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An Interactive Intuitionistic Fuzzy Method for Multilevel Linear Programming Problems

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Abstract: In this paper, we propose an interactive method for solving the multilevel linear programming problems based on the intuitionistic fuzzy set theory. Firstly, the membership function and the non-membership function are introduced to describe the uncertainty of the decision makers. Secondly, a satisfactory solution is derived by updating the minimum satisfactory degrees with considerations of the overall satisfactory balance among all levels. In addition, the steps of the proposed method are given in this paper. Finally, numerical examples illustrate the feasibility of this method.

Key words: intuitionistic fuzzy; multilevel linear programming; interactive method; satisfying degree

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0 Introduction

Multilevel programming (MP) was first proposed by Candler and Norton [1] in 1977. It is identified as mathematical programming which has a special hierarchical structure. There are multiple decision makers (DMs) in this structure and every DM has his/her objective function, decision variables and constraints, respectively. MP is very practical in the field of economic systems, engineering, transportation and so on. A lot of scholars have taken on this research since 1970s which leads to a rapid development in the theories, algorithms and applications of MP (Refs.[2-5]).

When taking into account some cooperation among the DMs, it is not appropriate to develop an algorithm for obtaining a Stackelberg solution to a multilevel programming problem. Then the interactive fuzzy methods have been developed in consideration of fuzziness of human judgment [6-12]. Sakawa *et al* [8] presented interactive fuzzy goal programming for multilevel linear programming problems. Wan *et al* [10] introduced an interactive fuzzy decision making method for bilevel programming with a common decision variable. Other fuzzy methods for solving multilevel programming problems can refer to Refs.[13-16].

The concept of intuitionistic fuzzy sets (IFS) developed by Atanassov [17,18] is a generalization of the fuzzy set theory. There have been some algorithms for the intuitionistic fuzzy optimization (IFO). Angelov [19] introduced a frame to solve optimization problems in intuitionistic fuzzy environment; Li [20] investigated multiattribute decision making using IFS theory; Liu and Wang [21] proposed an approach to multi-criteria decision making based on IFS; Mahapatra [22] used the IFO technique to solve multi-objective nonlinear programming problems.

There is no means to incorporate the lack of information with the membership degree in fuzzy sets, but IFS can be viewed as an approach to overcome the shortcoming of fuzzy set theory. In addition, IFO can reduce DMs' subjective consciousness as much as possible thus the practical problems can be reflected objectively. In this paper, we present an interactive method for multilevel linear programming based on IFS theory under the assumption of cooperative relationship among the DMs. Here, we consider the following multilevel linear programming:

$$\begin{cases} \min\limits_{x_1,\cdots,x_t} \ z_1(\boldsymbol{x}) = \sum\limits_j c_{1j}x_j \\ \min\limits_{x_2,\cdots,x_t} \ z_2(\boldsymbol{x}) = \sum\limits_j c_{2j}x_j \\ \quad\vdots \\ \min\limits_{x_t} \ z_t(\boldsymbol{x}) = \sum\limits_j c_{tj}x_j \\ \text{s.t.} \ \sum\limits_{k=1}^{t} \boldsymbol{A}_k \boldsymbol{x}_k \leqslant b \\ \quad x_j \geqslant 0, j = 1,2,\cdots,n \end{cases} \tag{1}$$

where, $\boldsymbol{x}_k$ and $z_k(\boldsymbol{x})$ are DM$_i$'s decision variable and objective function, respectively. DM$_i$ denotes the DM at ith level. $\bigcup_k \{\boldsymbol{x}_k | k=1,2,\cdots,t\} = \{x_1,x_2,\cdots,x_n\}$, $\boldsymbol{A}_k$ is $m \times n_k$ coefficient matrix, b is m-dimensional column vector, $k=1,2,\cdots,t$ and $n_1+n_2+\cdots+n_t=n$.

The rest of this paper is organized as follows. Section 1 briefly introduces the basic concepts of IFS. The interactive intuitionistic fuzzy method for problem (1) is established afterwards. Furthermore, numerical examples are given in Section 2 to illustrate the feasibility of this method. The last section gives a short conclusion for this paper.

1 Algorithm Formulation

1.1 Definitions and Properties of IFS

Definition 1[17, 23] Let X be a nonempty set of the universe. An intuitionistic fuzzy set (IFS) A in X is an object having the form : $A = \{\langle \boldsymbol{x}, \mu_A(\boldsymbol{x}), \nu_A(\boldsymbol{x}) \rangle | \boldsymbol{x} \in X\}$, where $\mu_A(\boldsymbol{x}): X \to [0,1]$ and $\nu_A(\boldsymbol{x}): X \to [0,1]$ define the degree of membership and non-membership, respectively, and for every $\boldsymbol{x} \in X$, $0 \leqslant \mu_A(\boldsymbol{x}) + \nu_A(\boldsymbol{x}) \leqslant 1$.

Definition 2 [17] $\pi_A(\boldsymbol{x}) = 1 - \mu_A(\boldsymbol{x}) - \nu_A(\boldsymbol{x})$ is called the degree of non-determinacy (or hesitancy) of the element $\boldsymbol{x} \in X$ to the IFS A. Obviously, for every $\boldsymbol{x} \in X$, $0 \leqslant \pi_A(\boldsymbol{x}) \leqslant 1$.

Definition 3 [18, 24] The intuitionistic fuzzy number (IFN) is defined as $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$, where $\mu_\alpha \in [0,1]$, $\nu_\alpha \in [0,1]$, $0 \leqslant \mu_\alpha(\boldsymbol{x}) + \nu_\alpha(\boldsymbol{x}) \leqslant 1$. Moreover, the function $s(\alpha) = \mu_\alpha(\boldsymbol{x}) - \nu_\alpha(\boldsymbol{x})$ is also defined to evaluate the degree of suitability that an alternative satisfies the DM's requirement.

Notice that maximization of the score function $s(\alpha)$ can decrease the DMs' hesitancy and their subjective consciousness can be reduced. Sakawa *et al* [8] proposed that taking fuzzy goals of the objective function and the decision variables may generate inconsistency between them, so we only consider that DMs have fuzzy goals for their objectives. If $z_i(\boldsymbol{x})$ is less than or equal to a value, the result is satisfied for DM$_i$; On the contrary, if $z_i(\boldsymbol{x})$ is greater than or equal to a value, DM$_i$ can't accept the result completely. Due to Ref.[22], we can elicit the membership function and non-membership function for every fuzzy objective by the following steps:

Step 1: For all $i=1,2,\cdots,t$, DM$_i$ solves the following problem (2),

$$\begin{cases} \min\limits_{x} \ z_i(\boldsymbol{x}) = \sum\limits_{j=1}^{n} c_{ij}x_j \\ \text{s.t.} \ \sum\limits_{k=1}^{t} \boldsymbol{A}_k \boldsymbol{x}_k \leqslant b \\ \quad x_j \geqslant 0, j=1,2,\cdots,n \end{cases} \tag{2}$$

Suppose that $\boldsymbol{x}_i^*$ is the optimal solution of problem (2), we call $\boldsymbol{x}_i^*$ the ideal solution of $z_i(\boldsymbol{x})$.

Step 2: DMs calculate the value of every objective function at the ideal point $\boldsymbol{x}_i^*$, $i,j=1,2,\cdots,t$, denote

$$L_i^a = z_i(\boldsymbol{x}_i^*) = \min_{j=1,2,\cdots,t} z_i(\boldsymbol{x}_j^*), U_i^a = \max_{j=1,2,\cdots,t} z_i(\boldsymbol{x}_j^*) \tag{3}$$

Moreover, we can assume that L_i^r and U_i^r are lower bound and upper bound of the non-membership function, respectively:

$$L_i^r = L_i^a + \varepsilon_i, \ U_i^r = U_i^a \tag{4}$$

where $\varepsilon_i = t_i(U_i^a - L_i^a)$, $0 < t_i < 1$, and ε_i is determined by DM$_i$, $i=1,2,\cdots,t$.

Step 3: We use the following linear membership function $\mu_i(z_i(\boldsymbol{x}))$ and non-membership function $\nu_i(z_i(\boldsymbol{x}))$ to describe the fuzzy goals of the DMs, respectively:

$$\mu_i\left(z_i\left(\boldsymbol{x}\right)\right)=\begin{cases}1, & z_i\left(\boldsymbol{x}\right)\leqslant L_i^a\\ \dfrac{U_i^a-z_i\left(\boldsymbol{x}\right)}{U_i^a-L_i^a}, & L_i^a\leqslant z_i\left(\boldsymbol{x}\right)\leqslant U_i^a\\ 0, & z_i\left(\boldsymbol{x}\right)\geqslant U_i^a\end{cases} \quad (5)$$

$$\nu_i\left(z_i\left(\boldsymbol{x}\right)\right)=\begin{cases}0, & z_i\left(\boldsymbol{x}\right)\leqslant L_i^r\\ \dfrac{z_i\left(\boldsymbol{x}\right)-L_i^r}{U_i^r-L_i^r}, & L_i^r\leqslant z_i\left(\boldsymbol{x}\right)\leqslant U_i^r\\ 1, & z_i\left(\boldsymbol{x}\right)\geqslant U_i^r\end{cases} \quad (6)$$

Rough sketch of the membership function and non-membership function for minimization type objective function are shown in Fig. 1. We can denote the score functions of DMs after eliciting membership and non-membership function according to the definition.

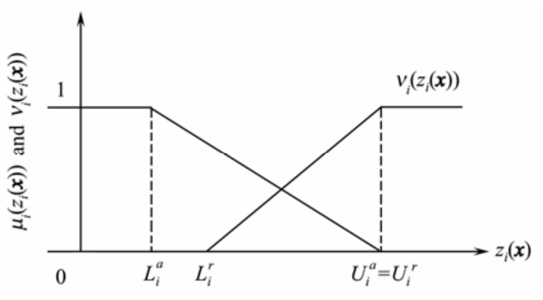

Fig. 1 Membership and non-membership functions of objective function

1.2 Interactive Intuitionistic Fuzzy Method

After eliciting the membership and non-membership function, DM$_i$ specifies a minimal satisfactory level $\delta_i\in[0,1]$ for the score function $s_i\left(z_i\left(\boldsymbol{x}\right)\right)$. DM$_t$ maximizes $s_t\left(z_t\left(\boldsymbol{x}\right)\right)$ under the existing constraints after getting the requirement of t–1 DMs at upper level, that is, DM$_t$ solves the following problem:

$$\begin{cases}\max\limits_{\boldsymbol{x}}\ s_t\left(z_t\left(\boldsymbol{x}\right)\right)\\ \text{s.t.}\ \sum\limits_{k=1}^{t}\boldsymbol{A}_k\boldsymbol{x}_k\leqslant b\\ \quad x_j\geqslant 0, j=1,2,\cdots,n\\ \quad s_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\delta_i, i=1,2,\cdots,t-1\\ \quad \nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant 0, i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\nu_i\left(z_i\left(\boldsymbol{x}\right)\right), i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)+\nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\leqslant 1, i=1,2,\cdots,t\end{cases} \quad (7)$$

If an optimal solution to problem (7) exists, it shows that DMs at upper level can obtain a satisfactory solution which has a satisfactory degree larger than or equal to the minimal satisfactory level specified by DMs. However, the larger the minimal satisfactory level of the upper level is, the smaller the satisfactory degree of DMs at lower level becomes. This may cause that the DM's satisfaction at each level is of great difference. Considering the overall satisfactory balance and the stability of decision, the DMs at upper level compromise with the lower DMs. So we define

$$\lambda=\lambda\left(\boldsymbol{x}\right)=\min\left(s_1\left(z_1\left(\boldsymbol{x}\right)\right), s_2\left(z_2\left(\boldsymbol{x}\right)\right),\cdots,s_t\left(z_t\left(\boldsymbol{x}\right)\right)\right)$$

then problem (7) convert to the problem (8):

$$\begin{cases}\max\limits_{\boldsymbol{x},\lambda}\ \lambda\\ \text{s.t.}\ \sum\limits_{k=1}^{t}\boldsymbol{A}_k\boldsymbol{x}_k\leqslant b\\ \quad x_j\geqslant 0, j=1,2,\cdots,n\\ \quad s_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\delta_i\geqslant\lambda, i=1,2,\cdots,t-1\\ \quad s_t\left(z_t\left(\boldsymbol{x}\right)\right)\geqslant\lambda\\ \quad \lambda\in[0,1]\\ \quad \nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant 0, i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\nu_i\left(z_i\left(\boldsymbol{x}\right)\right), i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)+\nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\leqslant 1, i=1,2,\cdots,t\end{cases} \quad (8)$$

The auxiliary problem of (8) is:

$$\begin{cases}\max\limits_{\boldsymbol{x},\lambda}\ \lambda\\ \text{s.t.}\ \sum\limits_{k=1}^{t}\boldsymbol{A}_k\boldsymbol{x}_k\leqslant b\\ \quad x_j\geqslant 0, j=1,2,\cdots,n\\ \quad s_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\lambda, i=1,2,\cdots,t\\ \quad \lambda\in[0,1]\\ \quad \nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant 0, i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)\geqslant\nu_i\left(\mathrm{z}_i\left(\boldsymbol{x}\right)\right), i=1,2,\cdots,t\\ \quad \mu_i\left(z_i\left(\boldsymbol{x}\right)\right)+\nu_i\left(z_i\left(\boldsymbol{x}\right)\right)\leqslant 1, i=1,2,\cdots,t\end{cases} \quad (9)$$

Sakawa *et al* [8] defined $\Delta_i=\dfrac{\mu_{i+1}\left(z_{i+1}\left(\boldsymbol{x}\right)\right)}{\mu_i\left(z_i\left(\boldsymbol{x}\right)\right)}$ to measure the overall interests, here we define the ratio of neighboring levels' score function value $\sigma_i=\dfrac{s_{i+1}\left(z_{i+1}\left(\boldsymbol{x}\right)\right)}{s_i\left(z_i\left(\boldsymbol{x}\right)\right)}$. For DM$_i$ and DM$_{i+1}$, $s_i\left(z_i\left(\boldsymbol{x}\right)\right)>s_{i+1}\left(z_{i+1}\left(\boldsymbol{x}\right)\right)$, $\sigma_i\in[0,1]$.

DM$_i$ sets the acceptable interval $\left[\sigma_i^{\mathrm{L}},\sigma_i^{\mathrm{U}}\right]$ for σ_i, if $\sigma_i<\sigma_i^{\mathrm{L}}$, it means that the lower level's satisfactory degree is low because the upper level's demand is too high. Then, DM$_i$ reduces his/her minimal satisfactory level δ_i; If $\sigma>\sigma_i^{\mathrm{U}}$, DM$_i$ increases δ_i.

The algorithm terminates if the solution $\boldsymbol{x}^*$ of problem (9) meets the following conditions:

① $s_i\left(z_i\left(\boldsymbol{x}^*\right)\right)\geqslant\delta_i$, for all $i=1,2,\cdots,t-1$.

② $\sigma_i\in\left[\sigma_i^{\mathrm{L}},\sigma_i^{\mathrm{U}}\right]$, for all $i=1,2,\cdots,t-1$.

If $\boldsymbol{x}^*$ can not satisfy both of the above conditions for some DM$_i$, then DM$_i$ needs to update the minimal satisfactory level of relevant objective function in accordance with the following criteria:

1) If condition ① is not satisfied, DM$_i$ decreases the minimal satisfactory level δ_i;

2) If $\sigma_i > \sigma_i^{\mathrm{U}}$, DM$_i$ increases δ_i; If $\sigma_i < \sigma_i^{\mathrm{L}}$, DM$_i$ decreases δ_i.

Note: If some DMs can not meet the termination conditions, update the DM's minimal satisfactory degree who located at the lowest level. So, the DMs at upper level can update their strategy according to the lower level's reaction such that a satisfactory solution can be obtained.

Suppose DM$_q$ is located at the lowest level among the DMs who don not satisfy the termination conditions, DM$_q$ adjusts the minimal satisfactory degree to δ_q', then we solve (10) with the updated δ_q':

$$\begin{cases} \max\limits_{x,\lambda} \ \lambda \\ \text{s.t. } \sum\limits_{k=1}^{t} \boldsymbol{A}_k \boldsymbol{x}_k \leqslant b \\ \quad x_j \geqslant 0, j=1,2,\cdots,n \\ \quad s_i\left(z_i(\boldsymbol{x})\right) \geqslant \lambda, i=1,2,\cdots,q-1,q+1,\cdots,t \\ \quad s_q\left(z_q(\boldsymbol{x})\right) \geqslant \delta_q' \\ \quad \lambda \in [0,1] \\ \quad \nu_i\left(z_i(\boldsymbol{x})\right) \geqslant 0, i=1,2,\cdots,t \\ \quad \mu_i\left(z_i(\boldsymbol{x})\right) \geqslant \nu_i\left(z_i(\boldsymbol{x})\right), i=1,2,\cdots,t \\ \quad \mu_i\left(z_i(\boldsymbol{x})\right)+\nu_i\left(z_i(\boldsymbol{x})\right) \leqslant 1, i=1,2,\cdots,t \end{cases} \tag{10}$$

Next, we test whether the solution to (10) satisfies the termination conditions or not. If not, the relevant DMs update the minimal satisfactory level until satisfactory solution is obtained. The above-mentioned algorithm is summarized as follows:

Step 1: For all $i=1,2,\cdots,t$, DM$_i$ elicits the membership function and non-membership function of the fuzzy goal of DM$_i$ in turn.

Step 2: DM$_i$ specifies the minimal satisfactory level δ_i, the lower and the upper bounds of σ_i, $i=1,2,\cdots,t-1$.

Step 3: DM$_t$ solves the problem (9), that is, it obtains the optimal solution $\boldsymbol{x}^*$ by maximizing the score functions of all the DMs. Then DMs calculate the value of $s_i\left(z_i\left(\boldsymbol{x}^*\right)\right)$ and $\sigma_i\left(z_i\left(\boldsymbol{x}^*\right)\right)$, $i=1,2,\cdots,t-1$.

Step 4: If the solution $\boldsymbol{x}^*$ satisfies the termination conditions, the algorithm terminates; Otherwise, go to Step 5.

Step 5: If DM$_q$ is located at the lowest level among the DMs who don't satisfy the termination conditions, updates δ_q according to the procedure of updating minimal satisfactory level and then solves problem (10).

Step 6: If the solution to (10) satisfies the termination conditions, the algorithm terminates; Otherwise, return to Step 5.

2 Numerical Examples

Example 1[10]:

$$\begin{cases} \min\limits_{x_1} z_1 = 18x_1 - 10x_2 - 11x_3 + 11x_4 - 23x_5 - 40x_6 \\ \min\limits_{x_2} z_2 = 35x_1 + 9x_2 - 20x_3 + 44x_4 - 10x_5 - 7x_6 \\ \text{s.t. } 47x_1 - 14x_2 - x_3 + 4x_4 + x_5 - 49x_6 \leqslant 1.5 \\ \quad -23x_1 + 2x_2 + 45x_3 - 35x_4 + 12x_5 + 41x_6 \leqslant 13.5 \\ \quad -9x_1 - 18x_2 + 12x_3 + 13x_4 + 37x_5 - 11x_6 \leqslant 5.5 \\ \quad 6x_1 - 19x_2 - x_3 - 2x_4 - 49x_5 - 11x_6 \leqslant -43.5 \\ \quad -31x_1 - 8x_2 + 2x_3 + 17x_4 + 47x_5 - 25x_6 \leqslant 6.3 \\ \quad 46x_1 + 3x_2 - 28x_3 + 17x_4 - 36x_5 - 3x_6 \leqslant 22.5 \\ \quad -45x_1 + 34x_2 - 44x_3 + 44x_4 + 16x_5 - 2x_6 \leqslant 17 \\ \quad 29x_1 - 13x_2 + 38x_3 + 19x_4 - 2x_5 + 7x_6 \leqslant 39 \\ \quad 13x_1 + 10x_2 + 27x_3 - 29x_4 - 49x_5 - 38x_6 \leqslant -38 \\ \quad x_i \geqslant 0, i=1,2,\cdots,6 \end{cases} \tag{11}$$

where $\boldsymbol{x}_1 = (x_1, x_2, x_3)'$, $\boldsymbol{x}_2 = (x_4, x_5, x_6)'$.

In our algorithm, we choose $\varepsilon_1 = 3, \varepsilon_2 = 4, \delta_1 = 0.5$, $\left[\sigma_1^{\mathrm{L}}, \sigma_1^{\mathrm{U}}\right] = [0.75, 0.9]$. The solution of problem (9) is $\boldsymbol{x}= (0.881\,520, 1.122\,045, 0, 0.066\,176, 1.040\,567, 0.520\,981)'$, $\lambda = 0.252\,975$, $z_1 = -39.397\,442$, $z_2 = 29.810\,803$, $\sigma_1 = 1$, $s_1(z_1) = 0.252\,975$, $s_2(z_2) = 0.252\,975$.

While $s_1(z_1) = 0.252\,975 < 0.5$ doesn't satisfy the termination condition (1), DM$_1$ changes $\delta_1 = 0.5$ to $\delta_1' = 0.27$. Then a problem corresponding to (10) is formulated as problem (12).

The solution of problem (12) is:

$\boldsymbol{x} = (0.890\,311,\ 1.125\,337,\ 0,\ 0.071\,434,\ 1.044\,841,\ 0.528\,990)'$, $\lambda = 0.228\,208$, $z_1 = -39.632\,923$, $z_2 = 30.280\,680$, $s_1(z_1) = 0.27 = \delta_1'$, $s_2(z_2) = 0.228\,208$, $\sigma_1 = 0.845\,215$.

$$\begin{cases} \max\limits_{x,\lambda} \ \lambda \\ \text{s.t. } \sum\limits_{k=1}^{t} \boldsymbol{A}_k \boldsymbol{x}_k \leqslant b \\ \quad x_j \geqslant 0, j=1,2,\cdots,6 \\ \quad s_1\left(z_1(\boldsymbol{x})\right) \geqslant 0.27 \\ \quad s_1\left(z_1(\boldsymbol{x})\right) \geqslant \lambda \\ \quad \lambda \in [0,1] \\ \quad \nu_i\left(z_i(\boldsymbol{x})\right) \geqslant 0, i=1,2 \\ \quad \mu_i\left(z_i(\boldsymbol{x})\right) \geqslant \nu_i\left(z_i(\boldsymbol{x})\right), i=1,2 \\ \quad \mu_i\left(z_i(\boldsymbol{x})\right)+\nu_i\left(z_i(\boldsymbol{x})\right) \leqslant 1, i=1,2 \end{cases} \tag{12}$$

The above solution satisfies all the termination conditions, so DMs obtain the satisfactory solution and the algorithm terminates.

$\boldsymbol{x} = (0.890\,311,\ 1.125\,337,\ 0,\ 0.071\,434,\ 1.044\,841,\ 0.528\,990)'$, $\lambda = 0.228\,208$, $z_1 = -39.632\,923$, $z_2 = 30.280\,680$, $s_1(z_1) = 0.27 = \delta_1'$, $s_2(z_2) = 0.228\,208$, $\sigma_1 = 0.845\,215$.

The above solution satisfies all the termination conditions, so DMs obtain the satisfactory solution and the algorithm terminates.

To demonstrate the feasibility of our method, we compare the results in Table 1. Our method is denoted by Method 1, the method of Wan *et al* [10] is Method 2, and the method of Zheng *et al* [11] is Method 3.

According to the Table 1, all the DMs' score function values obtained by Method 3 are better than that of Method 2; Though DM_1's score function value obtained by Method 3 is better than that of Method 1, DM_2's score function value is not ideal as that of Method 1. Furthermore, $\sigma = 0.722\,429$ is not in [0.75, 0.9] of Method 3, it means that there is a big difference between two DMs' satisfactory degree. By contrast, the parameter σ in Method 1 is much closer to 0.9. This guarantees not only the upper DM's advantage but also the satisfaction of both DMs. Consequently, our method is feasible.

Table 1 Comparisons of the results of Example 1

Method	(z_1, z_2)	$(s_1(z_1), s_2(z_2))$	σ
1	(–39.632 923, 30.280 680)	(0.270 000, 0.228 208)	0.845 215
2	(–39.613 400, 30.620 500)	(0.268 589, 0.210 059)	0.782 084
3	(–39.788 800, 30.591 800)	(0.293 190, 0.211 809)	0.722 429

Example 2[8]:

$$\begin{cases} \min\limits_{x_1,x_2,x_3} & z_1 = c_1\boldsymbol{x}_1 + c_2\boldsymbol{x}_2 + c_3\boldsymbol{x}_3 \\ \min\limits_{x_2,x_3} & z_2 = c_4\boldsymbol{x}_1 + c_5\boldsymbol{x}_2 + c_6\boldsymbol{x}_3 \\ \min\limits_{x_3} & z_3 = c_7\boldsymbol{x}_1 + c_8\boldsymbol{x}_2 + c_9\boldsymbol{x}_3 \\ \text{s.t.} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 + A_3\boldsymbol{x}_3 \leqslant b \\ & x_j \geqslant 0,\ j = 1,2,\cdots,15 \end{cases} \tag{13}$$

where $\boldsymbol{x}_1 = (x_1,\cdots,x_5)'$, $\boldsymbol{x}_2 = (x_6,\cdots,x_{10})'$, $\boldsymbol{x}_3 = (x_{11},\cdots,x_{15})'$. In addition, $c_1,\cdots,c_9$, A_1, A_2, A_3, b are showed in Table 5 of Ref.[8].

We choose $\varepsilon_1 = 10$, $\varepsilon_2 = 6$, $\varepsilon_3 = 2$, $\delta_1 = 0.95$, $\delta_2 = 0.85$, $[\sigma_1^{\mathrm{L}}, \sigma_1^{\mathrm{U}}] = [\sigma_2^{\mathrm{L}}, \sigma_2^{\mathrm{U}}] = [0.75, 0.9]$. The solution of problem (9) is

$\boldsymbol{x} = (2.580\,710,\ 0,\ 0.967\,857,\ 2.285\,659,\ 2.146\,568,\ 1.675\,894,\ 3.217\,242,\ 0,\ 0,\ 1.019\,755,\ 1.667\,012,\ 0,\ 0,\ 1.273\,604,\ 0.095\,182)'$, $\lambda = 0.581\,152$, $z_1 = -516.220\,154$, $z_2 = -451.706\,332$, $z_3 = -371.453\,935$, $s_1(z_1) = 0.803\,765 < \delta_1 = 0.95$, $s_2(z_2) = 0.595\,763 > \delta_2 = 0.85$, $s_3(z_3) = 0.581152$, $\sigma_1 = 0.741\,215$, $\sigma_2 = 0.975\,475$.

DM_1 and DM_2 are both not satisfied with the above solution, DM_2 changes $\delta_2 = 0.85$ to $\delta_2' = 0.65$. Then a problem corresponding to (10) is formulated as:

$$\begin{aligned} \max_{x,\lambda}\ & \lambda \\ \text{s.t.}\ & \sum_{k=1}^{t} \boldsymbol{A}_k\boldsymbol{x}_k \leqslant b \\ & x_j \geqslant 0,\ j = 1,2,\cdots,15 \\ & s_1(z_1(\boldsymbol{x})) \geqslant \lambda \\ & s_2(z_2(\boldsymbol{x})) \geqslant 0.65 \\ & s_3(z_3(\boldsymbol{x})) \geqslant \lambda \\ & \lambda \in [0,1] \\ & \nu_i(z_i(\boldsymbol{x})) \geqslant 0,\ i = 1,2,\cdots,15 \\ & \mu_i(z_i(\boldsymbol{x})) \geqslant \nu_i(z_i(\boldsymbol{x})),\ i = 1,2,\cdots,15 \\ & \mu_i(z_i(\boldsymbol{x})) + \nu_i(z_i(\boldsymbol{x})) \leqslant 1,\ i = 1,2,\cdots,15 \end{aligned} \tag{14}$$

The solution of problem (14) is $\boldsymbol{x} = (2.595\,286, 0, 0.964\,740, 2.266\,238, 2.173\,129, 1.664\,712, 3.272\,815,\ 0,\ 0,\ 1.010\,653,\ 1.694\,636,\ 0,\ 0,\ 1.265\,663,\ 0.059\,115)$, $\lambda = 0.558\,775$, $z_1 = -519.092\,639$, $z_2 = -453.212\,020$, $z_3 = -371.350\,523$, $s_1(z_1) = 0.865\,072 < \delta_1 = 0.95$, $s_2(z_2) = 0.65$, $s_3(z_3) = 0.558\,775$, $\sigma_1 = 0.751\,382$, $\sigma_2 = 0.859\,653$.

The value of $s_1(z_1)$ does not satisfy the termination condition (1), DM_1 changes $\delta_1 = 0.95$ to $\delta_1' = 0.91$ and solve the problem (15):

$$\begin{aligned} \max_{x,\lambda}\ & \lambda \\ \text{s.t.}\ & \sum_{k=1}^{t} \boldsymbol{A}_k\boldsymbol{x}_k \leqslant b \\ & x_j \geqslant 0,\ j = 1,2,\cdots,15 \\ & s_1(z_1(\boldsymbol{x})) \geqslant 0.91 \\ & s_2(z_2(\boldsymbol{x})) \geqslant 0.65 \\ & s_3(z_3(\boldsymbol{x})) \geqslant \lambda \\ & \lambda \in [0,1] \\ & \nu_i(z_i(\boldsymbol{x})) \geqslant 0,\ i = 1,2,\cdots,15 \\ & \mu_i(z_i(\boldsymbol{x})) \geqslant \nu_i(z_i(\boldsymbol{x})),\ i = 1,2,\cdots,15 \\ & \mu_i(z_i(\boldsymbol{x})) + \nu_i(z_i(\boldsymbol{x})) \leqslant 1,\ i = 1,2,\cdots,15 \end{aligned} \tag{15}$$

The solution of problem (15) is $\boldsymbol{x}=$ (2.608 886, 0, 0.961 929, 2.248 116, 2.197 912, 1.654 278, 3.324 670, 0, 0, 1.002 161, 1.720 411, 0, 0, 1.258 253, 0.025 461) , $\lambda = 0.537\,895$, $z_1 = -521.772\,923$, $z_2 = -454.616\,942$, $z_3 = -371.254\,030$, $s_1(z_1) = 0.91$, $s_2(z_2) = 0.700\,608$, $s_3(z_3) = 0.537\,894$, $\sigma_1 = 0.778\,453$, $\sigma_2 = 0.767\,755$.

By now, $s_1(z_1) = 0.91 = \delta_1'$, $s_2(z_2) = 0.700\,608 > 0.65 = \delta_2'$, moreover $\sigma_1 = 0.778\,453$, $\sigma_2 = 0.767\,755$ are all in the interval $[0.75, 0.9]$. That is to say, all the termination conditions of the proposed algorithm are satisfied, the DMs obtain the satisfactory solution.

3 Conclusion

This paper proposes an interactive intuitionistic fuzzy method for solving multilevel linear programming problems under the assumption of DMs' cooperative relationship. Considering the overall satisfactory balance, the DMs at upper level update the minimal satisfactory level continuously until a satisfactory solution is obtained. The numerical examples illustrate that not only the bilevel but also the multilevel linear programming problems can be solved by our proposed method.

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