



# Developing spatial mathematical skills through 3D tools: augmented reality, virtual environments and 3D printing

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## Abstract

In this paper, we show how to improve the mathematics teaching-learning process to enhance students' spatial visualization and orientation skills with the use of 3D tools; specifically, with augmented reality, virtual environments and 3D printing. Two of the tools presented in this work were especially developed for multivariable calculus courses. The inclusion of these materials in pedagogical activities integrates the senses of touch and sight to the learning process, favoring the understanding of important mathematical concepts related to three-dimensional space. This allowed instructors to present to students a natural way of modeling real world phenomena with proper mathematical language, thus achieving a significant increase in mathematics learning. Tests with control and experimental groups were conducted over four years, and students' final grades, failure rates and visualization-skills development were analyzed. Students and professors from several countries were interviewed and surveyed to assess perception and experience in the use of these tools. An analysis of variance with a sample of  $N = 993$  students and a significance level of  $\alpha = 0.01$  was performed, finding that the experimental group grades were seven points above those of the control group (on a 0–100 scale) and the failure rate dropped 14%. Moreover, from the spatial mathematical skills test with a sample of  $N = 442$  students, the experimental group obtained 15 points more than the control group, and the percentage of students achieving the minimum spatial skills level required to pass the course increased 36%. Our results reveal a positive impact in the use of these tools to develop spatial mathematical skills.

**Keywords** 3D printing · Augmented reality · Educational innovation · Mathematics teaching · Spatial visualization · Virtual environments

## 1 Introduction

Spatial abstraction, spatial visualization and spatial intelligence are different terms related to the essential skill that students need to develop in order to understand and implement important mathematical concepts, i.e., the skill

of mathematical visualization. Mathematical visualization includes a process where the student must be able not only to imagine three-dimensional objects together with their movements and transformations, but also be able to describe them using the proper mathematical language.

Several issues can be found in the visualization process. Most of them deal with the relationship between the real object (what the student perceives from the object) and the student's description of the object, first using her or his natural language and then a mathematical language. Traditional teaching of mathematics often leads students to limit the use of sight and touch, preventing them from developing important spatial visualization skills. It is because of the lack of these skills that many students do not thoroughly understand mathematical concepts and are thus unable to solve theoretical and applied problems in mathematics. Another issue with the traditional teaching of mathematics is the lack of intrinsic motivation in the student, mainly because mathematics is fre-

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quently introduced as a collection of formulas to memorize and not as a tool to understand and describe the real world.

In this paper, we show how to use augmented reality, remote virtual environments and 3D printing to improve the teaching-learning process of mathematics through the development of spatial visualization and problem-solving skills by tackling problems in a real context. Spatial visualization has been the concern of many mathematics educators. Most of the research about visualization has been conducted within a geometric context with children and teenagers. There has been little research about mathematical visualization concerning three-dimensional space, and even less in which the influence of technology on spatial visualization skills and mathematical performance are investigated.

This work consists of three parts. In the first part, we present a review of the principle research about spatial visualization, a discussion of the concept and its relationship with mathematics learning and problem-solving-skills development. We introduce mathematical visualization skills and establish how to develop them. In the second part, we describe two tools that have been especially developed to enhance students' spatial visualization skills; namely, AVRAM (an acronym in Spanish that translates as Remote Virtual Environments for the Learning of Mathematics) and ARC, Augmented Reality in Calculus. Moreover, we discuss the use of 3D printing in order to integrate the sense of touch towards the development of spatial visualization and problem-solving skills. Examples of in-class activities are shown in this section. In the third part, we present the methodology and report the results obtained in this research about the influence of these 3D tools in students' visualization skills and mathematical performance.

## 2 Theoretical framework

In H. Gardner's multiple intelligences theory [12], where spatial intelligence is established as one of these intelligences, Gardner claims that spatial thinking is essential for the development of scientific thinking, given the fact that it is used to represent and manipulate information in learning and in problem solving. It is estimated that most professions, such as engineering, architecture, design and medicine, and many scientific disciplines, such as chemistry, physics and mathematics, require professionals with highly developed spatial intelligence. People with a developed spatial intelligence can successfully tackle problems concerning location, orientation and space-data distribution.

Spatial intelligence, spatial thinking and spatial visualization are often considered as synonyms. Several authors argue about their differences, but, in the end, it is clear that all of them are related to the reasoning skills that the student needs to develop in order to imagine, describe, transform and

manipulate objects in three-dimensional space [6,11,16,22]. Spatial visualization and orientation have received a lot of attention in the mathematics education field [1,3,6,8,16,23,36,43]. Visualization involves evaluating an individual's processes and capacities to perform certain duties that require seeing or mentally imagining spatial geometric objects as well as his ability to relate these objects and perform specific operations or geometric transformations with them.

Ben-Chaim, et al. [4] and Nemeth [33] claim that spatial visualization capacity is not an innate ability but a competency that must be developed. This ability can only be developed when students participate in learning activities usually related to real-life experiences that engage them. Spatial visualization plays an important role in the development of students' mathematical thinking. Pittalis and Christou [35] stated that spatial skills are a strong predictor of student performance in 3D geometry. Hegarty and Waller [19] argued that, in general, spatial ability is required along with intelligence and visual perception to develop mathematical thinking. Revina et al. [37] in their study stated that spatial visualization tasks help students develop their conceptual understanding of volume measurement. Do-wns [10] assures that spatial thinking should be recognized as a fundamental part of K-12 education and as an integrator and facilitator for solving problems throughout the curriculum. Risma et al. [38] proposed that building block activities support the development of students' spatial visualization skills.

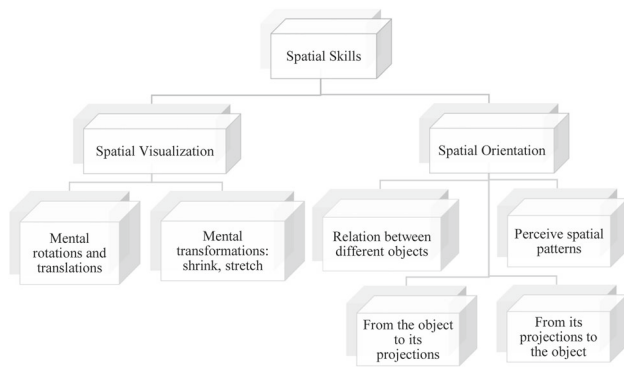
Bishop [6] identifies four processes that are applicable to visualization and mental images. They are (a) generate a mental image from certain information; (b) inspect the image to observe its position or the presence of parts of the elements; (c) transform the image with rotations, translations, scaling or decomposition, and (d) use the image to answer questions. The list of skills needed to process mental images can be very long. Previous investigations' results about spatial skills [6,26] can be summarized by describing ten skills grouped into two classifications:

### A. Spatial visualization skills

- (a) Skill to imagine rotating an object, unrolling a solid and changing the relative position of the object in the space.
- (b) Skill to visualize a configuration in which there are movements between its parts.
- (c) Skill to understand three-dimensional imaginary movements and manipulate objects with imagination.
- (d) Skill to manipulate or transform the image from a spatial pattern into another arrangement.

### B. Spatial orientation skills

- (a) Skill to determine relationships among different objects in the space.



**Fig. 1** Classification of spatial skills

- (b) Skill to recognize the identity of an object when it is seen from different angles or when the object is moved.
- (c) Skill to consider spatial relations where the orientation of the body's observer is essential.
- (d) Skill to perceive and compare spatial patterns.
- (e) Skill of having no confusion when there is a change in the orientation of the spatial object.
- (f) Skill of perceiving spatial patterns or maintaining orientation with respect to the objects in the space.

Figure 1 shows a classification of spatial skills together with examples of actions that each allow to produce.

Visualization skills are developed in three stages [5]. In the first stage, most children between 3 and 5 years old acquire two-dimensional topological skills. With these skills, they distinguish proximity, interior, exterior and set groupings. The second stage involves visualizing three-dimensional objects together with rotations and transformations in the space. People can visualize the concepts of area, volume and distance and combine these with rotations, translations, reflections and projections. Many freshmen students at the university exhibit a lack of development of stages two and three.

Sorby [39] presents a summary of the tests that have been used to measure a person's developmental level in the two first visualization stages. She describes about twelve tests. Most of them measure the skills in the first two stages. They are tests with the type of abstract reasoning exercises where it is asked what follows next, or blocks are drawn in order to see their projections on different planes and their movements in the three dimensional space. In many universities, abstract reasoning tests are considered important for measuring if a student will be successful in an engineering program [9,28]. Not many tests have been reported to measure skills in stage three. The few that are known are used to measure mental rotations, such as in Vanderberg's work [44]. In this paper, we present a way of measuring visualization skills in stages two and three within the context of calculus.

Technological development has allowed representations of three-dimensional objects to move beyond the flat plane and to have movement. We then can speak of dynamic visualization [18,36,41,46], which is an act formed by images in movement, either in the mind or in an external environment that the individual identifies with the object or processes in her or his mind. People possessing dynamic visualization skills have a specialized reasoning that allows them to solve mathematics problems with great ease. According to Tall and West [41], dynamic representation of mathematical processes allow more effective mental manipulations than could ever be achieved from a static text and pictures in a book. Other authors point out that dynamic visualization can be a very powerful tool to obtain a deeper understanding of many mathematical concepts and can be a resource for solving mathematical problems as well [14,18,36,41].

In recent years, many researchers and mathematics educators have acknowledged spatial visualization as an important component to develop mathematical thinking and problem solving skills [7,43]. Several studies show the relationship between visualization skills and relevant concepts in the engineering field [20,21,31,34,40,45]. Other studies even suggest that men have a better spatial orientation than woman, but that there is no significant difference regarding spatial visualization [15,17,25,32].

To develop spatial skills, several researchers have proposed diverse activities with blocks [38,39]. The types of blocks used go from plane images to augmented reality images. Alqahtani [1] reports that these techniques used to develop spatial visualization skills can be classified in software tutorials, flash courseware, e-learning modules, e-books, multitouch-screen technology, virtual reality, augmented reality, web 3D, the sketch-up software, colored 3D models and training websites. It is important and interesting to note that there are not many reports yet, as we show in this work, regarding the use of 3D printing to develop spatial skills.

Some of the most recent tools designed for developing spatial skills are: (1) Augmented reality, [24], where real objects can be virtually modeled and students are asked to visualize vertices, edges or, in an advanced stage, projections are given and the student creates a mental image of the object. (2) Virtual Mental Rotation Training [1] which is a semi-immersive system that allows selecting, rotating, zooming, and navigating with 3D models. This interaction is made by implementing the navigation with Six Degrees of Freedom (6DoF) in a virtual space. 6DoF allows seeing the object from six sides; namely, up-down; left-right; forward-back; pitch, yaw and roll, and it enables users to navigate and manipulate objects in virtual reality environments. In this paper, we present three 3D tools that have proven to have plenty of success in the development of visualization skills.

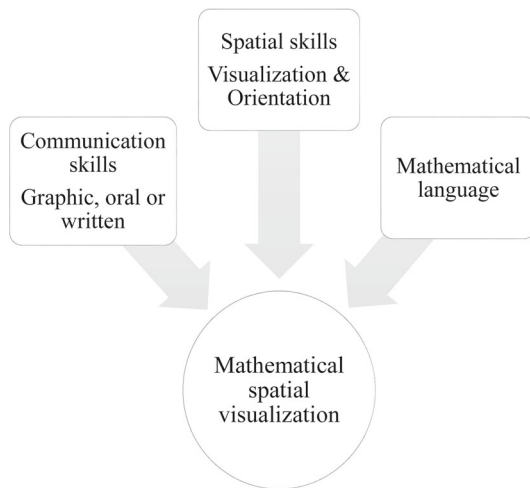


Fig. 2 Elements of mathematical spatial visualization

McKim [27] claims that visual thinking is accomplished by three types of images—the ones we see, the ones we imagine and the ones we draw. Expert visual thinkers use the three types of images in a flexible way. In mathematical visualization, it is equally important that students can develop spatial skills and that they be able to describe what they are imagining [29,30]. This description can be made by means of drawings, graphs or a natural language. A final stage leads to a description with a mathematical language. Figure 2 summarizes the elements of mathematical spatial visualization.

### 3 3D tools

In this section, we present the three 3D tools that have been used in calculus courses to develop mathematical spatial visualization skills. These tools have been a powerful ally of mathematics instructors to teach important mathematical concepts, improve the calculus teaching-learning process and develop projects that allow students to solve real-world problems.

#### 3.1 AVRAM: remote virtual environments for the learning of mathematics

AVRAM is a platform that allows authorship, manipulation and visualization of geometric models of surfaces in a remote 3D environment. It is formed by two elements; namely, a web server with an application responsible for synchronizing clients and generating 3D graphics, and an Android application that several mobile clients (using cell phones and tablets) can use to connect themselves to a virtual session where one of the clients from a mobile device, acting as the professor, coordinates and guides whatever is displayed. Figure 3 shows

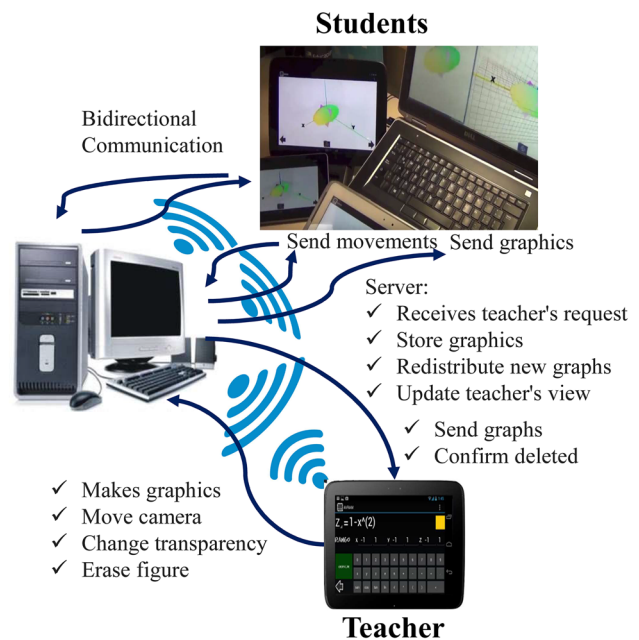


Fig. 3 AVRAM's components and several connected screens in a remote virtual environment

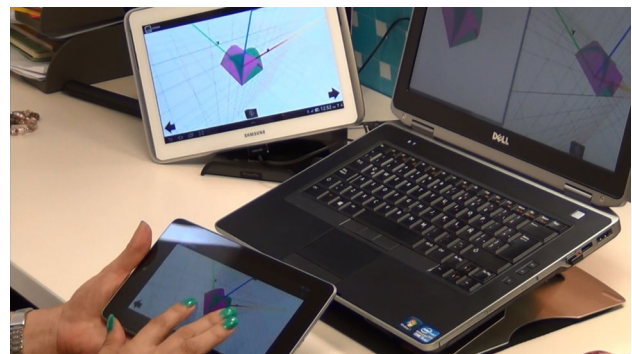


Fig. 4 AVRAM in action

a diagram of AVRAM's components and several connected screens, and Fig. 4 shows a picture of AVRAM in action.

The virtual environment is sent to several mobile clients through a local Wi-Fi network or a remote internet connection so that geometric models of surfaces can be visualized and manipulated by students individually and that they can answer the questions posed by the instructor. Moreover, students can navigate freely around the model in order to have different perspectives of the results.

AVRAM allows graphing explicit and implicit functions with transparency and deletion options. It has the option to see the coordinate axes' labels as well as to visualize the surfaces and movements displayed by the instructor. In addition, the number of surfaces that can be graphed in the same screen is unlimited. With the aid of transparency, students can visualize perfectly the intersections, unions and other operations among surfaces. Professors can generate the surfaces and

send them to the students' mobile devices. At any moment, students can manipulate the surface - scaling, shrinking and rotating it—by dragging it with their fingers. They can also look at its traces and projections on planes and axes. This manipulation allows students to solve the designed activity for the class. Students have the option to stop looking at the provided surface and to begin creating a surface of their own. A video describing AVRAM's main features can be found following the link: <https://www.youtube.com/watch?v=idnu25rdL5E&t=78s>

The type of activities that can be performed with AVRAM have the intention of showing how the instructor and the students look in the graphs for data and answers to the activities. It is certain that graphs can be generated using Mathematica, GeoGebra or any other graphers, but the advantage of a virtual environment is that a student can see how the instructor and her or his other classmates (perhaps those with more developed spatial skills) are working on the surface to find the answers in real time. If the instructor manipulates the surface, everyone sees what happens in real time. The same happens when working in teams. Everyone, instructors and students, can zoom the surface, look at it from different perspectives, enlarge, shrink, project, translate and rotate it.

Some examples of activities for quadric surfaces are the following:

1. Find the coefficients in the quadric surface equation that satisfy certain requirements, such as specific intersections with the coordinate axes, symmetries, rotations, etc.
2. Describe the characteristics of the largest sphere inscribed or the smallest sphere circumscribed in a given ellipsoid; find their equations and graph them all in the same screen or inscribe and circumscribe boxes in given ellipsoids.
3. Find surfaces limited by planes and quadric surfaces with a specific volume.

By solving these activities, students must use spatial visualization and orientation. They generate mental images that they graph using AVRAM and vice versa. From specific information, they inspect the object to observe its position or the presence of parts of the elements; they transform the object with rotations, translations and scaling; and, finally, they use the graph to answer the questions. Subsequently, they are asked to do the same, but without the aid of graphers.

Figure 5 shows students working the task of inscribing and circumscribing spheres in a given ellipsoid with AVRAM.

The AVRAM application has not yet lost its innovative role, despite the fact of being developed in the years 2013–2014. The interaction that can be achieved by using a virtual environment is still very attractive to students. AVRAM facilitates among students the development of the skills associated with exploring, communicating, analyzing, interpreting and

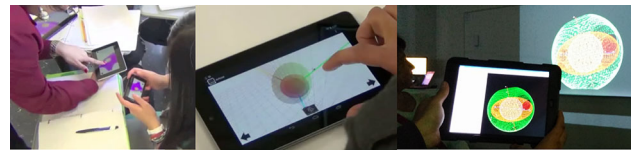


Fig. 5 Working with AVRAM in class

problem solving. Furthermore, the emotional connection that a personal immersive experience offers increases intrinsic motivation and contributes to a greater impact during the learning processes.

### 3.2 ARC: augmented reality in calculus

ARC is a mobile application with two sections that allows performing different exercises and multivariable calculus operations. The first section is focused on deploying surfaces from equations entered into the system. Students can input the equation of a quadric surface and interact with it afterwards. The platform allows rotating the surfaces in order to visualize them from different angles. They can also be visualized as solid surfaces or only as their mesh. Moreover, different set operations can be performed, such as intersections, unions or differences between two selected surfaces and visualizing the result of the operation. This tool can be used by professors to design their own activities by writing the equations to graph in a simple way and by specifying the type of desired operations and calculations. The tool permits dynamic visualization.

The second section of the application is composed of eight modules with a determined number of augmented reality cards that cover most of the topics of a multivariable calculus course and allow professors to design different activities for the students that can be performed inside or outside the classroom. Below is the description of each of the eight modules and the number of cards that each one requires:

1. *Module 1: Quadric surfaces.* This module comes with seven augmented reality cards. It allows graphing the main quadric surfaces and observing the traces and intersections with the coordinate planes. The equation coefficients can be modified to observe the changes in the graph. It also allows plotting any polynomial surface without rotation.
2. *Module 2: Level curves.* One augmented reality card is needed for this module. The student must associate the corresponding level curves with each of the surfaces. A correct association may show how the surface overlaps with its level curves.
3. *Module 3: Projectile launching.* One augmented reality card is needed for this module. A meteorite approaches and a projectile is launched. The direction vector of the line that describes the path of the projectile must be

calculated in order to destroy the meteorite. If the calculations are correct, the destruction of the meteorite will be observed.

4. *Module 4: Circular and spherical helices.* One augmented reality card is needed for this module. Two important curves in the space are observed, together with their parametric equations and their relationship with two surfaces.
5. *Module 5: Gradients, tangent planes and normal lines.* Six reality cards come with this module. Students can visualize the gradient vector, the tangent plane and the normal line of several surfaces at some points.
6. *Module 6: Maxima and minima with restrictions.* This module needs one augmented reality card. Students can observe critical points of some surfaces with and without restrictions.
7. *Module 7: Polar coordinates.* This module requires no augmented reality card. Students can see how the graph of a polar function is being drawn as the parameters change.
8. *Module 8: Cylindrical and spherical coordinates.* One augmented reality card needed for this module. Students can visualize the intersection of two surfaces, their projections on the coordinate planes and the complete surfaces.

Figure 6 shows a picture of how two quadric surfaces (a cone and an ellipsoid) are visualized as well as some level curves of a surface. A video showing ARC's modules can be found following the link: <https://www.youtube.com/watch?v=NhtKXwUUSjU>

Using augmented reality markers implemented with the RA Vuforia platform (AR for the Enterprise—Now Easier than Ever, 2016), the technological development team has adapted the video game creation platform Unity3D [42] to build and develop a mobile application capable of augmenting the video camera's traditional deploying of an electronic device with a three-dimensional graphic computational representation of geometric surfaces, which adapts in real time to the change in perspective by virtue of the active sensors' actions (such as accelerometers and gyroscopes) present in most of the modern electronic devices.

ARC with its augmented reality cards used in pedagogical activities has proven to be a highly beneficial resource

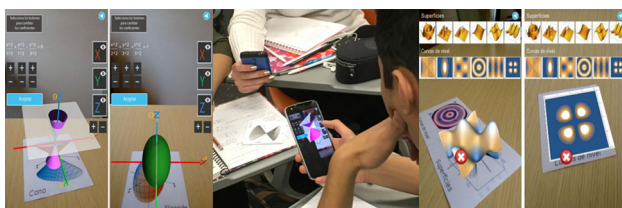


Fig. 6 Cone, ellipse and contour maps in ARC

to develop collaborative work in students. If the students are actively involved from the beginning, are guided in the use of the augmented reality application and have the appropriate devices at their disposal, they can learn by discovering mathematical concepts step-by-step and not just as a mere observer or spectator of the additional information that this technology presents. Furthermore, this is an incredibly attractive tool for students. The mathematics educators involved in this project have been captivated by this technology as well, because they have to think and design new pedagogical activities. They have also been benefited by team work with other colleagues.

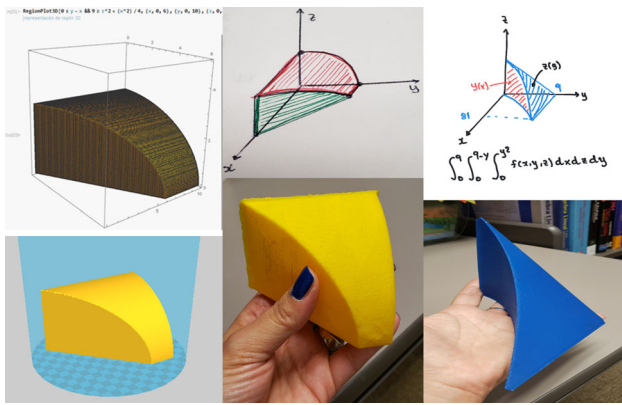
### 3.3 3D printing

3D printing has integrated one more sense to the calculus teaching-learning process: the touch. We have used DeltaBot, CubePro and LuzBot 3D printers with ABS and PLA filaments to print 3D models of surfaces in order to explain complex mathematical concepts. It allows students to see and touch what they have in their minds. It has an advantage over 3D graphers because it eliminates the possible confusion involved in representing a three-dimensional object in a lower dimension, such as the board. The DeltaBot 3D prints using cylindrical coordinates, while the CubePro and the LuzBot use rectangular coordinates. They can be used as a tool in the classroom to show how a surface is generated from its level curves. Figure 7 shows 3D printed models of surfaces used by the instructor to describe objects in the space that will lead the students to calculate surface areas and volumes. Other models are used to explain the concepts of critical points, maxima and minima, with and without restrictions and space curves, etc.

3D printed surfaces have been of two types: Some were generated using the software, Mathematica, from functions or equations describing three-dimensional regions. The second type was generated with a design software such as Rhinoceros. 3D printed surfaces are used by mathematics instructors to explain concepts and conduct in-class activities as part of the projects that students do. Figure 8 presents 3D models from a successful activity to develop spatial skills.



Fig. 7 3D printed models to work calculus concepts in class



**Fig. 8** 3D printed models to describe regions in the space and calculate volumes

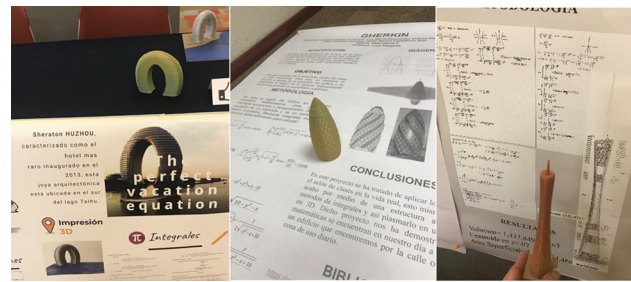
The activity illustrated consists of handing out a 3D printed object like the blue one shown in Fig. 8 to the student, and the task is the following:

1. Describe the object using your own words, commenting on the surfaces that limit it from the right, left, above and below.
2. Place the object in the three-dimensional space, locating as many sides of the object as possible with the coordinate axes.
3. Find an equation for each of the surfaces that limit the object.
4. Describe the object's projections on each of the coordinate planes, first with words and then with inequalities.
5. Describe the object using inequalities.
6. Find the surface area and the volume of the object.

This activity promotes the use of spatial visualization and orientation skills. Students must perform the four visualization processes that Bishop [6] mentions in order to reach step six adequately.

One of the projects conducted by students consisted of modeling a real building and calculating the surface area, the volume and the coordinates of the centroid, using multiple integrals. Students must execute several steps related to spatial skills:

1. Students must find a building that is formed with known surfaces (they know they will face the challenge of finding and analyzing the model).
2. Describe the building verbally (For example: “The building has three parts, the bottom part is a one sheeted hyperboloid, then it contains the frustum of a cone and it ends at the top with a paraboloid.”)
3. Locate the origin of the three-dimensional space at a convenient point of the building (The center of mass,



**Fig. 9** Students' posters and 3D printed models of the building

perhaps? The symmetry point? It is something that the students have to decide.)

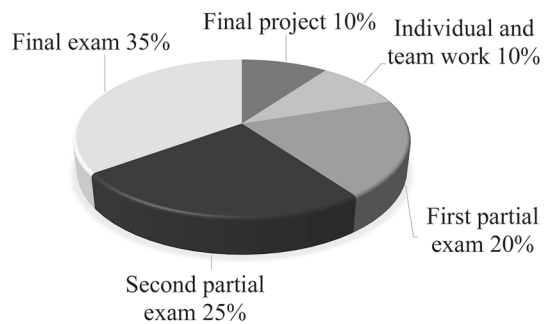
4. Conduct all the steps described in the previous activity.

We have changed the building of chess figures and perfume containers, among others. Figure 9 shows the poster and the 3D model of three teams of students for the building project.

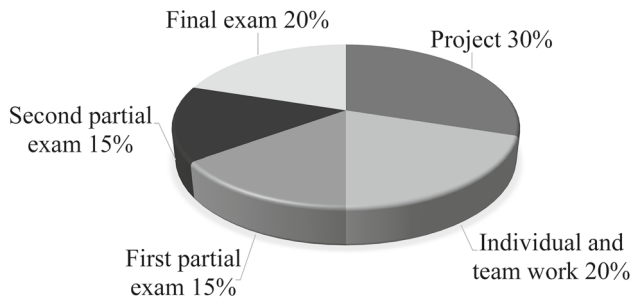
3D printing not only helps students to develop spatial skills; it also promotes self-engagement and enthusiasm in class. Students learn from the final result, but also from the 3D printing process itself. For the model to be 3D-printed properly, they must work with the ranges of the variables  $x$ ,  $y$  and  $z$ ; the width of the model; scales; supporting structures for the model; and, in general, with the print setup to create a good 3D model, which is not as easy as it looks. The process of 3D printing the model is more important than the model itself. Observing a 3D printer while working provides students with a deeper understanding of the concept of level curves through the layers that form the model and also of the cylindrical shells and disk methods for finding the volume of a solid of revolution. Students and professors are highly attracted by 3D printing. It has been an inspirational source for instructors in different disciplines, and they have started using them in their classes to teach important concepts.

## 4 3D tools implementation in the classroom

The investigation behind this report began in the year 2010 by trying to identify the necessary skills that an engineering student must develop from a multivariable calculus course in order to thoroughly understand mathematical concepts and implement them sufficiently to solve real-world problems that appear in their professional lives. Since a multivariable calculus course develops within the context of the geometry of three-dimensional space, spatial skills are fundamental. Several skills such as logical thinking or collaborative work are present, but we selected the problem-solving skill as part of this research because of its close connection to spatial skills.



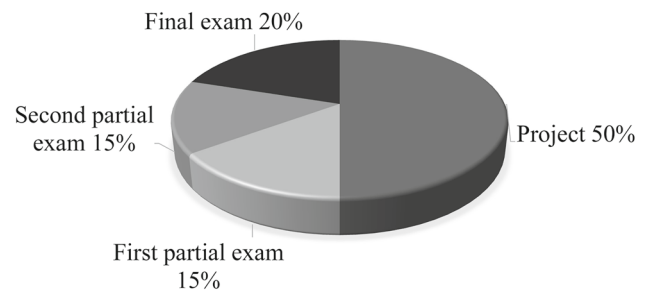
**Fig. 10** Traditional course



**Fig. 11** Transition course

Knowing that we needed to develop in students spatial skills and that the tools we knew and used at that moment were insufficient, we started to design and develop the tools that we presented in Sect. 3. The tool itself is not as important as the type of pedagogical activities that are designed to use it, nor the way in which these activities are implemented in class. That is why AVRAM was created in 2013 [29], along with a set of activities that were conducted in class and redesigned through the years to align with the objectives of the project. In the year 2016, ARC was developed to complement AVRAM with other types of visualization activities [30]. In the same year, we began using 3D printers; in the period of August–December, we implemented projects where students solved problems with objects that could be 3D printed. Moreover, it was in this period when the three 3D tools were implemented together with a new methodology. This methodology includes the use of the three tools, a project with 3D printing that is developed throughout the semester with POL (Project Oriented Learning) characteristics, and a process of continuous evaluation.

The main objective of this methodology is to develop spatial and problem solving skills. The impact on the change of methodology can be well appreciated in the evaluation. Figures 10, 11 and 12 show the evaluation changes for control and experimental groups and the next stage of the research. More precisely, Figs. 10 and 11 show the types of evaluation of the traditional and transition models, respectively, and they are compared in Fig. 12 with the model that will be implemented in the year 2019.



**Fig. 12** Course based on competencies and development of spatial skills

In the traditional course, the methodology is teacher-centered; the instructor exposes the mathematical concepts and makes use of software such as Mathematica, GeoGebra or other 3D graphers in the classroom. Collaborative work is carried out, and final projects are similar to those suggested at the end of each chapter of the principal calculus textbooks for engineers. Evaluation is performed with three written individual exams that are worth 80% of the final grade. Individual and team homework are worth 10%, while a final project is worth 10% of the final grade.

It was necessary to implement a transition model where the instructor stops being the center of the class and 3D printing activities become prominent to the students. In the transition model, the project is one that involves several topics of the course. It begins in the first weeks and ends in the final weeks with POL characteristics, and it is worth 30% of the final grade. Three written individual exams are taken, but now they are worth 50% and individual and team homework are worth 20% of the final grade. The transition model was also necessary in order to design and develop new evaluation instruments based on competencies. The implementation of the transition model was conducted in seventeen groups of the Mathematics III subject from summer 2016 to the present. Fifteen mathematics professors have been involved and more than 1500 students. In the year 2019, the model based on competencies and spatial skills development will be implemented (see Fig. 12).

The methodology presented in this paper encourages students to seek responsibly for proper mathematical knowledge by placing them in interactive learning environments where they do not only interact with the teacher, but also with each other, ensuring their full participation in the learning process. According to [13], there are several interactive approaches in teaching, among which there are creative tasks, social projects, use of new materials and solving tasks. The main activities designed in this project include these approaches with the use of 3D tools in cooperative student projects and activities having POL characteristics, where students use their creativity to design mathematical models, to describe them verbally and mathematically with inequalities



and equations, and finally to 3D print their models. In the case of AVRAM, the teacher and the students interact in real time in a remote virtual environment working activities in teams. For ARC, students are guided by the instructor in the use of the application, interacting with their classmates and developing collaborative work. Moreover, educators have been benefited by team work with other colleagues by designing activities with ARC. Finally, 3D printing promotes self-engagement and interaction inside and outside the classroom. This methodology strongly promotes the interaction between all parts, developing team working, argumentation and effective communication skills, along with problem solving and spatial mathematical skills.

## 5 Measurement, data and results

The study we have conducted has a sequential mixed-and-concurrent design. Data have been collected in different periods of time from August 2014 to June 2018. Quantitative variables of interest are final grades, failure rates and the results of a test presented at the end of the course designed to measure spatial and problem solving skills. Qualitative variables of interest were measured through individual and group interviews and surveys to students and professors.

In 2017, we performed the first measurement of the impact of the new methodology on the course final grades and the failure rate. The control group consisted of 20 Mathematics III groups with 593 enrolled students in the periods from August–December 2014 to January–May 2016. The experimental group consisted of 15 groups of the same subject with 400 students enrolled in the periods from June–December 2016 to August–December 2017. These courses have been taught by the same professors trying to homogenize course conditions in order to minimize bias.

It has been found that with the new methodology, the students' final grade average increased from 76.39 to 83.62 points on a 0–100 scale. Our results are based on an analysis of variance with a 99% confidence. Welch's unequal vari-

ances *t*-test produces the values of  $T = 98.98$  and  $p = 0$ ; thus, rejecting the null hypothesis of the equality of means.

Since Levene's test for the equality of variances with  $p = 0$  concludes that standard deviations of both groups are significantly different with a 99% confidence, the ANOVA is performed without assuming the equality of variances, and, in this case, the groups and Games-Howell levels tests are presented in order to prove the hypothesis of the difference between means.

Table 1 shows the test results for the final grades' quantitative variable. The confidence intervals do not overlap, the means do not share letters in the Games-Howell test and the difference is significant with a 99% confidence. Therefore, we can conclude that with the new methodology, the average of students' final grades increased seven points and that the experimental group presents less dispersion in the final grade.

However, due to the fact that data were collected in two different periods, it is natural to ask whether there is a difference in the students that caused this increase to occur in other subjects of the same period. As to this issue, we present the results in the same periods of time for the subject of Differential Equations, which is studied in the same semester as Mathematics III and, therefore, these two courses share a significant number of students. They also share two of the six instructors involved in the study. Table 2 shows the ANOVA with the test of the difference between means. Results show that there is no statistical difference in the averages of the final grades between the control group that took Differential Equations in the periods from August 2014 and May 2016 and the experimental group that took Differential Equations in the periods from June 2016 to December 2017.

It is also interesting to show that there has been a statistically significant increase in the percentage of students who pass the course. The average percentage rose from 81.8% to 95.3%. This represents for us an increase in the number of students who had developed the minimum spatial skills to pass the course. Table 3 shows the analysis of variance for the proportion of students with a passing grade.

**Table 1** Games-Howell test for the difference between the means of both groups final grades

	N	Mean	SD	CI 99%	Games Howell's simulation testing	
					Grouping	Level's difference 99%
Control group	593	76.398	12.668	(75.054, 77.742)	B	T = 9.95
Experimental group	400	83.618	10.118	(82.308, 84.927)	A	Adjust 0

**Table 2** Analysis of variance for the final grades in Differential Equations

	N	Mean	SD	CI 99%	Welch proof	
Control group	599	77.06	13.905	(75.592, 78.528)	F	P
Experimental group	443	75.788	13.727	(74.101, 77.475)	2.16	0.142

**Table 3** Analysis of variance for the proportion of students with a passing grade

	N	Proportion	SD	CI 99%	Games Howell's simulation testing	
					Grouping	Level's difference 99%
Control group	611	0.8183	0.3859	(0.7780, 0.8587)	B	T = 7.19
Experimental group	408	0.9534	0.211	(0.9264, 0.9805)	A	Adjust 0

We believe that this difference is due mainly because of the use of the new tools and the types of projects and activities, all designed with the sole purpose of developing spatial skills. Due to the large quantity of variables involved in this experiment, we require more specific measurements that show the development of spatial skills and its relationship with each one of the tools. Despite this fact, our research continues and is directed towards this goal. Meanwhile, we present the first tool that we have used to measure, in general, the development of mathematical spatial skills. It is a written individual test taken at the end of the course, and it consists of the following ten open questions:

1. A question where the student has to draw a quadric surface with some special characteristics and identify among several graphs the one that satisfies the given conditions.
2. Three exercises utilizing mathematical concepts and algorithms.
3. Two problems that the student must set up and solve using one of the methods studied in the course. The method is not specified.
4. Four multiple integrals where the task is not to evaluate the integral but to describe regions in the space in different ways:
  - (a) Set up an integral that represents the volume of a certain solid in the space. This implies having to describe a region in the space and decide the projection that will simplify the path for obtaining the result.
  - (b) Given a multiple integral, the student is asked to draw the region of integration (the object) and to describe it using another coordinate system. Here, the student has to draw the region in the space and use her or his natural language that will allow passing from one mathematical description to another in a different coordinate system.
  - (c) Given a multiple integral, the student is asked to change the order of integration, which implies knowing how to describe a region in the space by observing its projection on another plane. Here, the student has to draw or imagine the object, project it on the indicated plane and describe it with inequalities.

That is, 50% of the test is measuring mathematical-spatial skills, 20% is measuring problem-solving skills and 10%

the use and understanding of mathematical concepts and algorithms. To obtain these measurements, the control and experimental groups had the same instructor, and they have essentially taken the same test and exactly the same test by pairs. (Each exam that was taken by some of the groups included in the control group was taken without any variation by some of the groups included in the experimental group). The control group had 240 students who took the course with the traditional teaching methodology between the year 2015 and the first half of the year 2016. The experimental group had 179 students who took the course with the new methodology between the second half of the year 2016 and the first half of the year 2018. Note that the skills test given as a final exam is worth 35% of the control group's final grade, while the same test is worth 25% of the experimental group's final grade. If a student thinks she or he will fail the course, she or he can quit some weeks before the final exam. So if the student presents the final test, she or he can pass the course. The fact that the exam has more value for the control group plays against our research, because the student is more motivated or stressed to study.

Table 4 shows the results of the final exam's grades and the data statistical analysis. The variance analysis shows that there is a significant difference in the final exam's grades. The experimental group obtained, on average, 14 points more than the control group. The variance in the final grades is lower in the experimental group. An analysis by pairs (to whom the exact same exam was given) produced the result that the grades are statistically equal (in a group) or statistically greater in the other groups included in the experimental group. The greatest differences are found in the questions of type 1, 3 and 4, which are precisely those that deal with visualization skills, spatial orientation and problem solving skills.

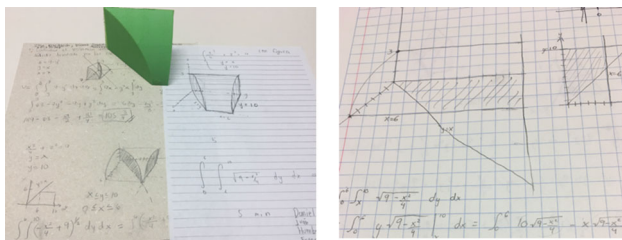
The percentages of students who passed the course also show a significant statistical difference. The experimental group has a passing grade percentage of 84.2%, whereas the control group has only 48.4%. With the new methodology, there was more than a 35% increase in the number of students who passed the test, which, as we have shown, measures spatial and problem solving skills. Table 5 shows the analysis of variance of the passing grade percentages. The variance of the experimental group is significantly lower than the variance of the control group.

**Table 4** ANOVA and confidence intervals for the skills test grades

	SS	DF	MS	F	Sig.
Between groups	22885.784	1	22885.784	72.971	0.000
Within groups	130783.29	417	313.629		
Total	153669.074	418			
	N	Mean	SD	CI 99%	
Control group	240	65.05	18.71	(62.10, 68.01)	
Experimental group	179	79.99	16.27	(76.57, 83.42)	

**Table 5** ANOVA and statistics of the proportion of students who pass the skills test

	SS	DF	MS	F	Sig.
Between groups	13.757	1	13.757	68.114	0.000
Within groups	88.867	440	0.202		
Total	102.624	441			
	N	Mean	SD	CI 99%	
Control group	258	0.4845	0.5007	(0.4121, 0.5569)	
Experimental group	184	0.8424	0.3654	(0.7567, 0.9281)	

**Fig. 13** Experimental group's drawing to the left and control group's drawing to the right

In the rest of the section, we present some qualitative results and students' perceptions, collected through opinion surveys, group panels and interviews. A very important result is shown in Fig. 13. Students are asked to find the volume of a certain region in the space by only using the equations of the surfaces that bound the region. To the left, below the green 3D printed model, one can find the drawings of two teams of the experimental group. The 3D printed model appears in the picture only to show how the drawings resemble reality, but students did not have it at the time they performed the exercise; however, the experimental group had had already an approach to the 3D tools described in Sect. 3. To the right, one can find the drawing made by one team of the control group. We can observe that the use of 3D tools significantly helps students to make better descriptions of regions in the space, either verbally, visually or algebraically; that is, by means of words, drawings or proper mathematical language.

Moreover, students are asked at the beginning of the course to make a video in which they explain a topic concerning quadric surfaces and where they need to describe regions

in the space. These initial videos do not show significant differences between the experimental group and the control group. The expressions used, the number of words and the depth of the explanations are essentially the same. At the end of the semester, students are asked to make another video in which they describe a more complicated region in the space. The control group uses fewer expressions and words than the experimental group; their descriptions are brief and scarce; and in some teams, the mathematical description is even incomplete. The experimental group has a more precise description; they use more appropriate words; they make comparisons and they describe more adequately a region in the space, using proper mathematical language.

Three group interviews were carried out, and students described the class as motivating, challenging and providing a significant amount of learning. They found the 3D tools very attractive and appropriate, and they craved that other science courses would implement activities that make use of these or similar tools. Professors commented that it was difficult to change from a traditional methodology of teaching mathematics to a new one; first, because of the continuous training of the use of the technology involved, but principally because of the large amount of time and imagination required to design new and engaging activities as well as their evaluations' rubrics.

In the individual interviews, students showed their satisfaction with the use of these 3D tools during the course. Many of them exhibited a big surprise seeing the grades achieved because the course is considered to be difficult among students.

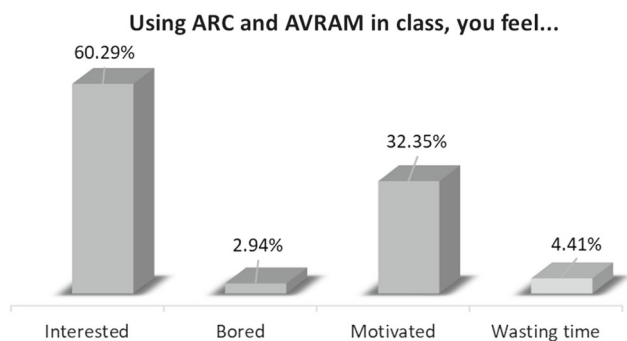


Fig. 14 Students’ answers to how they feel in class when using augmented reality (ARC) and virtual environments (AVRAM)

Surveys have been carried out since the year 2017. 282 students involved in experimental groups have answered these surveys. They have taken the course with one of the six professors involved in the research, and they come from two cities: Mexico City and Guadalajara.

Students were asked how they felt in the classroom while using virtual environments with AVRAM and augmented reality with ARC in class. More than 92% answered that they felt interested and motivated, about 3% felt bored and 4% felt they were wasting time. Most of the students show a great interest, they express awe and revelation in their faces, and they explored with the new tools until they found new purposes. A very small percentage of students preferred a teacher-centered traditional course. Figure 14 shows the results of this survey.

Furthermore, we asked students if 3D printing had helped them to learn some specific topics of the course related to spatial visualization and orientation. We also asked if it helped them to improve some spatial skills. Here we used a 1–5 scale, where 1 stands for a complete disagreement and 5 for a complete agreement. The results we obtained were that 95% of the students believe that 3D printing helped them to find the limits of integration of triple integrals, 90% believe it helped them describe regions in the space and visualize intersection curves between surfaces in order to obtain their parametric equations or to solve Lagrange’s optimization problems, 90% claimed to have increased their skills to generate mental images and used them to answer questions and 90% believed to have increased their spatial orientation skills. The results of this survey are shown in Fig. 15.

We also asked students how useful they found these 3D tools to visualize the three-dimensional space. Again, we used a 1–5 scale, where 5 stands for totally useful and 1 stands for totally useless. The results we obtained were that 97% of the students found 3D printing useful, while virtual environments (AVRAM) garnered 96% and augmented reality (ARC) 81%. Finally, 67% of the students found the traditional blackboard useful. Regarding the usefulness of the tools for describing regions in the space, students’ per-

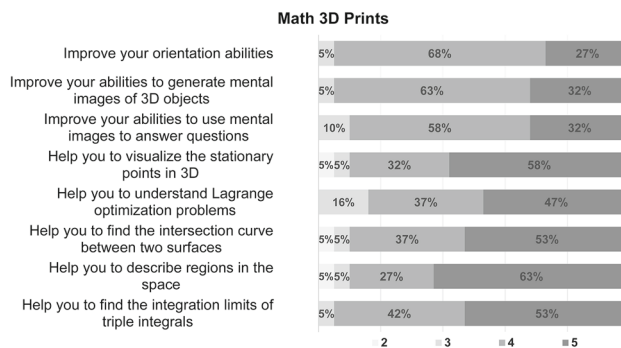


Fig. 15 Students’ answers about spatial skills and 3D printing

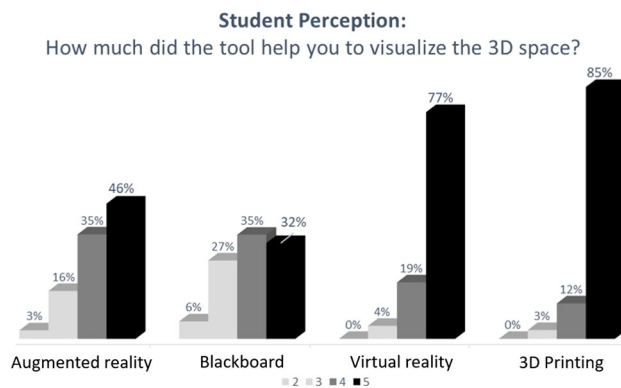


Fig. 16 Students’ perceptions about the usefulness of the tools to visualize the three-dimensional space

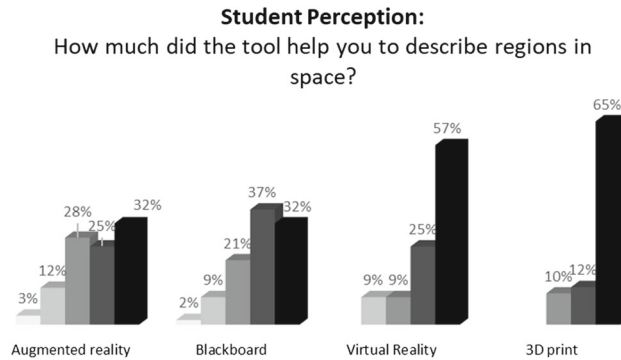


Fig. 17 Students’ perceptions about the usefulness of the tools to visualize the regions in the space

ceptions were that they found virtual environments and 3D printing really useful and augmented reality and the blackboard moderately useful. These students’ perceptions are described graphically in Figs. 16 and 17.

Using the same 1–5 scale, to the question of how much do these tools helped you to resolve problems, 78% of students found augmented reality useful, 84% of them liked virtual environments, 94% approved of Wolfram Alpha and 73% liked 3D printing. Figure 18 shows this student perception.

Furthermore, the institutional survey answered by students, where the performance of the instructor is assessed,

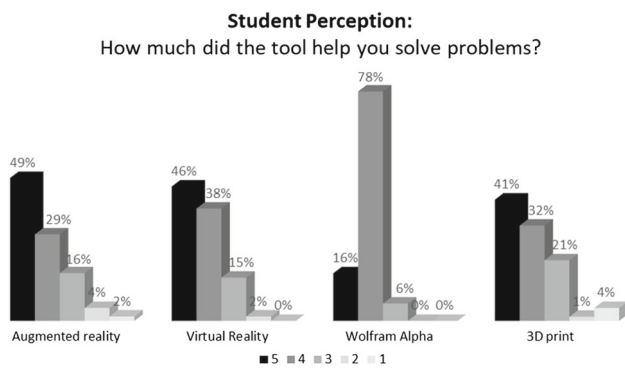


Fig. 18 Students’ perceptions about the usefulness of the tools to solve problems

has improved significantly. On average, an increase of 0.5 (in a 1–10 scale) in the performance of the instructors involved in the research has been observed. Some of the students’ comments in the survey are the following:

- The tools used in class helped me a lot to understand all the contents of the course.
- Technology implementation in subjects like mathematics is of great help, especially when we talk about visual aspects, such as graphs. These tools helped me understand and identify, with greater ease, the distinct parts that compose the graphs.
- The innovative teaching techniques of this course called my attention a lot. Undoubtedly, this is one of the best teachers that I’ve ever had.
- I think that augmented reality graphs are significant with respect to learning the contents of Mathematics III.
- Excellent class, I like that professors try to innovate so that the classes do not become a tedious routine.
- I liked the course a lot because the professor motivates students to learn more about the topics and does it in an interactive way.
- Excellent integration of 3D printing with Mathematics III.
- Excellent course, with the hope that they continue with this form of working.
- I’ve never learned that much in a mathematics class and, besides, having a good time!

Moreover, 102 Mathematics and Physics professors from four Mexican and six Latin American universities were surveyed. After watching videos on the three technological tools and their use in the classroom, they were asked how useful they would find these tools to use with their students in their classrooms. On a scale of 1–5, where 1 is not useful and 5 is very useful, they expressed that the tools can be very useful for teaching mathematics and physics and that they are will-

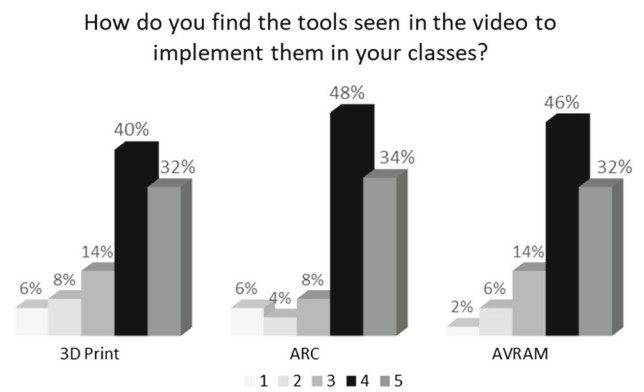


Fig. 19 Mathematics and physics professors’ answers about the usefulness of the tools

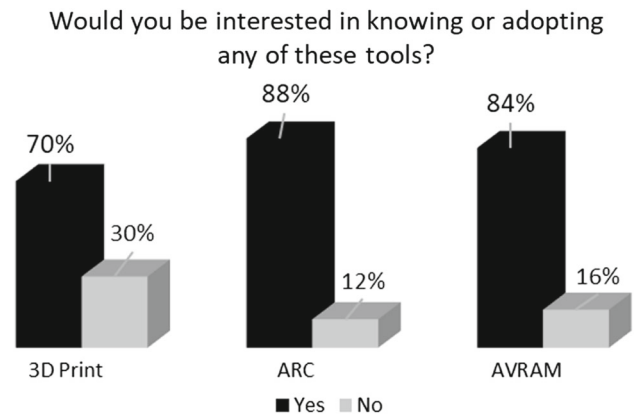


Fig. 20 Mathematics and physics professors’ answers about their interest in using the tools

ing to use them in their courses. These opinions are shown in Figs. 19 and 20 .

Some of these tools are currently being used by more than 25 professors in the Mathematics III courses in more than seven different cities in Mexico. They have proven to be very useful in the Mathematics II courses (integral calculus of a single variable) and in several physics courses as well.

## 6 Conclusions

In single variable and multivariable calculus courses, the absence of spatial skills in students and their development is evident. Students lack adequate development in the following skills:

1. Imagine the rotation of an object and visualize the relative changes in the position of an object in the space.
2. Understand imaginary movements in three dimensions and manipulate objects in imagination.
3. Identify the identity of an object when it is seen from different angles or when the object is moved.

#### 4. Describe the object using proper mathematical language.

Technology plays a fundamental role by allowing the reproduction of three-dimensional images through virtual environments or augmented reality. These images can be manipulated and transformed in ways that help students to visualize the space and to orientate themselves in it. The use of 3D printed models integrates one more sense to the teaching-learning process of the topics related to three-dimensional space: the sense of touch. The 3D tools that we have presented in this work not only help the students to acquire and develop spatial skills, but they also can be used as a motivating and engaging resource for the students of the current generation.

It has been found that with the use of these 3D tools and the accompanying methodology, spatial visualization and spatial orientation skills can be developed. Students have learned complex mathematical concepts together with their implementation in the formulation and solution of problems. Statistics show that students' final grades have increased and the failure rate decreased significantly. The skills test showed that, with the new methodology, students obtained a significantly higher grade. 50% of the test measures spatial mathematical skills, 20% measures problem solving skills and 30% measures concepts and algorithms. Questions where students have improved are precisely those related to spatial and problem-solving skills. Moreover, there is evidence that students feel more motivated in class when these tools are implemented in pedagogical activities in the classroom. The results show that with the new methodology, which makes use of these 3D tools, students have improved the way of describing objects in the space naturally, and they take the next step by describing the objects using the proper mathematical language. Students have increased spatial visualization and orientation by being able to transform objects, project them on different coordinate axes and planes, perform a detailed description of the region obtained when using a mathematical language and recover the object from its projections. Additionally, mathematical modeling projects that make use of 3D printing motivate students to solve problems in order to obtain a significant learning of mathematics.

The analyses on the final grades and on the failure rate in control and experimental groups conducted over four years have shown that the experimental group obtains significantly higher grades than the control group (7 points more on a 0–100 scale) and that the failure rate decreased 14%. The most surprising results were obtained in the spatial mathematical skills test, where, with a sample of  $N = 442$  students, the experimental group obtained about 15 points more than the control group, and the percentage of students who attained the minimum spatial skills required to pass the course increased by 36%. Our results show a positive impact in the use of these 3D tools to develop spatial skills.

A second part of the research is currently being conducted. Pre-tests and post-tests will be conducted with several of the tools frequently used in the literature about the topic in order to assess spatial skills before and after the activities that will use the 3D tools. We also want to measure the relationship between spatial skills development and problem-solving skills development. One of the formulated discussions in the literature is whether there is a significant difference with respect to gender in the acquisition and development of spatial orientation skills. We intend to study this phenomenon within the context of spatial mathematical skills. The tools to evaluate the development of spatial skills is a topic that continues to be of high interest among many educators. With the current technology, new assessment tools will be undoubtedly developed.

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