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Evolution of the Distribution of Neutron Exposures in the Galaxy Disc: An Analytical Model

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Abstract. In this work, based on the analytical model with delayed production approximation developed by Pagel & Tautvaišienė (1995) for the Galaxy, the analytic solutions of the distribution of neutron exposures of the Galaxy (hereafter NEG) are obtained. The present results appear to reasonably reproduce the distribution of neutron exposures of the solar system (hereafter NES). The strong component and the main component of the NES are built up in different epochs. Firstly, the strong component is produced by the s-process nucleosynthesis in the metal-poor AGB stars, starting from [Fe/H] ≈ -1.16 to [Fe/H] ≈ -0.66, corresponding to the time interval $1.06 < t < 2.6$ Gyr. Secondly, the main component is produced by the s-process in the galactic disk AGB stars, starting from [Fe/H] ≈ -0.66 to [Fe/H] ≈ 0, corresponding to the time interval $t > 2.6$ Gyr. The analytic solutions have the advantage of an understanding of the structure and the properties of the NEG. The NEG is believed to be an effective tool to study the s-process element abundance distributions in the Galaxy at different epochs and the galactic chemical evolution of the neutron-capture elements.

Key words. Nucleosynthesis—neutron exposures—abundances—stars: AGB stars: low-mass stars—galaxy: evolution.

1. Introduction

Many efforts have been made to explain the solar-system abundance of elements associated with the slow neutron-capture process (s-process). The s-process occurs mainly during hydrostatic He-burning phases of stellar evolution, which can be investigated either through nucleosynthesis computations in stellar models (Straniero *et al.* 1995; Gallino *et al.* 1998) or by the phenomenological models, mostly by the so-called classical model (Käppeler *et al.* 1989; Busso *et al.* 1999). The classical s-process was first outlined by Burbidge *et al.* (1957). Clayton *et al.* (1961) showed that the solar-system abundances of the s-only isotopes cannot be reproduced by a single neutron irradiation of an iron seed. Because the solar-system s-element composition was assumed to be the result of a superposition of different distributions of neutron exposures, a satisfactory solution was found by Seeger *et al.* (1965) assuming an exponential decreasing distribution of neutron exposures. When sufficiently detailed input data became available, it turned out that three different exponential distributions of neutron exposures are required for a complete description of the observed s-process abundance (Clayton & Rassbach 1967; Clayton & Ward 1974; Beer & Macklin 1985; Käppeler *et al.* 1982). The current understanding of the s-process is supported by many observational and theoretical works, indicating that the heavier s-nuclei, from Sr to Pb, belonged to the so-called main component. Below A$\approx$ 90, this component fails to describe the steep increase of the $\sigma_i N_i$ (*i.e.*, the products of neutron cross section times s-process abundance) curve towards the iron seed. Therefore, a weak component is added which is characterized by a smaller mean neutron exposure. Finally, a strong component is to be postulated in order to account for the abundance maximum at lead. Each of these distributions is expressed as

$$\rho(\tau) = \frac{f_i N_{Fe}(\mathrm{Z}_\odot)}{\tau_{0,i}} \exp(-\tau/\tau_{0,i}), \tag{1}$$

where f_i is the initial solar fraction of ^{56}Fe that has been irradiated, τ means the time-integrated neutron flux (*i.e.*, $\tau = \int_0^t n_n v_T dt'$, where n_n is the neutron density and v_T is the thermal velocity at temperature T), and $\tau_{0,i}$ is the mean neutron exposure. These three components will be distinguished by adding indices 1, 2 and 3 to the parameters f_i and $\tau_{0,i}$, respectively. An exponential distribution for $\rho(\tau)$ appears to provide for an excellent fit of the main and strong s-process components.

A physical justification for the choice of $\rho(\tau)$ seemed to appear when Ulrich (1973) showed that an exponential distribution of exposures was the natural consequence of repeated He-shell flashes during the AGB phase. If $\Delta\tau$ is the neutron exposure per pulse, and if r denotes the overlap of the Nth and the $(N-1)$th convective shell, after N pulses the fraction of material having experienced an exposure $\tau = N\Delta\tau \sim r^N \equiv \exp(-\tau/\tau_0)$, where $\tau_0 = -\Delta\tau/lnr$ is the mean neutron exposure. During the AGB evolution, stars experience enough helium shell flashes to establish an exponential distribution. There, the s-process was assumed to occur in convective thermal pulses, the classical analysis was considered to yield ‘effective’ conditions characterizing the stellar scenarios (Käppeler *et al.* 1990). In order to test the reliability of the classical model, Goriely (1997) has developed a new s-process model based on the superposition of a large number of canonical astrophysical events, the abundance predictions of the multi-event model are in good agreement with those of the widely used exponential model.

The knowledge of the last stages of evolution for AGB stars was improved by a series of investigations originally based on the activation of the ^{13}C(α, n)^{16}O reaction in low-mass stars (Straniero *et al.* 1997; Gallino *et al.* 1998). The model suggested that the main s-process component results from low-mass stars with masses between 1.5 and 3.0 M$_\odot$. According to the models, the ^{13}C neutron source is activated under radiative conditions during the intervals between subsequent He-shell burning episodes. The stellar models show that the distribution of neutron exposures is definitely non-exponential, and actually very difficult to be described analytically (Arlandini *et al.* 1995). The overabundances of elements heavier than iron observed at the surface of MS and S stars (Smith & Lambert 1990) clearly indicate that the s-process takes place during the AGB phase in the evolution of low- and intermediate-mass stars

($0.8 \leq M(\mathrm{M}_{\odot}) \leq 8$). Though the observations in the solar neighborhood exhibit a spread in the respective s-abundances, it is remarkable that the solar s-abundance distribution lays roughly at the center of the spread observed in MS and S stars (Busso *et al.* 1999). Most galactic disk AGB stars in the mass range $1.5 \leq M(\mathrm{M}_{\odot}) \leq 3.0$ can be considered as suitable sites for reproducing the main component. The mean physical conditions are found in AGB star models down to a metallicity slightly lower than $1/2\,\mathrm{Z}_{\odot}$ by Gallino *et al.* (1998).

In fact, the solar system chemical composition is the result of a complex galactic evolution mechanism, and depends upon the details of the stellar formation history, initial mass functions, chemical yields, etc. Since the s-process distribution and the neutron exposure distribution vary strongly for TP-AGB stars with different metallicity (Busso *et al.* 1999; Travaglio *et al.* 1999; Raiteri *et al.* 1999), the comparison with the solar distributions has to be complemented by the s-process distribution and the neutron exposure distribution for the TP-AGB stars with a fixed initial mass and metallicity. Recently, a quantitative calculation of the evolution of neutron exposure distribution in the Galaxy (NEG) has been carried out through solving a set of differential equations by Cui *et al.* (2007). The numerical models of the galactic chemical evolution attempts to combine ideas on the end-products of stellar evolution with ideas on the formation and evolution of galaxies in order to understand the NEG. As this involves many uncertainties, we prefer to adopt an analytical approach, which is to parameterize the problem as simply as possible, to study the characters of the NEG. This is somewhat unfashionable nowadays, but it has the advantage that one can immediately understand the structure and the properties of the NEG, which is not always the case in elaborate numerical models.

The distribution of neutron exposures plays a key role in the theory of s-process nucleosynthesis. It is a basis of qualitative and quantitative analysis as well as a good understanding of s-process abundance distribution. Therefore to investigate it more deeply is very necessary. In this work, we use the NEG $\rho_{\mathrm{Gal}}(\tau, t)$ to study the chemical evolution characteristic of the Galaxy for s-process elements. One of the main goals of this work was the development of an analytical model for the NEG, to provide stronger evolutionary support to the distribution of neutron exposures in the solar system. The paper is organized as follows: in section 2 we deduce the evolution equation of the NEG from the chemical evolution model developed by Pagel & Tautvaišienė (1995); in section 3 we discuss the distributions of neutron exposures in the AGB stars (hereafter the NEAGB); in section 4 we show the analytic solutions for the NEG and compare them with those of two parameterized models, namely the multi-event model (Goriely 1997) and the exponential model (Beer *et al.* 1997). Finally, in section 5 we summarize the main conclusions.

2. The basic equations and assumptions

In the chemical evolution model there are five variables: the total system mass M, the mass of 'gas' g, the mass existing in the form of stars (including compact remnants) s, the mass inflow rate F and the abundances X_i of the element(s). Instantaneous recycling cannot be used to describe the formation of elements, such as s-process elements, to which there is a significant contribution from stars that take a non-negligible time to complete their evolution. We adopt the analytical model with delayed production approximation developed by Pagel & Tautvaišienė (1995) for the disc of the Galaxy assuming no galactic wind and the inflowing material to be unprocessed, which works by the simple device of assuming that the delayed element or component of stars to be released at a single time Δ after the onset of stars formation and is then instantaneously recycled. This model has been adopted to investigate different aspects of the chemical evolution of the Galaxy. In the delayed production approximation, each generation of the AGB stars ejects a newly synthesized s-process element after a fixed time delay Δ, then the abundance of s-element in the gas, with the dimensionless time-like variable u introduced, is governed by (Pagel & Tautvaišienė 1995)

$$\frac{dX_i}{du} + \frac{F}{\omega g} = 0 \qquad \text{if } u < \omega\Delta, \tag{2}$$

$$= p_i(u - \omega\Delta)\frac{g(u - \omega\Delta)}{g(u)} \qquad \text{if } u \geq \omega\Delta, \tag{3}$$

where time-like variable u is defined by

$$u \equiv \int_0^t \omega(t')dt', \tag{4}$$

$\omega(t)$ is the transition probability for diffuse material ('gas') to change into stars in unit time at time t. p_i is the yields of s-process element i ejected from each generation of the AGB stars, which is given by an expression of the form

$$p_i = \frac{1}{\alpha}\int_{m_l}^{m_u} m_{Dup}x_i\phi(m)dm, \tag{5}$$

where the limits of integration are taken from $m_{l,\min} = 0.1$ to $m_U = 62\,\mathrm{M}_\odot$ (Miller & Scalo 1979), $\phi(m)$ is initial mass function (IMF) adopted from Miller & Scalo (1979) too, α is the lock-up fraction (assumed constant), m_{Dug} is the total mass dredged up from the intershell of an AGB star and x_i the mass fraction of species i in the intershell.

We use $\rho_{\mathrm{Gal}}(\tau, t)d\tau$ to represent the number of iron seed nuclei (per 10^{12} H atoms) that has received a neutron exposure between τ and $\tau + d\tau$ in the Galaxy before time t. The abundance of an s-process nucleus i at time t can be written as (Cui *et al.* 2007)

$$N_i(t) = \frac{1}{\sigma_i}\int_0^\infty \Psi_i(\tau)\rho_{\mathrm{Gal}}(\tau, t)d\tau, \tag{6}$$

where $\Psi_i(\tau)$ is the solution of a single exposure (see Clayton 1961). In addition, the abundance of an s-process nucleus i in the AGB star with the initial metallicity Z is given by an expression of the form

$$n_i(Z) = \frac{1}{\sigma_i}\int_0^\infty \Psi_i(\tau)\rho_{AGB}(\tau, Z)d\tau, \tag{7}$$

where $\rho_{AGB}(\tau, Z)$ is the distribution of neutron exposures for an AGB star (NEAGB). Combining equations (2) and (3) with equations (6) and (7), the rate of change of the NEG can be written as

$$\frac{d\rho_{\mathrm{Gal}}(\tau, u)}{du} + \frac{F}{\omega g}\rho_{\mathrm{Gal}}(\tau, u) = 0 \qquad \text{if } u < \omega\Delta, \tag{8}$$

$$= p'\rho_{AGB}(\tau, Z_{u-\omega\Delta})\frac{g(u - \omega\Delta)}{g(u)} \qquad \text{if } u \geq \omega\Delta, \tag{9}$$

p' is the fraction of ejection mass dredged up from AGB stars, which is given by the formula

$$p' = \frac{1}{\alpha} \int_{m_l}^{m_u} m_{Dup} \phi(m) dm. \tag{10}$$

Given a functional form for $\rho_{(AGB)}(\tau, Z)$ and the constant parameters α and $\omega\Delta$, equations (8) and (9) can be solved to give the NEG as a function of u. For simplicity, we hold on to the assumption of constant mass dredged up from AGB stars for s-process elements despite the possibility raised by the work of Straniero *et al.* (2003) that the mass depends on the initial metallicity.

3. The adopted NEAGB

3.1 *Radiative model: Case A*

We adopt the 25th pulse computed by Gallino *et al.* (1998), corresponding to a core mass $M_C = 0.70\ M_\odot$, as typical, and assume that all the pulses are identical. Then the distribution of the neutron exposure in the AGB stars can be obtained by three separated steps (Cui *et al.* 2007). Firstly, considering that the seed nuclei previously entered the He intershell at the same pulse would suffer different irradiation times after experiencing a fixed number of pulses, we can investigate the total fraction of seed nuclei in the He intershell with different irradiation times. Secondly, taking into account that the seed nuclei in different layers of the ^{13}C-pocket would achieve different neutron exposures per irradiation, by introducing the conception of multi-order distribution of neutron exposures and using the Monte Carlo method, we can calculate the distribution of neutron exposures for the seed nuclei that can experience any irradiation times. Finally, we can obtain the final distribution of neutron exposures in the He intershell of low-mass AGB stars. There are four parameters in this model: they are the overlap factor r, the mass fraction q, the temperature T and the maximum of neutron exposure $\tau_{\max}$ in the q layer. In this paper, since we are particularly interested in the exposure distribution which can reproduce the solar s-abundance distribution, we adopt the ^{13}C abundance profile indicated as standard (ST) by Gallino *et al.* (1998) (see their Fig. 1), and take $r = 0.45$, $q = 0.05$. Figure 1 adopted from Cui *et al.* (2007) shows the resulting asymptotic distribution of exposures in the ^{13}C burning scenario. As Fig. 1 shows, even in the above simplified scenario, the distribution of neutron exposures is much more complex than the usually exponential form.

3.2 *Convective model: Case B*

There is another possibility for the synthesis of s-process elements in the AGB stars, *i.e.*, with nucleosynthesis taking place during thermal pulses (Aoki *et al.* 2001). In this case, the neutron irradiation is derived primarily by the reaction $^{13}\mathrm{C}(\alpha, n)^{16}\mathrm{O}$, with a minor contribution from the marginal burning of ^{22}Ne. During the AGB evolution, stars experience enough He shell flashes to establish an exponential distribution

$$\rho_{AGB}(\tau, Z) = \frac{N(Fe, Z)}{\tau_0(Z)} \exp(-\tau/\tau_0(Z)). \tag{11}$$

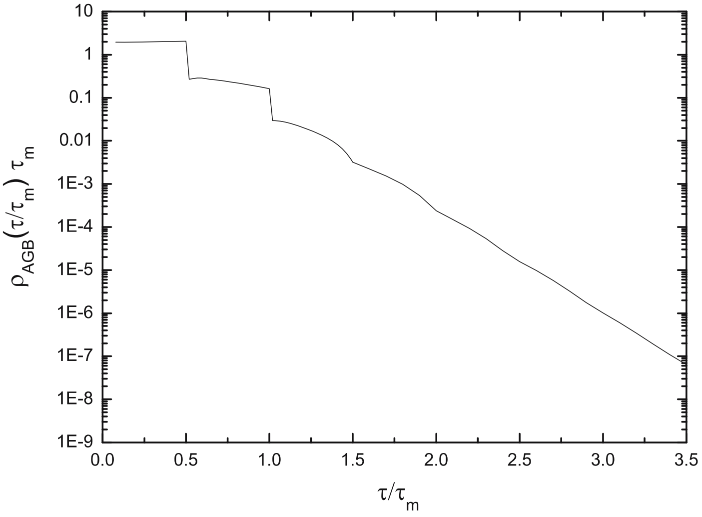

Figure 1. Distribution of the neutron exposures inside a single low-mass AGB star that can reproduce the main s-component in the solar system (adopted from Cui *et al.* 2007).

The resulting pattern of AGB nucleosynthesis, and its dependence on the initial metallicity of the star, have been discussed by Gallino *et al.* (1999). Since the ^{13}C neutron source is of primary nature, the typical neutron density in the nucleosynthesis zone scales roughly as $1/Z^{0.6}$ from $Z_\odot$ down to $1/50\ Z_\odot$. At lower metallicities, the effect of the primary poisons prevails (Busso *et al.* 1999).

4. Results of the evolution of NEG

The basic functions in our study, such as $F(u)$ and $g(u)$, are adopted from the analytical forms given by Pagel & Tautvaišienė (1995). In the present work, we concentrate on the influence of initial metallicity of the low-mass AGB stars to the NEG. More massive AGB stars, in the range 3–8 $M_\odot$, do not give a relevant contribution to the main and strong components (Travaglio *et al.* 1999). At the end of the TP–AGB phase, the s-process contributions to the ISM are determined by the amount of matter cumulatively dredged-up from the He shell to the surface and lost by stellar winds.

In general, the production factor of a given s-element varies with Z that is close to solar, the strong increase of the s-process yields with increasing neutron exposure dominates and the secondary nature of these nuclei is overcompensated, so that they actually increase for decreasing Z, instead of constant (Travaglio 1999, see their Fig. 2). Starting from AGB stars with nearly solar metallicity, first the s-process builds up the s-elements belonging to the Zr-peak. Then the Zr-peak yields decrease while the Ba-peak production increases, reaching a maximum at [Fe/H] $\simeq -0.6$. Despite the dependence of s-process yield on the initial metallicity of the AGB stars being very complex and non linear (Busso *et al.* 1999), the galactic disk AGB stars in the mass range $1.5 \leq M_\odot \leq 3.0$ can be considered as suitable sites for reproducing the main component, and the mean physical conditions of the main component are found in AGB star models down to a metallicity slightly lower than $1/2\,Z_\odot$ by Gallino *et al.* (1998). At lower metallicities, the neutron-flux skips Ba and feeds Pb, which reaches a maximum yields at [Fe/H] $= -1$ (Gallino *et al.* 1999). Eventually, yields of Pb also decrease which makes the secondary nature of the s-process evident. At very low metallicities, essentially all the Fe group seeds are converted to Pb. The origin of Pb and the site of the strong component of the s-process, should be attributed to these low-mass, metal-poor AGB stars. Below [Fe/H] $\simeq -2$ only a small s-signature is presented on heavy elements.

Based on the above discussion, we assume that the strong and the main components in the Galaxy are built up in different epochs. Firstly, the strong component is produced by the s-process nucleosynthesis in the metal-poor AGB stars, starting from [Fe/H] ≈ -1.16 to [Fe/H] ≈ -0.66, corresponding to the time interval $\omega\Delta < u < u_c$ where $u_c = 2.6$ is the time at which the contribution of the galactic disk AGB stars to the gas began. Secondly, the main component is produced by the s-process nucleosynthesis in the galactic disk AGB stars, starting from [Fe/H] ≈ -0.66 to [Fe/H] ≈ 0. The solutions of NEG are given by

$$\rho_{\mathrm{Gal}}(\tau, u) = 0 \qquad \text{if } u < \Delta, \tag{12}$$

$$= p' e^{\omega\Delta} \left(\frac{u_1 + u_0}{u + u_0}\right)^3 (u - \omega\Delta) \frac{N_{Fe1}}{\tau_{01}} e^{-\frac{\tau}{\tau_{01}}} \qquad \text{if } \omega\Delta \le u \le u_1 + \omega\Delta, \tag{13}$$

$$= p' \frac{e^{\omega\Delta}}{(u + u_0)^3} \left[(u_1 + u_0)^3 u_1 + \frac{(u + u_0 - \omega\Delta)^4 - (u_1 + u_0)^4}{4}\right] \frac{N_{Fel}}{\tau_{01}} e^{-\frac{\tau}{\tau_{01}}}$$

$$\text{if } u_1 + \omega\Delta \le u < u_c, \tag{14}$$

$$= p' \frac{e^{\omega\Delta}}{(u_c + u_0)^3} \left[(u_1 + u_0)^3 u_1 + \frac{(u_c + u_0 - \omega\Delta)^4 - (u_1 + u_0)^4}{4}\right] \frac{N_{Fel}}{\tau_{01}} e^{-\frac{\tau}{\tau_{01}}}$$

$$+ p' \frac{e^{\omega\Delta}}{(u + u_0)^3} \left[\frac{(u + u_0 - \omega\Delta)^4}{4} - \frac{(u_c + u_0 - \omega\Delta)^4}{4}\right] \frac{N_{Fe2}}{\tau_{02}} e^{-\frac{\tau}{\tau_{02}}} \quad \text{if } u \ge u_c. \tag{15}$$

The first term on the right of the solution (14) represents the contribution of the metal-poor AGB stars that formed during the early time of the Galaxy. In this case, the characteristics of s-process nucleosynthesis in the AGB stars are that the abundance of Fe seeds is lower but the neutron exposure is larger. The second term represents the contribution of the galactic disk AGB stars that formed during the later time of the Galaxy. The characteristics are that the abundance of Fe seeds is higher but the neutron exposure is lower. Obviously, the analytical model has the advantage that one can immediately understand the structure and the properties of the NEG, which is not the case in elaborate numerical models (Cui *et al.* 2007).

Gallino *et al.* (1998) have pointed out that the neutron density is relatively low, reaching $\sim 10^7\,\mathrm{cm}^{-3}$ in the q layer at solar metallicity and the temperature $T_9 = 0.1$ (in units of 10^9 K). According to the standard ^{13}C profile with solar metallicity given by Gallino *et al.* (1998), we take the maximum neutron exposure $\tau_{\max} \approx 1.79\,\mathrm{mbarn}^{-1}$ (kT $= 30$ keV) for the low-mass AGB stars with [Fe/H] $= -0.3$ in Case A. We choose $\tau_0 = 0.296(T_9/0.348)^{1/2}\,\mathrm{mbarn}^{-1}$ for low-mass AGB stars with [Fe/H] $= -0.3$ in Case B, which corresponds to a mean neutron exposure of the solar system. We use the primary nature of ^{13}C neutron source to calculate the neutron exposure per thermal

Table 1. The parameters of NEAGB.

	$\omega\Delta \leq u < u_c$	$u \geq u_c$
Case A	$\Delta\tau_{\max} = 10mb^{-1}$ [Fe/H] $= -1$	$\Delta\tau_{\max} = 1.79mb^{-1}$ [Fe/H] $= -0.3$
Case B	$\tau_0 = 8mb^{-1}$ [Fe/H] $= -1$	$\tau_0 = 0.296mb^{-1}$ [Fe/H] $= -0.3$

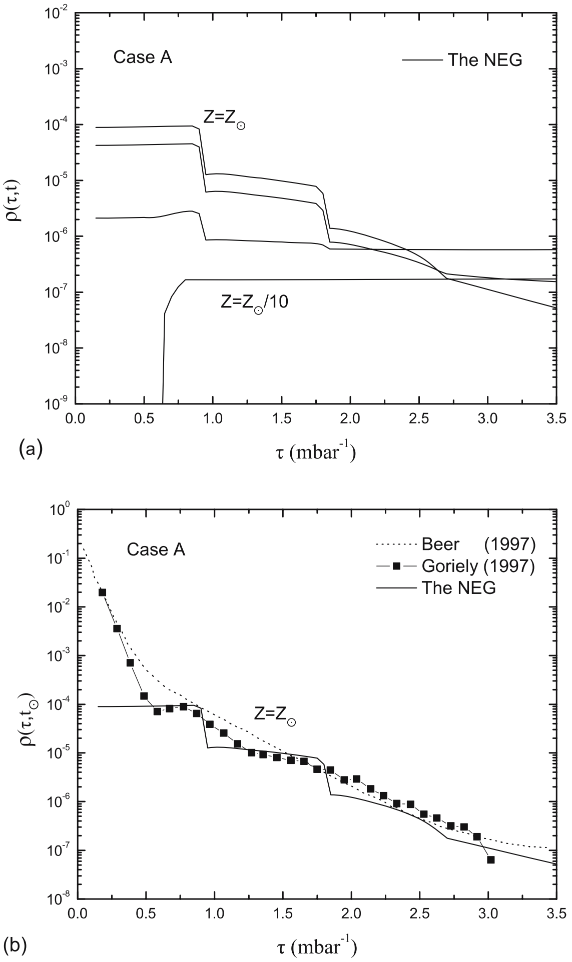

Figure 2. The evolution of the NEG as a function of neutron exposures (τ) for Case A. **(a)** The solid lines show respectively, the resulting NEG at $Z = 1/10$, $1/5$, $1/2$, $\mathrm{Z}_\odot$. **(b)** The NEG at $Z = \mathrm{Z}_\odot$. The analytical exposure distribution predicted by the exponential model of Beer *et al.* (1997) and the exposure distribution predicted by the canonical multi-event s-process model of Goriely (1997) are also shown for comparison.

pulse $\Delta\tau_{\max}$ and τ_0 at other metallicities. The basic parameters of the NGAGB adopted in our model are given in Table 1. Taking the total mass dredged up into the envelope of the AGB stars as $\Delta m_{Dup} = 10^{-2}$ $\mathrm{M}_\odot$ (Busso *et al.* 1992), the ejection mass fraction

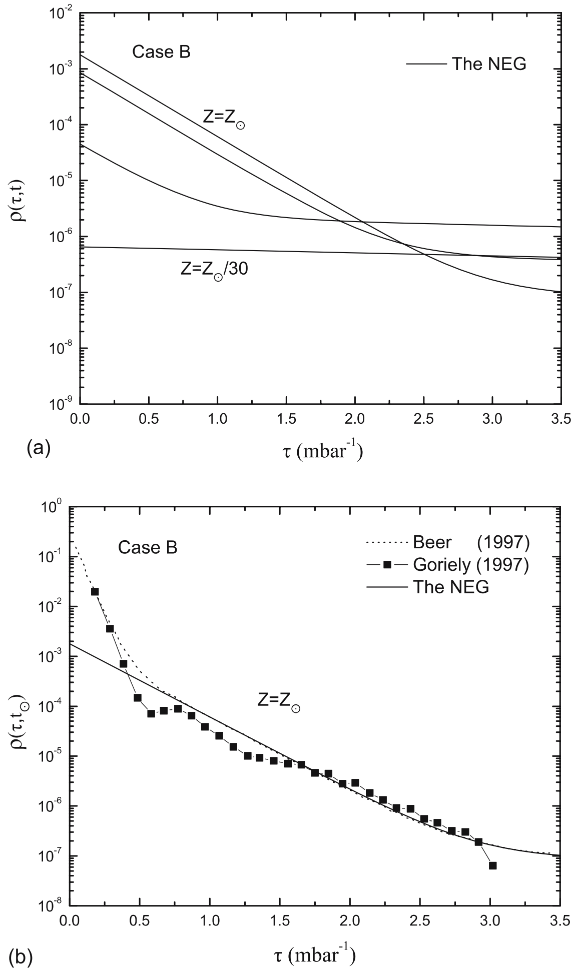

Figure 3. The evolution of the NEG as a function of neutron exposures (τ) for Case B. **(a)** The solid lines show respectively, the resulting NEG at $Z = 1/30$, $1/5$, $1/2$, $Z_\odot$. **(b)** The NEG at $Z = Z_\odot$. The analytical exposure distribution predicted by the exponential model of Beer *et al.* (1997) and the exposure distribution predicted by the canonical multi-event s-process model of Goriely (1997) are also shown for comparison.

$p' = 3.66 \times 10^{-4}$ is expected. The large parameter $\omega\Delta = 1.06$ for s-process elements is expected to reflect the time-scale for low-mass AGB stars.

In Figs. 2(a) and 3(a) we present our results for the evolution of the NEG as a function of metallicity for Case A and Case B. The [Fe/H] scale here is the indication of a time scale, albeit a non-linear one. From Fig. 2, we can see that there is a strong dependence of NEG on the metallicity or time of the Galaxy, the s-process contribution becomes important starting from [Fe/H] $= -1.5$ at lower values of [Fe/H], the contribution of s-process nucleosynthesis rapidly decreases due to the strong dependence of stellar yields on metallicity. The low-mass and metal-poor AGB stars are the dominant producers for the larger neutron exposures of the NEG, with a complex dependence on metallicity and a maximum efficiency at [Fe/H] $= -1$. The galactic disk AGB stars are the dominant producers for the lower neutron exposures of the NEG, with a maximum efficiency at [Fe/H] $= -0.6$. We also compare in Figs. 2(b) and 3(b), our results of NEG at $Z = \mathrm{Z}_\odot$ with the previous studies by Beer *et al.* (1997), obtained with the so-called classical approach and the results obtained by Goriely (1997) with multi-events model. Notice that our results are in agreement with the other results.

From the solutions of NEG and its definition, *i.e.*, $f(u)N_{Fe}(0) = \int_0^\infty \rho_{\mathrm{Gal}}(\tau, t)d\tau$, we can obtain the total number of iron seed nuclei in the Galaxy, which have been irradiated in all AGB stars, before time t

$$f(u)N_{Fe}(0) = 0 \quad \text{if } u < \Delta, \tag{16}$$

$$= p' \frac{e^{\omega\Delta}}{(u+u_0)^3} \left[(u_1+u_0)^3 u_1 + \frac{(u+u_0-\omega\Delta)^4 - (u_1+u_0)^4}{4} \right] N_{Fe1}$$

$$\text{if } u_1 + \omega\Delta \leq u < u_c, \tag{17}$$

$$= p' \frac{e^{\omega\Delta}}{(u_c+u_0)^3} \left[(u_1+u_0)^3 u_1 + \frac{(u_c+u_0-\omega\Delta)^4 - (u_1+u_0)^4}{4} \right] N_{Fe1}$$

$$+ p' \frac{e^{\omega\Delta}}{(u+u_0)^3} \left[\frac{(u+u_0-\omega\Delta)^4}{4} - \frac{(u_c+u_0-\omega\Delta)^4}{4} \right] N_{Fe2} \quad \text{if } u \geq u_c. \tag{18}$$

In Fig. 4 we present the calculated results for the evolution of the $f(u)N_{Fe}(0)$ as a function of time t. From the definition of the NEG, we can know that the factor $f(u_\odot)N_{Fe}(0)$ in the NES is the number of Fe seeds for the corresponding components that has been irradiated before the formation of the solar system. Our results are in agreement with the previous studies by Beer *et al.* (1997), obtained with the classical approach.

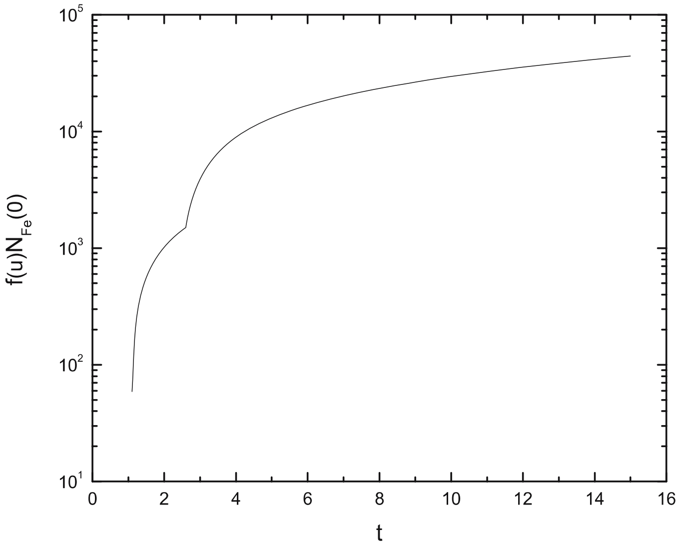

Figure 4. The evolution of the $f(u)N_{Fe}(0)$ as a function of time t.

It should be noticed that, for the lower neutron exposures, the NEG of Case A is slightly underestimated as compared to the results given by Beer *et al.* (1997), which is also obtained by the numerical model (Cui *et al.* 2007). The values of NEG for the lower neutron exposures are much more model dependent. The origin of discrepancies might be attributed to specific characteristics of the galactic evolution model (e.g., the age-metallicity relation). The uncertainty in this evaluation may depend on the set of prescriptions adopted to estimate the s-process yields from the AGB stars with varying metallicity and on the general prescriptions adopted in the galactic chemical evolution model.

5. Conclusions

Based on the analytical model with delayed production approximation developed by Pagel & Tautvaišienė (1995) for the Galaxy, the analytic solutions of the NEG are obtained. The results of the galactic evolution model, compared with the exposure distribution in the solar system, confirm the basic nucleosynthesis scenario outlined by Straniero *et al.* (1997) and Gallino *et al.* (1998). The ^{13}C neutron source, activated during the interpulse phases of low-mass TP-AGB stars, accounts for the exposure distribution in the solar system. From a comparison of the best fit by the classical analysis to the main component and the strong component by Beer *et al.* (1997) with the results of our NEG, one can obtain a clear indication that not a unique AGB stellar model, nor the classical analysis, was able to explain the main and strong component in the solar system, which must be considered as the outcome of different generations of AGB stars which are prior to the solar system formation.

The present results appear to reasonably reproduce the distribution of neutron exposures of the solar system. The strong component and the main component in the NES are built up in different epochs. Firstly, the strong component is produced by the s-process nucleosynthesis in the metal-poor AGB stars, starting from $[\mathrm{Fe/H}] \approx -1.16$ to $[\mathrm{Fe/H}] \approx -0.66$, corresponding to the time interval $1.06 < t < 2.6$ Gyr. Secondly, the main component is produced by the s-process nucleosynthesis in the galactic disk AGB stars, starting from $[\mathrm{Fe/H}] \approx -0.66$ to $[\mathrm{Fe/H}] \approx 0$, corresponding to the time interval $t > 2.6$ Gyr. As a matter of fact, our NEG calculations confirm, in a quantitative way, what was anticipated by Gallino *et al.* (1998), *i.e.*, that the role previously attributed to the strong component is actually played by the low-mass AGB stars with low metallicity. It is evident that the analytic solutions obtained in this work have the advantage of an understanding of the structure and the properties of the NEG, whereas numerical calculations can only supply evidence of the special case considered (Cui *et al.* 2007). From the definition of the NEG, we can know that the factor $f(u)N_{Fe}(0)$ in the NES is the number of Fe seeds for the corresponding component that has been irradiated before the formation of the solar system.

Despite the large number of approximations (the total amount of dredged-up material, the parameterization on the ^{13}C pocket and the dependence on the initial mass of low-mass AGB stars for the s-process nucleosynthesis, etc.) for the analytical model, the agreement of the model results with the NES provides a strong support to the validity of the analytical solutions of the NEG and the NEAGB prescriptions adopted in this work. The NEG is therefore believed to be an effective tool to study the s-process element abundance distributions in the Galaxy at different epochs and the galactic chemical evolution of the neutron-capture elements. It presents new features in comparison to the classical exponential model. Hopefully, the NEG can help us to improve our understanding of the exposure distribution in the solar system and open new perspectives in this direction.

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