

Phase Relationship Between Sunspot Number, Flare Index and Solar Radio Flux

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Abstract. To understand better the variation of solar activity indicators originated at different layers of the solar atmosphere with respect to sunspot cycles, we carried out a study of phase relationship between sunspot number, flare index and solar radio flux at 2800 MHz from January 1966 to May 2008 by using cross-correlation analysis. The main results are as follows: (1) The flare index and sunspot number have synchronous phase for cycles 21 and 22 in the northern hemisphere and for cycle 20 in the southern hemisphere. (2) The flare index has a noticeable time lead with respect to sunspot number for cycles 20 and 23 in the northern hemisphere and for cycles 22 and 23 in the southern hemisphere. (3) For the entire Sun, the flare index has a noticeable time lead for cycles 20 and 23, a time lag for cycle 21, and no time lag or time lead for cycle 22 with respect to sunspot number. (4) The solar radio flux has a time lag for cycles 22 and 23 and no time lag or time lead for cycles 20 and 21 with respect to sunspot number. (5) For the four cycles, the sunspot number and flare index in the northern hemisphere are all leading to the ones in the southern hemisphere. These results may be instructive to the physical processes of flare energy storage and dissipation.

Key words. Sun: sunspot—flare index—solar radio flux—solar cycle.

1. Introduction

The variations of sunspot number have well-established periods of about 11 years (Schwabe 1844; Hathaway *et al.* 2002). The period of magnetic activity cycle is twice as that of sunspot cycle, about 22 years on average (Hale *et al.* 1919). Moreover, solar activity represented by flare index also reveals the most pronounced 11-yr periodicity (Li *et al.* 2010).

The energy for solar flares is believed to derive from magnetic fields in the solar corona. Wheatland & Litvinenko (2001) presented a detailed model for energy

balance in the flaring corona. The assumptions of their model are that flares are powered by a coronal energy source and flares are the dominant mechanism for depleting these energetic sources. Moreover, there is a characteristic time-scale for flares to release the available free energy. The time-scale is the response time for the flaring corona. Under certain initial value of the energy supply rate, a threshold energy (Hudson 1991), and the characteristic flaring rate (Wheatland 2000), the model of Wheatland & Litvinenko (2001) predicted that both the flaring rate and the free energy of the system should lag behind the sunspot number by about 11 months. At the same time, they analyzed the monthly sunspot number and the monthly numbers of soft X-ray flares from 1976 to 1999 and found that there is a time lag of about six months between them.

Furthermore, Bachmann & White (1994) investigated the hysteresis phenomenon in solar cycle variations among seven solar activity indicators (sunspot number, F10, Ca II K, Mg II K, EUV(Ipe), He I and $L\alpha$). They found that F10 (Ca II K), Mg II K, EUV(Ipe) (He I) and $L\alpha$ have time lags of 20, 45, 85, 125 days with respect to sunspot number. H α emission essentially originates at the solar chromosphere (see review Joshi *et al.* 2012). Moreover, H α flare index is often used to describe the flare activity. Ozguc & Atac (2001) also found that there is a hysteresis phenomenon between flare index and total sunspot area, mean magnetic field and coronal index for solar cycle 21 and this hysteresis cannot be observed between flare index and total sunspot area for solar cycle 22. The relationship between soft X-ray flares and H α flares and sunspot number has been a matter of concern to some authors. For example, soft X-ray flares and hard X-ray flares have a time lag of about 2 and 3 years in relation to sunspot activity for solar cycle 21 (Bromund *et al.* 1995). Temmer *et al.* (2003) found that there is a characteristic time lag between flare activity and sunspot activity in the range of 10–15 months for odd-numbered solar cycles (19, 21 and 23) and no time delay for even-numbered solar cycles (20 and 22) by calculating the number of soft X-ray and H α flares. Tan (2010) compared the sunspot number with the distribution of the appearance rate of solar GOES flares and found that there is a time delay of one year for C-class and M-class flares and two years for X-class flares between the maximum value of sunspot number and the maximum value of annual numbers of soft X-ray flares. Yan *et al.* (2011) found that different sorts of smoothed monthly peak fluxes of soft X-ray flares showed different time lag or time lead with respect to the smoothed monthly sunspot number. However, there is a regular time delay for odd-numbered solar cycles and no time delay for even-numbered solar cycles between the smoothed monthly peak fluxes of soft X-ray flares and the smoothed monthly sunspot number considering the three types (C-class, M-class, and X-class flares) of soft X-ray flares together. In addition, the same result appeared in relationship between sunspot number and tilt angle of the heliosphere current sheet to cosmic-ray intensity (Nagashima & Morishita 1980; Van Allen 2000; Cliver & Ling 2001). The cosmic-ray cycle appears to lag behind the sunspot cycle by about 1 year for odd-numbered cycles such as 19 and 21, while the sunspot number and cosmic-ray intensity curves keep essentially in phase in even-numbered cycles (Cliver & Ling 2001).

In this paper, the phase relationship between flare index/solar radio flux at 2800 MHz and sunspot number is investigated in the northern and southern hemispheres and the entire Sun for four solar cycles. The phase relationship between solar radio flux at 2800 MHz and sunspot number is also investigated in the entire Sun for

four solar cycles. Our aim is to verify whether the flare index and solar radio flux at 2800 MHz lag behind sunspot number for the odd-numbered solar cycles and keep synchronous phase with the sunspot number for even-numbered solar cycles and to better understand the physical process of the storage and release of flare energy.

2. Data

The data used in this paper are as follows:

- (1) The monthly mean northern and southern hemispheric sunspot number from January 1945 to December 2004 were compiled by Temmer *et al.* (2006) (<http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/447/735>). The monthly mean northern and southern hemispheric sunspot number from January 1992 to December 2008 can be downloaded from NOAA's National Geophysical Data Center (NGDC) (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/HEMISPHERE_NUMBERS/). There is an overlapping time span from January 1992 to December 2004 with the Temmer's sunspot data. The first group of data generally matches the second group (for details, see Temmer *et al.* 2006). In this paper, we extract the sunspot number from January 1966 to May 2008 as our sample.
- (2) The flare index from January 1966 to May 2008 was downloaded from NOAA's National Geophysical Data Center (NGDC) (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA). A detailed definition of flare index can be found in Ozguc *et al.* (2003).

The quantitative flare index first introduced by Kleczek (1952), $Q = i \times t$, may be roughly proportional to the total energy emitted by a flare. In this equation, i represents the intensity scale of importance of a flare in H α and t represents the duration in H α (in min) of the flare. The flare index was calculated by T. Atac and A. Ozguc at Bogazici University Kandilli Observatory.

- (3) The data of solar radio flux at 2800 MHz from January 1966 to May 2008 were downloaded from NOAA's National Geophysical Data Center (NGDC) (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX/Penticton_Adjusted/).

To obtain a more accurate relationship between sunspot number and flare index, we separate the data of sunspot number and flare index into the northern and southern hemispheres and the entire Sun to investigate their respective phase shifts.

3. Results

Figure 1 shows the distribution of the monthly sunspot number and flare index in the northern and southern hemispheres from January 1966 to May 2008 with their 13-point smoothed values (red lines) superimposed. The period covers solar cycle 20 (January 1966–April 1976), cycle 21 (May 1976–July 1986), cycle 22 (August 1986–March 1996), and cycle 23 (April 1996–May 2008). The flare index has an

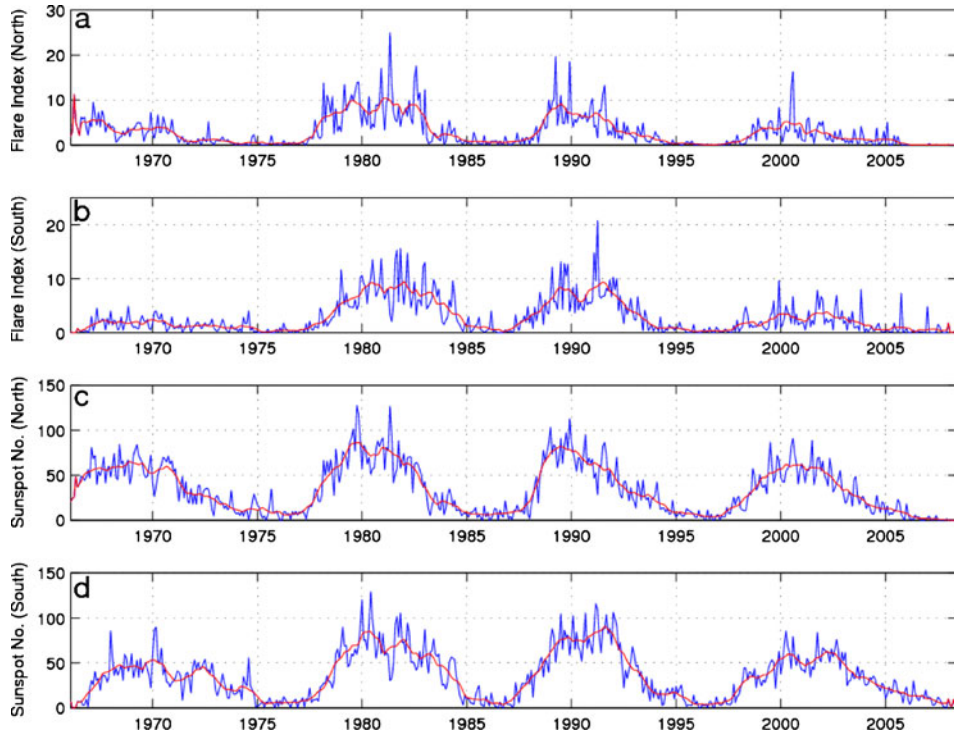


Figure 1. The distribution of the monthly flare index (**a**, **b**) and sunspot number (**c**, **d**) in the northern and southern hemispheres from January 1966 to May 2008 respectively with their 13-point smoothed values (red lines) superimposed.

obvious periodical variability as the sunspot number (see Fig. 1 a, b). To show the systematic time-lag (or time-lead) between the monthly sunspot number and flare index in a cycle, we have done a correlation analysis of the two time series. In order to eliminate the biasing due to large, flare productive regions, we use the 13-point smoothed monthly sunspot number and flare index to calculate the correlation coefficient and phase shift.

The smoothed monthly flare index has high positive correlation coefficients in relation to the smoothed monthly sunspot number (seen from Table 1). To calculate the cross-correlation between these solar activity indicators, we used the standard MATLAB code for cross-correlation. From the calculation of cross-correlation, we

Table 1. Correlation coefficients between the smoothed monthly sunspot number and flare index for the northern and southern hemispheres and the entire Sun in cycles 20, 21, 22 and 23.

	20	21	22	23
North	0.76	0.97	0.99	0.94
South	0.90	0.98	0.99	0.95
Entire	0.75	0.98	0.99	0.97

found that 12 correlation coefficients in Table 1 are significant at the 95% level. Figure 2 shows cross-correlation coefficients between the smoothed monthly flare index and sunspot number in the northern (a1–a4) and southern (b1–b4) hemispheres and the entire Sun (c1–c4) in solar cycles 20, 21, 22, and 23 respectively. The abscissa indicates the shift of the smoothed monthly sunspot number with respect to the smoothed monthly flare index, with negative values representing backwards shifts. The phase shifts correspond to the peak cross-correlation coefficients in the cycle. The vertical red lines in Fig. 2 indicate the phase shifts of the smoothed monthly sunspot number with respect to the smoothed monthly flare index. Table 2 shows the phase shifts between the smoothed monthly sunspot number and flare index for the northern and southern hemispheres and the entire Sun in cycles 20, 21, 22, and 23. We find that the smoothed monthly flare index have a time lead of one month and two months for cycles 20 and 23, no time lag or time lead for cycles 21 and 22 with respect to the smoothed monthly sunspot number in the northern hemisphere. In the southern hemisphere, the smoothed monthly flare index have no time lag or time lead for cycle 20, a time lag of one month for cycle 21, a time lead of one

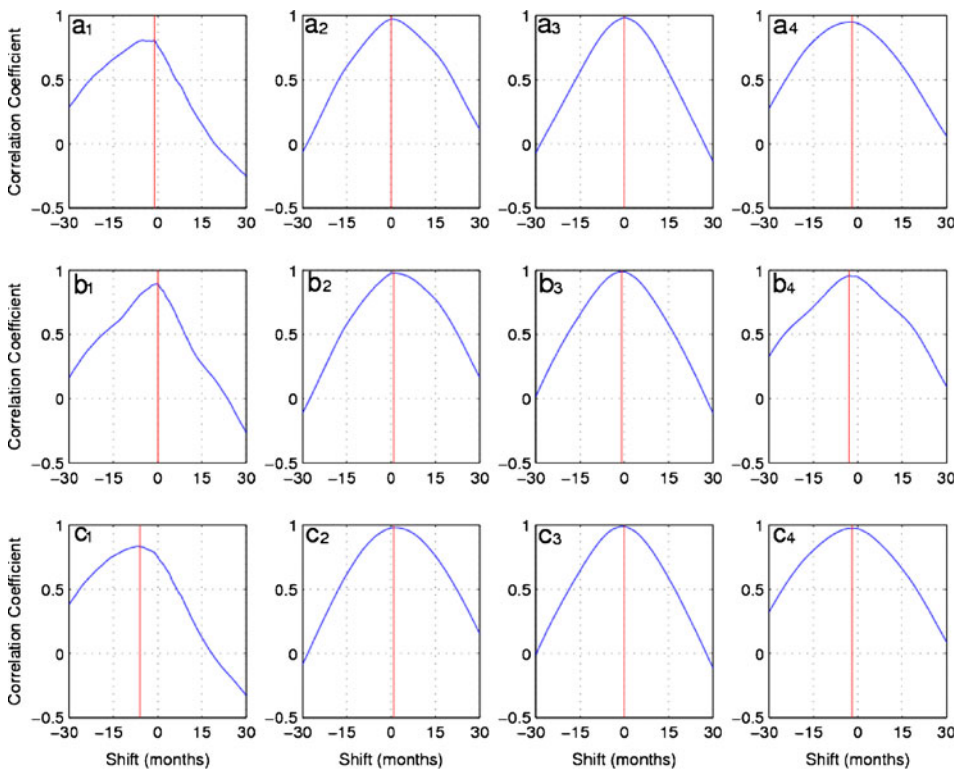


Figure 2. Cross-correlation coefficients between the smoothed monthly flare index and sunspot number in the northern (a1)–(a4) and southern (b1)–(b4) hemispheres and the entire Sun (c1)–(c4) in solar cycles 20, 21, 22 and 23 respectively. The abscissa indicates the shift of the smoothed monthly sunspot number with respect to the smoothed monthly flare index, with negative values representing backwards shifts. The vertical red lines indicate the phase shifts of the smoothed monthly sunspot number with respect to the smoothed monthly flare index.

Table 2. The phase shifts (in months) between the smoothed monthly sunspot number and flare index for the northern and southern hemispheres and the entire Sun in cycles 20, 21, 22 and 23. The positive (negative) values denote the smoothed monthly sunspot number leads (lags) to the smoothed monthly flare index.

	20	21	22	23
North	-1	0	0	-2
South	0	1	-1	-3
Entire	-6	1	0	-2

month and three months for cycles 22 and 23 with respect to the smoothed monthly sunspot number.

To compare with the previous results obtained by other researchers, we also considered the entire Sun. If we take the southern and northern hemispheres together, the smoothed monthly flare index have a very noticeable time lead of six months and two months for cycles 20 and 23, a time lag of one month for cycle 21, and no time lag or time lead for cycle 22 with respect to the smoothed monthly sunspot number.

Figure 3 shows the distribution of the monthly solar radio flux at 2800 MHz (blue line) for the entire Sun from January 1966 to May 2008 with their 13-point smoothed values (red lines) superimposed. Figure 4 shows cross-correlation coefficients between the smoothed monthly solar radio flux at 2800 MHz with respect to the monthly sunspot number in solar cycles 20 (a), 21 (b), 22 (c), and 23 (d) respectively. The abscissa indicates the shift of the smoothed monthly sunspot number with respect to the monthly solar radio flux at 2800 MHz, with negative values representing backwards shifts. The vertical red lines indicate the phase shifts of the smoothed monthly sunspot number with respect to the monthly solar radio flux at 2800 MHz. We also find that the smoothed monthly solar radio flux at 2800 MHz has a time lag

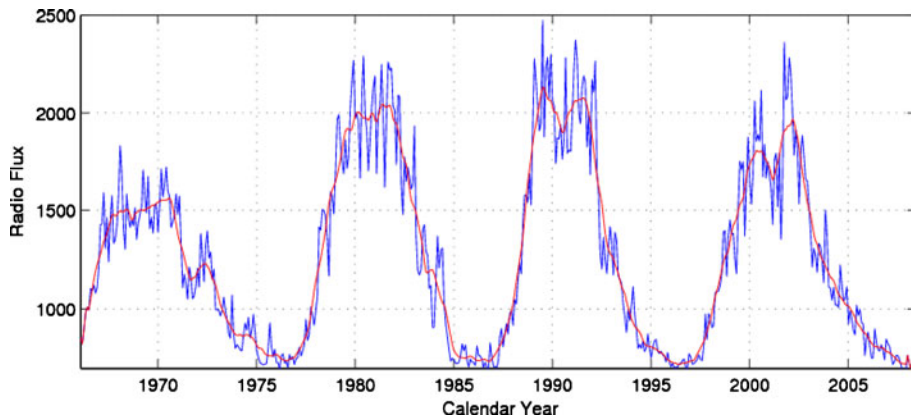


Figure 3. The distribution of the monthly solar radio flux at 2800 MHz for the entire hemisphere from January 1966 to May 2008 with their 13-point smoothed values (red lines) superimposed. The unit of solar radio flux is $10^{22} \text{ J} \cdot \text{s}^{-1} \cdot \text{m}^{-2} \cdot \text{Hz}^{-1}$.

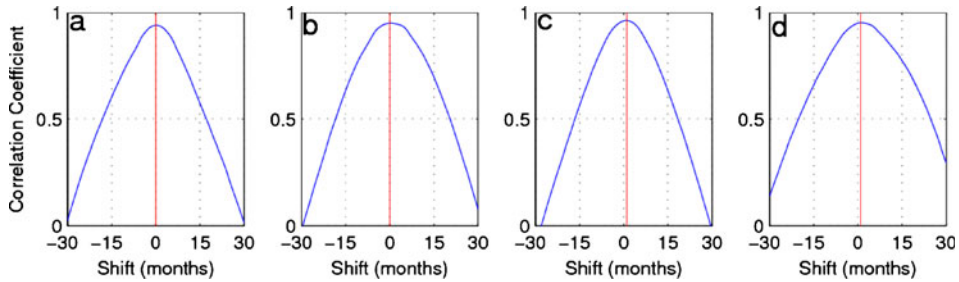


Figure 4. Cross-correlation coefficients between the smoothed monthly solar radio flux at 2800 MHz and sunspot number in solar cycles 20 (a), 21 (b), 22 (c) and 23 (d) respectively. The abscissa indicates the shift of the smoothed monthly sunspot number with respect to the monthly solar radio flux at 2800 MHz, with negative values representing backwards shifts. The vertical red lines indicate the phase shifts of the smoothed monthly sunspot number with respect to the monthly solar radio flux at 2800 MHz.

for cycles 22 and 23 and no time lag or time lead for cycles 20 and 21 with respect to the smoothed monthly sunspot number.

There is no regulation of time lag or time lead between the smoothed monthly flare index and sunspot number in the even-numbered and odd-numbered solar cycle for the two hemispheres and the entire Sun. In order to investigate the phase relationship between the activities of the two hemisphere, we calculated cross-correlation coefficients between the smoothed monthly sunspot number and flare index in the northern hemisphere and the ones in the southern hemisphere (Figs. 5, 6). We obtained similar results as Li (2009). The vertical red lines in Figures 5, 6 indicate the phase shifts of the smoothed monthly sunspot number (flare index) in the northern hemisphere with respect to the smoothed monthly sunspot number (flare index) in the southern hemisphere. The smoothed monthly sunspot number in the northern hemisphere lead to the smoothed monthly sunspot number in the southern hemisphere with 12, 7, 4 and 6 months for cycles 20, 21, 22 and 23 respectively. The same situation appears

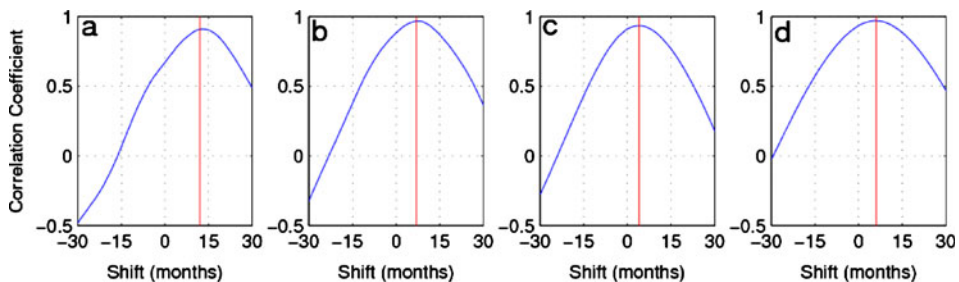


Figure 5. Cross-correlation coefficients between the smoothed monthly sunspot number in the northern hemisphere and ones in the southern hemisphere in solar cycles 20 (a), 21 (b), 22 (c) and 23 (d) respectively. The abscissa indicates the shift of the smoothed monthly sunspot number in the northern hemisphere with respect to the smoothed monthly sunspot number in southern hemisphere, with negative values representing backwards shifts. The vertical red lines indicate the phase shifts of the smoothed monthly sunspot number in the northern hemisphere with respect to the smoothed monthly sunspot number in the southern hemisphere.

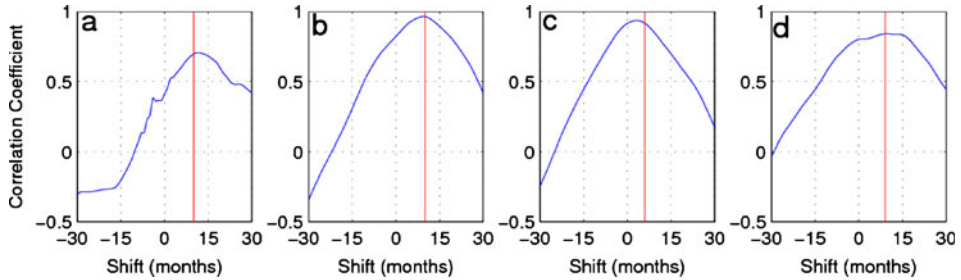


Figure 6. Cross-correlation coefficients between the smoothed monthly flare index in the northern hemisphere and the ones in the southern hemisphere in solar cycles 20 (a), 21 (b), 22 (c) and 23 (d) respectively. The abscissa indicates the shift of the smoothed monthly flare index in the northern hemisphere with respect to the smoothed monthly flare index in the southern hemisphere, with negative values representing backwards shifts. The vertical red lines indicate the phase shifts of the smoothed monthly flare index in the northern hemisphere with respect to the smoothed monthly flare index in the southern hemisphere.

with flare index. The smoothed monthly flare index in the northern hemisphere lead to the smoothed monthly flare index in the southern hemisphere with 10, 10, 6 and 9 months for cycles 20, 21, 22 and 23 respectively. For the four cycles, the smoothed monthly sunspot number and flare index in the northern hemisphere are all leading to the ones in the southern hemisphere.

4. Conclusions and discussions

In this paper, we present a study of phase relationship between sunspot number and flare index/solar radio flux from January 1966 to May 2008. We use the data of the smoothed monthly sunspot number, the smoothed monthly flare index and the smoothed monthly solar radio flux at 2800 MHz. The main results are as follows:

- (1) The smoothed monthly flare index has a very noticeable time lead of one month and two months for cycles 20 and 23, no time lag or time lead for cycles 21 and 22 with respect to the smoothed monthly sunspot number in the northern hemisphere.
- (2) The smoothed monthly flare index has no time lag or time lead for cycle 20, a time lag of one month for cycle 21, and a time lead of one month and three months for cycles 22 and 23 with respect to the smoothed monthly sunspot number in the southern hemisphere.
- (3) If we take the southern and northern hemisphere together, the smoothed monthly flare index have a very noticeable time lead of six months and two months for cycles 20 and 23, a time lag of one month for cycle 21, no time lag or time lead for cycle 22 with respect to the smoothed monthly sunspot number.
- (4) The smoothed monthly solar radio flux at 2800 MHz has a time lag for cycles 22 and 23 and no time lag or time lead for cycles 20 and 21 with respect to the smoothed monthly sunspot number.

Temmer *et al.* (2003) reported that the numbers of soft X-ray flares and H α flares have an obvious time lag in the odd-numbered solar cycles and no time lag in the

even-numbered solar cycles with respect to sunspot number for the entire Sun and both hemispheres. Wheatland & Litvinenko (2001) used the sunspot number as a proxy for the energy supply rate to the corona and found that the occurrence of soft X-ray flares lag behind the sunspot number by about 6 months. Yan *et al.* (2011) also found that the smoothed monthly peak fluxes of soft X-ray flares have an obvious time lag in the odd-numbered solar cycles and no time lag in the even-numbered solar cycles with respect to the smoothed monthly sunspot number for the entire Sun. However, we did not find the same results for flare index calculated from H α flares.

The radio flux at 2800 MHz, which originates from atmospheric layers high in the Sun's chromosphere and low in its corona, changes gradually from day-to-day, in response to the number of spot groups on the disk. Radio intensity levels consist of emission from three sources: from the undisturbed solar surface, from developing active regions, and from short-lived enhancements above the daily level. From the phase relationship between sunspot number and radio flux at 2800 MHz, different solar cycles have different phase shifts. Moreover, the phase shifts exhibit an odd-even pattern (there is no time lag or lead for cycles 20 and 21; there is one month time lag for cycles 22 and 23.). This result is consistent with that of Bachmann & White (1994).

Solar flares are strongly related with sunspot number as they generally occur in active regions lying over the sunspot regions. However, the flare activity described by H α flare index precedes sunspot activity for solar cycles 20 and 23. We give some explanation as follows: first, the unphysical definition of sunspot number may play a role here. Sunspots have a life time longer than one solar rotation. Wilson *et al.* (1987) think that the number of sunspots in an active region is one measure of the complexity of the magnetic structure of the region. Second, we adopt the maximum correlation coefficient to obtain the phase shift. Because there are some peaks in the sunspot number and flare index curve, the estimated uncertainty of time lead or time lag may exist. Third, perhaps the existence of many flares which play a role come from non-sunspot regions in a certain solar cycle.

In a cycle, solar activities are asymmetrical in amplitude regarding the northern and southern hemispheres (Li *et al.* 2002; Joshi & Joshi 2004; Joshi *et al.* 2006; Bankoti *et al.* 2010). Li (2009) found that sunspot activity does not synchronously occur in the northern and southern hemispheres, and there is a systematic time lag or time lead between the two hemispheres in a cycle. Joshi & Pant (2005) found that the northern hemisphere was more active in producing flares than the southern one after solar activity was minimum in 1996 and the dominance of northern hemisphere shifted towards the southern hemisphere after the solar maxima in 2000 and remained there in successive years. The study of N-S asymmetry indicates that the hemispherical asymmetry may show variations from one to another hemisphere during the course of cycle evolution (Joshi & Pant 2005). Therefore, it seems that the cyclic variation of dominant hemispheres of solar activity seems to have little or no relation with the cyclic variation of the systematic time delay of solar activity Li (2009).

Solar flares are closely related with sunspot number as they generally occur in active regions which have one or groups of sunspots (Harrison 1990). The appearance of the sunspots denotes the emergence of a strong magnetic flux. The energy supplying to the corona comes from the emergence of new magnetic flux or photospheric stressing of coronal fields. The sunspot numbers are the proxy of the energy

supply rate to the corona. The quantitative flare index is supposed to be approximately proportional to the total energy emitted by a flare. Consequently, these results may be instructive to the physical processes of flare energy storage and dissipation. As the period of the data is just four solar cycles, the results cannot be conclusive. We need forthcoming data and new methods to confirm these results.

All the previous studies take flare counts (mostly soft X-ray flares) as proxy for flare activity. However, in this study H-alpha flare index is taken which is proportional to the energy emitted by the flare. Further we should keep in mind that during solar flares, soft X-ray emission originates from the corona while H-alpha emission comes from the chromospheres. In this context, the time lead between flare index and sunspot activity likely indicates that the ascending phase of the cycle may contribute largely in terms of total energy released by flares. However, in terms of flare statistics (i.e., counts) this need not necessarily be true.

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