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Effect of Hall Current and Finite Larmor Radius Corrections on Thermal Instability of Radiative Plasma for Star Formation in Interstellar Medium (ISM)

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Abstract. The effects of finite ion Larmor radius (FLR) corrections, Hall current and radiative heat-loss function on the thermal instability of an infinite homogeneous, viscous plasma incorporating the effects of finite electrical resistivity, thermal conductivity and permeability for star formation in interstellar medium have been investigated. A general dispersion relation is derived using the normal mode analysis method with the help of relevant linearized perturbation equations of the problem. The wave propagation is discussed for longitudinal and transverse directions to the external magnetic field and the conditions of modified thermal instabilities and stabilities are discussed in different cases. We find that the thermal instability criterion gets modified into radiative instability criterion. The finite electrical resistivity removes the effect of magnetic field and the viscosity of the medium removes the effect of FLR from the condition of radiative instability. The Hall parameter affects only the longitudinal mode of propagation and it has no effect on the transverse mode of propagation. Numerical calculation shows stabilizing effect of viscosity, heat-loss function and FLR corrections, and destabilizing effect of finite resistivity and permeability on the thermal instability. The outcome of the problem discussed the formation of star in the interstellar medium.

Key words. Star formation—interstellar medium—thermal instability—FLR corrections—Hall current.

1. Introduction

Formation of stars in the interstellar medium is one of the most fascinating and important process in plasma astrophysics. The birth of stars is a vast field of research in modern astrophysics and cosmology. Thermal instability plays a very crucial role in the formation of stars and other astrophysical objects in interstellar medium. Thermal instability occurs when a positive temperature perturbation is completed in a thermal unstable medium, the perturbation produces and the emission rate reduces. This development is considered to be probable in a number of astrophysical circumstances such as in the solar corona, the gas in clusters of the galaxies and in the interstellar medium. The thing which is not so clear is the relative consequence of this development in an assortment of situations. Thermal instability has numerous applications in astrophysical conditions e.g. stellar atmosphere, star formation, a clumpy interstellar medium, globular clusters and galaxy formation, and many more situations. The instability may be determined by the radiative cooling of optically thin gas systems or by exothermic nuclear reactions.

In this connection the thermal and radiative instabilities arising due to various heat-loss mechanisms may be the cause of astrophysical condensation and the formation of large scale structures as well as small objects. Several authors investigated the phenomenon of thermal instability arising due to heat-loss mechanism in plasma. Field (1965) discussed the importance of thermal instability in the formation of solar prominences, condensation in planetary nebula and condensation of galaxies from the intergalactic medium. Hunter (1966) discussed the role of thermal instability in star formation. Raju (1968) emphasized the role of thermal instability in the formation of solar prominences. Aggarwal and Talwar (1969) investigated magnetothermal instability in a rotating gravitating fluid taking radiative heat-loss function. Ibanez (1985) studied the sound and thermal waves in a fluid with an arbitrary heat-loss function. Hoven and Mok (1984) carried out the problem of thermal instability in a sheared magnetic field. Bodo *et al.* (1985) investigated magnetohydrodynamic thermal instability in cool inhomogeneous atmosphere. Bora and Talwar (1993) discussed the magnetothermal instability with generalized Ohms law taking the effects of electrical resistivity, Hall current, electron inertia, thermal conductivity and radiative heat-loss function. Burkert and Lin (2000) pointed out the importance of thermal instability in the formation of clumpy gas clouds and they showed that thermal instability can lead to the breakup of large clouds into cold dense clumps. Shadmehri and Ghanbari (2001) discussed the problem of radiative cooling flows of self-gravitating filamentary clouds. Nejad-Asghar and Ghanbari (2006) discussed the formation of small-scale condensation in the molecular clouds via thermal instability. Fukue and Kamaya (2007) studied the thermal instability of partially ionized plasma taking radiative cooling function and two-fluid theory into account. Baruah *et al.* (2010) studied the thermal (radiative) instability in weakly ionized plasma with continuous ionization and recombination taking general heat-loss function. Recently, Prajapati *et al.* (2010) investigated the self-gravitational instability of the rotating viscous Hall plasma with arbitrary radiative heat-loss function and electron inertia. Thus thermal instability is important for star formation in interstellar medium.

In addition to this, the Hall current parameter is important in the dynamics of interstellar matter, magnetic reconnection processes, instability investigation of accelerated plasmas and in several other astrophysical situations. Tayler (1963) discussed a simple hydromagnetic stability problem involving finite conductivity, electron inertia and Hall effect. Fukue and Kamaya (1964) have investigated magneto-thermal instability of unbounded plasma with electron inertia and Hall effect. Sen and Chou (1968) carried out the investigation of thermal instability of plasma with Hall effect. Sharma and Sharma (1985) carried out the investigation of

Hall effect on thermal hydromagnetic instability of a partially ionized medium. Ali and Bhatia (1992) investigated thermal instability of partially ionized plasma in an oblique magnetic field incorporating the effects of Hall current and finite conductivity. Chhajlani and Parihar (1993) studied the stability of self-gravitating viscous magnetized Hall plasma with electrical and thermal conductivity through a porous medium. Shtemler *et al.* (2007) discussed the Hall instability of thin weakly ionized stratified Keplerian disk. Shaikh *et al.* (2008) discussed thermal instability of thermally conducting plasma taking the effects of magnetic field, viscosity, finite electrical conductivity and Hall current. Damiano *et al.* (2009) pointed out the effects of electron inertia and FLR on Hall magnetohydrodynamic waves. Uberoi (2009) discussed electron inertia effects on the transverse thermal instability incorporating the Hall current and rotation parameters. Kumar (2012) investigated the Hall current effect on thermal instability of compressible viscoelastic dust fluid in a porous medium. Recently, Aggarwal and Makhija (2014) studied the Hall effect on thermal stability of ferromagnetic fluids in a porous medium in the presence of horizontal magnetic field. More recently, Pant *et al.* (2016) investigated the problem of combined effect of Hall current and rotation on thermal stability of ferromagnetic fluids saturating in a porous medium under varying gravity field.

In the above discussed problems, the effect of finite ion Larmor radius is not considered. In many astrophysical plasma situations such as in solar corona, interstellar and interplanetary plasmas the assumption of zero Larmor radius is not valid. Roberts and Taylor (1962) and Rosenbluth *et al.* (1962) have shown the stabilizing influence of finite ion Larmor radius (FLR) effects on plasma instabilities. Hernegger (1972) investigated the stabilizing effect of FLR on thermal instability and showed that thermal criterion is changed by FLR for wave propagation perpendicular to the magnetic field. Sharma (1974) investigated the stabilizing effect of FLR on thermal instability of rotating plasma. Ariel (1988) discussed the stabilizing effect of FLR on thermal instability of conducting plasma layer of finite thickness surrounded by a non-conducting matter. Vaghela and Chhajlani (1989) studied the stabilizing effect of FLR on magneto-thermal stability of resistive plasma through a porous medium with thermal conduction. Bhatia and Chhonkar (1985) investigated the stabilizing effect of FLR on the instability of a rotating layer of self-gravitating plasma incorporating the effects of viscosity and Hall current. Vyas and Chhajlani (1990) pointed out the stabilizing effect of FLR on the thermal instability of magnetized rotating plasma incorporating the effects of viscosity, finite electrical conductivity, porosity and thermal conductivity. Marcu and Ballai (2007) showed the stabilizing effect of FLR on thermosolutal stability of a two-component rotating plasma. Kaothekar and Chhajlani (2014) investigated the problem of Jeans instability of self-gravitating rotating radiative plasma with finite Larmor radius corrections. More recently, Kaothekar *et al.* (2016) carried out the problem of Jeans instability of partially-ionized self-gravitating viscous plasma with Hall effect FLR corrections and porosity. Thus FLR effect is an important factor in the discussion of thermal instability and other hydromagnetic instabilities.

In the light of the above work, we find that in Vyas and Chhajlani (1990), Chhajlani and Parihar (1993), Bora and Talwar (1993), Prajapati *et al.* (2010) and Kaothekar *et al.* (2016), the joint influence of FLR corrections, Hall current, radiative heat-loss functions, viscosity, permeability, electrical resistivity, thermal conductivity and magnetic field on the thermal instability is not investigated. Therefore in the present work, thermal instability of viscous magnetized plasma with Hall current, FLR corrections, radiative heat-loss functions, thermal conductivity, finite electrical resistivity and permeability for thermal configuration is studied.

The paper is organized as follows. Section 2 contains the basic equations for a magnetothermal system. In section 3, linearized equations are derived for the first order approximation. Section 4 contains dispersion relation and the instability criterion for thermal and radiative modes is derived for longitudinal and transverse propagation. Numerical interpretation of the linear growth rate is done. Finally section 5 contains the summary and discussion of the results.

2. Equations of the problem

Let us consider an infinite homogeneous, thermally conducting, radiating, viscous plasma with Hall current, and finite electrical resistivity in the presence of magnetic field $\mathbf{H}(0, 0, H)$. The equations of the problem with these effects are written as

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla p - \frac{\nabla\cdot\mathbf{P}}{\rho} + \upsilon\left(\nabla^2\mathbf{v} - \frac{\mathbf{v}}{K_1}\right) + \frac{1}{4\pi\rho}(\nabla\times\mathbf{H})\times\mathbf{H}, \tag{1}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\mathbf{v}, \tag{2}$$

$$\frac{1}{\gamma-1}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\gamma}{\gamma-1}\frac{p}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho L - \nabla\cdot(\lambda\nabla T) = 0, \tag{3}$$

$$p = \rho R T, \tag{4}$$

$$\frac{\partial\mathbf{H}}{\partial \mathrm{t}} = \nabla\times(\mathbf{v}\times\mathbf{H}) + \eta\nabla^2\mathbf{H} - \frac{c}{4\pi Ne}\left\{\nabla\times[(\nabla\times\mathbf{H})\times\mathbf{H}]\right\}, \tag{5}$$

$$\nabla\cdot\mathbf{H} = 0, \tag{6}$$

where ρ, p, υ, T, $\mathbf{v}$ (v_x, v_y, v_z), η, λ, R, γ, c, N and e denote the fluid density, pressure, kinematic viscosity, temperature, velocity, electrical resistivity, thermal conductivity, gas constant and ratio of two specific heats, velocity of light, number density and charge of electron, respectively. Here $L(\rho, T)$ is the heat-loss function per gram of the material per second exclusive of thermal conduction and is, in general, a function of the local values of density and temperature. The operator $(\mathrm{d}/\mathrm{d}t)$ is the substantial derivative given as $(\mathrm{d}/\mathrm{d}t) = (\partial_t + \mathbf{v}\cdot\nabla)\cdot\mathbf{P}$, which is the pressure tensor taking into account the effect of finite ion gyration radius for the magnetic field along the z axis as given by Roberts & Taylor (1962),

$$\begin{aligned}
&P_{xx} = -\rho v_0\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right), && P_{yy} = \rho v_0\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right),\\
&P_{zz} = 0, && P_{xy} = P_{yx} = \rho v_0\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}\right),\\
&P_{xz} = P_{zx} = -2\rho v_0\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right), && P_{yz} = P_{zy} = 2\rho v_0\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right).
\end{aligned} \tag{7}$$

The parameter υ_0 has the dimensions of the kinematics viscosity and defined as $\upsilon_0 = \Omega_L R_L^2/4$, where R_L is the ion-Larmor radius and Ω_L is the ion gyration frequency.

3. Perturbation equations

The perturbation in fluid density, pressure, temperature, velocity, magnetic field, heat-loss function are given as $\delta\rho$, δp, δT, $\mathbf{v}$ (v_x, v_y, v_z), $\mathbf{h}$ (h_x, h_y, h_z) and L respectively. The perturbation state is given as

$$\rho = \rho_0 + \delta\rho, \quad p = p_0 + \delta p, \quad T = T_0 + \delta T, \quad \mathbf{v} = \mathbf{v_0} + \mathbf{v} \text{ (with } \mathbf{v_0} = 0),$$
$$\mathbf{H} = \mathbf{H_0} + \mathbf{h} \quad \text{and} \quad L = L_0 + L \text{ (with } L_0 = 0). \tag{8}$$

Suffix '0' represents the initial equilibrium state, which is independent of space and time.

Substituting the perturbation state into equations (1) to (6) and linearizing them by neglecting higher order perturbations, suffix '0' is dropped from the equilibrium quantities.

The linearized perturbation equations of motion for such medium are

$$\partial_t \mathbf{v} = -\frac{1}{\rho}\nabla\delta p - \frac{\nabla\cdot\mathbf{P}}{\rho} + \upsilon\left(\nabla^2\mathbf{v} - \frac{\mathbf{v}}{K_1}\right) + \frac{1}{4\pi\rho}(\nabla\times\mathbf{h})\times\mathbf{H}, \tag{9}$$

$$\partial_t \rho = -\rho\nabla\cdot\mathbf{v}, \tag{10}$$

$$\frac{1}{\gamma-1}\partial_t\delta p - \frac{\gamma}{\gamma-1}\frac{p}{\rho}\partial_t\delta\rho + \rho(L_\rho\delta\rho + L_T\delta T) - \lambda\nabla^2\delta T = 0, \tag{11}$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho}, \tag{12}$$

$$\partial_t\mathbf{h} = \nabla\times(\mathbf{v}\times\mathbf{H}) + \eta\nabla^2\mathbf{h} - \frac{c}{4\pi Ne}\left\{\nabla\times[(\nabla\times\mathbf{h})\times\mathbf{H}]\right\}, \tag{13}$$

$$\nabla\cdot\mathbf{h} = 0, \tag{14}$$

where L_T and L_ρ respectively denote partial derivatives $(\partial L/\partial T)_\rho$ and $(\partial L/\partial\rho)_T$ of the heat-loss function evaluated for the initial (unperturbed) state.

4. Dispersion relation

We seek plain wave solution of the form

$$\exp(ik_x x + ik_z z + \sigma t), \tag{15}$$

where σ is the frequency, k_x and k_z are the wave numbers of the perturbations along the x and z axes. The components of equation (13) may be given as

$$h_x = (iH/d)k_z v_x - (M/d)k_z^2 h_y,$$
$$h_y = (iH/d)k_z v_y + (M/d)(k_z^2 h_x - k_x k_z h_z),$$
$$h_z = -(iH/d)k_x u_x + (M/d)k_x k_z h_y. \tag{16}$$

Using equations (11), (12) and (15), we write

$$\delta p = \frac{\{(\gamma - 1)[T L_{\mathrm{T}} - \rho L_{\rho} + (\lambda k^2 T/\rho)] + \omega c^2\}}{\{(\gamma - 1)[(T\rho/p) L_{\mathrm{T}} + (\lambda k^2 T/p)] + \omega\}} \delta\rho. \tag{17}$$

Using equations (10)–(17) in equation (9), we may write the following algebraic equations for the components of equation (9) as

$$\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2 k^2 d}{(d^2 + M^2 k^2 k_z^2)}\right] v_x + \left[\upsilon_0 (k_x^2 + 2k_z^2) - \frac{V^2 k^2 M k_z^2}{(d^2 + M^2 k^2 k_z^2)}\right] v_y + \frac{i k_x}{k^2} \Omega_{\mathrm{T}}^2 s = 0, \tag{18}$$

$$-\left[\upsilon_0 (k_x^2 + 2k_z^2) - \frac{V^2 k^2 M k_z^2}{(d^2 + M^2 k^2 k_z^2)}\right] v_x + \left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2 k_z^2 d}{(d^2 + M^2 k^2 k_z^2)}\right] v_y - 2\upsilon_0 k_x k_z v_z = 0, \tag{19}$$

$$2\upsilon_0 k_x k_z v_y + \left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] v_z + \frac{i k_z}{k^2} \Omega_{\mathrm{T}}^2 s = 0. \tag{20}$$

Taking divergence of equation (9) and using equations (10)–(17), we obtain

$$\left[i k_x \frac{V^2 k^2 d}{(d^2 + M^2 k^2 k_z^2)}\right] v_x + \left[i \upsilon_0 k_x (k_x^2 + 4k_z^2) - i k_x \frac{V^2 k^2 M k_z^2}{(d^2 + M^2 k^2 k_z^2)}\right] v_y - \left[\omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2\right] s = 0. \tag{21}$$

The set of equations (18)–(21) can be written in the form.

$$\begin{bmatrix} N & P & 0 & \frac{i k_x}{k^2}\Omega_{\mathrm{T}}^2 \\ -P & N_1 & -2\upsilon_0 k_x k_z & 0 \\ 0 & 2\upsilon_0 k_x k_z & M_1 & \frac{i k_z}{k^2}\Omega_{\mathrm{T}}^2 \\ i k_x D & Q & 0 & -R \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \\ s \end{bmatrix} = 0. \tag{22}$$

We have made following assumptions:

$$V^2 = \frac{H^2}{4\pi\rho}, \quad \Omega_{\mathrm{T}}^2 = \frac{\Omega_I^2 + \omega\Omega_j^2}{B + \omega}, \quad \Omega_j^2 = c^2 k^2,$$
$$\Omega_I^2 = k^2 A, \quad i\sigma = \omega, \quad d = (\omega + \eta k^2),$$

$$A = (\gamma - 1)\left(TL_{\mathrm{T}} - \rho L_{\rho} + \frac{\lambda k^2 T}{\rho}\right), \quad B = (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right),$$
$$M = \frac{cH}{4\pi Ne}, \quad F = \upsilon_0(k_x^2 + 2k_z^2),$$
$$Q = i\upsilon_0 k_x(k_x^2 + 4k_z^2) - ik_x E, \quad N_1 = M_1 + \frac{V^2 k_z^2 d}{(d^2 + M^2k^2k_z^2)},$$
$$M_1 = \omega + \upsilon\left(k^2 + \frac{1}{K_1}\right),$$
$$R = \omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2, \quad D = \frac{V^2k^2 d}{(d^2 + M^2k^2k_z^2)},$$
$$E = \frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}, \tag{23}$$
$$N = M_1 + D, \quad P = F - E,$$

where $c = (\gamma p/\rho)^{1/2}$ is the adiabatic velocity of sound in the medium and $s = \delta\rho/\rho$ is the condensation of the medium.

The general dispersion relation can be obtained from the determinant of matrix of equation (21) as

$$\begin{aligned}
&\left[\omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2\right] \times \left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2k^2 d}{(d^2 + M^2k^2k_z^2)}\right] \\
&\times \left\{\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right]\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2k_z^2 d}{(d^2 + M^2k^2k_z^2)}\right] + 4\upsilon_0^2k_x^2k_z^2\right\} \\
&+ \frac{2\upsilon_0 ik_xk_z^2}{k^2}\Omega_{\mathrm{T}}^2\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2k^2d}{(d^2 + M^2k^2k_z^2)}\right] \\
&\times \left[i\upsilon_0k_x(k_x^2 + 4k_z^2) - ik_x\frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}\right] + \left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] \\
&\times \left[\omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2\right]\left[\upsilon_0(k_x^2 + 2k_z^2) - \frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}\right]^2 \\
&+ \frac{2\upsilon_0k_x^2k_z^2V^2d}{(d^2 + M^2k^2k_z^2)}\Omega_{\mathrm{T}}^2\left[\upsilon_0(k_x^2 + 2k_z^2) - \frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}\right] \\
&+ \frac{ik_x}{k^2}\Omega_{\mathrm{T}}^2\left[\upsilon_0(k_x^2 + 2k_z^2) - \frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}\right]\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] \\
&\times \left[ik_x\upsilon_0(k_x^2 + 4k_z^2) - ik_x\frac{V^2k^2Mk_z^2}{(d^2 + M^2k^2k_z^2)}\right] - \frac{V^2k_x^2d}{(d^2 + M^2k^2k_z^2)}\Omega_{\mathrm{T}}^2\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] \\
&\times \left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2k_z^2d}{(d^2 + M^2k^2k_z^2)}\right] - 4\upsilon_0^2k_x^4k_z^2\frac{V^2d}{(d^2 + M^2k^2k_z^2)}\Omega_{\mathrm{T}}^2 = 0.
\end{aligned} \tag{24}$$

The dispersion relation (24) represents the combined influence of viscosity, finite electrical conductivity, permeability, magnetic field, thermal conductivity, radiative heat-loss function, Hall current and FLR corrections on thermal instability of plasma. In the absence of FLR corrections, dispersion relation (24) is identical to the relation given by Prajapati *et al.* (2010) for non-rotational and non-gravitational case. In the absence of FLR corrections, viscosity and permeability, dispersion relation (24) is identical to the relation given by Bora and Talwar (1993) for non-gravitational case. In absence of radiative heat-loss function, thermal conductivity, finite electrical resistivity, viscosity and Hall current, the general dispersion relation (24) is identical to the relation given by Sharma (1974) for the non-rotational and non-gravitational case. In the absence of viscosity, finite electrical resistivity, FLR corrections, permeability, thermal conductivity and radiative heat-loss function, dispersion relation (24) is reduced to that obtained by Damiano *et al.* (2009). Also in the absence of FLR corrections, viscosity, finite conductivity, Hall current and thermal conductivity dispersion, relation (24) is reduced to that obtained by Field (1965). In the absence of Hall current, dispersion relation (24) is identical to Kaothekar *et al.* (2016) for non-gravitational and non porous case.

Now we discuss the general dispersion relation (24) for longitudinal and transverse wave propagation.

5. Discussion

5.1 *Longitudinal propagation* ($k_x = 0, k_z = k$)

In this case the perturbations are taken to be parallel to the direction of the magnetic field (i.e. $k_x = 0$, $k_z = k$). The dispersion relation (24) reduces to

$$\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] \times \left\{\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{V^2k^2d}{(d^2 + M^2k^4)}\right]^2 + \left[2\upsilon_0 k^2 - \frac{V^2Mk^4}{(d^2 + M^2k^4)}\right]^2\right\}\left[\omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2\right] = 0. \quad (25)$$

The first component of the dispersion relation (25) gives

$$\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) = 0. \quad (26)$$

This represents a damped mode modified by the presence of viscosity and permeability of the medium. Thus viscous force is able to stabilize the growth rate of the considered system. The above mode is unaffected by the presence of FLR correction, Hall current, magnetic field strength, finite electrical resistivity, thermal conductivity, radiative heat-loss function. This dispersion relation is identical to that of Prajapati *et al.* (2010).

The second factor of equation (25) on simplification gives

$$\omega^4 + 2\left[\eta k^2 + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right]\omega^3 + \left\{\left[\upsilon\left(k^2 + \frac{1}{K_1}\right) + \eta k^2\right]^2 + 2\left[V^2k^2 + 2\upsilon_0^2k^4 + +\eta k^2\upsilon\left(k^2 + \frac{1}{K_1}\right)\right] + M^2k^4\right\}\omega^2$$

$$+ 2\left\{\left[\upsilon\left(k^2+\frac{1}{K_1}\right)+\eta k^2\right]\left[V^2k^2+\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)\right]\right.$$
$$\left.+ M^2k^4\upsilon\left(k^2+\frac{1}{K_1}\right)+4\eta k^2\upsilon_0^2k^4\right\}\omega+\left\{\left[V^2k^2+\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)\right]^2\right.$$
$$\left.+ 4\upsilon_0^2k^4(\eta^2k^4+M^2k^4)+M^2k^4\upsilon^2\left(k^2+\frac{1}{K_1}\right)^2-4M\upsilon_0V^2k^6\right\}=0. \tag{27}$$

The above equation shows the dispersion relation for finitely conducting, viscous plasma including the effects of Hall current, FLR corrections, magnetic field and permeability. It is independent of radiative heat-loss functions and thermal conductivity. Hence the above dispersion relation represents the wave propagation. Equation (27) is a modified form of Vaghela and Chhajlani (1989) by the inclusion of Hall current in our case. Also equation (27) is a modified form of Chhajlani and Parihar (1993) by the inclusion of FLR corrections in our case.

In absence of viscosity, finite resistivity, FLR corrections and Hall current ($\upsilon = \eta = \upsilon_0 = M = 0$), equation (27) becomes

$$\omega^4 + 2V^2k^2\omega^2 + V^4k^4 = 0. \tag{28}$$

The roots of the above equation are

$$\omega_{1,2}^2 = -V^2k^2. \tag{29}$$

Equation (29) shows the Alfven mode in this mode there is no instability.

In the absence of viscosity, finite resistivity and Hall current ($\upsilon = \eta = M = 0$) equation (27) becomes

$$\omega^4 + (2V^2k^2 + 4\upsilon_0^2k^4)\omega^2 + V^4k^4 = 0. \tag{30}$$

The roots of the above equation are

$$\omega_{1,2}^2 = -(V^2k^2 + 2\upsilon_0^2k^4) \pm 2\upsilon_0k^2(\alpha V^2k^2 + \upsilon_0^2k^4)^{1/2}. \tag{31}$$

The above relation shows the modified form of Alfven mode by inclusion of FLR corrections. Thus FLR corrections modifies the mode by changing the growth rate.

In absence of viscosity, finite resistivity and FLR corrections ($\upsilon = \eta = \upsilon_0 = 0$), equation (27) becomes

$$\omega^4 + (2V^2k^2 + M^2k^4)\omega^2 + V^4k^4 = 0. \tag{32}$$

The roots of the above equation are

$$\omega_{1,2}^2 = \frac{-(2V^2k^2 + M^2k^4) \pm Mk^2(4V^2k^2 + M^2k^4)^{1/2}}{2}. \tag{33}$$

The above relation shows the modified form of Alfven mode by inclusion of Hall current. Thus Hall current modifies the mode by changing the growth rate. Equation (34) is modified form of Chhajlani and Parihar (1993) by the inclusion of

FLR corrections and modified form of Vaghela and Chhajlani (1989) by inclusion of Hall current in our case.

In absence of viscosity and finite resistivity ($\upsilon = \eta = 0$), equation (28) becomes

$$\omega^4 + (2V^2k^2 + M^2k^4 + 4\upsilon_0^2k^4)\omega^2 + 4\upsilon_0^2k^4M^2k^4 + V^4k^4 = 0. \tag{34}$$

The roots of the above equation are

$$\omega_{1,2}^2 = \frac{-(2V^2k^2 + M^2k^4 + 4\upsilon_0^2k^4) \pm [(M^2k^4 - 4\upsilon_0^2k^4)^2 + 4V^2k^2(M^2k^4 + 4\upsilon_0^2k^4)]^{1/2}}{2}. \tag{35}$$

The above relation shows the modified form of Alfven mode by inclusion of Hall current and FLR corrections. Thus the Hall current and FLR corrections modify the mode by changing the growth rate.

The third component of the dispersion relation (26) on simplifying gives

$$\begin{aligned}\omega^3 &+ \left[\upsilon\left(k^2 + \frac{1}{K_1}\right) + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right]\omega^2 \\ &+ \left[(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\upsilon\left(k^2 + \frac{1}{K_1}\right) + c^2k^2\right]\omega \\ &+ \left[k^2(\gamma - 1)\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] = 0. \end{aligned} \tag{36}$$

The above equation represents the combined influence of thermal conductivity, radiative heat-loss function, viscosity and permeability on the thermal instability of plasma, but there is no effect of Hall current, finite electrical conductivity, FLR corrections and magnetic field strength on the thermal instability of the considered system.

When the constant term of the cubic equation (36) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from the constant term of equation (36) is given as

$$\left[k^2(\gamma - 1)\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] < 0. \tag{37}$$

The above condition of instability is independent of FLR corrections, Hall current, finite electrical conductivity, magnetic field strength, viscosity and permeability. The above inequality (37) is reduced form of Bora and Talwar (1993). In the present case, we have considered the effects of FLR correction, viscosity and permeability but Bora and Talwar (1993) did not consider these effects. Thus the dispersion relation in the present analysis is modified due to the presence of FLR correction, viscosity and permeability, but the condition of instability is unaffected by the presence of FLR correction, viscosity and permeability. Thus we conclude that the FLR correction, viscosity and permeability of the medium have no effect on the condition of instability. Also it is clear that the growth rate of dispersion relation given by Bora and Talwar (1993) is getting modified due to the presence of FLR corrections, viscosity and permeability in our present case. Thus we conclude that FLR corrections, viscosity and permeability modify the growth rate of instability in the present case. Hence these are the new findings in our case.

Thus to argue the consequence of each limitation on the growth rate of instability we solve equation (36) numerically by initiating the following dimensionless amount:

$$\omega^* = \frac{\omega}{k_\rho c}, \quad \nu^* = \frac{\nu k_\rho}{c}, \quad k^* = \frac{k}{k_\rho}, \quad k_\lambda^* = \frac{k_\rho}{k_\lambda}, \quad k_{\rm T}^* = \frac{k_{\rm T}}{k_\rho}, \tag{38}$$

$$\omega^{*3} + \left[\upsilon^*\left(k^{*2} + \frac{1}{K_1^*}\right) + k_{\rm T}^* + k_\lambda^{*2}k^{*2}\right]\omega^{*2} + \left[\left(k_{\rm T}^* + k_\lambda^{*2}k^{*2}\right)\upsilon^*\left(k^{*2} + \frac{1}{K_1^*}\right) + k^{*2}\right]\omega^* + \left[\frac{k^2}{\gamma}\left(k_{\rm T}^* + k_\lambda^{*2}k^{*2} - 1\right)\right] = 0. \tag{39}$$

Figure 1 shows the effect of k_λ^* on the growth rate of thermal instability for fixed values of other parameters. From the figure, it is clear that as the value of k_λ^* increases both the peak value and the growth rate of thermal instability decreases. Thus the parameter k_λ^* moves the present system towards stabilization. In Fig. 2, we have plotted the growth rate of thermal instability against wave number for different values of parameter $k_{\rm T}^*$. From the figure, we conclude that as the value of $k_{\rm T}^*$ increases, the peak value of curves decreases and the area of growth rate also decreases. Hence, the presence of $k_{\rm T}^*$ also stabilizes the system. In Fig. 3, we have shown the effect of viscosity on the growth rate of thermal instability. Figure 3 displays that on increasing the value of viscosity, the growth rate of thermal instability also decreases. Hence, the presence of viscosity also stabilizes the system. In Fig. 4, we have shown the effect of permeability on the growth rate of thermal instability. Figure 4 displays that

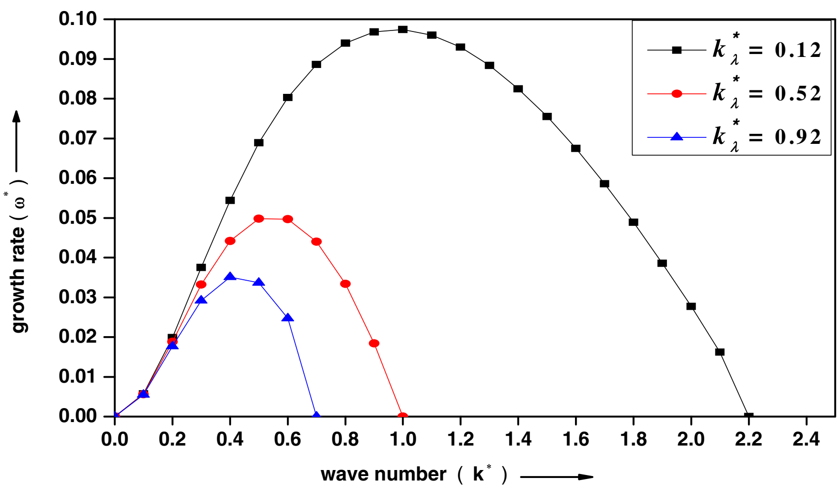

Figure 1. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of k_λ^* having $k_{\rm T}^* = 0.5$ and $K_1^* = \upsilon^* = \upsilon_0^* = 1$.

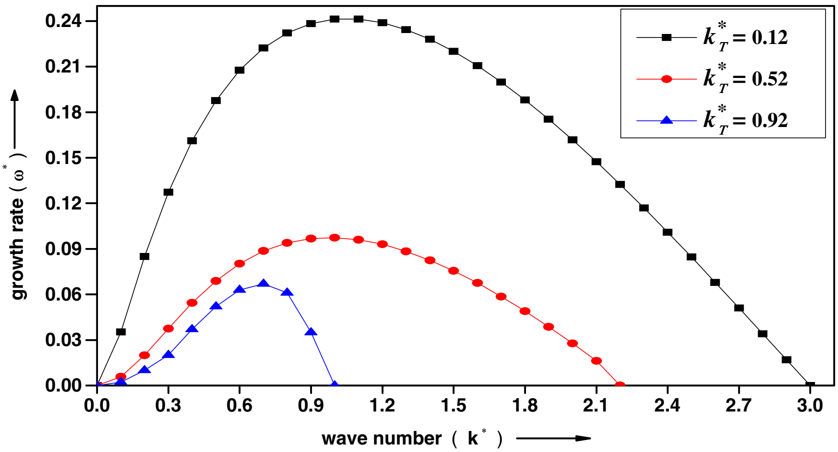

Figure 2. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of k^*_{T} having $k^*_{\lambda} = 0.1$ and $K^*_1 = \upsilon^* = \upsilon^*_0 = 1$.

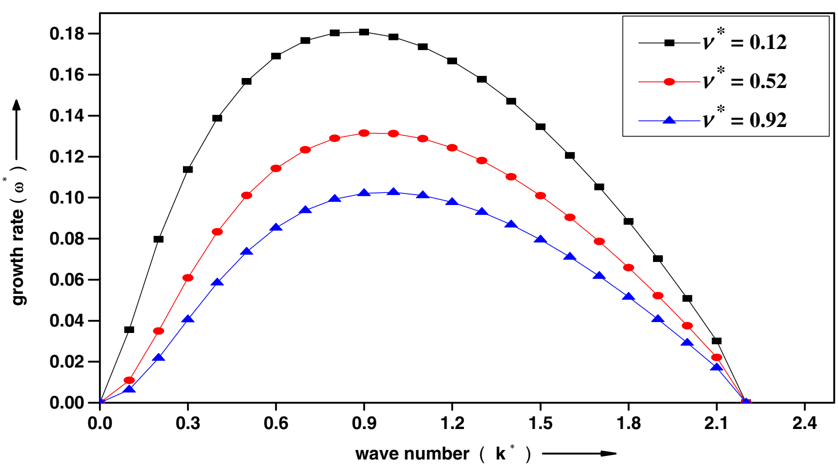

Figure 3. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of υ^* having $k^*_{\mathrm{T}} = 0.5$ and $k^*_{\lambda} = 0.1$, $K^*_1 = \upsilon^*_0 = 1$.

on increasing the value of permeability, the growth rate of thermal instability also increases. Hence, the presence of permeability also destabilizes the system. Therefore, the parameters k^*_{λ}, k^*_{T} and viscosity stabilize the system, whereas the parameter permeability destabilizes the system.

If the constant term of the cubic equation (36) is greater than zero, then all the coefficients of equation (36) must be positive. Equation (36) is a third degree in the power of ω having its coefficients positive, which is a necessary condition for

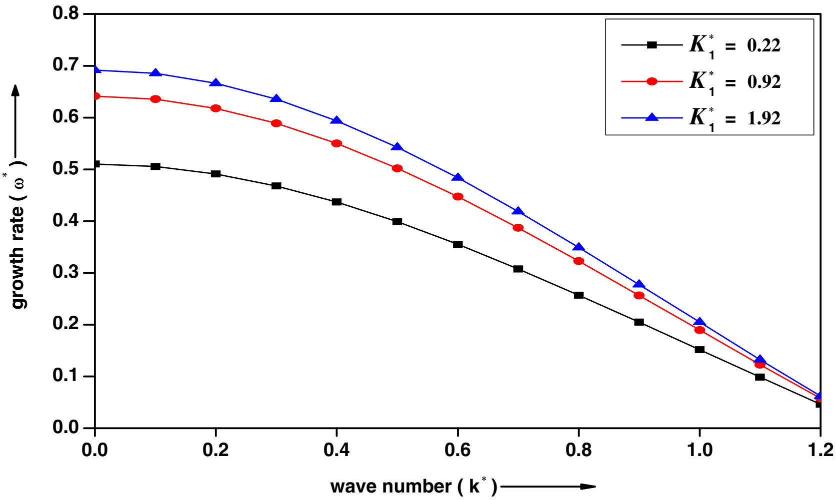

Figure 4. Growth rate (positive values of ω^*) against wave number k^* for different values of parameter K_1^* keeping the other parameters fixed having $k_{\mathrm{T}}^* = 0.5$ $k_\lambda^* = 0.1$, and $\upsilon^* = 1$, $\nu_0^* = 1$.

the stability of the system. To achieve the sufficient condition, the principal diagonal minors of Hurwitz matrix must be positive. The principal diagonal minors are

$$\Delta_1 = \left[\upsilon\left(k^2 + \frac{1}{K_1}\right) + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right] > 0,$$
$$\Delta_2 = \upsilon\left(k^2 + \frac{1}{K_1}\right)\left[\Delta_1(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + c^2 k^2\right] + (\gamma - 1)k^2 \rho L_\rho > 0,$$
$$\Delta_3 = \Delta_2\left[k^2(\gamma - 1)\left(T L_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] > 0. \tag{40}$$

Since $\Omega_j^2 > 0$, $\Omega_I^2 > 0$ and $\gamma > 1$, it is clear that all the Δ's are positive, hence the system represented by equation (36) is a stable system.

In the absence of thermal conductivity ($\lambda = 0$), dispersion relation (36) gives

$$\omega^3 + \left[\upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{\gamma L_{\mathrm{T}}}{c_p}\right]\omega^2 + \left[\upsilon\left(k^2 + \frac{1}{K_1}\right)\frac{\gamma L_{\mathrm{T}}}{c_p} + c^2 k^2\right]\omega + \frac{\gamma L_{\mathrm{T}}}{c_p}\left[k^2\left(c'^2 - \frac{p L_\rho}{T L_{\mathrm{T}}}\right)\right] = 0. \tag{41}$$

The condition of instability obtained from the constant term of the above equation is given as

$$\left[k^2\left(c'^2 - \frac{p L_\rho}{T L_{\mathrm{T}}}\right)\right] < 0. \tag{42}$$

Thus for longitudinal mode of propagation as given in equation (25), the system is unstable only for thermal condition, else it is stable. Also for longitudinal wave propagation, the thermal criterion of instability is unaffected by FLR corrections, viscosity, magnetic field, finite electrical resistivity and permeability but thermal conductivity and radiative heat-loss function modify the thermal expression and the fundamental thermal instability criterion becomes a radiative instability criterion.

5.2 *Transverse propagation* ($k_x = k, k_z = 0$)

In this case the perturbations are taken to be perpendicular to the direction of the magnetic field (i.e. $k_x = k, k_z = 0$). The dispersion relation (24) reduces to

$$\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right]^2 \left\{\left[\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right)\right] \times \left[\omega^2 + \omega\upsilon\left(k^2 + \frac{1}{K_1}\right) + \Omega_{\mathrm{T}}^2 + \frac{\omega \mathrm{V}^2 k^2}{d}\right] + \omega\upsilon_0^2 k^4\right\} = 0. \quad (43)$$

The first component of the dispersion relation (43) gives

$$\omega + \upsilon\left(k^2 + \frac{1}{K_1}\right) = 0. \quad (44)$$

This represents a stable viscous mode modified by the presence of permeability of the medium and discussed in equation (25).

The second component of the dispersion relation (43) on simplifying gives

$$\begin{aligned}
&\omega^5 + \left\{\left[2\upsilon\left(k^2 + \frac{1}{K_1}\right) + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right] + \eta k^2\right\}\omega^4 \\
&+ \left\{\left[2\upsilon\left(k^2 + \frac{1}{K_1}\right)(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + \upsilon^2\left(k^2 + \frac{1}{K_1}\right)^2 + \upsilon_0^2 k^4 + c^2 k^2\right]\right. \\
&\left. + \eta k^2\left[(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + 2\upsilon\left(k^2 + \frac{1}{K_1}\right)\right] + V^2 k^2\right\}\omega^3 \\
&+ \left\{\left[(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\upsilon^2\left(k^2 + \frac{1}{K_1}\right)^2\right.\right. \\
&+ (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) \\
&\left.\times\, \upsilon_0^2 k^4 \upsilon\left(k^2 + \frac{1}{K_1}\right)(c^2 k^2) + k^2(\gamma - 1)\left(T L_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] \\
&+ \upsilon\left(k^2 + \frac{1}{K_1}\right)\left[2\eta k^2(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + \eta k^2 \upsilon\left(k^2 + \frac{1}{K_1}\right) + V^2 k^2\right] \\
&\left. + \eta k^2[\upsilon_0^2 k^4 + c^2 k^2] + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + V^2 k^2\right\}\omega^2 \\
&+ \left\{\upsilon\left(k^2 + \frac{1}{K_1}\right)\left[\eta k^2 \upsilon\left(k^2 + \frac{1}{K_1}\right)(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right.\right. \\
&+ V^2 k^2(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + \eta k^2(c^2 k^2)
\end{aligned}$$

$$+\left[k^2(\gamma-1)\left(TL_{\mathrm{T}}-\rho L_\rho+\frac{\lambda k^2T}{\rho}\right)\right]\Bigg]+\eta k^2\upsilon_0^2k^4(\gamma-1)\left(\frac{T\rho L_{\mathrm{T}}}{p}+\frac{\lambda k^2T}{p}\right)$$
$$+\eta k^2\left[k^2(\gamma-1)\left(TL_{\mathrm{T}}-\rho L_\rho+\frac{\lambda k^2T}{\rho}\right)\right]\Bigg\}\omega+\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)$$
$$\times\left[k^2(\gamma-1)\left(TL_{\mathrm{T}}-\rho L_\rho+\frac{\lambda k^2T}{\rho}\right)\right]=0. \tag{45}$$

The above equation represents the combined influence of radiative heat-loss function, FLR corrections, finite electrical conductivity, thermal conductivity, viscosity, permeability and magnetic field on thermal instability of plasma. If we neglect the effect of FLR corrections equation (45) is identical to that given by Prajapati *et al.* (2010) for non rotational case. In the present case, we have considered the effects of FLR corrections, but Prajapati *et al.* (2010) have not considered this effect. Thus the dispersion relation in the present analysis is modified due to the presence of FLR corrections, but the condition of instability is unaffected by the presence of FLR corrections. Thus we conclude that FLR corrections have no effect on the condition of radiative instability, but the growth rate of the dispersion relation given by Prajapati *et al.* (2010) gets modified due to the presence of FLR corrections in our present case. Thus we conclude that FLR corrections modify the growth rate of radiative instability in the present case. Hence this is a new finding in our case.

When constant term of equation (45) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (45) is given as

$$\left[k^2(\gamma-1)\left(TL_{\mathrm{T}}-\rho L_\rho+\frac{\lambda k^2T}{\rho}\right)\right]<0. \tag{46}$$

The above inequality (45) is the reduced form of Bora and Talwar (1993). We solve equation (45) numerically by introducing the following dimensionless quantities:

$$\omega^*=\frac{\omega}{k_\rho c},\quad \nu^*=\frac{\nu k_\rho}{c},\quad k^*=\frac{k}{k_\rho},\quad k_\lambda^*=\frac{k_\rho}{k_\lambda},\quad k_{\mathrm{T}}^*=\frac{k_{\mathrm{T}}}{k_\rho},\ \eta^*=\frac{\eta k_\rho}{c},\ \upsilon_0^*=\frac{\upsilon_0k_\rho}{c}. \tag{47}$$

Using equation (47), we write equation (45) in non-dimensional form as

$$\omega^{*5}+\left\{\left[2\upsilon^*\left(k^{*2}+\frac{1}{K_1^*}\right)+k_{\mathrm{T}}^*+k_\lambda^*k^{*2}\right]+\eta^*k^{*2}\right\}\omega^4$$
$$+\Bigg\{\Bigg[2\upsilon^*\left(k^{*2}+\frac{1}{K_1^*}\right)(k_{\mathrm{T}}^*+k_\lambda^*k^{*2})+\upsilon^{*2}\left(k^{*2}+\frac{1}{K_1^*}\right)^2$$
$$+\upsilon_0^{*2}k^{*4}+k^{*2}\Bigg]+\eta^*k^{*2}\left[k_{\mathrm{T}}^*+k_\lambda^*k^{*2}+2\upsilon^*\left(k^{*2}+\frac{1}{K_1^*}\right)\right]$$
$$+V^{*2}k^{*2}\Bigg\}\omega^{*33}+\Bigg\{\Bigg[(k_{\mathrm{T}}^*+k_\lambda^*k^{*2})\upsilon^*\left(k^{*2}+\frac{1}{K_1^*}\right)^2$$

$$+ (k_T^* + k_\lambda^* k^{*2}) \upsilon_0^{*2} k^{*6} \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right) + \frac{k^{*2}}{\gamma} (k_T^* + k_\lambda^* k^{*2} - 1) \Bigg]$$

$$+ \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right) \Bigg[2\eta^* k^{*2} (k_T^* + k_\lambda^* k^{*2}) + \eta^* k^{*2} \times \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right)$$

$$+ V^{*2} k^{*2} \Bigg] + \eta^* k^{*2} [\upsilon_0^{*2} k^{*4} + k^{*2}] + (k_T^* + k_\lambda^* k^{*2}) + V^{*2} k^{*2} \Bigg\}$$

$$\times \omega^{*2} \Bigg\{ \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right) + \Bigg[\eta^* k^{*2} \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right) (k_T^* + k_\lambda^* k^{*2})$$

$$+ V^{*2} k^{*2} (k_T^* + k_\lambda^* k^{*2}) + \eta^* k^{*4} + \left[\frac{k^{*2}}{\gamma} (k_T^* + k_\lambda^* k^{*2} - 1) \right] \Bigg]$$

$$+ \eta^* k^{*2} \upsilon_0^{*2} k^{*4} (k_T^* + k_\lambda^* k^{*2}) + \eta^* k^{*2} \left[\frac{k^2}{\gamma} (k_T^* + k_\lambda^* k^{*2} - 1) \right] \Bigg\} \omega^*$$

$$+ \eta^* k^{*2} \upsilon^* \left(k^{*2} + \frac{1}{K_1^*} \right) \times \left[\frac{k^2}{\gamma} (k_T^* + k_\lambda^* k^{*2} - 1) \right] = 0. \quad (48)$$

In Figures 5–9, the dimensionless growth rate (ω^*) has been plotted against the dimensionless wave number (k^*) to see the effect of various physical parameters such as viscosity, radiative heat-loss function, resistivity and FLR corrections. From Fig. 5 we see that as the value of k_λ^* increases the growth rate decreases. Thus the effect of parameter k_λ^* is stabilizing. It is clear from Fig. 6 that the growth rate decreases with increasing parameter k_T^*. Thus the presence of k_T^* stabilizes the growth rate of

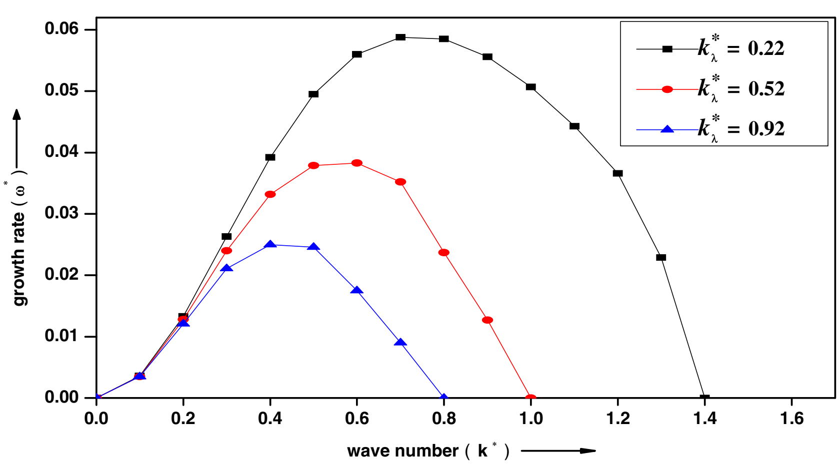

Figure 5. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of k_λ^* having $k_T^* = 0.5$ and $v^* = v_0^* = \eta^* = 1$.

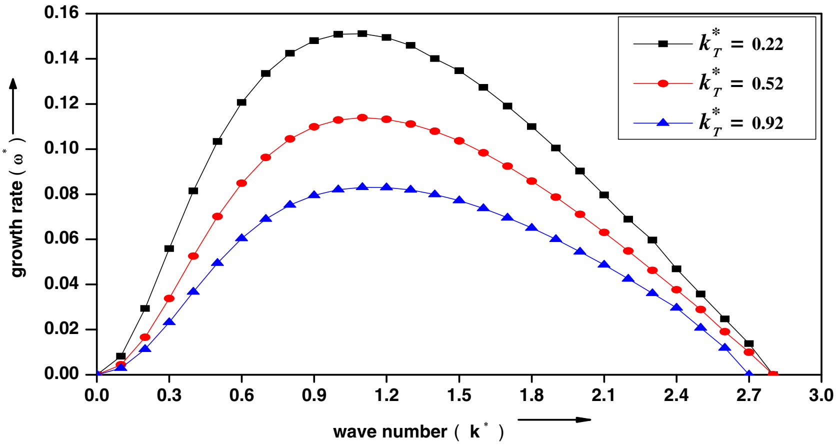

Figure 6. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of k_T^* having $k_\lambda^* = 0.5$ and $v^* = v_0^* = \eta^* = 1$.

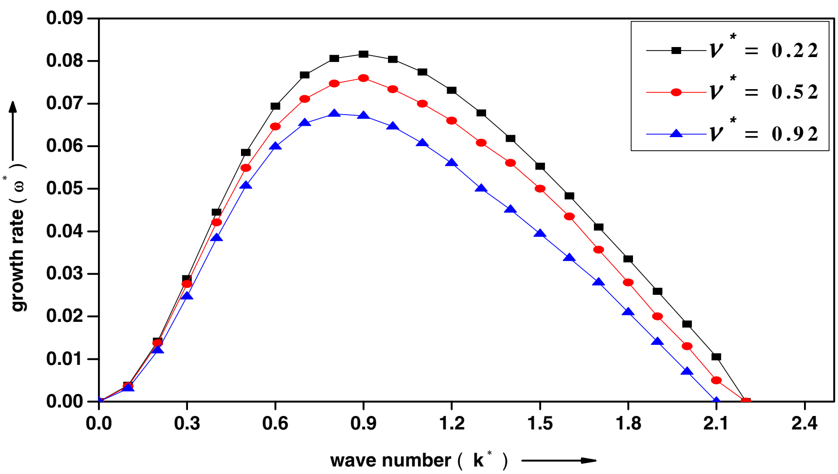

Figure 7. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of v^* having $k_T^* = 0.5$ and $k_\lambda^* = 0.1$, $\eta^* = v_0^* = 1$.

the system. From Fig. 7, we conclude that the growth rate decreases with increasing value of viscosity. Thus the effect of viscosity is stabilizing.

Figure 8 shows the influence of FLR corrections on the growth rate of thermal instability. From the figure, it is clear that the FLR corrections have a stabilizing effect on the growth rate of thermal instability. Figure 9 shows the influence of resistivity on the growth rate of thermal instability. From the figure, it is clear that the resistivity has a destabilizing effect on the growth rate of thermal instability. Therefore, the radiative heat-loss functions, viscosity and FLR corrections have a stabilizing influence on the system while the finite electrical resistivity has a destabilizing influence on the growth rate of the system.

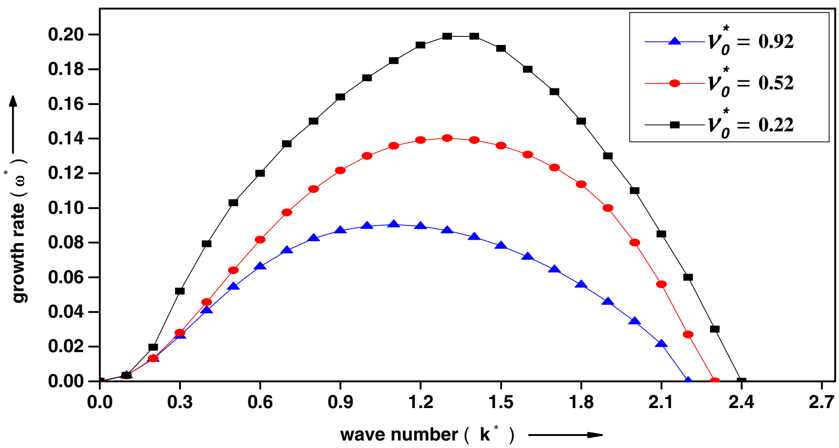

Figure 8. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of υ_0^* having $k_\lambda^* = 0.1$, $k_\mathrm{T}^* = 0.5$ and $\upsilon^* = \eta^* = 1$.

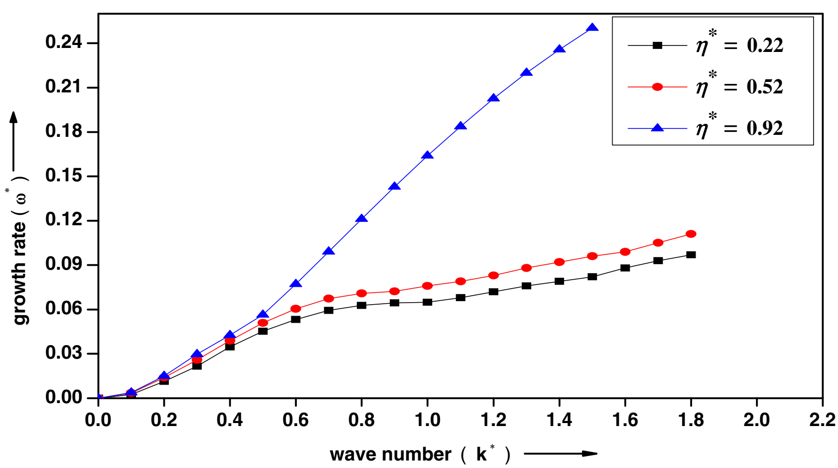

Figure 9. The normalized growth rate (ω^*) as a function of normalized wave number (k^*) for different values of η^* having $k_\lambda^* = 0.1$, $k_\mathrm{T}^* = 0.5$ and $\upsilon^* = \upsilon_0^* = 1$.

Now we wish to examine the effect of FLR corrections and radiative heat-loss functions on the considered system with some simplifications and at the same time we wish to investigate the physics involved in such simplifications in the present problem.

In absence of thermal conductivity ($\lambda = 0$), equation (45) reduces to

$$\omega^5 + \left\{\left[2\upsilon\left(k^2 + \frac{1}{K_1}\right) + \frac{\gamma L_\mathrm{T}}{c_p}\right] + \eta k^2\right\}\omega^4$$
$$+ \left\{\left[2\upsilon\left(k^2 + \frac{1}{K_1}\right)\frac{\gamma L_\mathrm{T}}{c_p} + \upsilon^2\left(k^2 + \frac{1}{K_1}\right)^2 + \upsilon_0^2 k^4 + c^2 k^2\right]\right.$$

$$+\eta k^2\left[2\upsilon\left(k^2+\frac{1}{K_1}\right)+\frac{\gamma L_{\mathrm{T}}}{c_p}\right]+V^2k^2\Bigg\}\omega^3$$
$$+\Bigg\{\Bigg[+\frac{\gamma L_{\mathrm{T}}}{c_p}\left(\upsilon^2\left(k^2+\frac{1}{K_1}\right)^2+\upsilon_0^2k^4\right)+\upsilon\left(k^2+\frac{1}{K_1}\right)$$
$$\times(c^2k^2)+\frac{\gamma L_{\mathrm{T}}}{c_p}\left[k^2\left(c'^2-\frac{pL_\rho}{TL_{\mathrm{T}}}\right)\right]\Bigg]+\upsilon\left(k^2+\frac{1}{K_1}\right)$$
$$\times\left[2\eta k^2\frac{\gamma L_{\mathrm{T}}}{c_p}+\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)+V^2k^2\right]+\eta k^2[\upsilon_0^2k^4+c^2k^2]$$
$$+\left(\frac{\gamma L_{\mathrm{T}}}{c_p}\right)V^2k^2\Bigg\}\omega^2+\Bigg\{\upsilon\left(k^2+\frac{1}{K_1}\right)\left[\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)\frac{\gamma L_{\mathrm{T}}}{c_p}+\eta k^2\right.$$
$$\left.+V^2k^2\frac{\gamma L_{\mathrm{T}}}{c_p}(c^2k^2)+\frac{\gamma L_{\mathrm{T}}}{c_p}\left[k^2\left(c'^2-\frac{pL_\rho}{TL_{\mathrm{T}}}\right)\right]\right]$$
$$+\eta k^2\left(\frac{\gamma L_{\mathrm{T}}}{c_p}\right)\left[\upsilon_0^2k^4+c'^2k^2-\frac{pL_\rho k^2}{TL_{\mathrm{T}}}\right]\Bigg\}\omega+\eta k^2\upsilon\left(k^2+\frac{1}{K_1}\right)\frac{\gamma L_{\mathrm{T}}}{c_p}$$
$$\times\left[k^2\left(c'^2-\frac{pL_\rho}{TL_{\mathrm{T}}}\right)\right]=0. \tag{49}$$

The condition of instability obtained from constant term of equation (49) is given as

$$\left[k^2\left(c'^2-\frac{pL_\rho}{TL_{\mathrm{T}}}\right)\right]<0, \tag{50}$$

and it is already discussed in equation (42). On comparing equations (45) and (49) we see that no new mode comes due to the inclusion of thermal conductivity, but the condition of instability and growth rate of instability get modified by the inclusion of thermal conductivity. Also on comparing equation (49) with equation (29) of Aggarwal and Talwar (1969), we conclude that the growth rate of radiative instability is modified by the inclusion of, FLR corrections and permeability in our case, but condition of instability is independent of and FLR corrections.

For infinitely conducting medium ($\eta = 0$), equation (45) becomes

$$\omega^4+\left\{\left[2\upsilon\left(k^2+\frac{1}{K_1}\right)+(\gamma-1)\left(\frac{T\rho L_{\mathrm{T}}}{p}+\frac{\lambda k^2T}{p}\right)\right]\right\}\omega^3$$
$$+\Bigg\{\Bigg[2\upsilon\left(k^2+\frac{1}{K_1}\right)(\gamma-1)\left(\frac{T\rho L_{\mathrm{T}}}{p}+\frac{\lambda k^2T}{p}\right)$$
$$+\upsilon^2\left(k^2+\frac{1}{K_1}\right)^2+\upsilon_0^2k^4+c^2k^2\Bigg]+V^2k^2\Bigg\}\omega^2$$
$$+\Bigg\{\Bigg[(\gamma-1)\left(\frac{T\rho L_{\mathrm{T}}}{p}+\frac{\lambda k^2T}{p}\right)\upsilon^2\left(k^2+\frac{1}{K_1}\right)^2$$

$$+ (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) \upsilon_0^2 k^4 \upsilon \left(k^2 + \frac{1}{K_1}\right)(c^2k^2)$$
$$+ k^2(\gamma - 1)\left(TL - \rho L + \frac{\lambda k T}{\rho}\right)\Bigg]$$
$$+ \upsilon\left(k^2 + \frac{1}{K_1}\right)[V^2k^2] \times (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + V^2k^2\Bigg\}\omega$$
$$+ \Bigg\{\upsilon\left(k^2 + \frac{1}{K_1}\right)\Bigg[V^2k^2(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)$$
$$+ \left[k^2(\gamma - 1)\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right]\Bigg]\Bigg\} = 0. \quad (51)$$

The condition of instability obtained from constant term of equation (51) is given as

$$\left\{\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) V^2k^2 + \left[k^2\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right]\right\} < 0. \quad (52)$$

This relation is the reduced form of Bora and Talwar (1993). From equation (52), we see that the magnetic field tries to stabilize the system. On comparing equations (45) and (51), we see that the one mode is increased due to the inclusion of finite resistivity. Also on comparing equations (46) and (52), we conclude that the inclusion of finite resistivity removes the effect of magnetic field from condition of instability and tries to destabilize the system. Also on comparing equation (52) with equation (29) of Aggarwal and Talwar (1969), we conclude that the condition of instability is modified by inclusion of FLR corrections and the growth rate of radiative instability is modified by the inclusion of FLR corrections and permeability in our case. Hence these are new results in our case.

In absence of viscosity ($\upsilon = 0$), equation (45) becomes

$$\omega^4 + \left[\eta k^2 + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right]\omega^3 + \Bigg[(\upsilon_0^2 k^4 + c^2k^2)$$
$$+ \eta k^2(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)$$
$$+ \eta k^2[\upsilon_0^2 k^4 + c^2k^2] + (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) V^2k^2\Bigg]\omega + \eta k^2\Bigg[(\gamma - 1)$$
$$\times \left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\upsilon_0^2 k^4 + k^2(\gamma - 1)\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\Bigg] = 0. \quad (53)$$

The condition of instability obtained from the constant term of equation (53) is given as

$$\left[\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\upsilon_0^2 k^4 + k^2\left(TL_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] < 0. \quad (54)$$

From equation (54), we see that FLR corrections try to stabilize the radiative instability.

For inviscid perfectly conducting medium ($\upsilon = \eta = 0$), equation (45) becomes

$$\omega^3 + \left[(\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right]\omega^2 + [(\upsilon_0^2 k^4 + c^2 k^2) + V^2 k^2]\omega$$
$$+ \left\{\left[\upsilon_0^2 k^4 (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + k^2(\gamma - 1)\right.\right.$$
$$\left.\left.\times\left(T L_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right] + V^2 k^2 (\gamma - 1)\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right\} = 0. \quad (55)$$

The above equation is the reduced form of Bora and Talwar (1993) in the absence of FLR corrections. The condition of instability obtained from the constant term of equation (55) is given as

$$\left\{\left[\upsilon_0^2 k^4\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right) + k^2\left(T L_{\mathrm{T}} - \rho L_\rho + \frac{\lambda k^2 T}{\rho}\right)\right]\right.$$
$$\left. + V^2 k^2\left(\frac{T\rho L_{\mathrm{T}}}{p} + \frac{\lambda k^2 T}{p}\right)\right\} < 0. \quad (56)$$

From equation (56) we see that FLR corrections and magnetic field tries to stabilize the radiative instability.

In the absence of viscosity, finite resistivity, thermal conductivity and radiative heat-loss function ($\upsilon = \eta = \lambda = L_{\mathrm{T},\rho} = 0$), equation (45) becomes

$$\omega^2 + [\upsilon_0^2 k^4 + c^2 k^2 + V^2 k^2] = 0. \quad (57)$$

The above equation (57) is the modified form of Uberoi (2009) by inclusion of FLR corrections in our problem. The condition of instability obtained from equation (57) is given as

$$[\upsilon_0^2 k^4 + c^2 k^2 + V^2 k^2] < 0. \quad (58)$$

From equation (58) we see that magnetic field and FLR corrections stabilize the system. On comparing equations (45) and (57), we see that dispersion relation given by Uberoi (2009) is modified by inclusion FLR corrections, radiative heat-loss function, thermal conductivity, viscosity, finite electrical resistivity and permeability in our case. Hence we have an improved result of Uberoi (2009).

Thus we conclude that for transverse wave propagation the thermal criterion is affected by FLR corrections, radiative heat-loss functions, viscosity, magnetic field strength, thermal conductivity and finite electrical resistivity. But there is no effect of Hall parameter in transverse mode. From curves, we find that FLR corrections, viscosity and heat-loss function have stabilizing influence on the growth rate of thermal instability, whereas permeability and finite electrical resistivity have a destabilizing influence on the thermal instability of plasma.

6. Conclusions

The thermal instability of an infinite homogeneous viscous thermally and electrically conducting, radiating fluid including FLR corrections and Hall current have been investigated for star formation in interstellar medium. We have considered longitudinal and transverse wave propagation to the direction of external magnetic field. We find that thermal criterion remains valid and gets modified because of radiative heat-loss function and thermal conductivity. We also find that for longitudinal wave propagation, Hall current, FLR correction, permeability, viscosity, magnetic field strength and finite resistivity have no effect on thermal criterion. But thermal and radiative effects independently as well as jointly modify the thermal criterion. Also FLR corrections and Hall current modify the growth rate of Alfven mode. For transverse wave propagation, FLR corrections, magnetic field strength, viscosity and finite resistivity affect the condition of radiative instability. FLR corrections stabilize the system in case of non-viscous medium. Also, magnetic field stabilizes the system but finite conductivity removes the effect of magnetic field thereby destabilizing the system. The Hall parameter has no effect on the transverse mode of propagation. Numerical calculation shows the stabilizing effect of heat-loss function, FLR corrections and viscosity and destabilizing effect of permeability of considered system.

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