# Linking Mathematics Education and Democracy: Citizenship, Mathematical Archaeology, Mathemacy and Deliberative Interaction

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**Abstract:** The relationship between mathematics education and democracy is discussed in terms of citizenship, mathematical archaeology, mathemacy and deliberative interaction. The first issue concentrates on the learner as a member of society; the second on the social functions of mathematics and on how to get to grips with mathematics in use; the third refers to an integrated kind of competence including different forms of reflection (mathematics-oriented, model-oriented, context-oriented and lifeworld-oriented reflections); the fourth issue considers the classroom as a micro-society and deals with the nature of the teaching-learning process. These four issues are discussed with reference to an example of educational practice, "Our Community", carried out among sixteen-year-old students as an interdisciplinary project including a one-week trainee service. Finally, it is indicated that a discussion of mathematics education and democracy is essential to a further development of social theory, as the notions of citizenship, mathematical archaeology, mathemacy and deliberative interaction become part of the discussion about modernity, reflexive modernity and other constructs from recent social theory.

**Kurzreferat:** *Eine Verknüpfung von Mathematikunterricht und Demokratie: Bürger in der Gesellschaft, mathematische Archäologie, "mathemacy" und "deliberative" Interaktion.* Die Beziehung zwischen Mathematikunterricht und Demokratie wird im Hinblick auf die Aspekte Bürger in der Gesellschaft, mathematische Archäologie, "mathemacy" (mathematische Bildung) und "deliberative" Interaktion diskutiert. Der erste Aspekt bezieht sich auf den Lernenden als Mitglied der Gesellschaft, der zweite auf die sozialen Funktionen der Mathematik. Bei dem dritten Aspekt geht es um eine umfassendere Kompetenz, die verschiedene Formen von Reflexion miteinschließt (mathematikorientierte, modellorientierte, kontextorientierte und lebensweltbezogene Reflexionen). Beim vierten Aspekt geht es um das Wesen von Lehr-Lern-Prozessen, wobei Unterricht als Mikrogesellschaft angesehen wird. Diese vier Aspekte werden im Hinblick auf ein unterrichtspraktisches Beispiel "Unsere Gemeinde" diskutiert, das einem fächerübergreifenden Projekt mit sechzehnjährigen Schülern entstammt. Schließlich wird gezeigt, daß eine Diskussion von Mathematikunterricht und Demokratie wesentlich für die Entwicklung einer Sozialtheorie ist, wenn der Bürger in der Gesellschaft, mathematische Archäologie, "mathemacy" und "deliberative" Interaktion Bestandteile der Diskussion um die Moderne, reflexive Moderne und andere Konstrukte der derzeitigen Sozialtheorie werden.

**ZDM-Classification:** A40, C60, D30

## Introduction

In *Reflexive Modernization*, Ulrich Beck, Anthony Giddens and Scott Lash discuss social development in terms of modernity, individualisation, post-tradition, reflexion, risk society, and several other constructs from recent social theory. In separate chapters each author presents an analysis; and in *The Reinvention of Politics*, Beck interprets the industrial society as an expression of modernity. However, the very success of industrialisation brings with it a development towards a different kind of modernity: "This new stage, in which progress can turn into self-destruction, in which one kind of modernisation undercuts and changes another, is what I call the stage of reflexive modernisation" (Beck, Giddens and Lash, 1994, p. 2). This development is not decided upon or monitored by any democratic institution. It is a social, "sub-conscious" process. In this way reflexive modernisation brings us into a risk society (Beck, 1992).

If we study the index of *Reflexive Modernization*, we do not find any reference to "mathematics". However, Beck's chapter contains the following observation: "Risks flaunt and boast with mathematics" (Beck, Giddens and Lash, 1994, p. 9). To me, this comment is crucial, but in *Reflexive Modernization* it is left as an insignificant side remark. A discussion of mathematics and mathematics education with respect to democracy does not appear to be considered significant for a deeper understanding of today's societies. Instead works like *The Theory of Communicative Action I–II* by Jürgen Habermas and *The Constitution of Society* by Anthony Giddens seem to subscribe to the idea that mathematics is "harmless"; a thesis which has been put forward by G. H. Hardy in A Mathematician's Apology. Hardy devoted his life to "pure" mathematics and did not find any link between this occupation and "worldly matters".

I shall maintain that the opposite is the case: An analysis of mathematics, mathematics education and democracy is essential to an interpretation of social development and to a discussion of, for instance, modernity, and reflexive modernisation. I shall return to this point in my conclusion. In what follows, however, I shall first of all concentrate on issues which relate mathematics education and the problems of democracy.

For this discussion, my involvement in a mathematics education project in South Africa has meant considerable inspiration to me.[1] In a significant way the development in this country has raised issues about democracy and education.[2]

## Democracy

The linguistic roots of "democracy" are found in Greek. *Demos* means "people", and *cratos* means "rule"; so, literally speaking, "democracy" means "ruled by people". This conceptual clarification, however, raises two fundamental questions: What does "ruling" mean? And: Who are the people? "Ruling" can refer to "legislation", but it can also refer to the actual process of carrying out decisions. In ancient Greece, the answer to the latter question was "free men"; slaves and women were not included.

Two extremes in the interpretation of "democracy" can be illustrated by the ideas of Joseph A. Schumpeter and Jean-Jacques Rousseau. In *Capitalism, Socialism and Democracy*, published in 1943, Schumpeter suggests an interpretation of democracy which is clear and simple. To him the role of the people is to produce a government, and he defines the democratic methods as "that institutional arrangement for arriving at political decisions in which individuals acquire the power to decide by means of a competitive struggle for the people's vote" (Schum-

peter, 1987, p. 269). This definition makes the American campaign for presidency an exemplary case of the democratic method. This method does not include people being involved in the actual discussion of political matters. The democratic method takes the form of a "competitive struggle", and the actual content of people's democratic activities is to cast their votes. According to Schumpeter, this elitist interpretation of democracy is important, because it is an illusion to assume that "the people" can actually take part in a rational process of deliberation and decision-making.

The opposite idea is exemplified by Rousseau's definition of democracy, put forward in *The Social Contract*, first published in 1762. Rousseau discusses the notion of "sovereignty": Who is the sovereign? In a monarchy the answer is: The king. But who, then, is the sovereign in a democracy? Rousseau's answer is simple and straightforward: The people! According to Rousseau, it should be possible for the people actually to participate in the ruling, and therefore his interpretation exemplifies direct democracy, while Schumpeter's definition represents the extreme of indirect democracy. In *The Social Contract*, Rousseau analyses conditions for democracy, and he takes the view that this form of governing can only exist in small communities. Democracy is not for "normal" societies.

In recent discussions, democracy certainly refers to formal procedures for electing – being of a government or of any other kind of ruling body. But it has also been emphasised that democracy is subject to the fulfilment of certain conditions: a fair distribution of social services and goods, equal opportunities and obligations for every member of society; opportunities for citizens to participate in discussions and decision-making.

The interpretation of democracy I want to suggest does not follow the line set out by Schumpeter. Instead it is inspired by the notion of direct democracy. To me, democracy concerns not only questions of governing a state but of governing institutions of any kind. Furthermore, "ruling" presupposes "citizenship" involving a variety of forms of "participation". Thus, "democracy" comes to mean "a way of life". This brings us to the notion of *deliberative democracy* which "refers to the idea that legitimate lawmaking issues from the public deliberation of citizens" (Bohman and Rehg (Eds.), 1997, p. ix). It "presents an ideal of political autonomy based on practical reasoning of citizens" (Bohman and Rehg (Eds.), 1997, p. ix). From this perspective it certainly makes new sense to explore the relationship between democracy and education.[3]

## Democracy and education

The Second World War raised the questions: How could Nazi barbarism emerge in the heart of Europe? Did education have a responsibility in not trying to prevent the development of authoritarianism? The claim of critical education is: Fundamental undemocratic developments must be challenged by education! In fact, the development of "critical education" can be seen as an educational attempt to provide a new foundation for education for citizenship.[4]

However, already in *Democracy and Education* from 1916, John Dewey integrates a broad perspective on democracy with an interpretation of education. He elaborates this perspective in subsequent writings. According to Dewey's pragmatic interpretation of science, nothing can be taken for granted. Science must refuse any form of dogmatism. Every question must be investigated with fresh eyes, and such a process, particularly as developed in empirical sciences, represents a process of inquiry. Basically, this process is initiated by a problem and based on experience. A research process can, however, also be interpreted as a process of learning. "It is a cardinal precept of the newer school of education that the beginning of instruction shall be made with the experience learners already have; that this experience and the capacities that have been developed during the course provide the starting point for all further learning" (Dewey, 1963, p. 74).

In his introduction to John Dewey's *On Education*, Reginald D. Archambault summarises Dewey's point of view concerning democracy, the scientific method and learning in the following way: "Dewey saw democracy as the political manifestation of scientific method, with its combination of purposiveness and objectivity, freedom and discipline, individual speculation and public verification. ... The aim of education is the development of reflective, creative, responsible thoughts. Hence, Dewey's whole conception of science, and its methods and its aims is directly relevant to education" (Dewey, 1974, p. xvii–xviii). The scientific method bridges over education and democracy. Education which organises itself in line with an inquiry process becomes "education for democracy".

That education has a social role to play can also be observed "negatively". During the apartheid period of South Africa, Fundamental Pedagogics was developed in order to help "justify" the apartheid system. The dictatorship found it essential to involve education in such a task, and Fundamental Pedagogics became education for non-democracy.[5] An important step to be taken in the education of the new South Africa is precisely to re-develop education as part of a democratic endeavour. Still, we have to keep in mind that whatever undemocratic aspects of global development we might refer to, the ultimate in barbarism against democratic life took place in central Europe.

Dewey makes two claims: (1) Education and democracy form aspects of the same discussion; and (2) the scientific method provides the important bridge between education and democracy. To me, Dewey's first claim is as actual as ever, certainly illustrated by the development in South Africa. However, I find Dewey's second claim problematic; also when we take a look at mathematics education.

## Democracy and mathematics education

If mathematics is interpreted as language, the speech act theory of language (see Austin, 1971, and Searle, 1969) will raise the question: What can be done by means of mathematics? Mathematics can be interpreted not only as a descriptive tool, but also as a source for decision making and action. This brings into focus the notion of "symbolic power", discussed by Pierre Bourdieu (1991), and the theme of "knowledge and power". Michel Foucault has concentrated much of his work on unmasking the interplay between "knowledge" and "power" (see, for in-

stance, Foucault, 1980, and Kelly (Ed.), 1994). I suggest that these more general discussions should be specified with mathematics in mind. A careful study of knowledge (in terms of mathematics) and power is still waiting to be carried out. This is, however, the programme of *aporism* (see Skovsmose, 1998).[6] Mathematics as a possible codification of power is not spelled out by the recent philosophy of mathematics, being neo-empirical or constructionist. While waiting for such a clarification we can, however, consider the thesis of the *formatting power of mathematics:* Social phenomena are structured and eventually constituted by mathematics.[7]

The claim is not that this thesis is true, but that the thesis expresses a possible truth, and that this possibility is important to consider when mathematics and mathematics education are investigated from a social and political point of view. Furthermore, the formatting power of mathematics is suggested to play a double role: Mathematics constitutes technological wonders, but technological catastrophes as well (D'Ambrosio, 1994). In this way, mathematics operates in the middle of social development.

Nor is the claim that the thesis is simple. Naturally, it does not make sense to claim that mathematics *per se* has a formatting power. The thesis concerns mathematics in context. Social, political and economic interests can be pursued by means of the powerful language of mathematics. In this way the thesis of the formatting power becomes a thesis of the existence of interplay between mathematics as a source of technological decision-making and action and other sources of social development.

The possibility of such an interpretation of mathematics has, as already mentioned, not been part of the general sociological discussion; here mathematics appears "harmless" and insignificant. To Dewey, the scientific method as such represents a unification of research, learning and democratisation, and therefore it certainly has a social role to play. However, Dewey's interpretation of this method and his whole perspective of science do not allow him to observe a possible double role of science. Dewey's interpretation of "education and democracy" is based on complete reliance on the scientific method as a faultless supporter of democratic thinking. This is a problematic assumption, certainly also when mathematics is considered. We have, therefore, to look for other ways of bridging over mathematics education and democracy than by means of scientific methodology.

Mathematics education might serve very different social functions, and fundamental questions have to be raised concerning the actual social organisation of mathematics education.[8] As already emphasised, it is not the actual truth of the thesis of the formatting power of mathematics which makes it relevant to the discussion of mathematics education. The relevance is produced by the challenge: Does mathematics education produce critical readers of the formatting? Or does mathematics education prepare a general acceptance of the formatting, independent of the critical nature of the actual formatting?

Essential functions in the technological society depend on how competence in mathematics is distributed by means of the educational system. Mathematics education can serve as a "blind" instrument for providing the mathematical competence in a form that "accommodates" to the present technological development. I call a social or political act "accommodating" in so far it does not challenge any aspects of the predominant distribution of power or any power relationships. The structure of the educational system can make sure that the mathematical competence is distributed in such a way that, for instance, the adequate number of people needed in developing the information technology in fact receive sufficient mathematical competence.

Mathematics education can also make sure that the "inverse competence" is in place and distributed in an "accommodated" way, meaning that a sufficient number of people come to understand that mathematics is not their business (see, for instance, Wedege, in print). Excluding a certain number of people from competence can also be "useful", as a potential group of critics is eliminated. In his address to the South African Senate in 1954 Hendrik Verwoerd claimed: "When I have control over Native education I will reform it so that the Natives will be taught from childhood to realise that the equality with Europeans is not for them ... People who believe in equality are not desirable teachers for Natives ... What is the use of teaching the Bantu mathematics when he cannot use it in practice" (Here quoted from Khuzwayo, 1997, p. 9).

Exclusion from mathematics can mean social and political suppression, and, ultimately, exclusion from society. In *Mathematics by All*, John Volmink writes: "Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an "objective" judge, in order to decide who in the society "can" and "cannot". It therefore served as *the* gate keeper to participation in the decision-making processes of society. To deny some access to participate in mathematics is then also to determine, *a priori*, who will move ahead and who will stay behind" (Volmink, 1994, p. 51–52).

In other words: Awareness of the politics of mathematics education leads to a discussion of conditions for equity and democratic life. Much research in mathematics education has, however, ignored this discussion. For instance, in *Mathematics Education as a Research Domain: A Search for Identity* we only find one reference to democracy. This, however, is a relevant one. In his chapter *A Postmodern Perspective,* Paul Ernest enumerates ten interdisciplinary themes which are a shared concern for both science and mathematics education, one of them being: "Critical citizenship, epistemological empowerment and enhanced democracy through science and mathematics for all" (Ernest, 1998, p. 82).

## "Our Community"

What I have tried so far is to indicate that it is possible, from a general perspective, to relate mathematics education and a discussion of democracy. However, what could such a linkage mean from the perspective of classroom practice? With reference to one example of educational practice, I shall discuss the following four issues:

- citizenship
- mathematical archaeology

– mathemacy
– deliberative interaction.

My intention is to illustrate how these issues, on the one hand, concern classroom practice in mathematics education, and, on the other hand, concern democracy.[9]

The project, "Our Community", took place in a comprehensive school in the village of Hinnerup in Denmark. Fifteen students, about sixteen years old, were involved in the project, which was planned, organised and carried out by two teachers, Jørgen Boll and Jørgen Vognsen. The students were all in the tenth form, which is the last form before they leave secondary school. The intention of the project was to give the students an idea of some of the conditions and possibilities, not only for living and working in a local community, but also for influencing its social and political life.

In Denmark it is quite normal for students to practise different sorts of jobs for a short period of time as part of their school education. Such a one-week trainee service introduced the project, "Our Community", and the students worked as porters, teachers, environmental supervisors, unskilled labourers, librarians, technical assistants, kindergarten assistants, etc. Besides participating in the jobs, the students had to clarify some general questions, formulated by the teachers, about their workplace, like: How is the workplace organised? Who makes the decisions? The idea was that the students should not only experience the actual nature of the work, but also get an impression of how the workplace was organised.

After the trainee service, the students, working in groups, concentrated on selected problems of Hinnerup. Two of them were: Studying the population forecast for Hinnerup; and making suggestions with respect to the economic situation of the local Music School.

*Population forecasting:* The students had gathered statistics specifying the number of people in the different age groups of the Hinnerup District. Based on these statistics they could produce their own forecast and compare it with the available official forecast. It became apparent that some decrease in the group of young people would take place, and that not every school in the Hinnerup District could expect to get a tenth form in the future. More general implications of the forecast concerned the programme of house and school building and the capacity of kindergartens and rest homes.

*The Financing of the Music School:* This school gives lessons to students interested in practising music. The Music School is organised as a private school, which means that the expenses are not covered by the Government but by the parents of the students who take lessons. However, in many other towns in Denmark the local authorities support the music schools financially. The students had got the necessary information about the economy of the Music School, and they made different budgets and experimented with alternatives forms of financing. The results of the calculations were organised as recommendations actually handed over to the head of the Music School. The students' investigations, however, continued. It was calculated what it would mean, in terms of increased taxes, if the authorities decided to support the Music School. It was discussed whether local organisations, other than the Music School, could also argue for the right to receive financial support. In this way, emphasis was placed on the fact that it does not make sense in a democracy to take care of an isolated case without discussing whether a particular decision has general consequences or not. It was also estimated what more extensive general financial support to different cultural activities in Hinnerup would mean for the level of taxes.When the project was about to finish, the Mayor visited the class. He was a former teacher, and as the Hinnerup District is small, his visit could be organised without many difficulties. Different topics of particular interest to young people in Hinnerup were raised and discussed. It was obvious that during the project work the students had gained much information relevant to such a discussion; they had more ideas about structures of decision-making in a local community and about economic possibilities and limitations.

**Citizenship**

The project ,"Our Community", shows that mathematics education for democracy could simply mean *learning about the local community* of which you are a member. Such a learning may provide *citizenship*. Learning for democracy does not only mean learning about constitutions, rules for election, etc. It also means being involved in democratic processes.

"Our Community" relates to the work of Marilyn Frankenstein (1989), who shows how a study of statistics and figures, concerning, for instance, unemployment, may develop an understanding of the society of which the students are members. She tries to show how relearning of mathematics can support people in developing a critical competence in living in a society, in which many decisions are made with reference to mathematics or attempts are made to justify them with reference to certain calculations, reliable or not.

That this is also the case in a small community, is illustrated by "Our Community". The students worked with population forecasting and observed how such figures could be used in justifying decisions about house building programmes and the planning of schools and kindergartens. The students also had the opportunity to get an impression of the reliability of such figures, as, for instance, they were based on certain expectations of people moving in or out of the district. The students saw how the organisation of the Music School and the kind of activities which it could offer depend on the policy of payment. They could make experiments, using spreadsheets, with alternative forms of payment including new assumptions about the number of students, the amount of fees, support from the district, etc. They could use mathematical tools to investigate alternatives, and in this way get an impression of the basis upon which actual decisions were made. The idea was to make mathematics, as a language of power, accessible to critical discourse.

The aim of "Our Community" was not to make the students accept social facts. Citizenship does not only imply being ready to live in and to face the "output" from authorities. It also means providing an "input" to authority,

a "talking back" to authority. Education for citizenship, therefore, also presupposes participation. In the project this was experienced in the most direct way, as the students in fact met with a person in power. That this person happened to be the Mayor is naturally sheer coincidence, but the general idea is clear: learning by doing makes sense, also when we talk about being engaged in a local community.

To me, citizenship means being involved in such activities as illustrated by "Our Community". But citizenship can also be analysed in more theoretical terms, one of these is "empowerment", another is *Mündigkeit*. The German word *Mündigkeit* has a double meaning. The term relates to legal theory referring to a person coming of age and thus acquiring an adult's rights and responsibilities in society. However, the term also has an informal meaning: that of having the capacity to speak for oneself. This notion was developed as part of a theoretical frame of critical education in order to contrast the idea that education should produce "followers" (see, for instance, Adorno, 1971). During the 1930's, too many followers were produced by European education. The present project shows that *Mündigkeit* can be given a specific interpretation, such as the students being able to participate in political discussions taking place in a local community. And, most important, *Mündigkeit* also includes competence in investigating decisions with mathematically formulated arguments.[10]

**Mathematical archaeology**

Social structuration[11], at macro or at micro level, often takes place with reference to mathematics. Such an interplay between knowledge and power is referred to by the thesis of the *formatting power of mathematics*. As mathematics is basically an "invisible" part of social structuration, we need strong analytical tools to capture the role of mathematics.

This leads to the notion of *mathematical archaeology*. By this activity, I understand the process of excavating mathematics which might be encapsulated in certain political arguments, technologies or administrative routines. As part of the ethnomathematical research programme, many instances of mathematical archaeology have been carried out. Ethnomathematical studies have emphasised that mathematics in many forms is present in "traditional" societies, and that mathematical competences are frozen in routines, techniques and handicrafts, in all kinds of ordinary life. (For an overview, see Gerdes, 1996 and Powell and Frankenstein (Eds.), 1997.) However, to me this does not seem to be a phenomenon that is specific to "traditional" societies. Different forms of mathematics are frozen in different societies, although the actual role of the frozen mathematics may be very different. A distinction can be made between a mathematical archaeology which tries to identify mathematics which has been "frozen" as part of a technological design, a procedure for decision-making, etc., and an archaeology which searches for mathematical patterns in activities not originally "formatted" by mathematics. The last type of studies has been a main focus for ethnomathematical research.

When mathematics exercises a formatting power, citizenship presupposes that this power is excavated, and a mathematical archaeology becomes a useful tool. This activity is illustrated by the project, "Our Community", both with respect to the population forecast and the budget for the Music School. The reliability of arguments for certain decisions concerning house building programmes became accessible to critical investigation only after mathematical archaeology had made it clear to the students what kind of mathematics-based argumentation was supposed to support the decisions.

**Mathemacy**

The notion of "literacy" has been developed by Paulo Freire to mean much more than just being able to read and write. Literacy also includes competence in interpreting social life. The notion of *mathemacy* can be developed in a similar way to mean more that an ability to calculate.[12]

In order to do this, it is important to pay attention to the notion of *reflection*. When a calculation, based on a mathematical task, is carried out, it is possible to reflect on the actual result. Such a reflection has mathematical concepts and algorithms as its objects. A question guiding such a *mathematics-oriented reflection* can be: Are the calculations made correctly?

When setting up a mathematical model, in order to solve a non-mathematical problem, we face many difficulties. Thus, any modelling process presupposes that certain simplifications are established. This means paying attention to certain aspects of "reality" and neglecting others. This is, for instance, what takes place when a population model is used for the purpose of forecasting. Reflections referring to a modelling process are of a different kind than mathematics-oriented reflections. While this kind of reflection has mathematics calculations as its object, a *model-oriented reflection* has the relationship between mathematics and an extra-mathematical reality as its object. A question guiding a model-oriented reflection can be: Is the output of the modelling process reliable? Model-oriented reflections are carried out with an interest in improving the model, i.e. with a technological interest. Such reflections concern the validity of the model.

A modelling can, however, also be considered from a more general perspective and with a different interest. The guiding questions could be: What is the actual purpose of carrying out the modelling? What, in fact, is the political and social function of applying mathematics to a certain situation? A reflection guided by such questions can be called a *context-oriented reflection*. "Context" is here understood as a political, social or cultural context. The mathematical archaeology, carried out as part of the project, "Our Community", illustrates how context-oriented reflections become possible, when the latent functions of the model become unearthed. Such reflections try to address the issue of "mathematics and power".[13]

Beck distinguishes between two processes: reflexion and reflection (see Beck, Giddens and Lash, 1994, p. 5–8). As mentioned above, reflexion refers to a social process which, so to speak, takes place outside the democratic

institutions of society. This is the process by means of which the successful industrial society produces an input to itself. This effective production means precisely that a new social form is created: a risk society. According to the thesis of the formatting power of mathematics, this reflexitivity is also an expression of a mathematical activity. Beck emphasises that reflection is different from reflexion. Reflection means reconsidering what is taking place. Reflection is a mental, a conscious or a theoretical activity. In my terms: It is a critical activity and a process of grasping basic processes of social development. If reflexive modernisation has mathematics as constituent, then reflections with respect to mathematics become of particular importance. This brings the notion of reflection into focus in an attempt to redefine mathemacy so that this competence gets a critical potential. Tentatively, mathemacy can be understood as a kind of competence in which mathematics-oriented, model-oriented and context-oriented reflections are brought together as an epistemic unit.

I want to constitute mathemacy in parallel with literacy, in such a way that mathemacy becomes a kind of competence for acting in the world structured by mathematics. In "Our Community", I saw students expressing this kind of competence when they were engaged in community problems and, in fact, were able to understand some of the principles in decision making and to challenge some of the decisions actually made. The notion of citizenship, then, becomes refined in terms of mathemacy.

A different kind of reflection, however, must also be considered. A study made by Mathume Bopape (1997) of children's attitudes towards mathematics in schools in South Africa shows an interesting phenomenon. Even though the children were considered "bad" students by the teachers and even though their school was located in a problematic area, the children expressed a very positive attitude towards learning mathematics. They simply considered it important to develop mathematical skills. Some of them referred to the fact that they might be able to help their parents in solving problems having to do with money and business. The children expressed a remarkable responsibility for learning mathematics. This can also be seen as a manifestation of a reflection related to mathematics, although not a mathematics-oriented, model-oriented or context-oriented reflection. This different kind of reflection could be termed a *lifeworld-oriented reflection.*[14] Also this kind of reflection forms part of the competence I call "mathemacy".

There is no point in trying to classify all different kinds of reflections. A variety of reflections do exist, and this *grand family of reflections* becomes an important aspect of mathemacy as a constituent of citizenship.

**Deliberative interaction**

It is not difficult to find examples of a teacher-student communication which is ritualised into fixed procedures: The teacher asks a question; the students raise their hands; the teacher asks a particular student; this student gives an answer; the teacher corrects the student if necessary; the teacher asks a new question...

*Classroom absolutism* refers to the phenomenon that communication between students and teacher is structured by the assumptions that mathematics (school mathematics) can be organised around exercises and questions which have one and only one correct answer, and that, ultimately, it is the teacher's job to make sure that mistakes are eliminated from the classroom (see Alrø and Skovsmose, 1996a). This absolutism can coagulate into a ritualised form of communication (which might appear to be efficient if, in fact, the job of the mathematics teacher is to eliminate mistakes).

Giddens's chapter in *Reflexive Modernization* has the title *Living in a Post-Traditional Society.* Here Giddens discusses the notion of tradition, and he defines a "formulaic notion of truth" (see Beck, Giddens and Lash, 1994, p. 63). Such truths have guardians. Considering mathematics education, we can re-interpret classroom absolutism in these terms, and a stereotype of communication is exercised when the guard makes sure that the students reproduce the formulaic truths. To me, classroom absolutism brings along a form of communication which is hostile to the development of a democratic culture in the classroom. Education for democracy cannot be based on stereotypes of teaching-learning practices dominated by guardians and formulaic truths. It is important to make possible an interaction in the classroom which supports dialogue and negotiation. *Deliberative interaction* should be a possibility.

With reference to Habermas, Joshus Cohen describes the "ideal deliberative procedure" in terms of freedom, reason, equality and consensus. This procedure captures "the notion of justification through public argument and reasoning among equal citizens" (Cohen, 1997, p. 72). Deliberative interaction can be defined as an interaction which resonates with an ideal deliberative procedure.

It is essential for the teacher to be aware of the student's good reasons in order to escape the paradigm of classroom absolutism. Even if an answer from a student might be different from what the teacher expects, the answer might be adequate when seen in relation to the student's preunderstanding. An awareness of this can be a useful starting point for a dialogue between teacher and student. In the article *The Student's Good Reasons,* Helle Alrø and I describe an *inquiry cooperation model.* This model is developed with reference to a teacher-student communication. The model includes different elements: active listening, where the teacher and the student *get in contact;* a process of *discovering* and *identifying* the student's good reasons for thinking the way he or she does; a process of the student *thinking aloud* and, in this way, puts his or her ideas and reasons forward in the dialogue; a *reformulation* by the teacher in order to make sure that he or she understands what the student says; a *challenge* of the student's good reasons; a *negotiation* of the teacher and student perspectives; a shared *evaluation* of the (possible) good reason. The inquiry cooperation model refers to a pattern of communication where the teacher and the student meet in a shared process of coming to understand each other. In this way the inquiry cooperation model exemplifies deliberative interaction.

A learning theory could take the form of a dialogical epistemology which could bring together two discussions. First, it could help to unite different forms of reflection and in this way provide a basis for analysing how mathemacy may be developed. Second, it could be a theory of democratic life in a classroom and therefore suggest how certain forms of communication might support the development of *Mündigkeit* and citizenship. This brings the notion of deliberative interaction to the heart of a learning theory of mathematics, and in this way we may come to understand the relationship between reflections (mathematics-oriented, model-oriented, context-oriented, lifeworld-oriented), mathemacy, citizenship and *Mündigkeit*.

Classroom communication does take place in a school setting which puts limitations to the structure of the communication. The students and the teacher are easily assigned particular roles, and the interaction between these actors becomes predescribed by a didactical contract. In order to make it possible for communication in the classroom to become a dialogue, it might be essential to frame the school situation in new ways. I have talked about "scene setting", acknowledging that the interaction in fact takes place in a school (see Skovsmose, 1994). "Our Community" can illustrate an effort in providing a different frame for learning. The students were actually getting a trainee training. They were learning about community by participating in the community life. They were, for a while, released from the roles of students. Back in school, they were facing "real" numbers, and they were interacting with real authority. Deliberative interaction becomes a real possibility.

## Conclusion

*Citizenship, mathematical archaeology, mathemacy* and *deliberative interaction* are all relevant issues when the relationship between mathematics education and democracy is considered. The first issue concentrates on the learner as a (new) member of society; the second on the social functions of mathematics and on how to get to grips with mathematics in use; the third refers to an integrated kind of competence; the fourth considers the classroom as a micro-society and deals with the nature of the teaching-learning process. As I have tried to illustrate, these four issues concern both mathematics education and democracy. In this way, I try to confirm Dewey's claim that it is important to link the discussion of education and democracy, but I also try to demonstrate that this linkage can never be established only in terms of a scientific method.

Let me now return to the question: Why is it important to discuss mathematics, mathematics education and democracy in order to get a deeper understanding of today's societies? According to the thesis of the formatting power, mathematics is an essential instrument when technological authority is exercised. Mathematics is part of technological empowerment, and, as already indicated, mathematics may play a double role in society, being a means for technological and economic development as well as a source of technological catastrophes. Industrialisation is linked to technologies integrating mathematics, and as industrialisation links with modernity so does mathematics. As the risk society can be associated with the further development of mathematically based technologies, the notion of reflexive modernisation can be analysed in terms of the formatting power of mathematics. Therefore, the discussion of citizenship, mathematical archaeology, mathemacy and deliberative interaction becoems important to a discussion of modernity, reflexivity, risk society, and other concepts from recent sociology.

Mathematics and mathematics education are put in the middle of social development, being within or outside the reach of democratic institutions, but nevertheless essential to a discussion of democracy. In my book *Towards a Philosophy of Critical Mathematics Education*, I describe the problem of democracy in a highly technological society referring to the difficulty of matching a critical competence which can match the actual social and technological development (Skovsmose, 1994, p. 40). This difficulty is also touched upon by Lash in his chapter *Reflexivity and its Doubles* in *Reflexive Modernization*. Here he writes: "Citizenship's rights in simple modernity, featuring equality before the law, political rights and social rights of the welfare state, have been transformed into reflexive modernity's rights of access to the information and communication structures. Reflexive modernity's new lower class, which is increasingly in many respects effectively an underclass, is deprived on both obligations and rights of what now is no longer social but predominantly cultural citizenship" (Beck, Giddens, Lash, 1994, p. 133). The question is: Can mathematics education provide an adequate response to this situation? Mathematics education is part of the development of reflexive modernity, and, as remarked by Volmink, mathematics education also serves as a gatekeeper: Who does and who does not get access to the information and communication structures? What role does mathematics education play in the production of "reflexive modernity's new lower class"? What other roles could mathematics education play?

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**Annotations**

[1] This project involves the following institutions: University of Durban-Westville, Aalborg University and the Royal Danish School of Educational Studies.

[2] This paper is partly based on my plenary lecture *Mathematics Education and Democracy* at the 6th Annual Meeting of the Southern African Association for Research in Mathematics and Science Education (SAARMSE), Pretoria, 14–17 January, 1998

[3] Paola Valero has drawn my attention to the notion of deliberative democracy as being of particular relevance to the discussion of mathematics education. See also the paper by Valero in part 2 of these analyses in Zentralblatt für Didaktik der Mathematik 1999/1. For a general discussion of "deliberative democracy", see Bohman and Rehg (Eds.), 1997.

[4] As part of the early discussion of critical education, inspired by Critical Theory, the notion of *politische Bildung* was crucial. However, *politische Bildung* cannot adequately be trans-

lated to "political education" as the German word Bildung has other connotations than "education". *Politische Bildung* refers to a wide range of issues concerning, for instance, "empowerment", "sociological imagination" and "autonomy". An important work in this area is Negt, 1964. See also Paffrath (Ed.), 1987. For a recent discussion of education and democracy see, for instance, Giroux, 1989 and Young, 1989.

[5] For a further discussion of Fundamental Pedagogics, see for instance Khuzwayo, 1997.

[6] This whole discussion of mathematics, mathematics education and power also has to be related to the "mathematical myths" as described by Dowling 1998.

[7] For a discussion of the formatting power of mathematics see Skovsmose, 1994. See also Keitel, 1989, 1993, Keitel, Kotzmann and Skovsmose, 1993, and the discussion of the "prescriptive use of mathematics" in Davis and Hersh, 1988.

[8] An essential step in developing the "politics of mathematics education" is taken by Mellin-Olsen (1987). See also Borba and Skovsmose, 1997, Fasheh, 1982, 1996, Niss, 1994, Noddings, 1993, Powell and Frankenstein (Eds.), 1997, Tate, 1996 and Woodrow, 1997.

[9] In my lecture at the SAARMSE conference I also referred to another example, described in Paras, 1997. At present I find one example sufficient to illustrate my points. It should be kept in mind, however, that many examples, developed in different political and cultural contexts, could serve the same purpose. A full description of the project "Our Community" is found in Skovsmose, 1994, p. 141–154.

[10] I must emphasise that I am not using the example, "Our Community", to make any empirically based claim about what the students involved in this projects actually did learn. I only use the references to the project to illustrate how a notion, like citizenship, can relate to both a discussion of democracy and a discussion of mathematics education. This is also the case for the next notions I want to discuss.

[11] According to Giddens: "The basic domain of study of the social sciences, according to the theory of structuration, is neither the experience of the individual actor, nor the existence of any form of societal totality, but social practices ordered across space and time" (Giddens, 1984, p. 2). And further: "Analysing the structuration of social systems means studying the modes in which such systems, grounded in the knowledgeable activities of situated actors who draw upon rules and resources in the diversity of action contexts, are produced and reproduced in interaction" (Giddens, 1984, p. 25).

[12] For a discussion of mathemacy, see D'Ambrosio, 1985. See also Skovsmose, 1994 and Skovsmose and Nielsen, 1996.

[13] In Christiansen, Nielsen and Skovsmose, 1997 (in Danish) a distinction is made between different kinds of reflections similar to the one made between mathematics-oriented, model-oriented, and context-oriented reflections.

[14] The notion of "lifeworld" is discussed in detail in Habermas, 1984, 1987. See also Skovsmose, 1994, p. 152–154, for a discussion of "reflective knowing" as an "open concept".

**Bibliography**


Adorno, T. W. (1971): Erziehung zur Mündigkeit. – Frankfurt am Main: Suhrkamp

Alrø, H.; Skovsmose, O. (1996a): On the Right Track. – In: For the Learning of Mathematics 16(1), p. 2–9, 22

Alrø, H.; Skovsmose, O. (1996b): The Students' Good Reasons. – In: For the Learning of Mathematics 16(3), p. 31–38

Austin, J. L. (1971): How to Do Things with Words. – Oxford: Oxford University Press

Beck, U. (1992): Risk Society. – London: Sage (First German edition 1986)

Beck, U.; Giddens, A.; Lash, S. (1994): *Reflexive Modernization*. – Cambridge: Polity Press

Bohman, J.; Regh, W. (Eds.) (1997): Deliberative Democracy: Essays on Reason and Politics. – Cambridge, MA: The MIT Press

Bopape, M. (1997): The Geometry Worksheet in the Air! – Copenhagen: Royal Danish School of Educational Studies, Department of Mathematics, Physics, Chemistry and Informatics (Research Report; 9)

Borba, M.; Skovsmose, O. (1997): The Ideology of Certainty – In: For the Learning of Mathematics 17(3), p. 17–23

Bourdieu, P. (1991): Language and Symbolic Power, edited with an introduction by John B. Thompson. – Cambridge: Polity Press

Christiansen, I. M.; Nielsen, L.; Skovsmose. O. (1997): Ny Mening til Begrebet Refleksion i Matematikundervisningen. – In: J. C. Jacobsen (Ed.), Refleksive Læreprocesser. Copenhagen: Politisk Revy, p. 173–190

Cohen, J. (1997): Deliberative and Democratic Legitimacy. – In: J. Bohman; Regh, W. (Eds.), p. 67–91

D'Ambrosio, U. (1985): Social-Cultural Bases for Mathematics Education. – Campinas: UNICAMP

D'Ambrosio, U. (1994): Cultural Framing of Mathematics Teaching and Learning. – In: R. Biehler; R. W. Scholz; R. Strässer; B. Winkelmann (Eds.), Didactics of Mathematics as a Scientific Discipline. Dordrecht: Kluwer Academic Publishers, p. 443–455

Davis, P. J.; Hersh, R. (1988): Descartes' Dream: The World According to Mathematics. – London: Penguin Books

Dewey, J. (1963): Experience and Education. – New York: Macmillan Publishing Company (First edition 1938)

Dewey, J. (1966): Democracy and Education. – New York: The Free Press (First edition 1916)

Dewey, J. (1974): On Education: Selected Writings, edited and with an introduction by Reginald D. Archambault. – Chicago: The University of Chigago Press

Dowling, P. (1998): The Sociology of Mathematics Education. – London: The Falmer Press

Ernest, P. (1998): A Postmodern Perspective on Research in Mathematics Education. – In: A. Sierpinska; J. Kilpatrick (Ends.), Mathematics Education as a Research Domain: A Search for Identity. Dordrecht: Kluwer Academic Publishers, p. 71–85

Fasheh, M. (1982): Mathematics, Culture and Authority. – In: For the Learning of Mathematics 3(2), p. 2–8

Fasheh, M. (1996): The Main Challenge: Ending the Occupation of our Minds; The Main Means: Building Learning Environments and Re-contextual Knowledge. – In: PDME III. Landås: Caspar Forlag, p. 13–26

Foucault, M. (1980): Power/Knowledge: Selected Interviews and Other Writings 1972–1977, edited by Colin Gordon et al. – Brigthon: The Havester Press and New York: Pantheon

Frankenstein, M. (1989): Relearning Mathematics: A Different Third R – Radical Maths. – London: Free Association Books

Gerdes, P. (1996): Ethnomathematics and Mathematics Education. – In: A. Bishop et al. (Eds.), International Handbook of Mathematics Education. Dordrecht: Kluwer Academic Publishers, p. 909–943

Giddens, A. (1984): The Constitution of Society: Outline of the Theory of Structuration. – Cambridge: Polity Press

Giroux, H.A. (1989): Schooling for Democracy: Critical Pedagogy in Modern Age. – London: Routledge

Habermas, J. (1984, 1987): The Theory of Communicative Action. – London: Heinemann and Cambridge: Polity Press

Hardy, G. H. (1967): A Mathematician's Apology, with a foreword by C. P. Snow. – Cambridge: Cambridge University Press

Keitel, C. (1989): Mathematics and Technology. – In: For the Learning of Mathematics 9(1), p. 7–13

Keitel, C. (1993): Implicit Mathematical Models in Social Practice and Explicit Mathematics Teaching by Applications. – In: J. de Lange et al. (Eds.), Innovation in Maths Education by Modelling and Applications. Chichester: Ellis Horwood, p. 19–30

Keitel, C.; Kotzmann, E.; Skovsmose, O. (1993): Beyond the Tunnel-Vision: Analysing the Relationship Between Mathematics, Society and Technology. – In: C. Keitel; K. Ruthven (Eds.), Learning from Computers: Mathematics Education and

Technology. Berlin: Springer, p. 243–279
Kelly, M. (ed.) (1994): Critique and Power: Recasting the Foucault/Habermas Debate. – Cambridge, MA: The MIT Press
Khuzwayo. H. (1997): Mathematics Education in South Africa: A Historical Perspective from 1948–1994). Copenhagen: Royal Danish School of Educational Studies, Department of Mathematics, Physics, Chemistry and Informatics (Research Report; 7)
Mellin-Olsen, S. (1987): The Politics of Mathematics Education. – Dordrecht: Reidel Publishing Company
Negt, O. (1964): Soziologische Phantasie und exemplarisches Lernen. – Frankfurt am Main: Europäische Verlagsanstalt
Niss, M.(1994): Mathematics in Society. – In: R. Biehler et al. (Eds.), Didactics of Mathematics as a Scientific Discipline. Dordrecht: Kluwer Academic Publishers, p. 367–378
Noddings, N. (1993): Politicizing the Mathematics Classroom. – In: S. Restivo et al (Eds.), Math Worlds: Philosophical and Social Studies of Mathematics and Mathematics Education. Albany, NY: State University of New York Press, p. 150–161
Paffrath, F. H. (Ed.) (1987): Kritische Theorie und Pädagogik der Gegenwart. – Weinheim: Deutscher Studien Verlag
Paras, J. (1997): Improving the Play Ground: A Fence Building project in Mathematics. – Copenhagen: Royal Danish School of Educational Studies, Department of Mathematics, Physics, Chemistry and Informatics (Research Report: 11)
Powell, A. B.; Frankenstein, M. (Eds.) (1997): Ethnomathematics: Challenging Eurocentrism in Mathematics Education. Albany, NY: State University of New York Press
Rousseau, J.-J. (1968): *The Social Contract*. – London: Penguin Books (First French edition 1762)
Schumpeter, J. A. (1987): Capitalism, Socialism and Democracy. – London: Unwin Paperbacks (First edition 1943)
Searle, J. (1969): Speech Acts. – Cambridge: Cambridge University Press
Sierpinska, A.; Kilpatrick, J. (Eds.) (1998): Mathematics Education as a Research Domain: A Search for Identity. – Dordrecht: Kluwer Academic Publishes
Skovsmose, O. (1994): Towards a Philosophy of Critical Mathematical Education. – Dordrecht: Kluwer Academic Publishers
Skovsmose, O. (1998): Aporism: Uncertainty about Mathematics. – In: ZDM Zentralblatt für Didaktik der Mathematik 30(3), p. 88–94
Skovsmose, O.; Nielsen, L (1996): Critical Mathematics Education. – In: A. Bishop et al. (Eds.), International Handbook of Mathematics Education. Dordrecht: Kluwer Academic Publishers, p. 1257–1288
Tate, W. F, (1996): Mathematizing and the Democracy: The Need for an Education that is Multicultural and Social Reconstructionist. – In: C. A. Grant; M. L. Comez (Eds.), Making Schooling Multicultural: Campus and Classroom. Englewood Cliffs, NJ: Prentice Hall, p. 185–201
Volmink, J. (1994): Mathematics by All. – In: S. Lerman (Ed.), Cultural Perspectives on the Mathematics Classroom. Dordrecht: Kluwer Academic Publishers, p. 51–68
Wedege, T. (in press): To Know – or Not to Know – Mathematics, that is a Question of Context. – In: Educational Studies in Mathematics, Special Issue
Woodrow, D. (1997): Democratic Education: Does it Exist – Especially for Mathematics Education? – In: For the Learning of Mathematics 17(3), p. 11–16
Young, R. (1989): A Critical Theory of Education. – New York: Havester Wheatsheaf

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## Vorschau auf Analysethemen der nächsten Hefte

Für die Analysen der Jahrgänge 31 (1999) bis 32 (2000) sind folgende Themen geplant:
– Mathematik und Deutsch
– TIMSS
– Mathematikdidaktische Forschung im Primarbereich
– Mathematik an Hochschulen lehren und lernen
– Analysis an Hochschulen
– Mathematik in der Ingenieurausbildung
– Theoretische Betrachtungen zu Schulbuchanalysen.

***Vorschläge für Beiträge zu o.g. Themen erbitten wir an die Schriftleitung.***

## Outlook on Future Topics

The following subjects are intended for the analysis sections of Vol. 31 (1999) to Vol. 32 (2000):
– TIMSS
– Research in primary mathematics education
– Teaching and learning mathematics at university level
– Calculus at universities
– Mathematics and engineering education
– Concepts and issues in textbook analyses.

***Suggestions for contributions to these subjects are welcome and should be addressed to the editor.***