



# An Efficient Certificateless Aggregate Signature Scheme

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**Abstract:** Aggregate signature can aggregate  $n$  signatures on  $n$  messages from  $n$  signers into a single signature that convinces any verifier that  $n$  signers sign the  $n$  messages, respectively. In this paper, by combining certificateless public key cryptography and aggregate signatures, we propose an efficient certificateless aggregate signature scheme and prove its security. The new scheme is proved secure against the two representative types adversaries in certificateless aggregate signature under the assumption that computational Diffie-Hellman problem is hard. Furthermore, from the comparison of the computation cost of the new scheme with some existing certificateless aggregate signature schemes in group sum computation, scalar multiplication computation, Hash computation and abilinear pairings computation, it concludes that the new scheme reduces the computation cost in scalar multiplication computation in half and maintains the same in the other computation costs.

**Key words:** digital signature; aggregate signature; certificateless aggregate signature; security; bilinear maps

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## 0 Introduction

To solve the key escrow problem in identity-based public key cryptography, a certificateless public key cryptography was proposed<sup>[1]</sup>. In certificateless public key cipher, the private key of one user consists of a partial private key generated by the key generation center (KGC) and the secret value selected by the user himself.

Aggregate signature<sup>[2]</sup> proposed by Boneh and colleagues can combine  $n$  signatures from  $n$  users on  $n$  messages into a signature. It is useful in secure routing and certificate chain compression. After the first aggregate signature scheme<sup>[2]</sup>, many aggregate signature schemes were proposed<sup>[3-9]</sup>. Several certificateless aggregate signature (CLAS) schemes are proposed<sup>[10-19]</sup>. In these schemes, Xiong *et al*<sup>[11]</sup> proposed a certificateless aggregate signature with constant pairing computations. However, Refs. [14, 15, 20] showed that Xiong *et al*'s scheme is subjected to several attacks. Also some improved schemes were proposed in Refs.[14, 15, 20]. Kang *et al*<sup>[19]</sup> also proposed a certificateless aggregate signature scheme. But the scheme needed high computations cost. In Ref.[21], the author also investigated the security of aggregate signatures.

In this paper, we propose an efficient certificateless aggregate signature scheme, and prove that the proposed scheme is existentially unforgeable under adaptive chosen-message attacks under the assumption that the computational Diffie-Hellman problem is hard. The new scheme combines aggregate signature with certificateless public key cryptography and improves the signature generating algorithm. Compared with some existing certificateless aggregate signature schemes in computation

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cost, the scalar multiplication computation cost of new scheme is reduced in half.

The rest of the paper is organized as follows. Section 1 introduces cryptographic hardness assumptions and the definition and security model of certificateless aggregate signature. In Section 2, a new certificateless aggregate signature scheme is proposed. Section 3 discusses the security of the proposed scheme. A comparison of computation cost is shown in Section 4. Finally, Section 5 concludes this paper.

## 1 Preliminaries

### 1.1 Bilinear Maps and Complexity Assumption

By Ref. [19], the bilinear map and related complexity assumption are depicted in following.

Let  $G_1$  and  $G_2$  be additive group and multiplicative group of the same prime  $q$  order, respectively. A map  $e: G_1 \times G_1 \rightarrow G_2$  is called a bilinear map if it satisfies the following properties:

- 1) Bilinear:  $e(aP, bQ) = e(P, Q)^{ab}$  for all  $P, Q \in G_1, a, b \in Z_q^*$ .
- 2) Non-degeneracy: There exists  $P, Q \in G_1$  such that  $e(P, Q) \neq 1$ .
- 3) Computable: There exists an efficient algorithm to compute  $e(P, Q)$  for any  $P, Q \in G_1$ .

**Computational Diffie-Hellman (CDH) Problem:** Given a generator  $P$  of an additive cyclic group  $G$  with order  $q$ , and given  $(ap, bp)$  for unknown,  $a, b \in Z_q^*$  to compute  $abP$ .

### 1.2 Definition and Security Model of Certificateless Aggregate Signature Scheme

The definition and security model of CLAS are identical to that of Refs.[1, 13, 19].

A certificateless aggregate signature scheme includes a KGC, an aggregating set  $U$  of  $n$  users  $U_1, \dots, U_n$  and an aggregate signature generator. There are six algorithms: Setup, Partial-Private-Key-Extract, UserKeyGen, Sign, Aggregate and Aggregate Verify.

There are two types adversaries  $A_1$  and  $A_2$  for CLAS. Type 1 adversary  $A_1$  does not have access to the master key, but it can replace the public key of any user. While type 2 adversary  $A_2$  has access to the master-key but cannot perform public key replacement. The security of CLAS scheme is modeled by two games<sup>[19]</sup> between a challenger  $N$  and an adversary  $A_1$  or  $A_2$ .

For more description, readers can refer to Refs. [1, 13, 19].

## 2 An Efficient Certificateless Aggregate Signature Scheme

In this section we proposed an efficient CLAS scheme. The new scheme consists of six algorithms:

**Setup:** Given a security parameter  $\tau$ , the KGC selects an additive cyclic group  $G_1$  and a multiplicative cyclic group  $G_2$  with the same order  $q$ , and chooses a bilinear map  $e: G_1 \times G_1 \rightarrow G_2$ . Then, KGC selects a generator of  $G_1$ ,  $P$  and random  $s \in Z_q^*$  as the master key and sets system public key  $P_{pub} = sP$ . KGC also picks four cryptographic Hash functions.

$$H_1, H_3, H_4: \{0, 1\}^* \rightarrow G_1, H_2: \{0, 1\}^* \times \{0, 1\}^* \times G_1 \times G_1 \rightarrow Z_q^*$$

The system parameter list is  $params = (G_1, G_2, e, P, P_{pub}, H_1, H_2, H_3, H_4)$ .

**Partial-Private-Key-Extract:** KGC generates the partial private key  $D_i$  for the user with identity  $ID_i$  as follows:

- 1) Calculate  $Q_i = H_1(ID_i)$ .
- 2) Output  $D_i = sQ_i$ .

**UserKeyGen:** This algorithm selects a random  $x_i \in Z_q^*$  as one user's secret value, and generates the user's public key as  $P_i = x_iP$ .

**Sign:** Given a state information  $w$ , one user  $U_i$  with identity  $ID_i$  and public key  $P_i$  signs a message  $m_i$  as follows:

- 1) Select a random number  $r_i \in Z_q^*$ , calculates  $R_i = r_iP$ .
- 2) Calculate  $h_i = H_2(m_i, ID_i, P_i, R_i)$ ,  $Z = H_3(w)$ ,  $F = H_4(w)$ .
- 3) Calculate  $T_i = h_iD_i + x_iZ + r_iF$ .
- 4) Output signature  $\sigma_i = (R_i, T_i)$ .

**Aggregate:** For  $n$  message-signature pairs  $((m_1, \sigma_1 = (R_1, T_1)), \dots, (m_n, \sigma_n = (R_n, T_n)))$  from  $n$  users  $U_1, \dots, U_n$  (they has the same state information), respectively, any aggregate signature generator can compute  $T = \sum_{i=1}^n T_i$  and output the aggregate signature  $\sigma = (R_1, \dots, R_n, T)$ .

**Aggregate Verify:** To verify an aggregate signature  $\sigma = (R_1, \dots, R_n, T)$  on messages  $m_1, \dots, m_n$  from  $n$  user  $U_1, \dots, U_n$  with identities  $ID_1, \dots, ID_n$ , corresponding public keys  $P_1, \dots, P_n$ , and same state information  $w$ , the verifier does the following steps:

- 1) Calculate  $Q_i = H_1(ID_i)$ ,  $h_i = H_2(m_i, ID_i, P_i, R_i)$ , for all  $i, 1 \leq i \leq n$ , and  $Z = H_3(w)$ ,  $F = H_4(w)$ .

$$2) \text{ Verify } e(T, P) \stackrel{?}{=} e(P_{\text{pub}}, \sum_{i=1}^n h_i Q_i) e(Z, \sum_{i=1}^n P_i) \\ e(F, \sum_{i=1}^n R_i).$$

3) If the equation holds, output true. Otherwise, output false.

### 3 Security Proof

**Theorem 1** The proposed certificateless aggregate scheme is existentially unforgeable against type 1 adversary under the assumption that the CDH problem is intractable.

**Proof** To prove the proposed scheme is existentially unforgeable against type 1 adversary, we show how a CDH attacker  $N_1$  uses type 1 adversary  $A_1$  to compute  $abP$  from  $(P, aP, bP)$ .

Setup:  $N_1$  sets system public  $P_{\text{pub}} = aP$  and params  $= (G_1, G_2, e, P, P_{\text{pub}}, H_1, H_2, H_3, H_4)$  and sends params to  $A_1$ .

$A_1$  executes the following types of queries in an adaptive manner.

$H_1$  queries: There is a list  $H_1^{\text{list}}$  to record  $H_1$  queries. When  $A_1$  queries  $H_1(\text{ID}_i)$ , the same answer will be given if the query can be found on  $H_1^{\text{list}}$ . Otherwise,  $N_1$  picks  $\varepsilon_i \in Z_q^*$  at random and flips a coin  $c_i \in \{0, 1\}$ . If  $c_i = 0$ ,  $N_1$  sets  $Q_i = \varepsilon_i(bP)$ , adds  $(\text{ID}_i, \perp, Q_i, c_i)$  to  $H_1^{\text{list}}$  and returns  $Q_i$ . Otherwise,  $N_1$  sets  $Q_i = \varepsilon_i P$ , adds  $(\text{ID}_i, \varepsilon_i, Q_i, c_i)$  to  $H_1^{\text{list}}$  and returns  $Q_i$ .

$H_2$  queries: There is a list  $H_2^{\text{list}}$  to record  $H_2$  queries for same query getting same answer. When  $A_1$  queries  $H_2(m_i, \text{ID}_i, P_i, R_i)$ , and the query cannot be found on  $H_2^{\text{list}}$ ,  $N_1$  picks a random  $\delta_i \in Z_q^*$ , adds  $(m_i, \text{ID}_i, P_i, R_i, \delta_i)$  to  $H_2^{\text{list}}$  and then returns  $\delta_i$ .

$H_3$  queries: There is a list  $H_3^{\text{list}}$  to record  $H_3$  queries for same query getting same answer. When  $A_1$  queries  $H_3(w_i)$ , and the query cannot be found on  $H_3^{\text{list}}$ ,  $N_1$  selects a random  $\phi_i \in Z_q^*$ , calculates  $Z_i = \phi_i P$ , adds  $(w_i, \phi_i, Z_i)$  to  $H_3^{\text{list}}$  and then returns  $Z_i$ .

$H_4$  queries: There is a list  $H_4^{\text{list}}$  of tuples  $(w_i, \mu_i, F_i)$  to record  $H_4$  queries for same query get-

ting same answer. Whenever  $A_1$  issues a query  $H_4(w_i)$ , and the query cannot be found on  $H_4^{\text{list}}$ ,  $N_1$  picks a random  $\mu_i \in Z_q^*$ , calculates  $F_i = \mu_i P$ , adds  $(w_i, \mu_i, F_i)$  to  $H_4^{\text{list}}$  and then returns  $F_i$ .

Partial-Private-Key queries:  $N_1$  keeps a list  $K^{\text{list}}$  for same query getting same answer. When  $A_1$  queries a Partial-Private-Key for  $\text{ID}_i$ , and the query can be found on  $K^{\text{list}}$ ,  $N_1$  first does an  $H_1$  query on  $\text{ID}_i$  and finds the tuple  $(\text{ID}_i, \varepsilon_i, Q_i, c_i)$  on  $H_1^{\text{list}}$ , then  $N_1$  does as follows:

1) If  $c_i = 0$ ,  $N_1$  aborts.

2) Else if there is a tuple  $(\text{ID}_i, x_i, D_i, P_i)$  on  $K^{\text{list}}$ ,  $N_1$  sets  $D_i = \varepsilon_i P_{\text{pub}}$ , and returns  $D_i$ .

3) Otherwise, calculate  $D_i = \varepsilon_i P_{\text{pub}}$ , set  $x_i = P_i = \perp$ , return  $D_i$  as answer and add  $(\text{ID}_i, x_i, D_i, P_i)$  on  $K^{\text{list}}$ .

Public-Key queries: To a Public-Key query on  $\text{ID}_i$ , if the query can be found on  $K^{\text{list}}$ , the same answer will be given. Otherwise,  $N_1$  does as follows:

1) If there is a tuple  $(\text{ID}_i, x_i, D_i, P_i)$  on  $K^{\text{list}}$  (in this case, the public key  $P_i$  of  $\text{ID}_i$  is  $\perp$ ), choose  $x'_i \in Z_q^*$ , compute  $P'_i = x'_i P$ , return  $P'_i$  as answer and update  $(\text{ID}_i, x_i, D_i, P_i)$  to  $(\text{ID}_i, x'_i, D_i, P'_i)$ .

2) Otherwise, select  $x_i \in Z_q^*$  at random, calculate  $P_i = x_i P$ , return  $P_i$  as answer, set  $D_i = \perp$  and add  $(\text{ID}_i, x_i, D_i, P_i)$  to  $K^{\text{list}}$ .

Secret-Value queries: On receiving a Secret-Value query on  $\text{ID}_i$ , firstly,  $N_1$  makes public-key query on  $\text{ID}_i$ , then finds  $(\text{ID}_i, x_i, D_i, P_i)$  on list  $K^{\text{list}}$ , returns  $x_i$  (Note that the value of  $x_i$  maybe  $\perp$ ).

Public-Key-Replacement queries: When  $N_1$  receives a Public-Key-Replacement query,  $N_1$  first finds  $(\text{ID}_i, x_i, D_i, P_i)$  on list  $K^{\text{list}}$ , if such a tuple does not exist on list  $K^{\text{list}}$  or  $P_i = \perp$ ,  $N_1$  first makes Public-Key query on  $\text{ID}_i$ , then updates  $P_i$  to  $P'_i$ .

Sign queries: When  $N_1$  receives a Sign query on  $m_i$  by one user with identity  $\text{ID}_i$ , firstly  $N_1$  recovers  $(\text{ID}_i, \varepsilon_i, Q_i, c_i)$  from  $H_1^{\text{list}}$ ,  $(m_i, \text{ID}_i, P_i, R_i, \delta_i)$  from  $H_2^{\text{list}}$ , then does as follows:

1) If  $c_i = 0$ , select  $k_i, n_i \in Z_q^*$ , set  $R_i = k_i^{-1} (n_i P - P_i - P_{\text{pub}})$ ,  $Z = \delta_i Q_i$ ,  $F = k_i \delta_i Q_i$  and record  $(w_i, \perp, \delta_i Q_i)$  on list  $H_3^{\text{list}}$ , and  $(w_i, \perp, k_i \delta_i Q_i)$  on list

$H_4^{\text{list}}$ , then compute  $T_i = n_i \delta_i Q_i$ , output  $\sigma_i = (R_i, T_i)$ . Here  $\delta_i$  is found in  $(m_i, \text{ID}_i, P_i, R_i, \delta_i)$  from  $H_2^{\text{list}}$ .

2) If  $c_i = 1$ , randomly select  $R_i \in G_1$ , sets  $T_i = \varepsilon_i \delta_i P_{\text{pub}} + \phi_i P_i + \mu_i R_i$ , output  $\sigma_i = (R_i, T_i)$ . Here  $\phi_i$  is found from  $(w_i, \phi_i, Z_i)$  on  $H_3^{\text{list}}$ ,  $\mu_i$  is found from  $(w_i, \mu_i, F_i)$  on  $H_4^{\text{list}}$ .

Forgery:  $A_1$  outputs a forged aggregate signature  $\sigma^* = \{R_1^*, \dots, R_n^*, T^*\}$  under a set  $U$  of  $n$  users with identities set  $L_{\text{ID}}^* = \{\text{ID}_1^*, \dots, \text{ID}_n^*\}$  and the corresponding public keys set  $L_{\text{PK}}^* = \{P_1^*, \dots, P_n^*\}$ , messages set  $L_m^* = \{m_1^*, \dots, m_n^*\}$ , and a state information  $w^*$ . There exists  $I \in \{1, \dots, n\}$  such that  $A_1$  has not asked the partial private key for  $\text{ID}_I$ , and the sign query on  $(m_I^*, \text{ID}_I^*, P_I^*)$ . Let  $I=1$ . Then the forged aggregate signature satisfies

$$e(T^*, P) = e(P_{\text{pub}}, \sum_{i=1}^n h_i^* Q_i^*) e(F^*, \sum_{i=1}^n R_i^*) = e(Z^*, \sum_{i=1}^n P_i^*) \quad (1)$$

where

$$Q_i^* = H_1(\text{ID}_i^*), \quad h_i^* = H_2(m_i^*, \text{ID}_i^*, P_i^*, R_i^*), \\ Z^* = H_3(w^*), \quad F^* = H_4(w^*).$$

$N_1$  finds  $(\text{ID}_i^*, \varepsilon_i^*, Q_i^*, c_i^*)$  from  $H_1^{\text{list}}$ ,  $(m_i^*, \text{ID}_i^*, P_i^*, R_i^*, \delta_i^*)$  from  $H_2^{\text{list}}$ ,  $(w_i^*, \phi_i^*, Z_i^*)$  from  $H_3^{\text{list}}$  and  $(w_i^*, \mu_i^*, F_i^*)$  from  $H_4^{\text{list}}$  for all  $i, 1 \leq i \leq n$ .  $N_1$  proceeds only if  $c_1^* = 0$ ,  $c_i^* = 1$  for all  $i, 2 \leq i \leq n$ . Otherwise,  $N_1$  aborts.

From

$$e(T^*, P) = e(P_{\text{pub}}, \sum_{i=1}^n h_i^* Q_i^*) e(F^*, \sum_{i=1}^n R_i^*) e(Z^*, \sum_{i=1}^n P_i^*) \quad (2)$$

It holds

$$e(P_{\text{pub}}, h_1^* Q_1^*) = e(T^*, P) (e(P_{\text{pub}}, \sum_{i=2}^n h_i^* Q_i^*) \\ \cdot e(Z^*, \sum_{i=2}^n P_i^*) e(F^*, \sum_{i=2}^n R_i^*))^{-1} \quad (3)$$

But,  $Q_1^* = \varepsilon_1^* (bP)$ ,  $h_1^* = \delta_1^*$ ,  $Z^* = \phi^* P$ ,  $F^* = \mu^* P$ , and for all  $i, 2 \leq i \leq n$ ,  $Q_i^* = \varepsilon_i^* P$ ,  $h_i^* = \delta_i^*$ . So,  $N_1$  can calculate

$$abP = (\delta_1^* \varepsilon_1^*)^{-1} (T^* - \sum_{i=2}^n \delta_i^* \varepsilon_i^* P_{\text{pub}} - \sum_{i=1}^n (\phi^* P_i^* + \mu^* R_i^*))$$

**Theorem 2** The proposed certificateless aggregate scheme is existentially unforgeable against type 2 adversary under the assumption that the CDH problem is intractable.

**Proof** To prove the proposed scheme is existentially unforgeable against type 2 adversary, we show how a CDH attacker  $N_2$  uses type 2 adversary  $A_2$  to compute  $abP$  from  $(P, Ap, bP)$ .

Setup: Firstly,  $N_2$  picks a random  $\eta \in Z_q^*$  as the master-key, and sets system public key  $P_{\text{pub}} = \eta P$  and system parameters

$$\text{params} = (G_1, G_2, e, P, P_{\text{pub}}, H_1, H_2, H_3, H_4)$$

Then he sends params and the master key  $\eta$  to  $A_2$ . Since  $A_2$  has access to the master-key, there is no need to handle  $H_1$  as random oracle.

To carry on attack,  $A_2$  does the following types of queries in an adaptive manner.

$H_2$  queries:  $N_2$  keeps a list  $H_2^{\text{list}}$  for same query getting same answer. When  $A_2$  queries  $H_2(m_i, \text{ID}_i, P_i, R_i)$ , and the query cannot be found on  $H_2^{\text{list}}$ ,  $N_2$  picks a random  $\delta_i \in Z_q^*$ , and adds  $(m_i, \text{ID}_i, P_i, R_i, \delta_i)$  to list  $H_2^{\text{list}}$ , then returns  $\delta_i$ .

$H_3$  queries:  $N_2$  keeps a list  $H_3^{\text{list}}$  for same query getting same answer. When  $A_2$  queries  $H_3(w_i)$ , and the query cannot be found on  $H_3^{\text{list}}$ ,  $N_2$  picks a random  $\phi_i \in Z_q^*$ , calculates  $Z_i = \phi_i (aP)$ , adds  $(w_i, \phi_i, Z_i)$  to  $H_3^{\text{list}}$ , then returns  $Z_i$ .

$H_4$  queries: There is a list  $H_4^{\text{list}}$  for same query getting same answer. When  $A_2$  queries a query  $H_4(w_i)$ , and the query cannot be found on  $H_4^{\text{list}}$ ,  $N_2$  picks a random  $\mu_i \in Z_q^*$ , calculates  $F_i = \mu_i P$ , adds  $(w_i, \mu_i, F_i)$  to  $H_4^{\text{list}}$ , and returns  $F_i$ .

Public-Key queries: To answer a Public-Key query on  $\text{ID}_i$ , if the request can be found on  $K^{\text{list}}$ , the same answer will be given. Otherwise,  $N_2$  picks  $x_i \in Z_q^*$  and flips a coin  $c_i \in \{0, 1\}$ . If  $c_i = 0$ ,  $N_2$  returns  $x_i (bP)$ , adds  $(\text{ID}_i, \perp, D_i, P_i, c_i)$  to list  $K^{\text{list}}$ . Otherwise, it calculates  $P_i = x_i P$ , and adds  $(\text{ID}_i, x_i, D_i, P_i, c_i)$  to  $K^{\text{list}}$  and returns  $P_i$ .

Secret-Value queries: To answer Secret-Value query on  $\text{ID}_i$ ,  $N_2$  first finds the tuple on  $K^{\text{list}}$ . If  $c_i = 0$ ,  $N_2$  aborts, otherwise, returns  $x_i$ .

Sign queries: To answer Sign query on  $m_i$  by one user with identity  $\text{ID}_i$ .  $N_2$  firstly finds  $(\text{ID}_i, x_i, P_i, c_i)$  on  $K^{\text{list}}$ ,  $(w_i, \phi_i, Z_i)$  from  $H_3^{\text{list}}$   $(w_i, \phi_i, Z_i)$ , and  $(w_i, \mu_i, F_i)$  from  $H_4^{\text{list}}$ , then does as follows:

1) If  $c_i = 0$ ,  $N_2$  randomly selects  $R_i \in G_1$ , and  $n_i \in Z_q^*$ , sets  $Z = n_i P$ , adds  $(w_i, n_i, n_i P)$  on  $H_3^{\text{list}}$ , and calculates  $T_i = \eta \delta_i H_i(\text{ID}_i) + n_i x_i (bP) + \mu_i R_i$ , outputs  $\sigma_i = (R_i, T_i)$ .

2) If  $c_i = 1$ ,  $N_2$  executes the standard sign algorithm to generate and outputs  $\sigma_i = (R_i, T_i)$ .

Forgery: Finally,  $A_1$  returns a forged aggregate signature  $\sigma^* = \{R_1^*, \dots, R_n^*, T^*\}$  under a set  $U$  of  $n$  users with identities set  $L_{\text{ID}}^* = \{\text{ID}_1^*, \dots, \text{ID}_n^*\}$ , the corresponding public keys set  $L_{\text{PK}}^* = \{P_1^*, \dots, P_n^*\}$ , messages set  $L_m^* = \{m_1^*, \dots, m_n^*\}$ , a state information  $w^*$ . There exists  $I \in \{1, \dots, n\}$  such that  $A_1$  has not asked the partial private key for  $\text{ID}_I$ , and sign query on  $(m_I^*, \text{ID}_I^*, P_I^*)$ . Let  $I = 1$ . The forged aggregate signature satisfies

$$e(T^*, P) = e(P_{\text{pub}}, \sum_{i=1}^n h_i^* Q_i^*) e(F^*, \sum_{i=1}^n R_i^*) e(Z^*, \sum_{i=1}^n P_i^*) \quad (4)$$

Where  $Q_i^* = H_1(\text{ID}_i^*)$ ,  $h_i^* = H_2(m_i^*, \text{ID}_i^*, P_i^*, R_i^*)$ ,  $Z^* = H_3(w^*)$ ,  $F^* = H_4(w^*)$ .

$N_2$  finds  $(m_i^*, \text{ID}_i^*, P_i^*, R_i^*, \delta_i^*)$  from list  $H_2^{\text{list}}$ ,  $(w_i, \gamma_i, Z_i)$  from list  $H_3^{\text{list}}$  and  $(w_i, \mu_i, F_i)$  from list  $H_4^{\text{list}}$  for all  $i, 1 \leq i \leq n$ .  $N_2$  proceeds only if  $c_1^* = 0$ ,

$c_i^* = 1$  for all  $i, 2 \leq i \leq n$ . Otherwise,  $N_2$  aborts.

From

$$e(T^*, P) = e(P_{\text{pub}}, \sum_{i=1}^n h_i^* Q_i^*) e(F^*, \sum_{i=1}^n R_i^*) e(Z^*, \sum_{i=1}^n P_i^*) \quad (5)$$

It holds

$$e(Z^*, P_1^*) = e(T^*, P) e(Z^*, \sum_{i=2}^n P_i^*) e(P_{\text{pub}}, \sum_{i=1}^n h_i^* Q_i^*) \cdot e(F^*, \sum_{i=1}^n R_i^*)^{-1} \quad (6)$$

But,  $h_1^* = \delta_1^*$ ,  $Z^* = \phi_1^*(aP)$ ,  $P_1^* = x_1^*(bP)$ ,  $F^* = \mu^* P$ , and for all  $i, 2 \leq i \leq n$ ,  $h_i^* = \delta_i^*$ ,  $Z^* = \phi_i^* aP$ ,  $P_i^* = x_i^* P$ . Hence,  $N_2$  can calculate

$$abP = (\phi_1^* x_1^*)^{-1} (T^* - \sum_{i=1}^n (\eta \delta_i^* Q_i^* + \mu^* R_i^*) - \sum_{i=2}^n \phi_i^*(aP))$$

## 4 Comparisons

In this section, we compare the proposed scheme with Refs. [14, 18] in computation cost. We laid stress on the comparisons of the computation in Partial-Private-Key-Extract Algorithm, Sign Algorithm, and Aggregate Verify Algorithm. The comparison result is show in Table 1. Obviously, the proposed scheme needs low computation cost.

**Table 1 Comparisons of the computation cost**

Scheme	P1	P2	P3
Cheng <i>et al</i> [14]	1H+1S	2H+5S+4D	2nH+2nS+2nD+3B
Chen <i>et al</i> [18]	1H+1S	4H+4S+3D	(3n+2)H+2nS+3nD+4B
Our scheme	1H+1S	3H+4S+3D	(2n+2)H+nS+3nD+4B

P1: Partial-Private-Key-Extract Algorithm; P2: Sign Algorithm; P3: Aggregate Verify Algorithm;

D: Group sum computation; S: Scalar multiplication computation; H: Hash computation; B: bilinear pairings computation

## 5 Conclusion

In this paper, a new certificateless aggregate signature scheme is proposed. It is proved that the new scheme is existentially unforgeable under adaptively chosen-message attacks assuming the computational Diffie-Hellman problem is hard. Furthermore, a comparison of the new scheme with some existing certificateless aggregate signature schemes indicates that the new scheme is more efficient. But, in Hash and bilinear pairings computation cost, the new scheme has insufficient predominance.

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