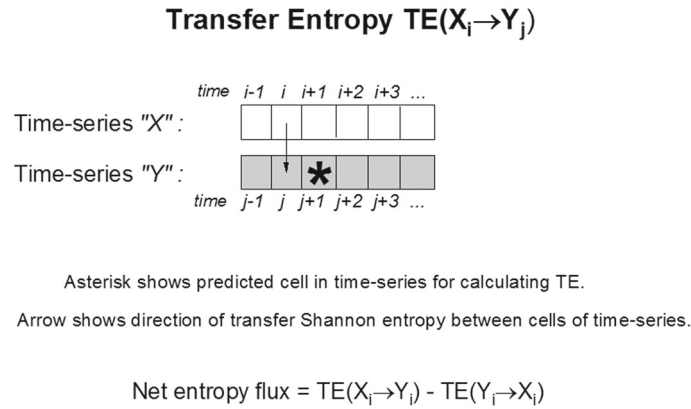
The values of mode energy density are invariant for any coordinate system on the solar surface.

Below, we will denote a particular magnetic mode as  $(l; m)$ , for example, mode (5; 2).

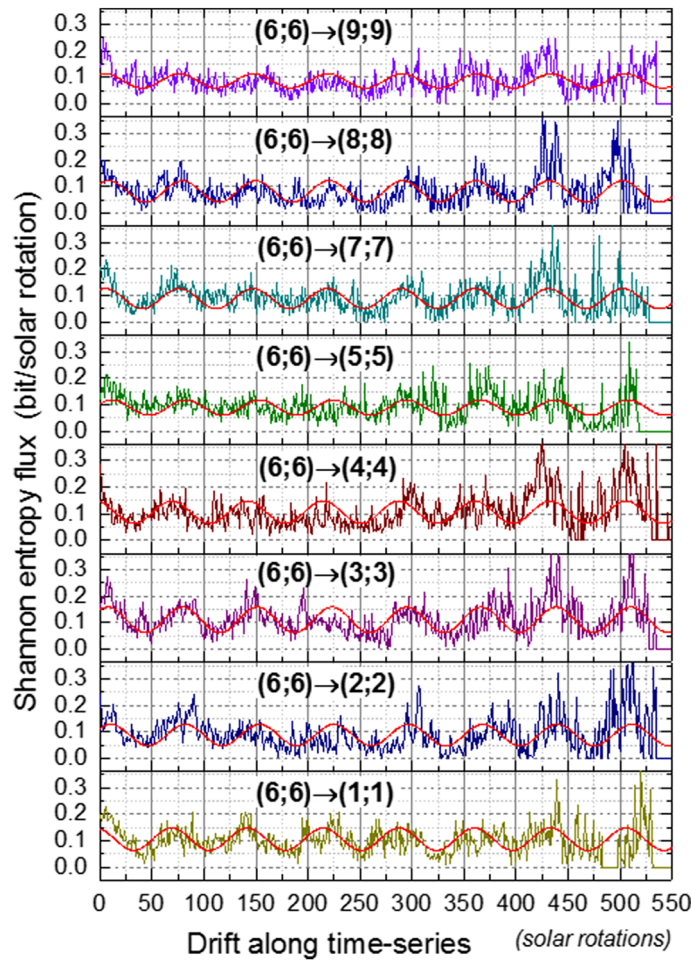
In this study, we have used Wilcox Solar Observatory data which have been obtained via the website: <http://wso.stanford.edu>, courtesy J. T. Hoeksema. So, our investigation of dependencies has been made for 54 solar magnetic modes, from (1; 0) through (9; 9). The data of magnetic mode energy densities have been organized into individual time-series covering a period of four solar cycles (564 solar rotations).

## 2. Shannon's entropy transferring

Schreiber (2000) developed a promising measure of the coupling strength between two time-series—transfer entropy. Transfer entropy (TE) quantifies the amount of information transfer from one variable to the other. In a data stream of two variables, e.g.,  $X(t) = \{x_1, x_2, \dots, x_N\}$  and  $Y(t) = \{y_1, y_2, \dots, y_N\}$ ,  $\text{TE}(X \rightarrow Y)$  from time-series  $X(t)$  to  $Y(t)$  is the information shared between  $X$ 's past and  $Y$ 's present, given the knowledge of  $Y$ 's past. Let index  $i$  indicate a given point in the time-series. Then



**Figure 1.** Transfer entropy between time-series of physical values. When cell's indexes are equal ( $i = j$ ), it is possible to estimate net entropy flux. During our calculations, TE indexes have been drifted along time-series.

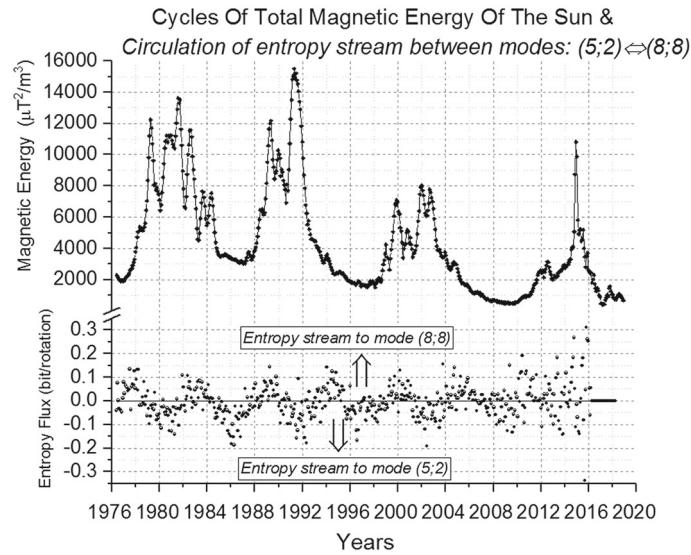


**Figure 2.** Modulation of TE from sector mode (6; 6) to other sector modes during four solar cycles.

$$T_{X \rightarrow Y} = \sum_{y_i, y_{i-1}, x_{i-1}} P(y_i, y_{i-1}, x_{i-1}) \times \log_2 \left[ \frac{P(y_i | y_{i-1}, x_{i-1})}{P(y_i | y_{i-1})} \right], \quad (2.1)$$

For pure Markov systems,  $T_{X \rightarrow Y} = 0$  because conditional probabilities in (2.1) are equal, i.e.,  $P(y_i | y_{i-1},$

$x_{i-1}) = P(y_i | y_{i-1})$ . Intuitively,  $TE(X \rightarrow Y)$  estimates how much better one predicts  $y_i$  using both  $x_{i-1}$  and  $y_{i-1}$  over using  $y_{i-1}$  alone. A nonzero value of the TE for long time-series ( $N \rightarrow \infty$ ) certainly implies a kind of influence of  $X(t)$  on  $Y(t)$ . Also, TE can be described as conditional mutual information, with



**Figure 3.** The cycles of magnetic energy density of the Sun and the circulation of net Shannon entropy flow between solar magnetic modes (5; 2) and (8; 8).

the history of the influenced variable in the condition. Generally, it gives a quantitative estimation of the difference of investigated dynamic system from the Markov’s one.

It should be noted here that some critical remarks have been made about understanding of ‘influence’ (James *et al.* 2016). They are as follows:

- Is this influence necessarily via information flow?
- Is influence necessarily direct?

In short, the answer is that the transfer entropy (for real non-infinite time-series) might both overestimate information flow and underestimate influence. This should be kept in mind if appropriate results are discussed. Nevertheless, such potential ‘overestimation’ or ‘underestimation’ cannot destroy the existed or created fake dependencies, if same dependencies appear on the base of different time-series.

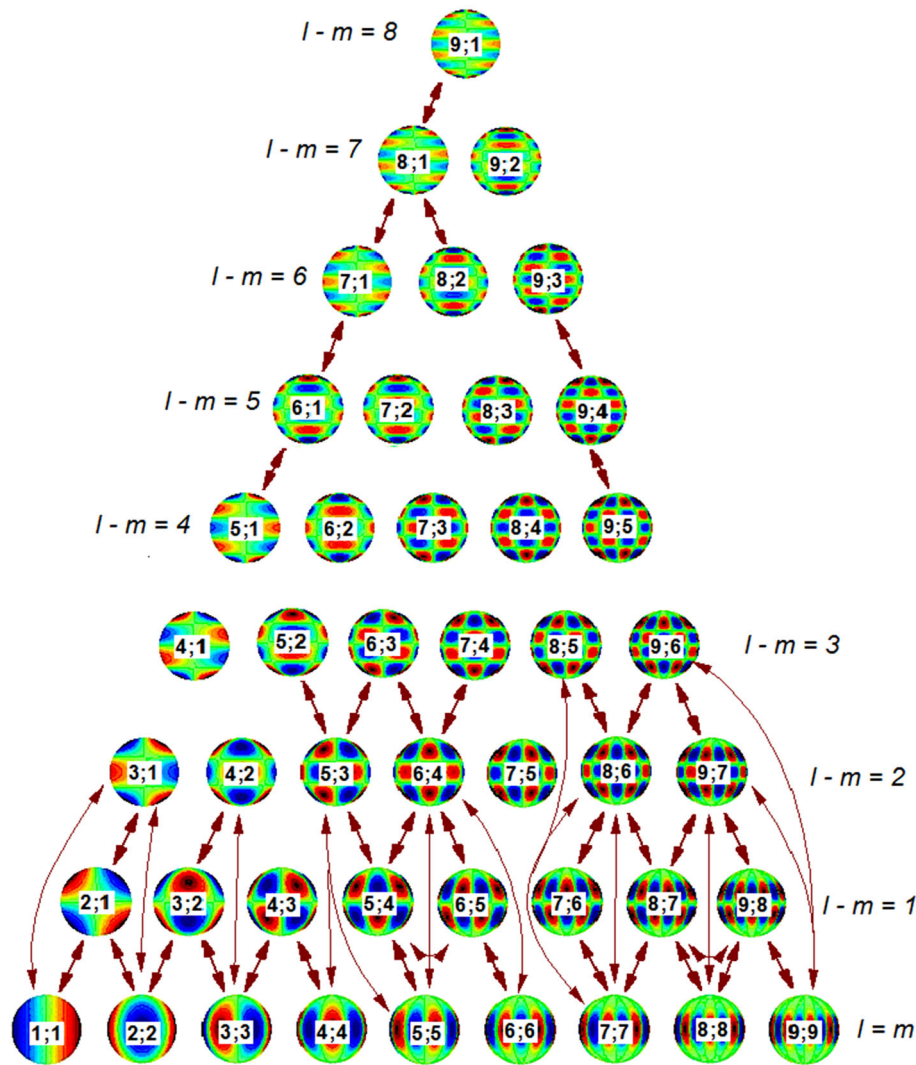
In Figure 1, there is an illustration of TE between concrete cells of time-series. The entropy transfer difference values (Net TE) have been added to our investigation additionally. The net TE flux can be received if two entropy streams (direct and back) between cells are compared. Because of each cell’s state relates to certain time—forward and back entropy flows must correspond to same time and cells.

We have used appropriate Shannon entropy transferring estimation software, which is described and explained in Lee *et al.* (2012). The most creative and useful software for us was the ‘Partition’ software. It has been chosen after taking into account precision, computational time and the difficulty associated with parameter selection for probability calculations.

### 3. Main results

#### 3.1 Modulation of Shannon entropy transferring processes

We have found that Shannon entropy transferring process between most mode energy time-series has been subjected to steady harmonic modulation with period of about 72 solar rotations ( $\approx 5.38$  years). These modulations have occurred independently from intensity and phase of the solar magnetic energy cycles. In correspondent Fourier spectra of the TE processes, a period of about 72 solar rotations was the only one having reached a peak of power above the level of three variances. As an example, in Figure 2, eight such cases of TE modulations are demonstrated, where entropy has been transferred from sector mode (6; 6) to other sector modes. The ordinates show values of entropy flux in bit/solar rotation. The abscissa axis is solar rotations (years 1976.4 through to 2017). Inside the composite figure, the correspondent TE pairs of modes are written. It can be seen that fitting made by sine function is in coordination with each TE data set. The amplitudes of TE modulations have not exceeded 0.1 bit/rotation. Physically, it can mean that influence of states of the sector mode (6; 6) to the other sectors happens after 10 solar rotations approximately. It should also be mentioned that some increase in modulation amplitudes toward the end of time-series in Figure 2 may be due to drifting along time-series that decreases statistics.



**Figure 4.** The coupled solar magnetic modes arranged in accordance with their magnetic structure. Colors correspond to the magnetic field polarities (blue–red). The arrows indicate pairs of the coupled modes. Modes in the pyramid were distributed in accordance with a number of latitudinal neutral lines, which increases toward the top of the pyramid. Some distant couples are hidden here.

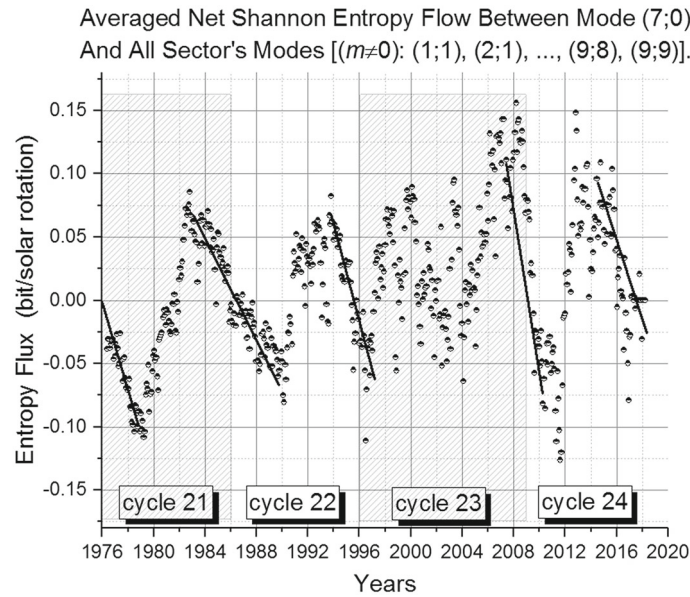
Some exceptions in TE modulation have happened for several zonal magnetic modes ( $m = 0$ ). They have not revealed any periodicity at all.

### 3.2 Circulation of Shannon entropy transferring

Besides modulations, circulations of net entropy flux between modes are also found. The periods and amplitudes of such circulations were similar to the modulation ones. Mostly, they have appeared for sector modes ( $m \neq 0$ ) and among them especially for tesseral modes and modes of  $(l - m) = 2$ . Circulation of net entropy flux means that such mode states must be coupled. It is like kinetic and potential energies for pendulum. In Figure 3, we demonstrate cycles of magnetic energy

density of the Sun (top part of Figure 3), together with entropy flux circulation between modes (5; 2) and (8; 8) (bottom part of Figure 3).

The periodicity of entropy flow direction can clearly be seen due to the wavy distribution of flux points (from positive to negative values and vice versa). Comparison of top and bottom figures shows that coupling between modes has occurred independently from intensities and phases of the solar magnetic energy density cycles. In other words, the solar surface magnetic fields have not been affected by entropy circulation between their modes. These circulations have not felt variations in solar magnetic field configurations. The entropy circulation periods were close to 72 solar rotations, as a rule.



**Figure 5.** Irregular Shannon entropy exchange between mode (7; 0) and all sector modes ( $m \neq 0$ ). Straight lines demonstrate that when entropy flow has changed direction, the new sunspot cycles have appeared.

In order to show for which pairs of modes circulation of entropy flux has been found, we have composed all data in the pyramid figure in according to magnetic polarity zonal structure on the solar surface. This result is shown in Figure 4. At the bottom of the pyramid, purely sector magnetic modes are found. Above them are the tesseral modes, then modes with  $(l - m) = 2$ , etc. Towards the top of the pyramid, the magnetic modes become more zonal structured. It can be seen that when the number of magnetic field zonal bands exceeds four, the entropy flux circulations between the correspondent modes practically disappear. This illustration demonstrates the concentration of TE circulations toward purely sector mode and less zonal mode. But entropy circulation in the horizontal direction in Figure 4, i.e., among purely sector modes, tesseral modes, and so on, are absent as a rule. This very new result is awaiting physical explanation.

### 3.3 Specificity of the seventh-order zonal magnetic mode TE

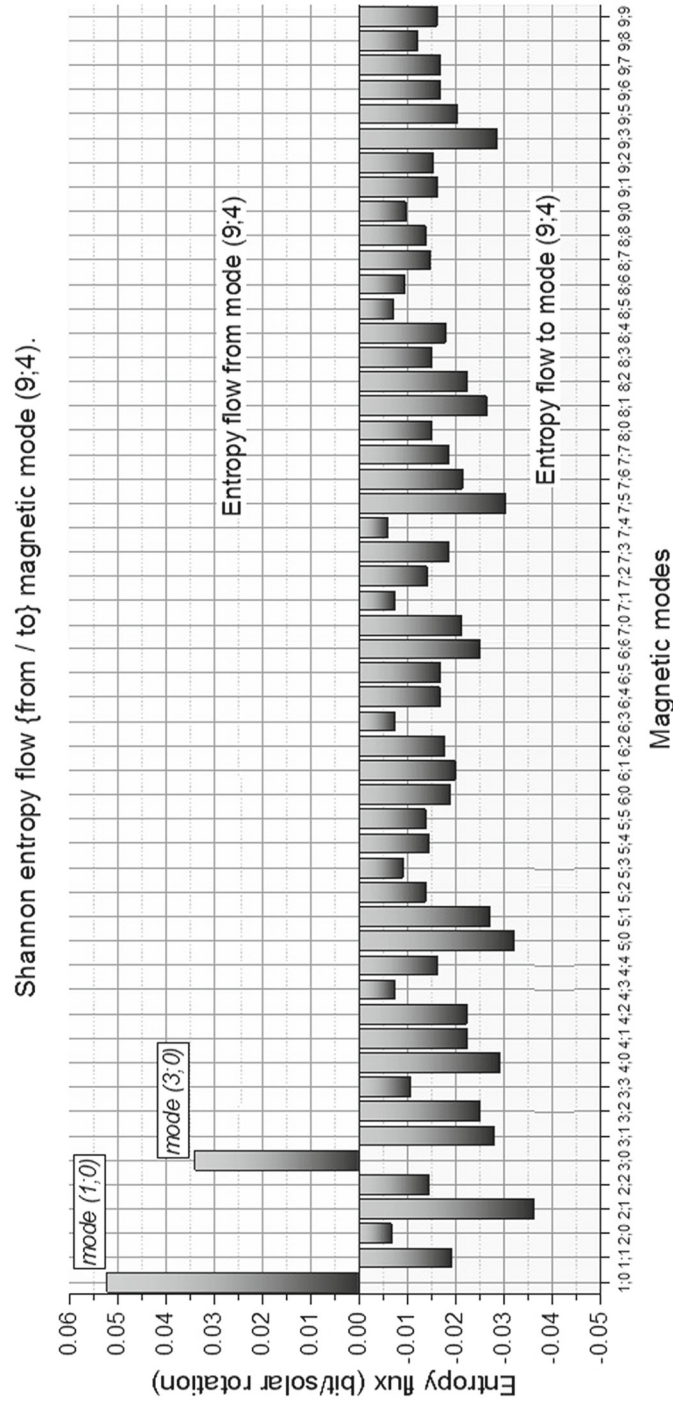
The zonal mode ( $l = 7; m = 0$ ) has demonstrated specific irregular TE circulations in close correspondence to the phase of the sunspot cycles. This mode has shown a peculiar Shannon's entropy transferring. It is highly likely that TE from mode (7; 0) depends on the phase of the solar sunspot cycles. The limited length of the time-series did not allow clearing this question.

In Figure 5, the averaged Shannon entropy exchanges between mode (7; 0) and all sector modes ( $m \neq 0$ ) are illustrated.

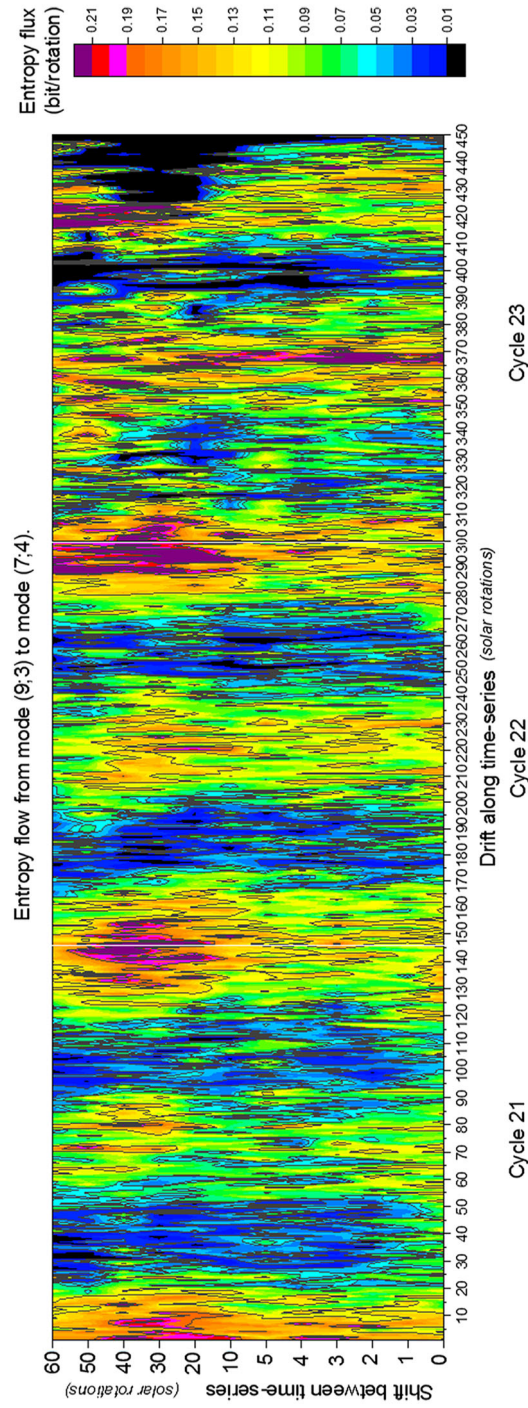
It can be concluded that Shannon entropy flow from mode (7; 0) begins shortly after it reaches the maximum of sunspot cycle and ends when a new cycle appears. Cycle 23 was unusual as can be seen in Figure 5 of the TE process. The reason for such extraordinary behavior is unclear yet.

### 3.4 Sequence of transferring of the Shannon entropy

The average values of net Shannon entropy fluxes for time series have revealed another aspect of the phenomenon studied here. It is noteworthy that even zonal modes (8; 0), (6; 0), (4; 0) were entropy transmitters (all mean fluxes were mainly positive for these modes), but odd zonal modes (1; 0), (3; 0) were pure entropy receivers. The arrangement of all solar modes according to averaged net entropy flux values creates a sequence of entropy flow over modes. At the beginning of the sequence there are modes with mostly positive average entropy fluxes (entropy transmitters), and at the end, there are modes with negative fluxes (entropy receivers). Thus, the zonal modes of even orders (8; 0), (6; 0), (4; 0) become the 'sources' of entropy flow and the odd orders (1; 0), (3; 0) become the final 'absorbents' of entropy flow. All sector modes ( $m \neq 0$ ) are situated between them. The sequence of sector modes begins from mode (2; 1) as entropy flow 'source', and ends by modes



**Figure 6.** Average values of net Shannon entropy flux between mode (9; 4) and all other modes calculated for years (1976)–(2017). Here, the negative values mean that entropy flux directs to mode (9; 4).



**Figure 7.** Two-dimensional map of Shannon entropy transferring from mode (9; 3) to mode (7; 4). The ordinates are a shift between the time-series, the abscissas—drift along the time-series. The scale of entropy flux is shown at the right. The results of the three solar cycles are shown.

(7; 4) and (9; 4) as entropy flow ‘targets’. The illustration of definitions of the entropy ‘receiver’ or the ‘target’ is shown in Figure 6. For mode (9; 4), the ordinate axis shows values of average entropy flux. The abscissa axis shows modes taken part in the TE process. It can be seen that only two zonal modes (1; 0) and (3; 0) were ‘target’ for entropy transferring from mode (9; 4), but this mode itself was ‘target’ for all the other modes. In bulk, this sequence connects two kinds of zonal magnetic fields (even and odd orders) and inside this entropy chain, all sector modes are arranged.

The main role of the even zonal modes as entropy transmitter is unexpected and will be significant for solar magnetism. The domination of tesseral magnetic mode (2; 1) as entropy transmitter is also remarkable.

### 3.5 Time of Shannon entropy transportation between solar magnetic modes

While calculating TE, it is possible to make drift along the time-series or, also make delay in time between the time-series. In the last case, it is not able to receive net TE flux, but it is possible to estimate the delay-time needed to get a maximum influence of one mode to the other. As a result, the estimation of such delay-time has given (20–40) solar rotations. In Figure 7, the estimation of influence delay of mode (9; 3) as ‘source’ on to the mode (7; 4) as ‘target’ is shown.

It can be seen that the maximum peaks (or minimum values) of entropy flux are repeated every 72 solar rotations for all shifts. This is a manifestation of the modulation process described above. New here are the locations of maxima of Shannon entropy flux (yellow–pink areas), which are grouped mainly around the time-series shifts of (20–40) solar rotations with mean step along the abscissa axis of 72 rotations. Physically, it can mean that this time is needed to get maximum influence of mode (9; 3) to mode (7; 4). In other words, it is the Shannon entropy transportation time. It should be mentioned that such estimations of entropy transportation time along the same time-series (like  $TE[(9; 3)_t \rightarrow (9; 3)_{t+\Delta}]$ ) have given similar results. So, the present state of mode magnetic energy has highest influence

to the future one that will appear after about 20 solar rotations.

It should be noted that the modulations make dependencies between modes (or inside mode time-series itself) weaker or stronger every 36 solar rotations, approximately. But the same time is needed for transportation of Shannon entropy. For this reason, it cannot be excluded that the two processes may have one common pulsated source.

## 4. Conclusion

We have no doubts that this research has not covered everything all in detail, and hence must be continued. Because TE modulation process has been found for most of the solar magnetic modes, two very important questions arise: What are the mechanisms of entropy modulations and entropy exchange between them? What is the reason of periodic (half a sunspot cycle) strengthening and weakening of dependencies? Such cause must have no connections with the solar magnetic field configurations. Apparently, besides the sunspot dynamo, solar magnetic fields must be subjected by additional generation mechanism or mechanisms. It is highly likely that the above-mentioned zonal *even* modes as the transmitters of Shannon entropy have relations to solar torsion oscillations. But modulations and circulation of entropy flows in sector modes can be originated by Rossby waves (Löptien *et al.* 2018).

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