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ORIGINAL PAPER

Optimal catalogue selection and custom design of belleville spring arrangements

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Abstract A Belleville spring is usually used in mechanical systems to obtain large loads for small distances of travel. Such a spring has a non-linear load-length curve. Moreover, Belleville springs are commonly stacked together in series or in parallel or both. Thus, defining a design that uses Belleville springs is not an easy matter. The existing assistance tool only checks whether a proposed design is acceptable or not. To improve design assistance, we show how optimization processes can be used in order to build a tool that interactively proposes an optimal design directly from the designer's requirements. The tool presents both an optimal design using a Belleville spring taken from a catalogue and an optimal custom design. An illustrative example is presented to highlight the benefits of using such an assistance tool.

Keywords Optimization · Design · Component selection Belleville spring · Conical disc spring

List of symbols

D_e	Outside diameter (mm)
D_i	Inside diameter (mm)
d	Density
E	Young's modulus (MPa)
F_1	External load at point 1 (N)
F_2	External load at point 2 (N)
i	Number of alternating discs in stacked column
l_0	Overall height of a single spring (mm)
L_0	External free length (mm)
L_1	External length at point 1 (mm)
L_2	External length at point 2 (mm)
M	Mass of the arrangement (g)
n	Number of discs arranged in parallel (nested)
s_1	Deflection at point 1 for a single disc (mm)
s_2	Deflection at point 2 for a single disc (mm)
$s_{\max}$	Maximum allowable deflection to be compared to $s_1; s_2$
S_1	External deflection at point 1 (mm)
S_2	External deflection at point 2 (mm)
S_h	External deflection (mm)
t	Spring thickness (mm)
V_e	Overall volume of the arrangement (mm^3)
α_s	Safety factor related to stress at point I
δ	Diameter ratio $(= D_e/D_i)$
μ	Poisson's ratio
σ	Maximum allowable stress (MPa)

1 Introduction

Designers commonly exploit elastic components to store energy in mechanical systems. When the allowable travel is reduced, there are several kinds of springs that can replace the usual cylindrical helical spring, as studied by Parise [12]. In such cases, Belleville springs (or conical disc springs) are often used. Details on designing Belleville springs can be found in Wahl's book [22], which is considered as the reference for spring design. More recently, Curti [6] has studied the influence of friction in the calculation of Belleville

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springs, and Atxaga [3] has analyzed the failure of a set of Belleville springs.

To help designers, a first level of assistance has been proposed by Carfagni [5] who has developed CAD software for the automated checkout and design of Belleville springs. Recently, optimization methods have been applied to define assistance tools for designers. An example is the 'Advanced Spring Design 7' software from the Spring Manufacturers Institute [19] and Universal Technical Systems [20], which was designed in a full graphic environment with convenient automatic unit conversion and easy access to dynamic plots and reports. ASD utilises the TK Solver collaborative math engine with a process known as backsolving to solve a variety of combinations of input and output variables. Consequently, it is useful for design verification.

Many strategies can be exploited to develop decision support systems in preliminary design [18]. One of them has been proposed to be applied to spring design [13]. Thus, synthesis tools have been proposed for cylindrical compression [14] and traction [16] springs, or conical springs [17] to provide enhanced assistance. Such tools make use of optimization processes in order to propose a spring that directly satisfies the designer's needs. The research work performed on optimal compression spring design is now included in the 'Spring CAD software packages' distributed by the Institute of Spring Technology [8]. Such capabilities would be useful for the custom design of Belleville springs.

To reduce costs, instead of defining a custom design, designers often prefer to exploit an arrangement of Belleville springs selected from a catalogue. Many spring manufacturers have on-line pdf catalogues [2,10] but do not propose selection procedures. The component selection process has been studied by Bradley [4]. According to this work, three different sorts of component may be distinguished: the simple-to-select component, the routinely selected component and the difficult-to-select component. Belleville springs belong to the third category. They can be used in both series and parallel arrangements. Thus, it is not easy to define the adequate arrangement and use of a given Belleville spring for a given set of specifications. Paredes [15] has proposed a method for optimal selection of cylindrical compression springs, which could be extended to the selection of Belleville springs.

There is a lack of assistance tools, both for optimal catalogue selection and custom design of Belleville spring arrangements and the work presented here is intended to help satisfy the need by combining industrial and mathematical knowledge.

We first recall the notions to be considered in a design that includes Belleville springs. Section 3 describes the proposed assistance tool and gives details of the various resolution processes. Finally, Sect. 4 presents an illustrative example.

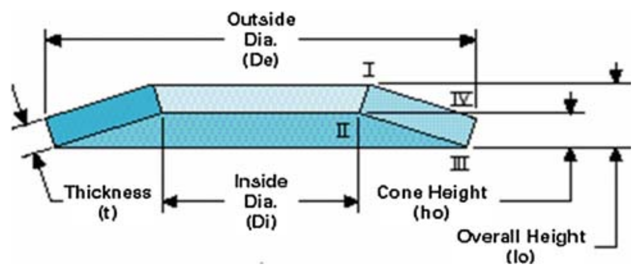


Fig. 1 Geometry of a Belleville spring

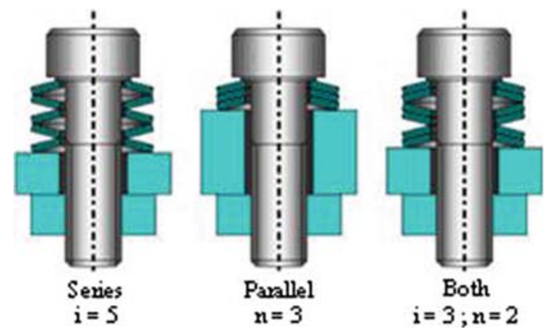


Fig. 2 Examples of arrangements

2 Notions to be considered for Belleville springs

2.1 Notions for a single Belleville spring

The geometry of a Belleville spring is shown in Fig. 1.

The parameters that commonly define the spring geometry are D_e, D_i, t and l_0.

2.2 Arrangements of Belleville springs

Belleville springs can be arranged in series, in parallel or both as presented in Fig. 2. An arrangement is defined by the number of discs in series (i) and the number of discs in parallel (n).

The Load-Length curve of the arrangement is non-linear, as can be seen in Fig. 3. The load, F, for a given external deflection, S, can be calculated as follows:

$$F = \frac{4nESt}{i\alpha(1-\mu^2)D_e^2}\left[\left(h_0 - \frac{S}{i}\right)\left(h_0 - \frac{S}{2i}\right) + t^2\right] \quad (1)$$

$$\text{with } \alpha = \frac{1}{\pi}\frac{\left(\frac{\delta-1}{\delta}\right)^2}{\frac{\delta+1}{\delta-1} - \frac{2}{\ln\delta}}$$

$$\delta = \frac{D_e}{D_i}$$

The operating range of an arrangement is defined by the least compressed state (state 1) and the most compressed state (state 2) that are achieved.

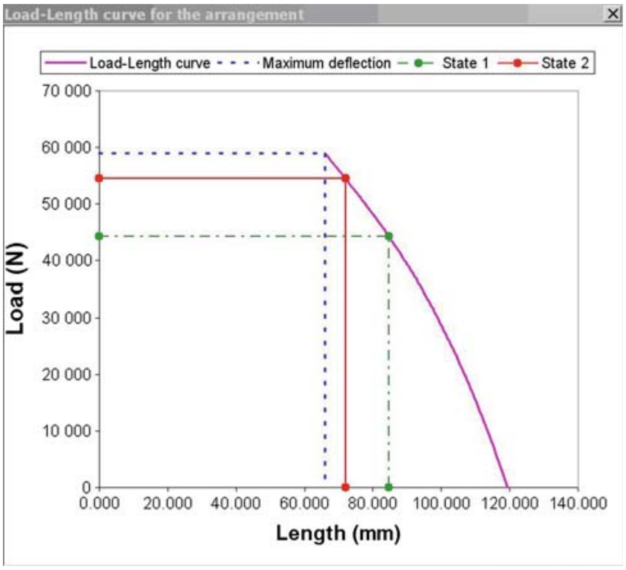


Fig. 3 Load-Length curve of the arrangement

The use of an arrangement can be defined by the values to be taken, by choosing among loads (F_1; F_2), lengths (L_1; L_2) or deflections ($S_h = L_1 - L_2$; $S_1 = L_0 - L_1$; $S_2 = L_0 - L_2$).

2.3 Other notions

Other notions may be considered by designers. The overall volume provides an appreciation of the space occupied:

$$V_e = L_0 D_e^2 \frac{\pi}{4} \quad \text{with} \quad L_0 = i(h_0 + nt) \tag{2}$$

The mass of the arrangement can also be of great interest:

$$M = dt\left(D_e^2 - D_i^2\right)\frac{\pi}{4000} \tag{3}$$

The safety factor related to stress is calculated by comparing the maximum allowable stress σ with the maximum stress obtained at point I (see Fig 1). σ depends on the material; it is about 1,400 MPa for common steel and can reach 3,000 MPa for high resistance steel.

$$\alpha_s = \frac{i\sigma\alpha\left(1-\mu^2\right)D_e^2}{4ES\left[\beta\left(h_0 - \frac{S}{2i}\right) + \gamma\, t\right]} \tag{4}$$

$$\text{with} \quad \beta = \frac{6}{\pi \ln \delta}\left(\frac{\delta - 1}{\ln \delta} - 1\right)$$

$$\gamma = \frac{3(\delta - 1)}{\pi \ln \delta}$$

3 Proposed assistance tool

The proposed tool was implemented using the Excel software. This proved to be an efficient way to build mock tools that could be easily distributed and tested in industry [1].

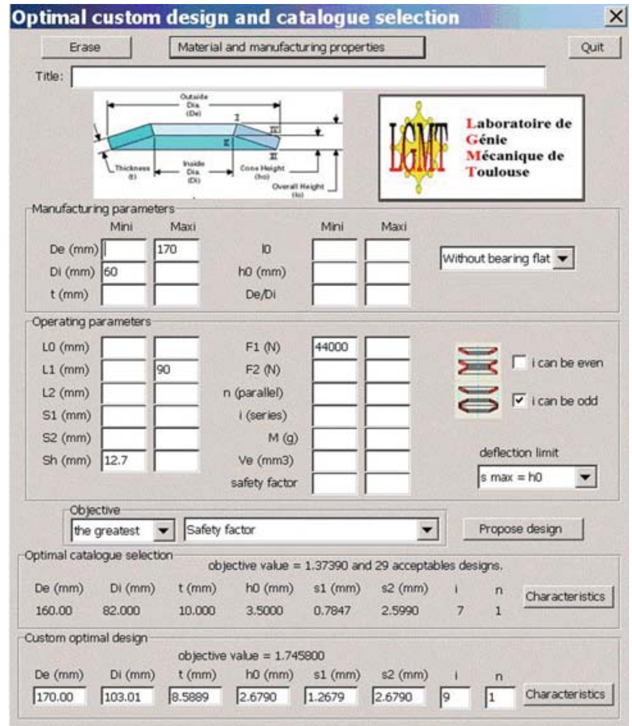


Fig. 4 Main window interface

3.1 User-friendly interface window

The main interface window, shown in Fig. 4, is composed of five areas.

The first one allows the manufacturing parameters to be defined to characterize the geometry of a single spring. The second defines the characteristics of the arrangement and all the other notions considered.

All the parameters taken into account are defined using interval values. This has proved to be an efficient interface [14,15] for a tool as it makes it usable at any design stage. In the early design stages, there are always a large number of parameters that are yet to be set. It is thus difficult to provide fixed values for a problem and it is more convenient to define parameters through their possible lower and/or upper limits.

In addition to the parameters mentioned above, the designer can define whether the spring has bearing flats or not, if parameter i has to be even or odd, and the rule for the calculation of the maximum allowable deflection $s_{\max}$.

Finally, for the best arrangements to be proposed, an objective function has to be selected in the third area. The objective can be any of the previously considered parameters that can be minimized or maximized.

The "propose design" button launches the calculation process and the results are presented in the fourth and fifth areas. One area gives the optimal arrangement that can be made from stock springs (from the catalogue) and the other shows the optimal custom design.

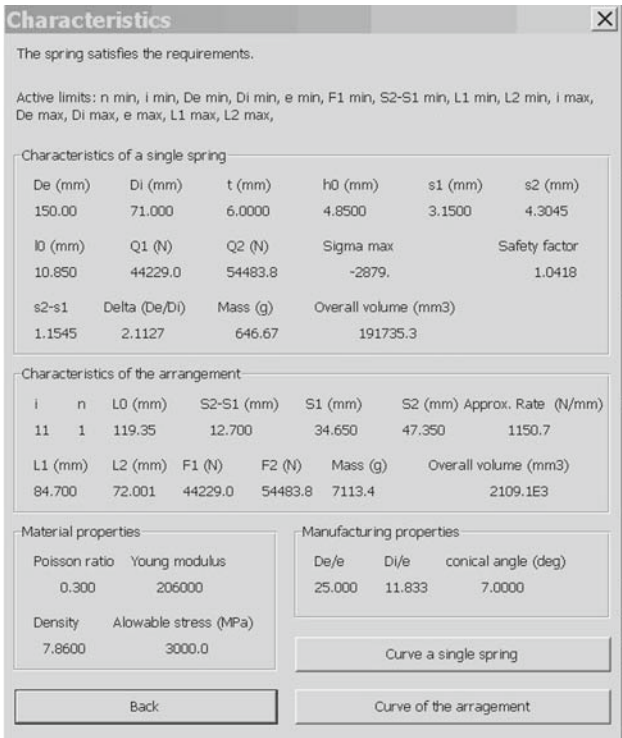


Fig. 5 Detailed characteristics

Detailed characteristics of each result can be obtained by using the "characteristics" button as shown in Fig. 5.

The Load-Length curves of either a single spring or of the arrangement can be displayed as shown in Fig. 3.

An additional window also enables the material and manufacturing properties to be defined. It is presented in Fig. 6. The manufacturing limits are only exploited when defining an optimal custom design as Belleville springs from the catalogue are, of course, expected to be ready for use.

3.2 Catalogue selection process

Finding the best arrangement of Belleville springs for a given specification is not an easy matter. On the one hand, if the specifications are imprecise, there is a wide range of available designs and it is difficult to choose the best one. On the other hand, when the specifications are very precise, it becomes difficult to find an acceptable design.

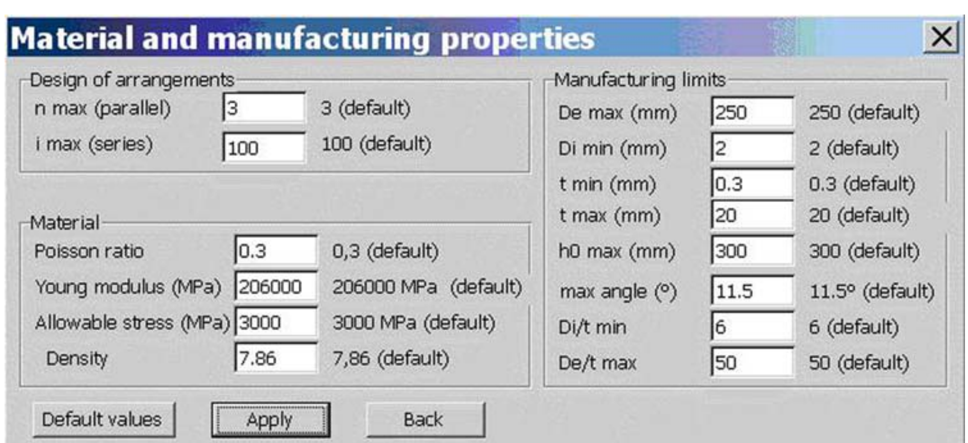
The proposed method is an extension of the one presented by Paredes [15] for cylindrical compression springs.

The idea is to test each potential arrangement of each Belleville spring of the catalogue. To do this, all the potential combinations of parameters i and n are tested for each spring.

The next step consists of finding the optimal way to use the current arrangement in relation to the specifications. Finally, the designs considered (arrangement and associated use) are compared and the best one is proposed as the result. If no design satisfies the requirements, then the closest to the specifications is proposed as the best result. The associated algorithm is presented in Fig. 7.

The process for calculating the optimal use of an arrangement will now be given in detail. We decided to define the use of an arrangement by the two overall lengths L_1 and L_2. Finding the best L_1 and L_2 values consists in solving an optimization problem where the objective function is defined by the user in the main interface. When the objective does not depend on the working parameters (i.e. the mass or the external diameter), maximizing the safety factor is taken as an alternate objective. As the optimization problem only involves managing two variables, the graphical resolution process proposed by Johnson [9] can be used. Interval arithmetic [11] is first exploited to refine the allowable bounds on L_1 and L_2 by merging the constraints related to S_1, L_1, F_1, α_S and S_2, L_2, F_2, α_S, respectively. Finding the length associated with a given load F requires solving Eq. 1,

Fig. 6 Material and manufacturing properties



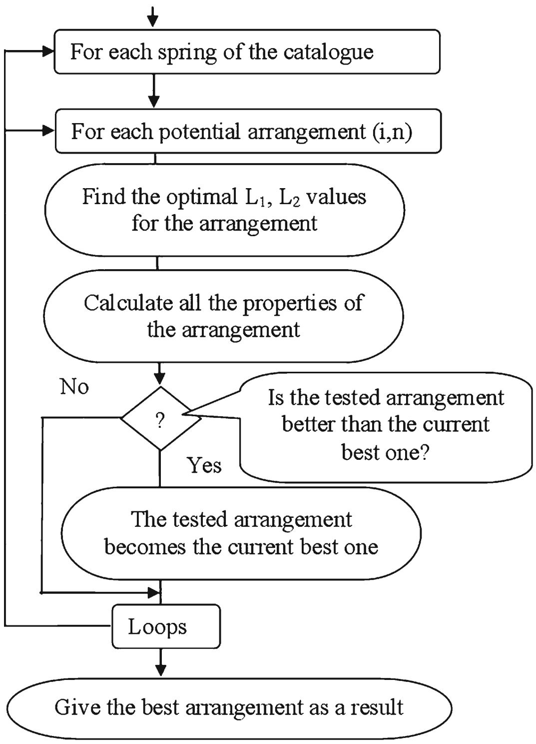


Fig. 7 Selection algorithm

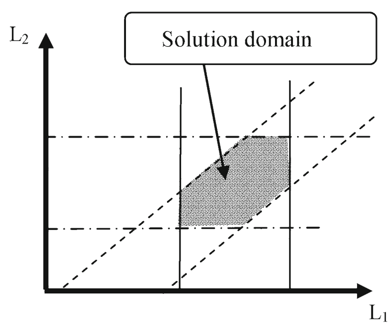


Fig. 8 Solution space for L_1 and L_2

which is a third order polynomial. It is solved using Cardan's method. The length associated with a given safety factor is obtained by solving the second order polynomial Eq. 4. The resulting solution space can be represented as shown in Fig. 8. It is defined by the refined bounds on L_1 and L_2 and by the initial bounds on the travel S_h.

Each point inside the solution domain represents L_1, L_2 values that lead to a use that satisfies the requirements. Depending on the objective function, an algorithm that tests all the boundaries of the solution domain enables the best (L_1, L_2) values to be chosen. If the domain does not exist, i.e. there is no acceptable use for the tested arrangement, then an alternative method is used in order to give (L_1, L_2) values that are close to the requirements.

Table 1 Constraints for the custom design problem

Parameter	Upper limit	Lower limit
D_e	✓	✓
D_i	✓	✓
t	✓	✓
h_0	✓	✓
δ	✓	✓
n	✓	✓
i	✓	✓
L_0	✓	✓
L_1	✓	✓
L_2	✓	✓
S_1	✓	✓
S_2	✓	✓
S_h	✓	✓
F_1	✓	✓
F_2	✓	✓
M	✓	✓
V_e	✓	✓
α_s	✓	✓
$D_e t$	✓	
$D_i t$		✓
Conical angle	✓	

3.3 Optimal custom design process

When the number of Belleville springs to be manufactured is large, it may be advantageous to draw up a custom design. Finding the optimal custom design related to the requirements can be defined as an optimization problem.

The optimization function is selected by the designer in the main interface window.

The chosen variables are $[D_e, D_i, \ t, \ h_0, n, i, s_1, \ s_2]$. They enable the designer to define the spring geometry, the arrangement and the use of the arrangement. $D_e, D_i, t, h_0, \ s_1$ and s_2 are 6 continuous variables. n and i are 2 discrete variables.

A large set of constraints is taken into account to satisfy not only the design requirements but also the manufacturing constraints. Details of the constraints are shown in Table 1. As the problem has to be treated automatically, default values (0 for a lower limit and 10^6 for an upper limit) are set by the program if the associated cells are left empty in the interface window.

We thus obtain a mixed discrete–continuous optimization problem with 6 variables and 39 constraints which has to be solved automatically.

The goal was to find a fast, reliable and complete method for solving our problem automatically for any requirements.

As both continuous and discrete variables had to be managed, we decided to implement a direct method [21] associated with a branch and bound process [7].

The Excel solver proposes a generalized reduced gradient solution process with two optional methods, a conjugate gradient method or a quasi-Newton approach (BFGS). Considering the small number of variables, the BFGS method was chosen.

Direct methods are based on a displacement on the solution space. This kind of property is useful here as the tool is more likely to provide the designer with an adequate though non-optimal solution if the resolution process is interrupted before completion (where it proves difficult to obtain full convergence).

Direct methods require a starting point inside or close to the solution area. The closer the starting point is to the final solution, the more likely the algorithm is to converge towards the optimal solution. This is especially true here, considering the large number of constraints.

For this reason, we decided to take the proposed optimal catalogue design (see Sect. 3.2) as a starting point for the custom design optimization process.

4 Illustrative example

4.1 Presenting the design problem

The design problem under consideration here deals with finding a Belleville spring arrangement for a hydraulic braking system of a ropeway. The mechanism is expected not only to keep the ropeway stationary when it is parked but also to stop the ropeway in case of an emergency.

The principle is shown in detail in Fig. 9. The mechanism comprises two arms (1 and 1') that are linked to two clogs (2 and 2') that can press the master wheel (6). The Belleville spring arrangement (5) gives the load acting at the opposite sides of the two arms. To leave the master wheel free to rotate, hydraulic pressure is used in the acting cylinder (3 and 4) to compress the Belleville spring arrangement. In case of a lack of hydraulic pressure, the Belleville spring arrangement is released and the load is transmitted to stop the master wheel. Figure 10 presents an example of such a braking system.

In the case under study, the Belleville arrangement has to provide a minimum load of 44,000 N even when the brake lining is worn. The brake lining is considered as fully worn when the travel of the Belleville spring arrangement is increased by 7.7 mm. Moreover, a minimum travel of 5 mm is required between the loaded and unloaded positions when the brake lining is in mint condition. Thus the final minimum travel required is 12.7 mm.

The allowable space for the arrangement induces $D_e \leq 170$ and $D_i \geq 60$ mm. The maximum available axial length is 90 mm.

4.2 Initial design

Considering the problem, a designer tries to find a solution without computer assistance. The catalogue he uses contains 270 springs classified by increasing values of D_e from 7.75 to 250 mm.

The designer thus first selects a Belleville spring that fits the requirements related to geometrical properties which are easily defined in the catalogue (D_e and D_i). Then, by hand, he has to find an arrangement and a use that enable the other requirements to be fulfilled. If the selected arrangement does not lead to a satisfactory design, another arrangement and/or spring is tested until an acceptable design is found. This tedious trial and error process leads to the design presented on Fig. 11. The arrangement obtained is composed of 11 Belleville springs stacked in series. The characteristics of the selected Belleville springs are listed in Table 2.

The proposed arrangement has to be exploited with $L_1 = 84.7$ and $L_2 = 72$ mm. This leads to $F_1 = 44229$ N and $\alpha_s = 1.04$ (Fig. 5 presents the full characteristics of the design and Fig. 3 the associated load-length curve). The proposed design

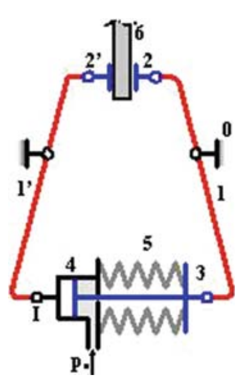


Fig. 9 Principle of the hydraulic braking system

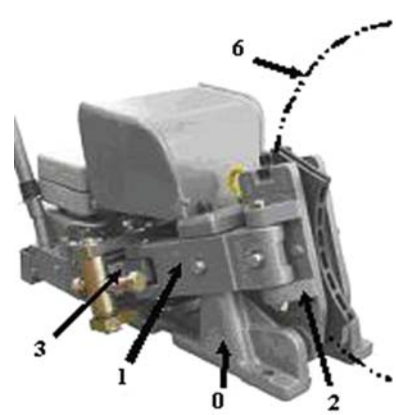


Fig. 10 Example of a hydraulic braking system

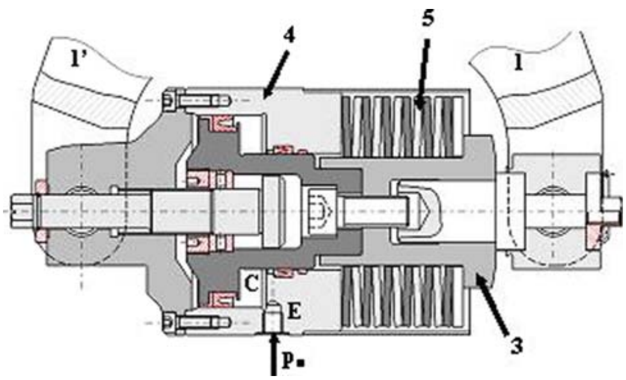


Fig. 11 Initial design

Table 2 Characteristics of the chosen Belleville spring

D_e(mm)	D_i(mm)	T (mm)	h_0(mm)
150	71	6	4.85

can thus be considered as satisfactory although offering a very small safety factor.

4.3 First run of the proposed assistance tool

The proposed tool is then run to see if a better design can be found. The goal is thus to propose a design offering the greatest safety factor. The requirements and the associated results are presented in the main window (see Fig. 4).

The tool demonstrates that only 29 arrangements can fit the requirements. It is thus easy to understand the difficulty of finding an acceptable design by hand.

The characteristics of the optimal catalogue design are shown in Fig. 12. It can be seen that the proposed design fits the requirements and that the lower limits on F_1 and S_h are active. We thus have an optimal design that does not reach all the limits given in the specification sheet. This kind of result is often obtained with such a tool and clearly shows that, when defining the specifications, the designer does not have to worry about whether a given limit is expected to be active or not but only needs to enter the data as it comes and let the tool manage all the constraints simultaneously.

The tool has thus selected another spring from within the catalogue and defined an arrangement and a use that enables the requirements to be satisfied while having a safety factor of 1.37, which is significantly better than the design obtained by hand.

The tool also proposes a custom design. Its properties are shown in Fig. 13. The safety factor has been improved to reach $\alpha_s = 1.74$ and there are more active constraints. This shows that the optimization process has benefited from defining a custom design.

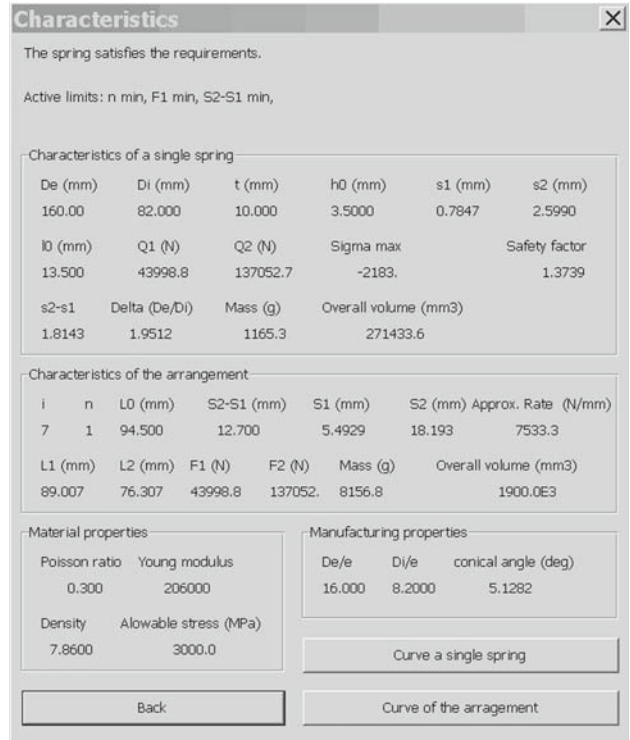


Fig. 12 First optimal catalogue design

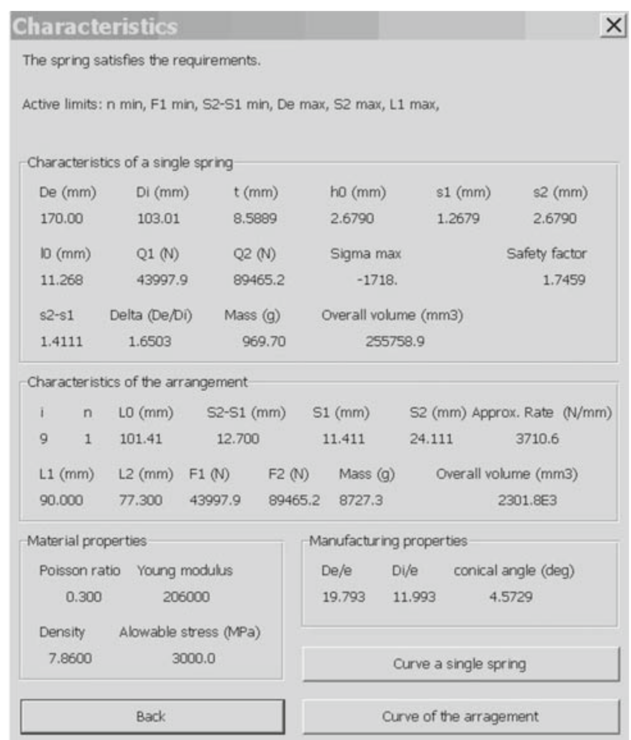


Fig. 13 First optimal custom design

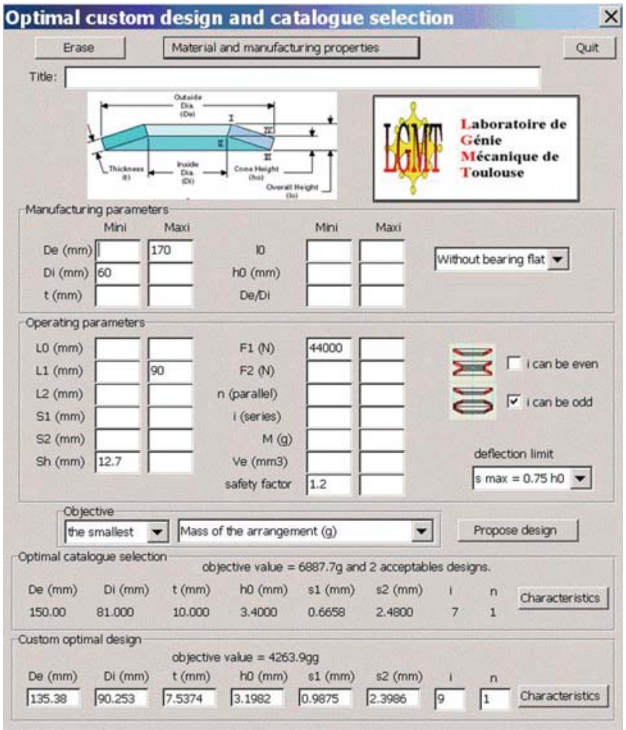


Fig. 14 Main interface window for second run

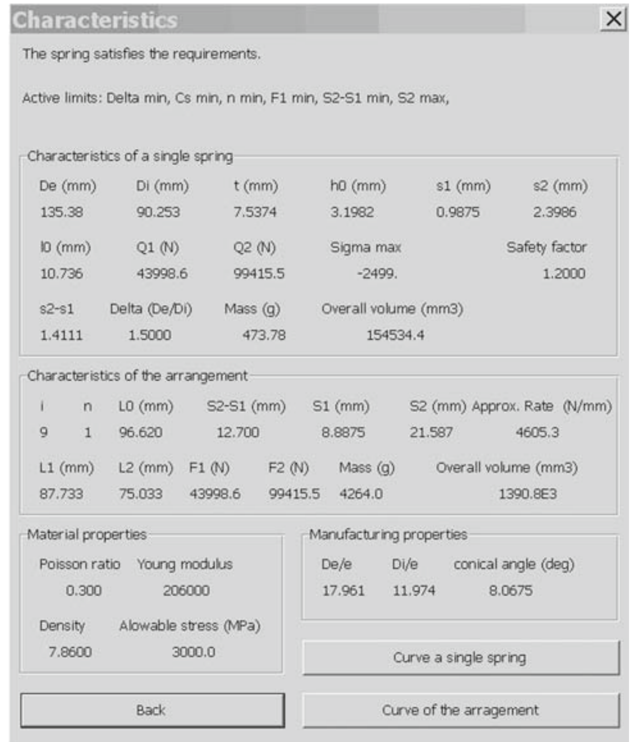


Fig. 15 Second optimal custom design

4.4 Second run of the proposed assistance tool

It can be seen that the proposed designs lead to arrangements with significant mass:

- 7.1 kg for the manual design
- 8.1 kg for the optimal catalogue design
- 8.9 kg for the optimal custom design

A second run is then performed to see if a design with a reduced mass can be achieved. The requirements are improved in order to benefit from the first run by requiring a minimum safety factor value of 1.2 for stress at point I and by exploiting the conventional limit for deflections of $s_{\max} = 0.75 h_0$.

The goal is now to find the arrangement with the lowest mass.

The requirements and the associated results are presented on Fig. 14.

It can be seen that the new requirements have reduced the solution space as there are only two catalogue designs available. The lightest has a mass of 6.9 kg and $\alpha_s = 1.26$.

We can thus consider that, in that case, compared to the manual design, the optimal catalogue selection process leads to a major gain on the safety factor.

The custom design leads to a significant improvement of the objective function by proposing an arrangement with a mass of only 4.2 kg. The full characteristics are presented in Fig. 15. The active limits presented show that some constraints related to the designer's requirements are reached: α_S; F_1; S_h; and $s_{\max}$, and that the manufacturing constraint related to δ is implemented. This clearly shows the benefit of automatically managing both manufacturing and design requirements.

5 Conclusion

An assistance tool dedicated to Belleville springs has been presented. It is able to interactively propose Belleville spring arrangements directly from the designer's requirements.

The requirements are entered using a specification sheet where data are defined by interval values. This enables the tool to be used at any step of the design process.

As a result, the tool proposes an arrangement that includes a spring from within a catalogue. An optimal custom design is also proposed which automatically takes the manufacturing capabilities into account. The designer can thus easily compare the two proposed solutions and see if he can draw advantage from a custom design particularly when mass production is considered.

Moreover, any parameter that is considered in the specification sheet can be used as the objective function that is to be minimized or maximized. This enables the designer to test several design options easily while exploring the design space.

This work could be enhanced by including fatigue life validation. Another improvement could be to take the manufacturing tolerances into account so as to be able to minimize them.

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