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Possible Effect of the Earth's Inertial Induction on the Orbital Decay of LAGEOS

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Abstract. The theory of velocity dependent inertial induction, based upon extended Mach's principle, has been able to generate many interesting results related to celestial mechanics and cosmological problems. Because of the extremely minute magnitude of the effect its presence can be detected through the motion of accurately observed bodies like Earth satellites. LAGEOS I and II are medium altitude satellites with nearly circular orbits. The motions of these satellites are accurately recorded and the past data of a few decades help to test many theories including the general theory of relativity. Therefore, it is hoped that the effect of the Earth's inertial induction can have any detectable effect on the motion of these satellites. It is established that the semi-major axis of LAGEOS I is decreasing at the rate of 1.3 mm/d. As the atmospheric drag is negligible at that altitude, a proper explanation of the secular change has been wanting, and, therefore, this paper examines the effect of the Earth's inertial induction effect on LAGEOS I. Past researches have established that Yarkovsky thermal drag, charged and neutral particle drag might be the possible mechanisms for this orbital decay. Inertial induction is found to generate a perturbing force that results in 0.33 mm/d decay of the semi major axis. Some other changes are also predicted and the phenomenon also helps to explain the observed changes in the orbits of a few other satellites. The results indicate the feasibility of the theory of inertial induction i.e. the dynamic gravitation phenomenon of the Earth on its satellites as a possible partial cause for orbital decay.

Key words. Inertial induction—LAGEOS—orbital decay.

1. Introduction

The theory of inertial induction was first proposed by Sciama (1972) as a quantitative route to Mach's principle. However, the theory resulted in only partial success. Later, when a velocity-dependent component of dynamic gravity was added by Ghosh (1984, 1986a) to the Newtonian static component and Sciams's acceleration dependent term, the combined model could explain many astrophysical and cosmological phenomena. Since then the model of this proposed inertial induction, based upon extended Mach's principle, has been applied to a number of problems with success. All the results have been published earlier. LAGEOS, the Laser Geodynamics Satellite, was launched by NASA with the main objective of accurately determining the position of the satellite with respect to Earth. It is a passive satellite at an altitude of 5900 km and an inclination of 110°, in an almost circular orbit around the Earth. It is at a reasonably high altitude and experiences considerably less atmospheric drag, which makes it the satellite of choice to test many theories, including the general theory of relativity. However, accounting for all the known forces acting on the satellite, there still remains a residual along-track acceleration which acts as a drag with a mean value of -3.4×10^{-12} m s^{-2}. It causes a gradual decay of the semi-major axis at a rate of 1.3 mm/d. Earlier, several mechanisms have been put forward to explain this decay (Rubincam 1982), but none of the theories could completely explain the phenomenon. Studies have suggested that the perturbation produced by the Schach effect, Pontyng–Robertson effect (Robertson 1937) and the terrestrial radiation pressure cannot explain the whole of the secular decay and, apparently, it was assumed that the charged particle drag might be the possible mechanism for this average decay. However, later it was found that charged particle drag has a minor contribution. Rather, the thermal drag due to infrared radiation from the Earth (Rubincam 1987) is the principal mechanism for the decay of the LAGEOS' orbit, which explains about 47% of the observed average drag value. Further analysis with a better model (Rubincam 1988) suggests that this thermal drag can explain up to 70% of the average decay, mostly depending on the spin axis position of LAGEOS, while in this model it was considered the spin axis of the LAGEOS to be fixed in space. Finally, it was summarized that Yarkovsky thermal drag along with charge and neutral particle drag can explain almost the entire observed drag (Rubincam 1990; Scharroo *et al.* 1991; Afonso *et al.* 1985). But these three effects entailed certain approximations, which, although feasible, lacked evidence to support the underlying assumptions. Consequently, these two proposals are more in the realm of possibilities than proven facts. So later on a considerable amount of work has been performed to interpret the spin axis orientation and spin axis direction of Lageos (Bertotti & Iess 1991; Farinella *et al.* 1996; Andrés *et al.* 2004), under the influence of forces and torques due to the Earth's gravitational and magnetic fields. For the measurement of spin parameters with accuracy, LageOS Spin Axis Model – LOSSAM (Andrés *et al.* 2004) have been developed. Further, by frequency analysis of the SLR data for a certain span of time, an exponential decrease of the spin rate and increase of the spin period was indicated (Kucharski *et al.* 2013). Considering this change of spin axis orientation as per LOSSAM, a numerical analysis of the thermal forces on Lageos was done (Andrés *et al.* 2006). This shows a large difference in acceleration obtained while considering its spin axis to be fixed, as assumed earlier. The mechanism of thermal drag has also been investigated to find out its impact on the Lageos node and inclination (Farinella *et al.* 1990). Recent work also showed that there can be a possible effect on frame-dragging test due to decay of the semi major axis of Lageos (Iorio 2016). This encouraged the authors to test the model of velocity-dependent inertial induction for the problem of the orbital decay of such satellites to explain the residual amount of the observed decay.

The model of inertial induction (Ghosh 1984, 1986a, 2000) suggests that, the total gravitational force between two interacting bodies depends on their relative velocity and acceleration along with the static Newtonian gravitational pull. In this paper, a system comprising the Earth and a polar satellite is considered and the effect on the satellite due to the velocity-dependent inertial induction from the Earth is estimated. First, a general expression for the radial, normal and tangential components are deduced and next, using a standard data of the Earth and LAGEOS, changes in the orbital parameters are evaluated. After allowing for approximations in the inclination and eccentricity of the LAGEOS' orbit, the result found from the inertial induction model is quite interesting and explains around 25% of the decrease of the semi-major axis. Apart from a change in the semi-major axis, changes in the other orbital elements can also be identified with the Gaussian perturbation equations. In a further investigation of the Earth's inertial induction, a similar analysis is also performed on the satellite Stella, with an inclination of 98.6° and an altitude of 800 km. Thus, the expectation that the effect of Earth's dynamic gravitational effect, i.e. inertial induction, can be detected in the form of excess decay rate of its satellites appears to be vindicated.

2. Model of velocity-dependent inertial induction

The model of inertial induction was first proposed by Ghosh (1984) which suggests that the total interactive force between the two particles depends not only on their relative distance but also on their relative velocity and acceleration. Therefore force $\mathbf{F}$ on m_1 due to only the velocity-dependent inertial induction from m_2 can be expressed as follows:

$$\mathbf{F} = \frac{Gm_1m_2}{c^2r^3} v_{\rm rel}^2 \mathbf{r} \cos\alpha\, |\cos\alpha|\,, \tag{1}$$

where G is the universal gravitational constant, c is the speed of light in vacuum, $\mathbf{r}$ is the position vector of m_2 with respect to m_1 and α is the angle made by $\mathbf{v}_{\rm rel}$ with $\mathbf{r}$ as shown in Fig. 1. The inertial induction mechanism has been able to explain the secular acceleration of Phobos (Ghosh 1986a, 2000) whose value is found to be 1.5×10^{-3} deg yr^{-2} and also certain other unexplained phenomena in celestial mechanics, worth mentioning among those are the secular retardation of the Earth's

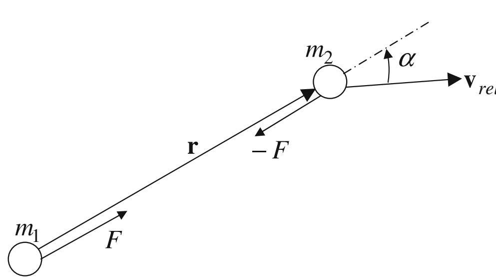

Figure 1. Force due to velocity dependent inertial induction.

rotation due to velocity-dependent inertial induction (Ghosh 1986a), transfer of solar angular momentum (Ghosh 1988), excess red shifts and flat rotation curves of spiral galaxies (Ghosh 1997; Ghosh *et al.* 1988).

3. Perturbing force due to inertial induction of the Earth

In order to estimate the effect of inertial induction on the motion of a polar satellite, it is first necessary to determine the perturbing force due to the velocity-dependent inertial induction. A satellite of mass m_{S}, which revolves around the Earth in a circular polar orbit of radius R with angular velocity ω_{S} is considered as shown in Fig. 2. Now, an elemental mass δm_{E} of the Earth is taken for interaction based on inertial induction, with the satellite. Therefore, the force on the satellite due to the velocity dependent inertial induction from δm_{E} can be expressed as

$$\mathbf{dF} = -\frac{G(\delta m_{\mathrm{E}})m_{\mathrm{s}}}{c^2 S^2} v_{\mathrm{rel}}^2 \cos\alpha \cdot |\cos\alpha| \ \hat{\mathbf{S}}, \tag{2}$$

where $\cos\alpha = \hat{\mathbf{S}} \cdot \hat{\mathbf{v}}_{\mathrm{rel}}$, $\hat{\mathbf{S}}$ and $\hat{\mathbf{v}}_{\mathrm{rel}}$ are the unit vectors along $\mathbf{S}$ and $\mathbf{v}_{\mathrm{rel}}$ respectively, $\mathbf{v}_{\mathrm{rel}}$ is the relative velocity of the satellite with respect to the elemental mass of the Earth δm_{E}, which is equal to $\rho(r)(r^2\cos\theta)\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r$ and $\rho(r)$ is the density of δm_{E} at that location. So, the relative velocity of the satellite with respect to the elemental mass is

$$\mathbf{v}_{\mathrm{rel}} = \mathbf{v}_{\mathrm{S}} - \ \mathbf{v}_{\mathrm{E}}.$$

$\mathbf{v}_{\mathrm{S}}$ is the velocity of the satellite and $\mathbf{v}_{\mathrm{E}}$ is the velocity of the elemental mass δm_{E}. Therefore $\mathbf{v}_{\mathrm{S}} = \omega_{\mathrm{S}} \times \mathbf{R}$ and $\mathbf{v}_{\mathrm{E}} = \Omega_{\mathrm{E}} \times \mathbf{r}$, where Ω_{E} is the spin angular velocity of

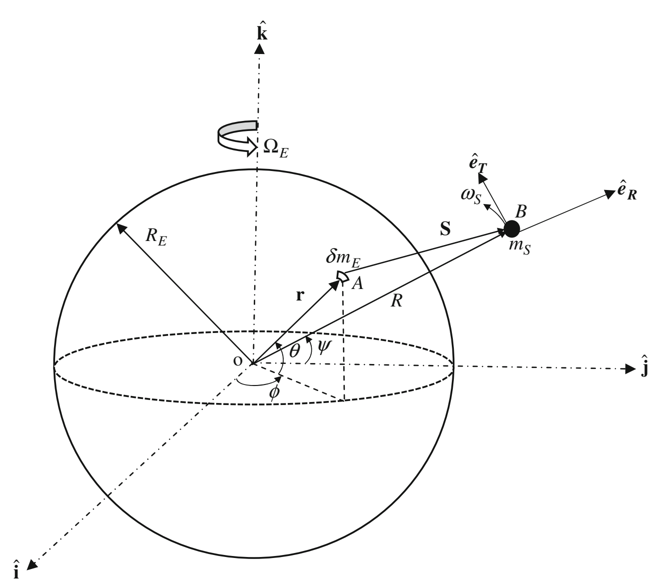

Figure 2. Schematic diagram of the Earth satellite system.

the Earth, ω_S is the angular velocity of the satellite and $\mathbf{R}$, $\mathbf{r}$ are the position vectors of the satellite and elemental mass, respectively, where

$$\mathbf{r} = \hat{\mathbf{i}}\, r\cos\theta\cos\varphi + \hat{\mathbf{j}}\, r\cos\theta\sin\varphi + \hat{\mathbf{k}}\, r\sin\theta,$$

$$\mathbf{R} = \hat{\mathbf{j}}\, R\cos\psi + \hat{\mathbf{k}}\, R\sin\psi,$$

where ψ is the argument of latitude of the satellite. Using the above relations, $\mathbf{v}_{\mathrm{rel}}$ becomes

$$\begin{aligned}\mathbf{v}_{\mathrm{rel}} \;=\; & \hat{\mathbf{i}}\,(\Omega_{\mathrm{E}} r\cos\theta\sin\varphi) - \hat{\mathbf{j}}\,(R\omega_{\mathrm{S}}\sin\psi + \Omega_{\mathrm{E}} r\cos\theta\cos\varphi)\\ & + \hat{\mathbf{k}}\,(R\omega_{\mathrm{S}}\cos\psi)\,.\end{aligned}$$

Now, the relative distance between A and B can be expressed as

$$\begin{aligned}\mathbf{S} \;=\; & \mathbf{R} - \mathbf{r} = -\hat{\mathbf{i}}\,(r\cos\theta\cos\varphi) - \hat{\mathbf{j}}\;(r\cos\theta\sin\varphi - R\cos\psi)\\ & -\hat{\mathbf{k}}\;(r\sin\theta - R\sin\psi)\,.\end{aligned}$$

Therefore,

$$\cos\alpha = \frac{\mathbf{S}\cdot\mathbf{v_{rel}}}{S\cdot v_{\mathrm{rel}}} = \frac{Rr\cos\theta\cos\psi\;\;(\omega_{\mathrm{S}}\tan\psi\sin\varphi - \Omega_{\mathrm{E}}\cos\varphi - \omega_{\mathrm{S}}\tan\theta)}{S\;v_{\mathrm{rel}}}. \quad (3)$$

Using the expression of $\cos\alpha = \mathbf{S}{\cdot}\mathbf{v_{rel}}/S{\cdot}v_{\mathrm{rel}}$ in equation (2), the force on the satellite by the elemental mass due to velocity dependent inertial induction is expressed as follows:

$$\mathbf{dF} = -\frac{Gm_{\mathrm{S}}}{c^2}[\{\rho\,(r)\,r^2\cos\theta\,(\mathbf{S}\cdot\mathbf{v}_{\mathrm{rel}})\cdot|\mathbf{S}\cdot\mathbf{v}_{\mathrm{rel}}|/S^4\}\,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r]\;\hat{\mathbf{S}}. \quad (4)$$

This force acts along the line joining the A and B as in Fig. 2, which is resolved into its X, Y and Z components in the Earth-centered inertial frame of reference as

$$\begin{aligned}F_X \;=\; & \frac{Gm_{\mathrm{S}}}{c^2}\int_0^{R_{\mathrm{E}}}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}[\{\rho\,(r)\;\,r^3\cos^2\theta\cos\varphi\;\,(\mathbf{S}\cdot\mathbf{v}_{\mathrm{rel}})\\ & \cdot|\,\mathbf{S}\cdot\mathbf{v}_{\mathrm{rel}}|/S^5\}]\;\mathrm{d}\varphi\;\mathrm{d}\theta\;\mathrm{d}r,\end{aligned} \quad (5a)$$

$$\begin{aligned}F_Y = & \frac{Gm_{\mathrm{S}}}{c^2}\int_0^{R_{\mathrm{E}}}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}[\{\rho\,(r)r^2\cos\theta\,(\mathbf{S}{\cdot}\mathbf{v}_{\mathrm{rel}})\\ & \cdot|\mathbf{S}{\cdot}\mathbf{v}_{\mathrm{rel}}|(r\cos\theta\sin\varphi - R\cos\psi)\;/S^5\}]\;\mathrm{d}\varphi\mathrm{d}\theta\,\mathrm{d}r,\end{aligned} \quad (5b)$$

$$\begin{aligned}F_Z = & \frac{Gm_{\mathrm{S}}}{c^2}\int_0^{R_{\mathrm{E}}}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}[\{\rho(r)r^2\cos\theta(\mathbf{S}{\cdot}\mathbf{v}_{\mathrm{rel}})\\ & \cdot|\mathbf{S}{\cdot}\mathbf{v}_{\mathrm{rel}}|\;(r\sin\theta - R\sin\psi)\;/S^5\}]\;\mathrm{d}\varphi\mathrm{d}\theta\,\mathrm{d}r\end{aligned} \quad (5c)$$

With the help of the following transformation matrix, the radial, tangential and normal components of the force are obtained as shown below:

$$\begin{bmatrix} F_{\mathrm{R}} \\ F_{\mathrm{T}} \\ F_{\mathrm{N}} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} F_Y \\ F_Z \\ F_X \end{bmatrix},$$

$$F_{\mathrm{R}} = F_Y \cos\psi + F_Z \sin\psi, \tag{6a}$$
$$F_{\mathrm{T}} = -F_Y \sin\psi + F_Z \cos\psi, \tag{6b}$$
$$F_{\mathrm{N}} = F_X. \tag{6c}$$

4. Effect on LAGEOS

LAGEOS' motion is affected by simultaneous gravitational and non gravitational perturbations along with some general relativity effects like Lense–Thirring effect (Lucchesi 2007) and Schwarzschild effect. So, in order to find out the effect of Earth's velocity dependent inertial induction as mentioned in the above section, LAGEOS is considered as a polar satellite, neglecting the deviation of the inclination angle of the orbital plane which does not introduce any significant error in the order of magnitude of the force components. Using the expression of force components, as per Gauss perturbation equations, rate of change of semi major axis can be expressed as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{n\sqrt{1-e^2}}[(e\sin\psi)\, f_{\mathrm{R}} + (1 + e\cos\psi)\, f_{\mathrm{T}}], \tag{7}$$

where f_{R} and f_{T} are the radial and tangential force per unit mass, e is the eccentricity of the orbit of the LAGEOS, which is assumed to be zero for simplification. Therefore,

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{n} f_{\mathrm{T}}.$$

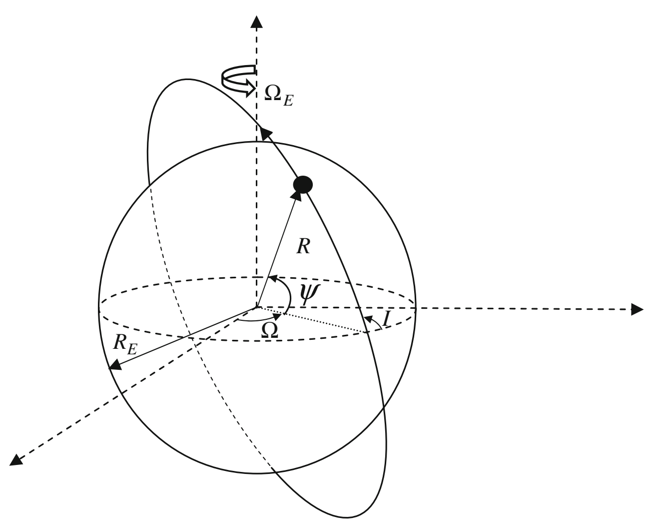

Figure 3. Orbit of Lageos.

The average rate of change of the semi-major axis can be obtained by integrating the expression of $\dot{a}$ over the whole orbit from 0 to 2π for the true anomaly.

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)_{\mathrm{avg}} = \frac{1}{\pi n}\int_0^{2\pi} f_{\mathrm{T}}\mathrm{d}\psi. \tag{8}$$

Similarly, other changes in orbital elements like the inclination angle I, longitude of ascending node Ω, eccentricity e, as shown in Fig. 3, due to this phenomenon, can also be estimated by using the general perturbation equations. The numerical values of these changes are tabulated section 5.

5. Results and discussion

The values of all the parameters which have been applied in the problem are listed in Table 1.

The density variation of the Earth can be expressed as follows:

$$\begin{aligned}
&\rho(r) = (18 - 10\zeta) \times 10^3, && \text{for } 0 \le \zeta \le 0.2,\\
&\rho(r) = (13.143 - 5.714\zeta) \times 10^3, && \text{for } 0.2 \le \zeta \le 0.55,\\
&\rho(r) = (9.667 - 6.667\zeta) \times 10^3, && \text{for } 0.55 \le \zeta \le 1,
\end{aligned}$$

where $\zeta = r/R_{\mathrm{E}}$.

Using the above functions and the data from Table 1, after numerical computation, change in the semi major axis of LAGEOS is found to be

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)_{\mathrm{avg}} = -\ 0.33 \text{ mm/d}.$$

It is a well-known fact that if a perturbing force acts on the satellite, apart from the change in semi-major axis, there will be changes in other orbital elements as well, although the order of magnitude of such a change will be very small. Using the expression of f_{R}, f_{T} and f_{N} which are respectively the radial, tangential and normal perturbation force per unit mass on the satellite, the changes thus found are tabulated along with the change in the semi major axis (Table 2).

Table 1. Values of various parameters used in the paper.

Symbol	Numerical value
Universal gravitational constant, G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Speed of light, c	3×10^8 m s^{-1}
Spin rate of the Earth, Ω_{E}	7.27×10^{-5} rad s^{-1}
Radius of the Earth, R_{E}	6.378×10^6 m
Semi major axis of LAGEOS, a	1.227×10^7 m
Eccentricity of LAGEOS' orbit, e	0.004
Inclination of LAGEOS' orbit, I	110°
Mean motion of LAGEOS, n (ω_{S})	4.65×10^{-4} rad s^{-1}
Mass of LAGEOS, m_{S}	411 kg
Altitude of LAGEOS, R	12278 km

Table 2. Secular changes in orbital parameters due to inertian induction.

Parameters	Change due to velocity-dependent inertial induction
$\dot{a}$	-0.33 mm/d
$\dot{\Omega}$	0.6 mas/yr
$\dot{I}$	1.6 mas/yr
$\dot{e}$	-2.5×10^{-17} s^{-1}

As mentioned earlier, the observed change in the semi-major axis of LAGEOS is 1.3 mm/d while our model gives us a result of 0.33 mm/d. A brief analysis over the past works reveal that although Yarkovsky thermal drag can explain most of the observed drag, but it entirely depends on the spin axis position and there will be no drag when the spin axis of the LAGEOS is normal to the orbital plane. Further the fluctuations in the observed along-track acceleration residuals are not clearly explained by this mechanism, during the period when LAGEOS orbit intersects the Earth's shadow, which indicates that there may be other forces acting on the satellite. There is also some uncertainty in the potential of the satellite and value taken for the drag coefficient in calculating neutral particle drag, since the direct measurement of the density at LAGEOS altitude is not possible. Apart from the change in semi major axis, there is also an extra drift of the orbital plane, as predicted by Lense–Thirring effect, at a rate of 31 mas/yr (mas = milliarc seconds). Similarly, this mechanism also predicts change in orbital inclination and eccentricity of LAGEOS. In order to have a better realization, similar analysis is done for another satellite, STELLA. It shows that there is a secular decrease of 8.8 m/yr in the altitude of STELLA, while the observed data (Krzysztof *et al.* 2013) available for the same is 30 m/yr.

6. Conclusion

With reference to the previous works, this paper once again shows that inertial induction is a model worth considering while dealing with problems of celestial mechanics. It has been also noted that the rotation of the Earth will have comparatively less effect in this phenomena due to the fact that the velocity of the satellite is relatively much higher than the velocity at any point on the Earth, although it has been observed that there will be a drift of the orbital frame mainly due to rotation of the Earth as explained by the Lense–Thirring effect. Better accuracy could have been achieved if the exact inclination of the orbit of LAGEOS and the eccentricity of the orbit were considered. However, the added complexity in the equations due to such considerations would probably not be worthwhile, because of the small value of the eccentricity as well as the small deviation of the inclination of the orbit, which has been used in the model. With more data available, it would have been possible to match the changes in the other orbital parameters as well. A similar investigation with the satellite STELLA produces convincing results and explains a part of its decay in the semi-major axis too. Similarly effect on other orbital parameters of STELLA can also be calculated from the same process as described in this paper. Some other interesting results due to the inertial induction are given in Ghosh (1993, 1986b, 1995) and the partial explanation of the orbital decay of LAGEOS provides further support in favour of the model of inertial induction. The application of the model to other problems relating to the planets and satellites can, hopefully, explain some of the unexplained observed phenomena.

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