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SHORT ORIGINAL PAPER

Augmented reality and solids of revolution

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Abstract In this paper is discussed an approach for the use of augmented reality in mathematics education that takes advantage of affordances in this technology to offer a tangible experience when interacting with solids of revolution. A multidisciplinary team that must get involved in the field of interaction design and human computer interaction supports the integration of this technology. After giving some elements to understand the perspective created for innovation in the pedagogy of mathematics, the perspective used for the design of the augmented reality application will also be explained. The decisions to perform a vision-based use that involves dynamical visualizations are supported with the idea of the creation of an immersive experience. The solids of revolution application will then be exposed focusing in the affordances that are considered in the learning design. Conclusions about the changes that the use of this technology could bring for spatial visualization development will be suggested.

Keywords Augmented reality · Visualization · Calculus · Learning design

1 Introduction

Nowadays, the perception of students about the learning of mathematics can be improved with the visualization and gesture elements that the new emergent digital technologies offer. In particular with the technology of augmented reality (AR) new ways for the interaction with mathematical knowledge could bring to the fore 3D visual perspectives and give the opportunity to show their potential to promote the development of mathematical reasoning [1].

However, the integration of digital resources into the learning process of mathematics raises a research problem that involves several issues. One of them is the resilience of a curriculum that remains oblivious to the rapid development of new technologies. Another question is to know about the cognitive processes related to learning Mathematics, especially with the difficult issues that this science presents dealing with symbolic representations. In addition there must be considered the technical problems that represent the design, development and production of didactic resources with digital technologies.

Certainly, having an extensive training in Educational Mathematics allows supporting the design of a didactic resource using emerging technologies, but this is not enough to produce those digital resources. The introduction of technology must be based on pertinent educational research carried out by a multidisciplinary team. The authors' experience certify the importance of working together with experts in digital design and programming in order to pursue a common goal: transform the way students interact with mathematical knowledge and appreciate its impact when learning mathematics [2].

In this paper we seek to highlight the need to form multidisciplinary research groups holding the commitment to deepen in the field of learning design and Human Computer Interaction to produce digital didactic resources designed to make a difference in the interaction with mathematical knowledge. In this sense, the perspective of an innovation in the pedagogy of mathematics will be discussed in the first section. This includes the integration of digital technology in order to promote a visual perception. In the second section we

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clarify our sight about the contribution we make to interactive design development. The third section summarizes some type of uses reported about AR in order to place the perspective that this paper proposes. The fourth section is dedicated to expose the application in AR that has been built in order to improve the spatial visualization skill when learning about solids of revolution in calculus. Finally, conclusions will be delivered recalling the importance of enhancing the affordances that emergent technologies could bring in education and its impact on interactive design.

2 Mathematics education and the integration of digital technologies

Results of educational research today make us recognize serious limitations for the learning of mathematics in the school classroom that affect even in an attitude of rejection towards the courses of Mathematics. "Math anxiety" expresses a sense of tension that interferes with course activities and blocks students in their reasoning and memory. Furner and Gonzalez-DeHass [3] point out that underlying causes of this relate to teachers' instructional practices, including examinations; however, even if the teacher can lower stress levels, researchers should continue to point out the need for a change in teacher training that promotes a change in how mathematics courses are taught.

The slow change in school curriculum at all levels of education leaves the impression that an immovable curriculum establishes that mathematical knowledge as a reality to which students have only access through memorization. Several research reports have led us to question whether this is the only option available to make Mathematics a learning object. According to the study by Jackson [4], most interviewees think it is acceptable to admit that they have difficulty with mathematics and have negative perceptions of mathematics, including emotional and physical manifestations. The mathematical knowledge we have shared with students in the school environment leaves the impression of being a product alien to reality and in which they have little or nothing to say. Ashcraft and Krause [5] evidence that students avoid taking math classes and interacting with situations where mathematics is necessarily required, even this conditions the choice of the career they wish to pursue.

No one can dare to doubt about the usefulness of mathematics and it is assumed that learning math develops a logical thought. Nevertheless, problems with the learning of mathematics are widely accepted and the feeling it arouses is that math seems so outside of our daily world. Is math learning a necessary evil? What is wrong with math teaching that denies mathematics as a product of our intuitions? In the end, the abstract nature of mathematics results from real experiences as a human species. Humans are privileged with the capability to generalize in a symbolic way; thus, creating symbols is part of the human nature. A virtual world is being built from those symbols and perhaps this provides the feeling of being an autonomous world, as stated by Moreno-Armella [6]. This is a feasible reason for the world of mathematics (a symbolic system par excellence) to be so distant from students.

Mathematics education has been providing significant progress with the creation of frameworks that allow us to understand problems dealing with the symbolic representations of mathematics. Duval [7] argues about the learning difficulties in terms of a paradoxical nature of mathematical knowledge because of the mode of access to this science. Unlike other sciences, he points out, in mathematics there is not access by perception or instruments for mathematical objects. The only way to access it is using signs, words or symbols, expressions or drawings, but at the same time, mathematical knowledge cannot be confused with those semiotic representations. Mathematics has involved the development of several semiotic systems, from the duality of image and language to the cognitive modes that are bound to receptors on sensory information of sight and hearing. Symbolic notation is derived from written language and has led to the algebraic writing, and the display has moved from the construction of plane figures with tools to the figures in perspective, and then graphics "translate" curve equations. All of this leads to significant problems of cognitive coordination.

Particularly in [7] Duval states a key insight to analyze cognitive processes involved in mathematical thinking: you have several systems of semiotic representations that must be coordinated during mathematical activity. There caution should be exercised as it involves cognitive requirements, for processing mathematical and semiotic representations and for the transformation between different semiotic representations. It is important to develop flexibility working with the algebraic, graphical and numerical representations, looking to favor the transition between representations. The authors in this paper believe that today, the mathematical symbolic power of digital technologies enhance a visual and tangible perception of the mathematical representations (numeric, algebraic and graphical) and this allows a current way to bring mathematics closer to the students.

Introducing digital technology in the teaching and learning process of mathematics has been conditioned to the current topics in formal curriculum and the current practice in teaching. There is an underlying idea about technology to improve what already was done without technology. However, from a historical perspective, Moreno-Armella [6] argues about the co-evolution of knowledge and tools. This feature of the human development applies particularly in mathematical thinking (knowledge) and symbolic technology (tool). In the mixture of mathematical and computer symbols, the author states the existence of an internal mathematical universe that works as a field of significant mathematical reference which may be achievable through the computer screens. This invites to consider the idea of "the truth" in mathematics as a product of human activity increasingly sophisticated. Implications for the mathematics education should emerge from this bringing the opportunity for students to perceive mathematics through the exploration of digital technologies. New ways of thinking about the meaning that students develop with the computer interaction should encourage new research questions to expand the educational research agenda, and allowing a new way to conceive the mathematical activity performed by students.

At present, there are some ways to analyze the kind of intervention with digital technologies that is practiced in education. One is the SAMR Model (Substitution, Augmentation, Modification and Redefinition) proposed by Puentedura [8] to help teachers to understand the level at which they are using technology in classroom. The common practice is to use digital technology as a direct tool substitute, with or without any functional improvement. As Kirkland [9] states, these are *enhancement* tasks that correspond to levels of Substitution and Augmentation in SAMR Model for learning with technology. On the other side, the *transformation* tasks include Modification and Redefinition levels. There technology allows a significant redesign of tasks, or the creation of new ones, previously inconceivable.

Based in the authors' practice, the Substitution level may be exemplified by providing materials through a technological platform instead of using paper, and the Augmentation level could be associated to the incorporation of a scientific calculator to perform the operations in the same activities used before without it. Those are enhancement tasks for mathematics education for sure. However, the transformation tasks add changes to the conventional practices; it assumes a new design using technology. For instance, Modification level could be exemplified by the using of graphing software that allows to draw the graph of a mathematical function and analyze their features visually; before these technologies the goal was to draw the graph by hand. Meanwhile the Redefinition level it implies that, through technology, the student is motivated to ask, to build questions that promote a learning situation. This may be exemplified with activities using some graphing tools that allow performing a curves dynamic animation as a result of changes in the algebraic representation of a mathematical function. This promotes in students to ask why the changes are happening and by looking for the answer, the algebraic and numeric representations are evoked. There lies a good opportunity for learning, performing processes of analysis and synthesis motivated by that visualization process.

The Modification and Augmentation levels in SAMR Model set a new perspective for the integration of digital technology in education looking to transform the conventional practices. This is the perspective from which to understand the present paper; the integration of emergent digital technologies for the learning of mathematics inquires to transform curriculum. The main intention is to take advantage of the potential of emergent technologies to interact with mathematical representations and understand mathematics in a different way. It is worth to say that in order to transform the learning of mathematics through the integration of digital technologies we should take into account the framework provided by Moreno-Armella and Hegedus [10] and Moreno-Armella and Sriraman [11]. They have coined the idea of *co-action* as performed between the user and the technology. Co-action is possible because of the power of technology to *execute* the intentional action or input from the user. In mathematics education, this idea is a key element that brings to the fore the role of technology, not just to support but, above all, bringing new information to the student to think about. The mathematical symbolic power of digital technology allows the visual and tangible perception of the mathematical representations (numeric, algebraic and graphical) and this is a current way to bring closer mathematics to students. Technology allows the mediation of mathematical symbols and through technology it is possible now to *execute* tasks with those symbols. This kind of interaction with mathematics makes sense to the students and brings meaning to their learning.

3 Interactive dimensions

The Association for Computing Machinery (ACM) since 1992 characterized the Human Computer Interaction (HCI) as a discipline dealing with interactive computer systems and the phenomena surrounding them. HCI is concerned with the design, evaluation and implementation of those systems for human use; thus, it draws from supporting knowledge on both the machine and the human side. HCI is an interdisciplinary area; it has science, engineering and design aspects. It is emerging as a specialty including computer science, psychology, sociology, anthropology and industrial design [12].

The authors' commitment with mathematics education mediated through digital technology has led them to assess their knowledge within the HCI field. Thus, they have been working interactively with students having programmer and design skills in order to create an AR interactive application through which students could perceive solids of revolution in a better way. The AR application is a solution for mathematics learning related to the spatial visualization skill. It is offered to students by means of a tablet device with an intentional learning goal in mind. There, the features of AR technology are exploited not just to see the solid but also to interact with it considering different dynamic processes to build the solid.

Löwgren [13] provides a reference for the origin of *interaction design* as a new discipline. The author recalls the words of Moggridge about the opportunity to create imaginative and attractive solutions in a virtual world through the design of behaviors, animations and sounds. The notion of interaction design started to gain popularity beyond pure utility and efficiency, extending to also influence aesthetic qualities of use. Capturing the heritage of the original definition of interaction design, but looking to highlight potential edges of the discipline, Löwgren gives a simple formulation: Shaping digital things for people's use. Thus, referring to shaping interactive products and services with a specific focus on their use.

In this paper, the authors wanted to position their own educational research agenda for the learning of mathematics, one with the aim of providing students with didactical digital resources that rest on mathematics education and could bring them a digital version of mathematical processes. Thus, they have been shaping digital things for students to understand mathematics. The solids of revolution application described here is an example of the kind of digital products designed in order to bring the students an opportunity to develop their spatial visualization skill in an interactive way. In this solution the dimensions for interaction are manifold; from the design process of the AR application, to the students interaction through the GUI, and also by means of the interaction due to the way to hold and turn the tablet device to manipulate the AR scene. The scenes could promote the visualization skill because of the AR features. This is a product meant for the AR technology; the kind of interaction it offers is not possible without it.

4 Augmented reality in education

Augmented reality technology allows the combination of virtual and real environments; through AR the perception of a virtual object in a real place and time is possible. The production of a rich content built from computer-generated three-dimensional models is a promising tool in learning that could be reached through compact technologies like mobile devices, which support the delivering of AR experiences. This motivates the effort to apply AR in classrooms [14].

It is important to distinguish two forms of AR currently available for educators: *location-aware* and *vision-based*. The location-aware use of AR refers to showing digital media to learners as they move through a physical area. Meanwhile, the vision-based use of AR presents digital media to learners after they point the camera in their mobile device at an object. Dunleavy and Dede [15] explain the difference in both forms of use of AR describing the scenario of a life science student. Outside, walking in a garden, the student passes by an oak tree and there is a video in her smartphone that shows animal images that could be found near the tree; that is the location-aware kind of use of AR. But also, when pointing the camera at the base of the tree, it triggers a 3-dimensional model that illustrates de anatomical structure of the oak; that is a vision-based use of AR. The authors provide a literature review that focuses on AR for learning using mobile location-aware technologies, and state that there are few studies about vision-based form, which by the way could bring a greater degree of digital immersion.

Through AR technology it is possible to create immersive learning experiences. Dede [16] clarifies *immersion* as the subjective impression that one is participating in a comprehensive realistic experience. A digital immersive learning experience involves a willing suspension of disbelief based on sensory, actions and symbolic factors. It involves the empowering of the user to initiate actions that are not possible in the real world. This author belongs to a research team with great experience in virtual reality and they are currently studying augmented reality. Their immersive simulations use location-aware AR; the students move around a physical location and find digital objects and virtual people superimposed on real space. Through video, audio and text files, they receive navigation and collaboration cues and academic challenges. Their research included a control group of students playing a similar engaging board game. The high levels of students' engagement as well as educational goals were similar in both groups. This result made the author express that further design-based research is needed to know the scope of AR experiences for more powerful learning outcomes emerging from them. Research questions should give place to studies about the affordances that immersive technologies offer for learning and about the learning design that best fits with the technology.

According to Diaz et al. [17] there are two different ways to visualize AR; it could be a *static* way or a *dynamic* way. A static AR could include text and 3D models without variation; meanwhile, a dynamic AR includes animations or videos showing a continuous flow of images in motion. The authors produced an AR application for an Electronics course, and they found that students perceive learning easier when using dynamic AR contents comparing with static AR objects. Differences between static and dynamic content in AR applications for education should be taken into account.

Up here it has been pointed out two forms of use of AR (location-aware and vision-based) and two ways to visualize AR (static and dynamic). The authors in this paper assume a research agenda for the learning of mathematics that focus in placing together the advantages of a vision based and dynamic visualization of AR applications to promote immersive learning experiences. They take for granted that combining virtual and physical objects in the same space and time is the main affordance that AR technology offers. It brings the power to emphasize the strengths of real and virtual objects. AR affords a natural interaction because of the engagement with immersive virtual content. The user can move around and change perspective or move closer or farther to change scale. It results natural to point to objects and reach out to touch and move. Digital interactive visual representations of a dynamic object bring the opportunity of a new kind of educational products, especially for mathematics. The goal to pursue with the approach suggested above is to stand out the mathematical content, as well as the strategy that includes AR technology for the integration of a dynamic visualization and gesture into the mathematics learning process. In terms of Martín-Gutiérrez et al. [18], through AR technology it is possible to perform small virtual animations to see invisible abstract concepts in real world; definitely a main strength offered by this technology for the learning of mathematics.

Dobozy [19] argues about the urgent need for new developments in pedagogy including effective technology-enhanced and mediated learning design that must be student-centered and highly personalized. The author offers an alternative description for the concept *Learning Design* (LD) in order to support researchers. She stresses that making explicit the way LD is conceptualized is prior to engage in LD as a process. Learning Design is a conceptual construct useful for designers of learning sequences that make explicit epistemological and technological integration attempts. Meanwhile, LD conceived as a process illustrates the learning intent including subject-specific content that is practiced and opens for adaption, adoption and enhancement.

Based on this conceptualization of LD, the authors in this paper can make it clear the kind of effort applied to date in order to integrate AR technology for the learning of mathematics. There has been challenges emerged by the multidisciplinary tasks that could lead to the innovation of practices in math education. Technology changes the way students learn, and the fast transformation of how knowledge could be delivered because of the emergent technologies, it requires the conjunction of academic researchers, designers and software developers working together. The technological product that results from this multidisciplinary collaboration gives evidence of an epistemological and technological integration with a learning goal in mind.

Taking into account that mathematics deals with symbolic representations, and that technology allows the mediation of mathematical symbols, it results quite suitable to inquire about LD and AR affordances. According to Kaptelinin [20] the concept of *affordances* was proposed by James Gibson as part of his ecological approach to visual perception. Gibson's approach opposed strongly to the traditional cognitive psychology, which conceived *perception* as a process of developing representations. From a traditional point of view, sensory data have no meaning; nevertheless, data combined with information stored in memory is interpreted, and eventually become meaningful. The key idea underlying in Gibson's approach is the mutuality between the animal (including humans) and the environment; animal and environment are two parts of a whole system. The purpose of perception is to efficiently get information that has significance to acting in the environment. He argues that humans (as any animal) directly pick up meaningful information from the environment, without requiring developing internal representations of their environments. That meaningful information is about *affordances*, that is, action possibilities that the environment offers, like a chair offers sitting, but not to any animal.

The design community adopted the concept of affordances suggested by Norman [21] in order to make immediately obvious the possible uses of their products. Soon the concept become useful for interaction design even when it was a little imprecise, as the same Norman acknowledged. The concept came to play a central role in Human-Computer Interaction (HCI). Kaptelinin [20] gives an overview of some of the key conceptual explorations in HCI research making an attempt to clarify the meaning of affordances and relate this to specific agenda of HCI research and practice. Particularly he names contributions about extending the "possibility for action" to the application software and not just to the physical aspect of the system. There are also key advances by speaking of *cognitive affordance* in the sense that it helps users with their cognitive actions. This way the concept of affordance allows valuing the AR technology not just from the physical affordances it offers, but also because of the cognitive affordances that the Learning Design could provide.

5 An augmented reality learning design for solids of revolution

The design and development of the AR application was performed during 2014 in its first attempt. It required the work of a multidisciplinary team including academic researchers, designers and software developers. Periodic work sessions allowed the team to make decisions of the progress acquired about the design and software development process. The interface design was constructed from the initial sketches to the mock-ups that the developers used to scaffold the graphics user interface (GUI) in the cross-platform game engine named Unity. In parallel to this process, the 3D models and animations were created in 3D authoring tools Autodesk Maya and Rhino. They were imported to Unity to generate the packages required for Android and iOS to be available for both operative systems and increase the scope of devices that could be used in the classroom.

The theme of solids of revolution was chosen because of the difficulties that the students generally present in classroom with spatial visualization. The aim of the AR application is to produce an external visualization that functions

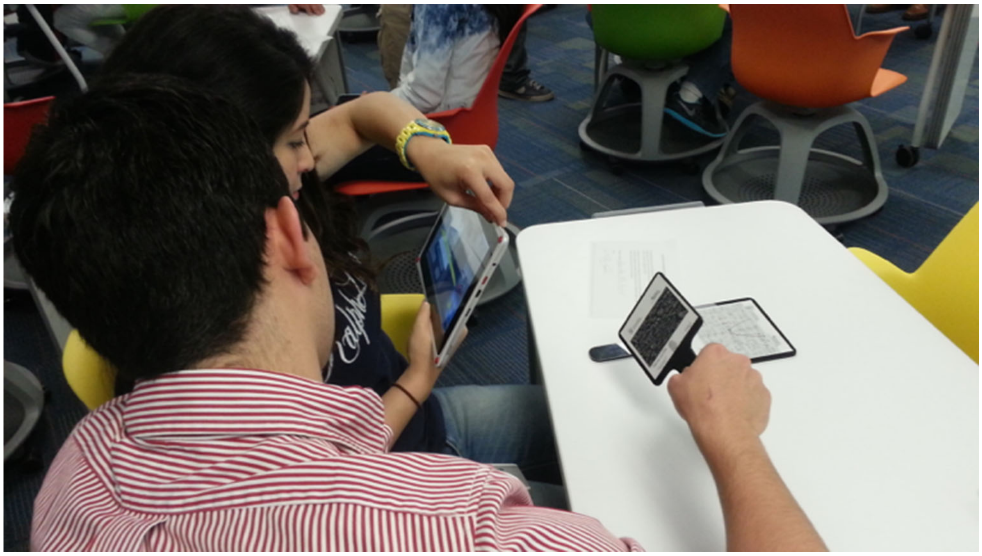

Fig. 1 Students interacting with the vision-based AR environment

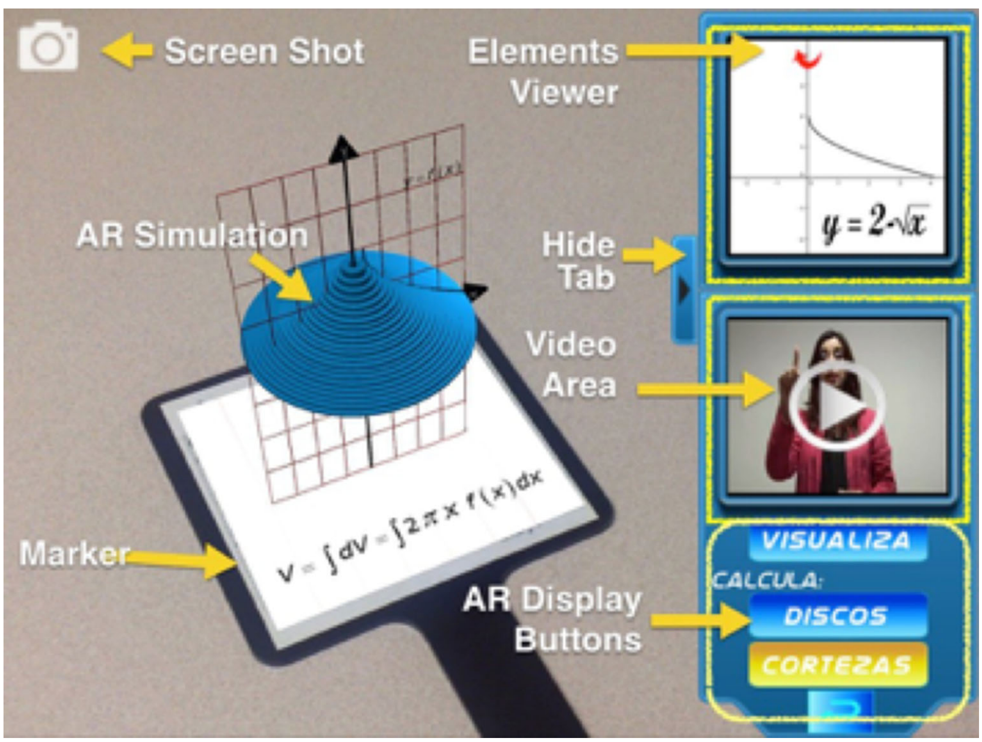

Fig. 2 AR environment showing elements of the AR learning design

as a cognitive affordance that supports the internal visualization of the process of construction of the solid. There is also the physical affordance of a small paddle with the mark (image) that displays the virtual object over the paddle when pointing there with the device camera. The paddle allows the student to move and rotate it and see from different perspectives the dynamic virtual simulation. Students handle a small paddle with the mark to display the augmented reality simulation. The AR application is already installed in the tablet and students work in pairs to have a better interaction with the product and promote collaboration. Figure 1 shows two students interacting in one of the sessions we have been performing as pilot studies testing the usability of the application. Students work together with the tablet and the paddle

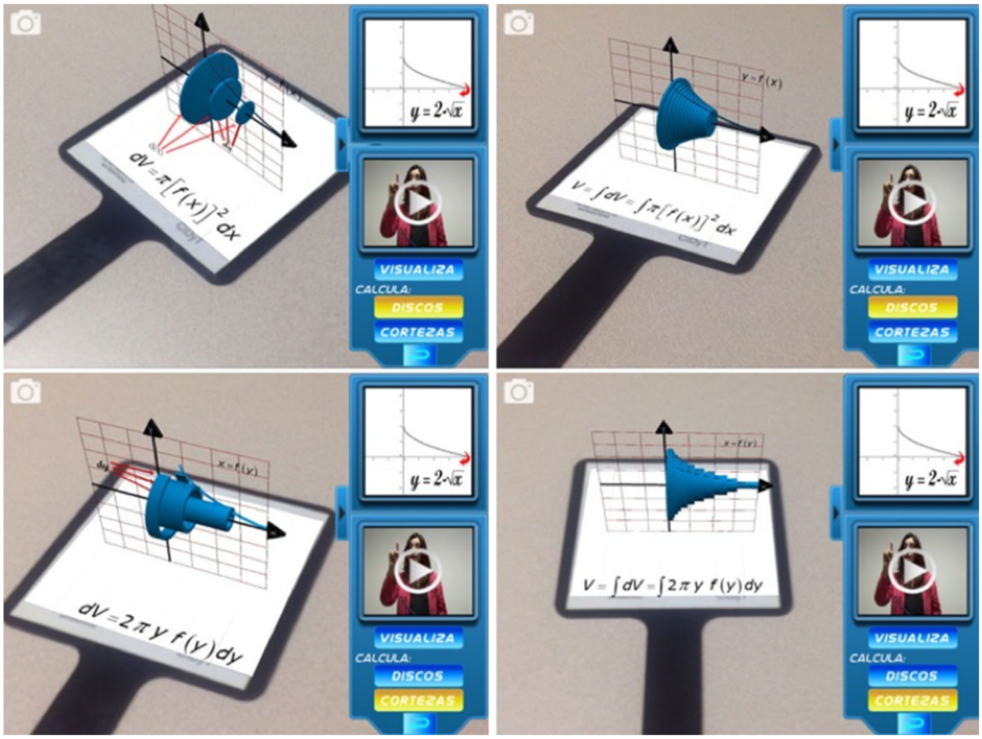

Fig. 3 Scenes from the dynamic visualization of the AR showing the different methods to calculate volume

showing how visual perception and gesture elements are main components for the interaction with AR environment making a tangible mathematical content.

The Learning Design includes the visual perception as a main access road to mathematics supports the use of a vision-based form of AR for solids of revolutions. Through the dynamic animations of processes performed to build the solid, the user is invited to perceive different ways to conceive the solid. In Fig. 2 there is an image showing the elements included in the AR environment. There appears the AR simulation embedded in the marker on the paddle that is seen through the tablet camera. Moving the paddle involves gesture elements to have different perspectives of the AR. There is also a zone with a small video explaining the dynamical visualization; this element hides by clicking the available tab to have a wider view of the AR. The zone also shows the graphical representation of a mathematical function that produces the solid, and the display buttons to produce the different dynamical visualizations. There are three kinds of dynamic visualizations included in the application. The first one simulates the production of the solid of revolution by the rotation of an area that is bounded in part by the graphic of the mathematical function. The second one simulates de accumulation of volume differentials following the known as *discs method* for calculating the solid volume. Meanwhile, the third one simulates the accumulation of volume differentials but now illustrating the *barks method*. The corresponding integral is the algebraic representation that stands over the marker to give meaning to these mathematical processes for calculating the volume of the solid of revolution.

The design takes into account the affordances of this technology to show the virtual animations. A key idea is to propose first the visualization of the solid in the most natural way possible, this means to produce the solid of revolution by the rotation of an area around the axis. Several animations take place in order to promote the visualization of the solid being constructed. Visualizing the rotation in real place and time helps to imagine the form of the solid. This first display is followed by two more corresponding to the two methods for calculating the volume of the solid of revolution by means of a definite integral. The visualizations of the two mathematical procedures are different from that first natural visualization of an area rotating. Therefore, it is included the animation of the accumulation of disks with its corresponding integral, and the accumulation of cylindrical shells with its corresponding integral. Figure 3 presents the same solid of revolution, which is visualized the way that each of the methods requires calculating its volume.

An extra element in learning design highlights the visual perception of the different solids of revolution that should be considered given the features of the graphical representation of the mathematical function delimiting the area to rotate. As shown in Fig. 4, it is possible to distinguish the area in terms of an *increasing* or *decreasing* function; and at the

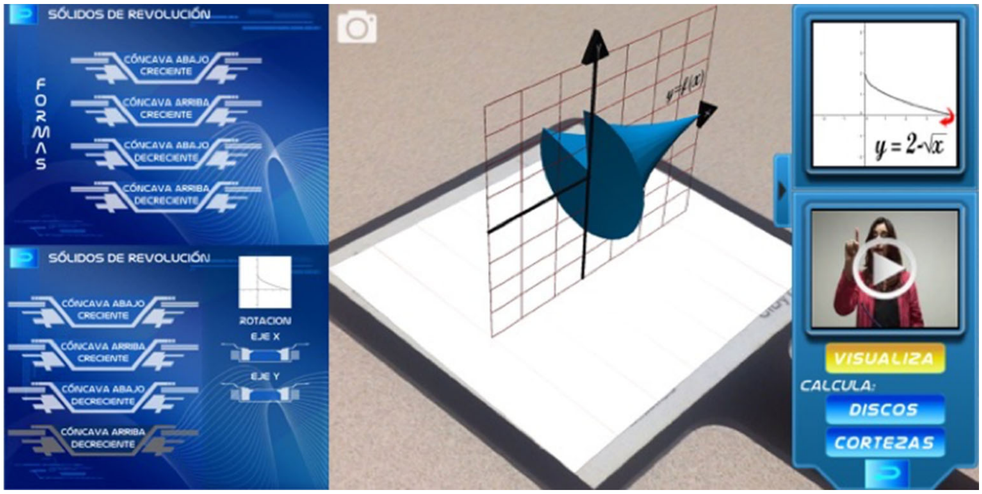

Fig. 4 Four different forms in the graphical representation generating eight solids of revolution

same time, distinguishing which is *concave upward* or *concave downward*. Combining both features there were chosen four mathematical functions to represent each case: increasing and concave upward, increasing and concave downward, decreasing and concave upward, and finally, decreasing and concave downward. Each of these cases generates two solids corresponding to the rotation around each of the axes (x or y); thus, there are eight solids of revolution in total. Considering the generation of each solid through both methods (discs and barks) it requires sixteen dynamic visualizations built for the AR application. There is also the natural generation of each solid through the rotation of a specific area. This gives a total of 24 dynamic visualizations composing this vision-based AR application.

6 Conclusion

In this paper the authors seek to share their approach to research in mathematics education integrating AR technology. It has been discussed that the aim is to transform the kind of experience that students could have with mathematical content. Far from repeating the same curriculum with technology, the goal is to identify the kind of content that could be resized through technology. By knowing the conventional curriculum of calculus and looking for a new curriculum assisted by technology, the spatial visualization skill is a main content that deserves to be developed in students bringing a special access to the graphical symbolic representation in mathematics.

Through AR this has been a great opportunity to make solids of revolution tangible to students; they can interact with them. The affordances of the technology allow the recreation of visual processes in 3D that are happening in real place and time. The involvement of perception while the dynamical visualization occurs is a key factor for this approach. Perception is possible through the interactive design embedded in the AR product; students can move the paddle and visualize, from different perspectives, the animation scene building the solid. During the pilot proofs of the application it generally feels this is an enjoyable situation for students. There is no doubt about this promoting motivational aspects that could be enough to foster and active the learning process. But besides that, the research agenda managed, it includes the progress of affordances theory in HCI particularly addressed to new challenges in learning design (LD). Affordances denote action possibilities provided from the environment, and the authors are searching for the AR environment that could promote cognitive actions, supported by a dynamical visualization of mathematical content. The solids of revolution AR application offers students to interact with it, and through it they could develop their spatial visualization skill. It is an interactive product seeking the visualization in space of solids emerging from figures contained in a plane.

Greenfield [22] argues how informal learning environments (like television or video games and Internet) affect learners' cognitive skills. Media technologies are producing a new profile of cognitive skills; some changes led to looses but others to gains. This author points out that formal education must adapt to those changes. Particularly it is quite interesting to know about the continuing global rise in IQ performance over more than 100 years. It has been found evidence of this change comparing records of British people in Raven Standard Progressive Matrices Test in 1942 and 1992. This is a nonverbal IQ test that provides a measure of visual intelligence. The items present a geometric pattern with a hole that should be filled with one of 6 figures. It is important to consider that the Raven Test displays in two dimensions (2D). It has been found that average performance increased during those 50 years for all age groups, and cognitive aging has also been reduced. Thus, in 1942 average performance was smaller, and with increasing age in groups, it decreased. Meanwhile, in 1992 average performance increased and it seems almost the same along the increasing age groups.

There are several reasons to understand those changes in visual intelligence, and for sure, advances in technology as part of the human culture should be taken into account with each of them. In the present paper it has been shown AR technology bringing a 3D virtual object embedded in real world, and this is a feature without precedent. It is quite sure that in the future the uses of this technology will increase offering positive changes for the development of the spatial visualization skill. The authors in this paper are convinced of the benefits that this represents for the learning of mathematics and are committed to take advantage of AR technology for making explicit epistemological and technological integration attempts right now, as the one shown here for understanding about the calculation of volume of solids of revolution.

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