Deliberative Mathematics Education for Social Democratization in Latin America

Paola Valero, Bogotá (Colombia)

Abstract: Latin America is committed to build more democratic social relationships as a part of its current democratization process. Mathematics education is a relevant set of social practices that could contribute to the consolidation of democratic social relationships in the school. This dimension of social interaction in mathematics education as a source of democratization is explored conceptually and is given a practical meaning through the discussion of an inservice teacher education program, which illustrates a deliberative democratic ideology of mathematics education.

Kurzreferat: *"Deliberative" mathematische Erziehung für eine soziale Demokratisierung in Lateinamerika.* Lateinamerika ist dabei, im Rahmen des gegenwärtigen Demokratisierungsprozesses demokratischere soziale Verhältnisse aufzubauen. Mathematische Erziehung ist ein relevanter Teil der sozialen Praktiken, die zur Festigung demokratischer sozialer Verhältnisse in der Schule beitragen können. Diese Dimension sozialer Interaktion im Mathematikunterricht als mögliche Quelle für Demokratisierung wird untersucht. Ein praktisches Beispiel zur Veranschaulichung der "deliberativen" demokratischen Ideologie mathematischer Erziehung wird durch die Diskussion eines entsprechenden Lehrerfortbildungsprogramms gegeben.

ZDM-Classification: A40, B50, C60

Introduction

One of the current concerns in Latin American countries is democracy. Democracy can be defined as an ideal way of social organization that establishes a series of political, juridical, economic and cultural values, norms and behaviors aiming at providing a better living for the whole population of a given state. This definition highlights a conception of democracy not as an actual reality, but as a goal to reach (Dahl 1989). This distinction allows discerning between theoretical or normative formulations, and the real conditions of social organizations. In this sense, democracy is "what we cannot have but, still, we cannot stop desiring" (Zemelman 1992). This definition also considers four different dimensions of democracy. The political dimension includes the series of procedures to form governments by means of regular, free elections as the corner stone of representative democracy. The juridical dimension sets and protects the different basic legal human rights and duties. The economic dimension deals with the material conditions of living and the organization of the economy by the state. And the socio-cultural dimension which considers the space where democratic values are embedded and embodied in people' s interactions (Murillo & Valero 1996).

Since the late 80's, Latin America entered the internationalization and globalization processes. Economically, they have forced the insertion of national economies in the international market and the adoption of several neoliberal policies. Politically, they have led to a transition to democracy – for e.g. in Chile and Argentina – and to a democratic progression – for e.g. in Colombia and Costa Rica. But given the negative impact of globalization and internationalization on society, recent democratization efforts have emphasized the spreading and embedding of democratic values in the cultural sphere where social relationships and practices occur. Therefore, my concern in this paper is the socio-cultural dimension of democracy, the sphere of social interactions among people in their everyday life, where common citizens communicate and build their living conditions. This discussion steps on the assumption that mathematics education can contribute to democracy in its different dimensions (Skovsmose 1994, Niss 1996, Mora 1998). But in contrast to other arguments, I claim that special attention should be paid to the basic cultural sphere of interactions among people in the social practices of mathematics education. Since it is precisely in these interactions where values, beliefs and behaviors forge ideologies that transmit and reproduce democratic (or anti-democratic) socialization patterns. In what follows, I will present a reflection about what mathematics education means when connected to democracy. Then, I will present a particular view about how mathematics education relates to democracy. And finally, through the example of an inservice teachers' professional development program, I will discuss the notions of collective, transformative, deliberative and coflective mathematics education.

Mathematics education: social practices and field of knowledge

The term "mathematics education" has at least two different connotations: one designates the social practices where the teaching and learning of mathematics actually occur, and the other refers to the field of knowledge where the scientific study of those social practices is carried out (Ernest 1998). A widespread definition of mathematics education as a field of scientific research and knowledge considers that it is the area "covering the practice of mathematics teaching and learning at all levels in (and outside) the educational system in which it is embedded" (Sierpinska & Kilpatrick 1998, p. 29). This statement also defines mathematics education as practices focused on the didactical relationship between teacher and students, which take place mainly in the context of the classroom and which have mathematical content as its constitutive element: "Thus, mathematics and its specificities are inherent in the research questions from the outset. One is looking at mathematics learning and one cannot ask these questions outside of mathematics" (p. 26). The definition given to the field of knowledge determines which practices constitute mathematics education as much as the focus that research has to adopt.

These two definitions are problematic when mathematics education for democracy is approached both from the perspective of the field of knowledge and from the realm of social practices. First, the justifications to connect mathematics education to democracy are not only found in the mathematical content, but also and mainly in the social and political factors that constitute the learning and teaching relationships in the classroom, in the school and in society. Second, and as a consequence of the latter, it is necessary

to study the context of the practices and its components. By doing so, we could gain a better understanding of what mathematics education for democracy means in other instances where the social relationships that constitute and shape mathematics teaching and learning are built. Thus, a definition of the social practices of mathematics education should include not only all the institutionalized relationships among teachers, students and mathematics at the different levels of schooling, inside and outside the educational system, but also the activity of policy makers that at a national level deal with the design of curricular guidelines for the teaching of mathematics (Woodrow 1997); the activity of writing mathematics textbooks (Dowling 1998); the complex relationships that configure the teaching of mathematics within the organizational structure of educational institutions (Perry et al. 1998); the spaces of teacher education both in its initial (Vithal et al. 1997) and further stages (Mora 1998); as well as the configuration processes of social conceptions about the role of mathematics education in society (Valero 1997). All these practices together should be potential and legitimate objects of study if we aim at understanding and, at the same time, promoting a mathematical education for democracy.

Mathematics education for democracy: an ideological nature

To further explore the nature of the social practices of mathematics education in society, I argue that they are "ideological". Since they are attached to well-organized, action-oriented belief systems, which contain both empirical claims about the nature of social practices and normative propositions about how they should be (Morrow and Brown 1994). In contrast to the negative connotation of the term in Marxism or Postmarxism, ideology here refers to these sets of ideas that are embodied and manifested in social relationships, and that express "modes of existence" through which people experience their relation to each other and the world (Held 1980, p. 186).

The social practices of mathematics education cannot be apprehended without considering the belief systems about mathematics, its teaching and learning. Such systems give an account of the nature and normativity of mathematics education. This ideological dimension is present in all the different scenarios of practice where mathematics education takes place. It is assembled historically and socially as a result of the interaction of the people who participate in mathematics education in its different scenarios. It also determines the way that the different participants in those social practices engage others and their own activity.

The connection between mathematics education and democracy can be seen from this perspective. Firstly, democratic mathematics education is an ideology that opposes a *traditional ideology* about mathematics, its teaching and learning. Tradition in mathematics education has been defined as a particular kind of interaction among teacher, students and mathematical knowledge, where mathematics is mainly procedures, the teaching is an information transmission controlled by the teacher, and learning is an acquisition of information and a mechanical training (Gregg 1995, p. 443). However, this ideology does not only assemble in the classroom, but also in all the different spheres where the social practices of mathematics education occur. Practices such as preservice and inservice mathematics teacher education, textbook writing, school organization and policy making contribute to the reproduction of that traditional situation. Secondly, this traditional ideology contradicts democratic aims because it justifies mathematics education only in terms of the importance of knowing mathematics *per se*; it generates and reinforces an absolutist view of mathematics and school mathematics (Borba & Skovsmose 1997); and it creates authoritarian relationships between the possessors of knowledge and the de-possessed.

On the contrary, democratic ideologies, in general, adopt a position in which mathematics education has a purpose that goes beyond the classroom because it provides student-citizens with tools to perform outside the school. They also admit a view of mathematics and school mathematics in connection to society both in its construction process and also in its close impact and use in social activities. And they present a view of teaching and learning as dialogical processes between teachers and students to build mathematical knowledge. Although these general features may sound reasonable, a careful analysis of some of the existing democratic ideologies in mathematics education put in evidence problems both in their underlying conception of democracy and in its implications on mathematics education.

One of those ideologies is the *elevating ideology*. It claims that it is important to raise the mathematical knowledge and capacities of people, in order to have well informed citizens who can exercise their individual rights, express their opinions and interests through elections and impact the government and society (Niss 1996). This ideology is anchored on a liberal democratic conception that defends individualism, liberty and equity (p. 27–28). The problem of this ideology is that it reinforces a view of society as a sum of free, rational individuals who interact motivated by their own will and benefit and who have the right to form part of a democratic society. This individualistic position does not suit the current Latin American needs because it promotes the increase of social unbridgeable gaps and prevents collective action from playing a role in achieving social equity. Furthermore, the mere teaching of more mathematics does not necessarily result in a more socially aware citizenry. Mathematics in itself does not possess a social critical dimension (Skott 1992). Finally, teaching more mathematics has led to extensive content-based curricula where the amount of mathematics covered in the school forces a pace of teaching and learning which goes beyond the possibilities of all learners (Serrano 1997). Therefore, the purpose of mathematics for all may end in an excluding circumstance instead of an empowering situation.

Another is the *critical ideology*. It defends the idea that, since mathematics exercises a formatting power in highly technological societies, mathematics education should contribute to build a "democratic competence" or the capacity to adopt a critical position in face of the rulers' actions and decisions. If citizens are to participate in a democratic system, they should develop the capacity to

recognize and criticize the effects of the use of mathematics in society (Skovsmose 1994). This ideology can also be examined from a Latin American perspective. The critical ideology overemphasizes the role of mathematics in society. In Latin America, the power structure has lead to a clientelist political system where decisions are made based on personal loyalty of clients to patrons, political convenience, power of conviction through the use of language or violent and physical imposition. In this "rationality", mathematics does not necessarily constitute a formatting power that greatly influences decision-making. Assuming the predominance of this power in Latin American societies could reinforce the myth of reference, or the spread idea that, when social phenomena are looked at through mathematics, they can be described in its terms and, therefore, become mathematics; and the myth of participation, or the conviction that people are handicapped to participate in society if they do not understand and are not able to use mathematics in a critical way (Dowling 1998, p. 4–11).

As an alternative to these two, a *deliberative ideology* could be proposed. In contrast to the liberal democratic tradition based on the idea that free, autonomous, productive monads – called individuals – constitute society and politics by means of a social contract (Macpherson 1982), I argue that human beings are real, historical beings who interact in order to collectively produce and transform their material living conditions (Marx and Engels 1968). This premise views human beings as social in nature, not in the sense of the liberal individuals who decide to live together, but in the sense of human beings acting, participating and existing only in connection to a collectivity or community to which they belong. Individuals do not exist without or outside society since their entire life makes part of the intricate, multilateral relationships established among people to create their material conditions of living. As Lave (1996, p. 149) states: "being human is a relational matter, generated in social living, historically, in social formations whose participants engage with each other as a condition and precondition for their existence".

The core of this notion of human beings is *action* since it is what provokes the production of living conditions. We could characterize this action by describing its main features. First, it is *collective* since it requires the collaboration and equal participation of the people engaged and committed to the process of constructing the world were they live. Second, it is *transformative* given the fact that it has as its main purpose to constantly forge such a world, and that such a continuous production implies modifying previous conditions to generate new ones. Third, it is *deliberative* because it is mediated by a communicative process in which people interact to consider attentively and carefully the pros and cons of their decisions before making them, and the reasons or lack of reasons for their opinions and judgements, before actually expressing them (Valero 1995). And finally, it is *coflective* because it rests on a collective process of reflection. "Reflection" comes from the Latin reflexus which has two components: the prefix "re" which means back or again and the word "flexio" that means bending. Reflection as a whole, then, means bending back. A connotation of the word is related to the individual meta-thinking process by means of which one bends on one's thinking and actions in a conscious way. It is an individual process since it is the person herself who bends back her thinking about her own self. Coflection – "co"-"flection" – is the word that refers to the meta-thinking process by means of which people, together, bend on each other's thoughts and actions in a conscious way. That is, people together think about the actions they undertook, but also adopt a critical position towards them.

If we adopt this assumption about the nature of human beings and the characteristics of human actions, we could define the cultural dimension of democracy in the sphere of people's or citizens interactions. Then, democracy is a type of social relationship where collective, transformative, deliberative and coflective actions are undertaken by people in order to generate and improve their material and social living conditions. As a central part of these democratic relationships, there is the construction of a communicative process, which allows the establishment of a shared language for which all participants are responsible and in whose creation and modification all participants have a role to play.

This idea is a resource for democracy in Latin America, in contrast to the current predominant relationships of exclusion, suppression, inequality, violence and even physical extermination that predominate in our countries. And it comes to be so, because precisely this sense of collective production of well-being still needs to be emphasized in societies where the state has not built the minimum appropriate living conditions for the majority of the population, but where most of the people themselves have to provide these basic material and social conditions.

What does this concept of democracy mean to mathematics education? The reason why mathematics education is relevant in the search for democratic social relationships is not only because mathematics has a crucial role to play in societies, and therefore in the student-citizens' competencies, but also and mainly because its social practices have predominantly been shaped according to a traditional ideology. As a result, they reproduce undemocratic interaction patterns in all the different scenarios where they take place. In other words, the justification of the connection between mathematics education and democracy is not to be found exclusively in mathematics and its impact in society. Since mathematics does not exercise a strong formatting power in Latin American societies, the justification for the connection will come precisely from the practices of mathematics education themselves, understood as that complex and broad set of social activities, suggested earlier. The interactional dimension, as a basic constituent of mathematics education practices, has to be brought into focus because it is the sphere where general ideologies are forged and, with them, social values, beliefs and behaviors connected to the teaching and learning of mathematics in the school.

Deliberative teacher education: a case of teachers' professional development in Colombia

My intention now is to illustrate how the deliberative ideology came into existence in an inservice teacher education program. I have chosen this set of practices since it plays a relevant role in the reproduction and maintenance of antidemocratic, traditional ideologies in mathematics education. If teachers do not live transformative educational experiences that challenge their traditional ideologies, democratic mathematics teaching and learning will keep on being exceptional and not wide-spread, regular practices in schools.

PRIME I: A professional development program in a context of reform

PRIME I was an inservice teacher education program carried out by a team of five teacher educators from "una empresa docente", the research center on mathematics education at the Universidad de los Andes in Bogotá, Colombia. It aimed at designing, developing and evaluating a professional development strategy for mathematics teachers and administrators in Colombian schools. It also intended to support the current national educational reforms in the teaching of mathematics, which demand the creation of critical school communities that can interpret the general curricular guidelines stated by the Ministry of National Education and undertake the design and development of institutional curricula according to the particular necessities of their learners. This professional development strategy was conceived as an alternative to traditional inservice mathematics teacher training where the passive reception of information, the delivery of effective teaching formulas and the increase of teachers' mathematical knowledge are emphasized. In contrast, it opened an interaction between a pair of secondary mathematics teachers from each of the 15 public and private participant schools, the whole group of the 30 participant teachers and the 5 teacher educators.

This interaction was based on carrying out a small-scale action research project on the teachers' practice in their schools. Action research, conceived as a methodology to carry out a critical, systematic inquiry about one's teaching, in order to understand and modify it, was a powerful tool for teachers to get engaged in a learning process based on didactical problem solving about their own practices. Supporting this inquiry process, teachers and teacher educators got involved in a series of seminars and advisory meetings and in the writing of a short paper reporting the experience. In their action research projects, each pair of teachers chose a topic from the syllabus of their courses at that time, the teaching of which they wanted to improve. During eight months, each pair of teachers completed a curricular design for a maximum of three class sessions, put it into practice, observed its implementation, and evaluated it. Teachers' projects focused on topics like the teaching of linear equations in one unknown (Moreno & de Castellanos 1998, Rivera & Barón 1998), approaching the concept of variable with eighth grade students (Mena & Moreno 1998), geometrical representation of algebraic expressions (Nieto & Oliveros 1998), and the teaching of trigonometrical functions to tenth grade students (Lascano & Ramirez 1998).

Now, let us analyze the assumptions of such a professional development strategy and its practical implications, and how it reflects the key principles of the democratic deliberative ideology.

Collective and transformative action

An important assumption of this professional development strategy was that teacher qualification does not happen in individual isolation. In other words, the increase of individual teacher's capacities does not necessarily imply an improvement in the teaching of mathematics in the school since this teaching is the result of a series of connected institutional factors. First, although teachers have a given professional knowledge, a determined set of beliefs about mathematics, its teaching and learning, and a particular commitment to their practice, these features depend on the professional culture of the group of mathematics teachers in the school. This culture, which is both activated and consolidated through collective activities such as professional interaction, professional qualification and curricular design, is influenced by the administrators' leadership for opening opportunities for teachers to effectively engage in that collective activity (Perry et al. 1998). Therefore, the teaching of mathematics in a school is the result of the activity of a "community of practice" (Lave 1996, p. 150) that builds the conditions for teachers' teaching and learning, in a continuous process of professional interaction focused on the specific issues of the didactics of mathematics.

This principle meant that collective action was central to the development of the teacher education strategy. This collective action began inside each school, with the pair of teachers who had the task of undertaking their small action research project. Following the characteristics of this type of inquiry as suggested by Kemmis & Mctaggart (1992), teachers selected their research question according to their real needs and shaped its definition until reaching a very concrete and precise aspect of their teaching that they could approach within their time and resource restrictions. The pair of teachers went through the inquiry process as a team. This basic team in some cases also attracted the attention of the rest of the mathematics teachers in the school, who participated and got involved directly or indirectly in the project. Finally, the interaction among all the 30 participant teachers in the professional strategy also provided a discussion space to give and receive help from colleagues with similar professional interests.

The cooperation that occurred in the schools by means of the inservice education program was in many cases the activation point of professional interaction inside the group of mathematics teachers (Castro et al. 1997). As Mena & Moreno (1998) expressed: "... we would like to highlight that, from the point of view of the institution, this research work helped consolidating a work team that decided to formalize meetings to coordinate, plan and implement professional development activities and to follow up and evaluate the teaching-learning of mathematics in the school" (p. 110)

In fact, the main goal of inservice education aiming at professional development is to consolidate a professional

community of practice, which embodies a professional culture in which values, views and behaviors are shared. It connects with the deliberative ideology since it breaks with the idea that teachers' learning is mainly individual instead of a social cognitive process happening inside the school.

Deliberative and coflective interaction

Since teachers' actions were the paramount of the project, the 5 teacher educators agreed on the following principle. Inservice teachers hold a rich didactical knowledge, more or less explicit, which comes from their teaching experience. Therefore, teacher educators did not aim at imposing neither theoretical knowledge, nor the reading of specialized bibliography. The need for those two things was expected to arise from the teachers' desire to clear out their own doubts (Valero et al. 1998, p. 19). Given the fact that teachers' knowledge was privileged, the relationship teacher educator/teachers did not reproduce the typical relationship of knowledge-possessors/knowledge-deprived, which primes in traditional teacher education environments. Teacher educators consciously adopted the role of equal partners whose function was to question and promote coflection among teachers. As a teacher expressed: "[In] the interaction with the teacher-educators ... I had the opportunity to ask questions and to raise issues to which, most of the time, there was no answer; but I finally understood that it has to be like that if the main contribution really has to be made by us [the teachers]" (Perry et al. 1996, p. 28).

These assumptions manifested in practical terms when teachers faced a process of didactical problem solving, the activity that teachers consciously carry out in order to tackle the difficulties found in their interaction with the students when building mathematical knowledge in the classroom (Perry et al. 1998). This problem solving does occur not only in the sphere of the individual teacher's practice in the classroom, but also inside the institutional community of practice to which teachers belong. This type of activity promoted deliberation and coflection among teachers. The whole inquiry process invited the pair of teachers to make decisions, analyze the pros and cons of possible courses of action, find justifications to undertake a particular path to a solution, and negotiate the final actions to be carried out. Moreno & de Castellanos (1998), describing the process to identify their research problem, say:

> "At the beginning of the project a diagnostic test was applied to the tenth grade students in order to identify the typical mistakes that show the possible difficulties students have in solving systems of linear equations in three unknowns, using the substitution method. The choice of the topic considered the request of the physics teacher who commented on the necessity of reinforcing this solution method and provide the students with tools that could improve the application of it in solving problems in that school subject. Analyzing the students' answers, we observed that the process to solve these systems was applied correctly, but there were mistakes in finding the unknowns and precisely those led the students to an incorrect solution to the system. Therefore, we chose to redefine the object of our study as 'Solving first grade equations with one unknown' " (p. 89–90)

This fragment exemplifies part of the deliberative process that teachers had to constantly engage in. As they had to justify their choices not only to each other, but also to their colleagues and to the teacher educators given the permanent discussion among all the participants of the pairs' work, they needed to make explicit all the assumptions they where stepping on and the reasons why they decided to change their original ideas. After an analysis of what they found in the students, these two teachers had a strong evidence to support a transformation of their initial topic which has been suggested as a result of a previous preoccupation.

Another important element going hand in hand with deliberation is coflection. This thinking together on the own actions was a source for both questioning and, consequently, transformation of previous decisions. A teacher said: "Some activities (de)stabilized us; but this was good because they allowed us to think about our teaching practices and to remember some things that because of the routine and the lack of time we forgot when interacting with our students ... [we understood] what our students may feel in a similar situation" (Perry et al 1996, p. 27). An example of how coflection happened is one of the strategies teacher educators used to discuss the teachers' curricular design to be applied during the implementation phase of their projects. When discussing a particular teaching activity or sequence designed by the teachers, a teacher educator played the role of a low-skilled student and tried to express her thinking aloud for the couple to realize a possible reaction from a student. Then, a coflection was set among the three people (the two teachers and the teacher educator) about the reasons why the student could come with such an answer. Then, they built together a critique to what had originally been proposed and suggested possible alternatives that, again, were analyzed until reaching a final, supported decision. This type of interaction turned out to be a trigger for coflection since "from the teacher educator's reflections (playing the role of a student), that allowed to examine minutely the project, arose doubts and strategies to tackle our problem" (p. 32).

Connected to the problem of democratic social relationships, deliberative and coflective interactions among the teachers helped raising a critical consciousness about the practices of mathematics teaching in the schools. Together with the realization of the need for collective action, teachers began questioning their beliefs about mathematics, its teaching and learning and this is considered to be an important step not only in teachers' professional development, but also in the beginning of a transition from traditional to deliberative democratic ideologies about school mathematics education.

A day to become true?

A final reflection about the ideas presented will address a question: Does mathematics education have a "day to become true" (Valero 1997) in Latin American countries? If so, two main issues deserve some critique: one is whether experiences as those described above are sustainable and, another is whether adopting a deliberative ideology of mathematics education and acting accordingly, could be an alternative for a democratic mathematics education.

A partial view of the professional development strategy implemented in PRIME I was presented above. It is partial since it is highlighting some characteristics, which could be different from the traditional style of inservice education that predominates in Colombia. The intention of the example was not to show a "perfect", unproblematic model for teacher education. Nevertheless, this presentation does not intend to suggest that implementing this alternative view was an easy and absolutely successful task. The program faced tensions having to do with the expectations of all the participants on what a teacher education program should be; with the participants' view of research and inquiry as the support of a collective, coflective and deliberative interaction process and with the balance between the interaction itself and the content of the interaction (Valero et al. 1998, p. 27–33). These tensions, when solved inappropriately by the teacher educators, led some teachers to make the decision of quitting their action research projects and, consequently, the PRIME I project. This point suggests that it is problematic to put into practice an innovative, deliberative inservice teacher education as this. There are obstacles arising from the time constraints that all participants face to make the necessary collective work required, the school organization restrictions to allow more permanent professional interaction among teachers, the demands that funding agencies who support these inservice programs want to see as the product of their investment and the teacher educators themselves (Gómez et al. 1998). Therefore, we could question whether large-scale applications of this kind of programs could be sustained for a long time in a large number of schools and teachers.

This leads us to the other critical point of whether spreading a deliberative ideology about mathematics education and acting consistently could contribute to more democratic social relationships in Latin America. The main obstacle to overcome is precisely the dominance of traditional ideology in all the spheres of mathematics education in our countries. The close connection between, on the one hand, the socially shared values and beliefs about mathematics and its teaching-learning in the school, and, on the other hand, the actual behavior of people in what constitutes mathematics education practices establishes an almost unbreakable wall. Social processes, in this sense, seem to be more "reflexive" – a social, subconscious process that is not decided upon or monitored by any democratic institution (Skovsmose 1998) – than really "reflective" and maybe even less "coflective". Then, it seems that most of the discourses about mathematics education for democracy, as well as the general discussion of democracy in its political, juridical, economic and cultural dimensions could easily stay in the realm of an idealistic philanthropy that is far from becoming true in reality. Nevertheless, proposing concrete actions that could shake traditional ideologies and move the actors of the social practices of mathematics education towards more deliberative, democratic ideologies is necessary. And even more necessary is the study of the mechanisms of these social processes in order to understand why and how mathematics education can be democratized. This could be a small contribution to make, from mathematics education both as practices and as a scientific discipline, to the several problems and challenges of democracy in Latin America.

Acknowledgments

I would like to thank R. Vithal, P. Perry, P. Gómez, V. Mesa, M. Castro, O. Skovsmose, J. Skott, K. Jess, D. Mora and E. Emborg for their comments and suggestions. This paper is part of the research initiated by the Centre for Research in Learning Mathematics, Denmark.

References

Borba, M.; Skovsmose, O. (1997): The ideology of certainty in mathematics education. – In: For the Learning of Mathematics 17(No. 3), p. 17–23

Castro, M.; Perry, P.; Valero, P.; Gómez, P.; Agudelo, C. (1997): La interacción profesional como elemento dinamizador en el desarrollo profesional de los profesores de matemáticas. – In: R. Farfán (Ed.), Resúmenes de la Reunión Latinoamericana de Matemática Educativa – RELME I. Morelia (México), July 14–19, 1997, p. 74

Dahl, R. (1989): La Poliarquía: Participación y Oposición. – Madrid: Tecnos

Dowling, P. (1998): The Sociology of Mathematics Education: Mathematical Myths/Pedagogic Texts. – London: Falmer Press

Ernest, P. (1998): A postmodern perspective on research in mathematics education. – In: A. Sierpinska; J. Kilpatrick, Mathematics Education as a Research Domain: A Search for Identity. Dordrecht: Kluwer, p. 71–86

Gómez, P; Perry, P; Castro, M; Valero, P. (1998): Los profesores de matemáticas como investigadores. La problemática de la formación permanente. – In: Memorias del III Congreso Iberoamericano de Educación Matemática CIBEM III. Caracas, July 26–31, 1998

Gregg, J. (1995): The tensions and contradictions of the school mathematics tradition. – In: Journal for Research in Mathematics Education 26(No. 5), p. 442–466

Held, D. (1980): Introduction to Critical Theory. Horkheimer to Habermas. – Cambridge: Polity

Kemmis, S; Mctaggart, R. (1992): Cómo Planificar la Investigación Acción. – Barcelona: Laertes

Lascano, M.; Ramírez, R. (1998): Una estrategia didáctica para la enseñanza de la función coseno. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 125–134

Lave, J. (1996): Teaching, as learning, in practice. – In: Mind, Culture, and Activity 3(No. 3), p. 149–164

Macpherson, C. (1982): La Democracia Liberal y su Epoca. – Madrid: Alianza

Marx, K.; Engels, F. (1968): A contribution to the critique of political economy. – In: Selected Works in One Volume. London: Lawrence and Wishart

Mena, M.; Moreno, E. (1998): Aproximaciones al concepto de variable. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 101–111

Mora, D. (1998): Probleme des Mathematikunterrichts in Lateinamerikanischen Ländern – explorative empirische Studie zur Entwickung didaktischer und curricularer Innovationsansätze im Kontext der Educación Popular am Beispiel Nicaragua und Venezuela. – Universität Hamburg, Doctoral Dissertation (http://www.sub.uni-hamburg.de/disse/05/)

Moreno, I.; de Castellanos, L. (1998): Secuencia de enseñanza para solucionar ecuaciones de primer grado con una incógnita. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 89–100

Morrow, R.; Brown, D. (1994): Critical Theory and Methodology. – London: Sage, Vol. 3

Murillo, G.; Valero, P. (1996): De una democracia restringida hacia una participativa: el peligro de la contra-reforma y la regresión en Colombia. – In: E. Diniz (Ed.), O Desafio da Democracia na América Latina. Rio de Janeiro: IUPERJ, p. 493–511

Nieto, P.; Oliveros, M. (1998): Representación geométrica de expresiones algebraicas. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 112–116

Niss, M. (1996): ¿Por qué enseñamos matemáticas en la escuela? – In: L. Puig; J. Calderón (Eds.), Investigación y Didáctica de las Matemáticas. Madrid: Visor – MEC, p. 19–30

Perry, P.; Valero, P.; Castro, M; Gómez, P.; Agudelo, C. (1996): Reporte de Investigación del Proyecto PRIME I. – Bogotá: Universidad de los Andes, "una empresa docente"

Perry, P.; Valero, P.; Castro, M.; Gómez, P.; Agudelo, C. (1998): La Calidad de las Matemáticas en Secundaria. Actores y Procesos en la Institución Educativa. – Bogotá: "una empresa docente"

Rivera, H.; Barón, G. (1998): Una propuesta para la enseñanza de ecuaciones algebraicas de primer grado. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 118–124

Serrano, H. (1997): Educación y democracia. – In: Matemáticas y Educación No.1, p. 1–6

Sierpinska, A.; Kilpatrick, J. (1998): Mathematics Education as a Research Domain: A Search for Identity. – Dordrecht: Kluwer

Skott, J. (1992): Matematiske Modeller i Fagdidaktisk Belysning. Et Rids af en Didaktisk Debat. – Danmarks Lærerhøjskole, Pædagogiske Kandidat Opgave

Skovsmose, O. (1994): Towards a Philosophy of Critical Mathematics Education. – Dordrecht: Kluwer

Skovsmose, O. (1998): Linking mathematics education and democracy: citizenship, mathematical archaeology, mathemacy and deliberative interaction. – In: ZDM Zentralblatt für Didaktik der Mathematik 30(No. 6), p. 195–203

Valero, P. (1995): Ciudadanía Deliberativa: Hacia una Educación Cívica en Colombia. – Universidad de los Andes, Tesis de pregrado

Valero, P. (1997): A day to be true. Mathematics education for democracy in Colombia. – In: Chreods No. 11, p. 49–61

Valero, P.; Perry, P.; Castro, M.; Gómez, P.; Agudelo, C. (1998): Desarrollo profesional de directivos y profesores: motor de la reforma de las matemáticas escolares. – In: P. Perry (Ed.), Experiencias de Desarrollo Profesional en Matemáticas. Un Apoyo para la Reforma en la Escuela Secundaria. Bogotá: "una empresa docente", p. 3–38

Vithal, R; Parras, J; Zuma, Z; Desai, S.; Ramdas, R.; Samsukal, A.; Gcabashe, E. (1997): Student teachers doing project work in primary mathematics classrooms. – In: P. Kelsall; M. de Villiers (Eds.), Proceedings of the Third National Congress of the Association for Mathematics Educators of South Africa. Durban, July 7–11, 1997

Woodrow, D. (1997): Democratic education: does it exist – especially for mathematics education? – In: For the Learning of Mathematics 17(No.3), p. 11–16

Zemelman, H. (1992): La democracia limitada y los excesos teóricos. – In: Pablo González; Marcos Roitman (Eds.), La Democracia en América Latina: "Actualidad y Perspectivas". Madrid: Universidad Complutense de Madrid – Centro de Investigaciones Interdisciplinarias en Humanidades de la U.N.A.M. México, p. 91–102

Author

Valero, Paola, The Royal Danish School of Educational Studies, 101 Emdrupvej, DK-2400 Copenhagen NV, Denmark.
E-mail: paola@dlh1.dlh.dk