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ORIGINAL PAPER

The role of digital technologies in mathematics education: purposes and perspectives

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Abstract
Over the last decades, digital technologies (DTs) have become ubiquitous in mathematics education. Still, their integration into classroom teaching and learning varies enormously. In this narrative overview, we focus on the different purposes for which DTs are used in mathematics education in order to study how the effectiveness of DTs depends on researchers' underlying goals and perspectives. We set up an experience- and literature-based framework including five different purposes. Applying this framework gave rise to the following results: (1) there is evidence for the benefit of using DTs for mathematics learning; (2) research on DTs leads to new theoretical developments and (3) to new design paradigms; (4) issues of equity with respect to access to and use of DTs are important but under-researched; and (5) DTs challenge curricula and teaching and assessment practices. While early research on the use of DTs focused on questions such as "does it work?" or "does it work better?", the maturing of the field has shifted to more nuanced questions. As a future research agenda, we recommend further study of how the use of DTs in mathematics education impacts the time required for learning as well as the temporality of teaching and learning, how it changes the nature of doing mathematics and the relation to basic skills and higher-order skills in particular, how curricula, teaching practices, and assessment might change due to the availability of sophisticated mathematical tools, how DTs and other resources might be combined in teaching and learning, and how they may help to address equity issues in education. These questions will prompt the development of new theoretical constructs and approaches.

Keywords Digital technologies · Mathematics education · Purpose

1 Introduction

For millennia, humans have been developing and using tools to facilitate the execution of tasks. Such tools have changed the practices they were designed for. From a historical point of view, digital technologies (DTs) have emerged only recently, but they have quickly and drastically impacted our lives and our society. It is hard to imagine that only 3 decades ago, many of us did not use computers, had no email access and did not have a smartphone offering important functionalities beyond making phone calls.

Clearly, mathematics education is not excluded from these developments. What have been considered as key mathematical skills for a long time—drawing graphs, solving algebraic equations, making geometrical constructions, representing statistical data—can now be outsourced to sophisticated DTs offering graphs, symbolic computation, dynamic geometry, and data representation. Mathematical knowledge—including skills that were considered sophisticated or even 'magical', such as finding antiderivatives—has become embedded in DTs, much like navigating has become embedded in smartphones. Technological developments continue at high speed, as the recent generative language models for mathematics using artificial intelligence show, even if the explanations are still open for improvement.

The accelerated development of DTs in mathematics education is not only manifest in artificial intelligence tools, but also in multitouch technology, tools for augmented, virtual and mixed reality, 3D printers, and software for distance teaching, which proved valuable during the school lockdowns caused by the COVID-19 pandemic. While recent studies have summarized trends in research on DTs in

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mathematics education (Chen et al., 2020; Herfort et al., 2023; Inglis & Foster, 2018), these developments raise many questions for the field of mathematics education: how to integrate them in mathematics teaching? How do they impact learning? How will they change curricular goals and how will they change the rationale for teaching mathematics? In this narrative review, we focus on an overarching question that often goes unaddressed in research papers, namely: why would one want to use DTs in mathematics education? What are the main purposes of doing so, and what are the underlying assumptions being made about the nature of mathematics, the nature of learning and the goal of education?

2 Theoretical background

To address this "why" question, let us consider the notion of purpose in some more detail. In several studies, Ainley and colleagues have theorized the notion of purpose from a student perspective (Ainley & Pratt, 2002; Ainley et al., 2006; Ainley, 2012). The authors highlight the importance of learners experiencing the purposeful nature of their mathematical activity, where the purposefulness might relate to the activity having a meaningful outcome or the mathematical concepts and techniques being of utility. Similarly, applying the notion of purposefulness to the case of DTs, it would be important that students experience DTs as providing a meaningful outcome, being useful or enabling them to reach their goals.

It is clear, however, that teachers may have different objectives for using DTs than those of their students. Their purposes for integrating DTs into their teaching practices may be strongly related to their views and beliefs about the nature of mathematics and the goals of mathematics education. For example, Thurm and Barzel (2022) highlight that an important purpose of using DTs in mathematics education for teachers is to make multiple representations accessible to students, with the underlying belief that a multi-representational view is an important aspect of the targeted mathematical knowledge. To further elaborate this point about the beliefs or perspectives that underpin purposes, Bray and Tangney (2017) developed the following classification of purposes of interventions using DTs: Change in attitude, improved performance, development of conceptual understanding, skills-focused, support teachers, and collaboration and discussion. This classification shows the wide range of purposes teachers may have in including DTs in their teaching. For any given teacher, the effectiveness of DTs will be modulated by the underlying purposes.

Acknowledging that teachers and students, but also parents and employers, may perceive the purposes of using DTs in mathematics education differently, our focus in this article is that of research: we wonder how the notion of purpose is addressed in the research literature. The guiding research question, therefore, is: What purposes do mathematics education researchers have—based on what can be gleaned from the literature—for using DTs?

Inspired and informed by the literature on students' purposes and teachers' beliefs mentioned above, by our own experience of the field, and by an initial literature inventory, we set up a new, five-dimensional framework to reflect the range of purposes that motivates the use of DTs in mathematics education research. This framework is not necessarily exhaustive and may indeed change in the future (for example, a purpose that may emerge soon is whether the use of DT contributes to students' understanding of societal issues such as climate change and immigration), but we believe it captures current and significant purposes:

i. to improve mathematics learning;
ii. to understand mathematics learning;
iii. to design for mathematics learning;
iv. to provide equitable access to mathematics learning;
v. to change mathematics curricula and teaching and assessment practices.

These five purposes—which are different from the ones identified by Bray and Tangney (2017) even if the idea is similar—structure our attempt to answer the research question.

3 Methods

In line with the invitation to contribute to this Special Issue, we address the topic at stake through a narrative review, or, more precisely, a narrative overview (Green et al., 2006) of the field from the perspective of the proposed purposes framework. To do so, we focus on research published over the past 6 years (2017–present), highlighting articles that we consider particularly interesting with respect to each purpose. Naturally, their interest to us cannot be separated from the history of research on DTs and our own specific research foci and experiences.

As a starting point to explore current mathematics education research literature, we searched the SCOPUS database for papers that include the terms "digital technology" and "mathematics education" (or equivalent), and were published in 2017 or later. This led to the identification of 299 studies (March 2023), whose title, keywords and abstract we studied, as well as the reference list, aiming to identify the purposes that were addressed. We acknowledge that this search was limited to publications written in English and that this might narrow the diversity of perspectives considered in this review.

As a second step, we used snowballing, based on both the references in the SCOPUS corpus and our own knowledge of the field, to extend our corpus. Also, we identified key publications, based on citations and on our own perception of their relevance, in the light of the purpose identification.

Finally, we synthesized these results into the narratives for each of the five purposes included in the framework, that form the Results section.

4 Results

In this section, we will successively present the findings for each of the five purposes.

(i) To improve mathematics learning through digital technology

One purpose of integrating DTs in mathematics education is to improve learning. This is of course an appealing idea and has been a dominant one historically: DTs can offer new means to foster learning, to make learning more efficient and to invite deep learning. The perspective underlying this purpose seems to be the view that mathematics essentially remains unaffected through the availability of DTs, so the nature of doing and learning mathematics does not really change: while also being clearly distinct, doing mathematics using DTs is more or less similar to doing mathematics using non-digital technologies. Rather, DTs just offer new avenues to learning that may be more effective, for example in terms of learning gains, test results, or time efficiency.

Does the use of DTs work? Does it lead to learning gains, and even to higher learning gains compared to the regular teaching practices? To address this question globally, we briefly discuss two meta-review studies. Even if they are recent, they refer to older publications and are therefore helpful in summarizing what has been found with respect to learning gains in the period before our time frame of the previous five years.

The first study was a second-order meta-analysis published by Young (2017). It reports a significant positive effect of the use of DTs in mathematics education with a small to moderate average effect size of $d = 0.38$. Variations were found for different didactical roles: 0.47, 0.42, and 0.36, for computation enhancement technologies, for instructional delivery enhancement technologies, and for presentation and modelling enhancement technologies, respectively. Young (ibid.) remarks that the effect sizes seem to decrease as the quality of the meta-analyses increases, where quality refers to both the meta-analysis itself and the studies included in it. Also, the effect sizes do not significantly increase over time, despite technological tools improving, probably along with teachers' ability to use them in teaching (Drijvers et al., 2021).

A second, more recent meta-analysis on learning outcomes was published by Hillmayr et al. (2020). This study addresses both mathematics and science learning. It reports a significant medium positive effect of the use of digital tools on student learning outcomes, with an overall effect size of $g = 0.65$. This is a bigger effect compared to the Young (2017) study. The authors suggest that this increased effect might result from a further development of DTs and learning programs over recent years. For small-sample studies, larger effect sizes were found than for large-scale studies, which suggests that scaling up effective use of DTs in education is not straightforward. The provision of teacher training on digital tool use was identified as the most important success factor. Also, dynamic mathematical tools were mentioned as effective, more than hypermedia technology. Finally, the use of DTs was more effective when it was combined with other (more traditional?) instruction methods, rather than as a substitute to them, and when students worked in pairs rather than individually.

To summarize, the two meta-analyses show that the use of DTs improves learning outcomes in mathematics education, as is evidenced by significant positive effect sizes. Interestingly, the focus lies on deeper understanding and improved learning gains, rather than on more efficient teaching and learning strategies in terms of time investment, while the latter seems to be an important element in the discussion on DTs use in education in general.

Even if these meta-analyses are useful in providing an overview and synthesizing the quantitative results from many studies, they only include studies with an experimental design, and cannot provide details on why, how, when or for whom DTs worked well. This limitation, and the global perspective taken, risks putting all DTs into one single category and ignoring the circumstances under which DTs are being used. In the next section, therefore, we focus on research whose purpose is provides insight into the why, how, when or for whom, which is increasingly done through theory development.

(ii) To understand how digital technology affects mathematics learning and teaching

We can quite clearly see *that* a student or a teacher is using DTs. Theories enable researchers to dig into the "how and why and when" by guiding them to focus on certain phenomena but not others, thus acting as a tool for making some aspects of teaching and learning more visible. Theories also provide hypotheses that help to define or conceptualise phenomena—for example, what exactly do we mean by learning? In the current literature the most frequently used theories in papers published in 2017 or later can be grouped into three main categories.

First, there are *instrumental approaches* to using DTs in mathematics education, which have as a key assumption that learning processes take place in interaction with the environment through using artefacts, which are shaped by the learner but also shape the learner (Artigue, 2002; Trouche, 2004). This view on learning has consequences for teaching, since an important task for the teacher is to set up so-called instrumental orchestrations that foster students' instrumental genesis (Drijvers et al., 2010). In recent years, this instrumental approach has been further elaborated in different ways. It forms a basic framework to study the "objectification" of mathematical procedures once they are carried out by digital technology (Jankvist et al., 2019) and it has been applied to other topics, such as programming (Gueudet et al., 2022).

Papers of interest include Mithalal and Balacheff (2019), who introduced the notion of computational deconstruction to pinpoint a key point in instrumental genesis: how can one "break down" a problem in such a way that it can be outsourced to a device? Haspekian et al. (2023) elaborate on the notion of instrumental distance: the qualitative gap between the ways of approaching school mathematics in the paper-and-pencil environment and in digital environments. Soury-Lavergne (2021) writes of duos of artefacts, highlighting the interplay between using different types of tools, such as physical and digital artefacts.

The second most prevalent theory is based on the work by Papert (1980) and Noss and Hoyles (1996), and highlights the importance of viewing learning as creating, making, constructing, doing. This *constructionist view* invites us to benefit from the opportunities that DTs offer for engaging students in such creation processes. It tends to focus on student learning, coming from a constructivist set of epistemological assumptions, and less on classroom-level learning or on the teacher's role. As was the case for the instrumental approach, this constructionist approach has also evolved (e.g., see Ng & Tsang, 2021). Taking a "maker perspective", Ng and Ye (2022) describe how 11–12-year-old students explored the properties of prisms and pyramids through making these solids with a 3D printing pen. The results of the post-test, focusing on the solid's numbers of edges, vertices and faces, reveal a significant improvement compared to the pre-test with a large effect size ($d = 1.121$, $p < 0.001$). The analysis of the qualitative data suggests that the making of the solids in 3D enabled by the printing pen, in combination with the embodied nature of the activity, might explain this learning outcome. For example, students used gestures in their explanations that seem based on their concrete experiences with the 3D pen. Recent research on constructionism also focuses on the creativity students need to engage in making and problem solving (Karavakou & Kynigos, 2022).

The third most prevalent theory zooms in on *models of teacher practice and teacher learning* and professional development. The most cited one is the TPACK model on teacher learning (e.g., see Mishra & Koehler, 2006). With its components of technological, pedagogical, and content knowledge, it is an extension of Shulman's (1986) PCK model. Rather than providing a new view on learning, TPACK highlights the need for the professional development teachers will be wanting to use DTs in their mathematics teaching. Other models of teacher activity are the Instrumental Orchestration model mentioned above, and the Structuring Features of Classroom Practice framework (Bozkurt & Ruthven, 2017). Clark-Wilson and Hoyles (2019) use the Mathematical Pedagogical Technology Knowledge model to design and evaluate online teacher professional development courses. Trouche et al. (2020) stress the opportunities and issues of having so many digital and non-digital resources available, which makes a teacher's course preparation an interesting but not an easy task. Thurm and Barzel (2022) argue that teachers' practices of using DTs in their courses not only depend on their knowledge, but also on their self-efficacy beliefs and epistemological beliefs.

In addition to these three theoretical lenses, other approaches to understanding learning in a context of using DTs focus on the meaning-making process during (inter) actions with the digital tools. One approach to meaning making is through a semiotic lens. In their overview of embodiment in mathematics education, Radford et al. (2017) discuss the observation of a grade-5 student working on properties of a regular prism in terms of an embodied meaning-making process. They highlight that mathematical meanings are multimodal and stress the cognitive role of embodiment and semiotics. The latter is elaborated in the notion of semiotic bundles, in which multimodal signs come together, and which can be analyzed in synchronic way (the bundle at one moment) and in a diachronic way, that is, in their development over time. Together, such a semiotic analysis offers insights on the process of meaning making, and in the mediating role of the DTs involved. This is in line with work by Mariotti and Montone (2020), who build on the theory of semiotic mediation (Bartolini Bussi & Mariotti, 2008) to introduce the notion of synergy to study how students connect different contexts or tools/artefacts, and manage the transitions while moving back-and-forth between digital and physical (paper-and-pencil) technologies. Clearly, this resonates with the "duo of artefacts" approach mentioned above, which also has been elaborated for the case of taking historical artefacts as a starting point for learning (Maschietto et al., 2019).

Finally, some materialist theories have also emerged (de Freitas & Sinclair, 2014), which seek to question the dualisms that are inherent to most constructivist and sociocultural theories, and elaborate bodies as distributed and extended and fully implicated in socio-material ecologies of learning. Recent research includes de Freitas et al.'s (2019) study of the significance of affect in student-DTs interactions (with Wii graphs), with a particular focus on the notion of sympathy. Dimmel et al. (2021) study immersive virtual environments, focusing on the movement of participants, spatial inscriptions and environment in the production of diagrams—this approach posits DTs less as a mediator through which access to mathematical concepts is possible and more as the device that produces the concept.

To summarize, the theoretical views on learning mathematics using DTs form a diverse and somewhat scattered landscape. Sinclair et al. (2022) provide a map of some of the key influences, assumptions and elaborations of this landscape, which illustrates the dynamicity of the research field. Clearly, research on DTs whose purpose is to understand learning can also contribute to thinking about how DTs can be put into practice, to be explored and validated. This leads to the question of how to design innovative learning arrangements benefiting from DT, which we will address now.

(iii) To use digital technology to design for mathematics learning

DTs may be used for the purpose of innovative educational design. The perspective here is that DTs offer room for the design of new approaches and activities to traditional or new mathematical topics. The design can be inspired by new technological tools, as well as by new design paradigms.

As for the new tools, some DTs such as graphing calculators, dynamic geometry environments and computer algebra systems, have received much research attention and been widely implemented in classrooms over the past 40 years. There are also emerging DTs, such as multi-touch technology, motion sensors, and augmented and virtual reality devices, that are not yet common practice in regular classroom learning but can prompt exploratory "proof-of-concept" studies. These DTs may push the envelope on what counts as mathematical, or what the modalities of learning look like. As an example, Ferrara and Ferrari (2022) exploited the opportunities multi-touch technology offers for the understanding of number from a learning assemblage perspective. In their design, the DTs and the activities invite an ordinal rather than a cardinal view on number. Another type of use of multi-touch technology in geometry is described by Hegedus and Otálora (2023). As a second DT example, Nemirovsky et al. (2020) designed student activities using Wii technology to explore early algebra. As a third type of DT, several authors designed activities in which virtual reality is used (Dimmel et al., 2021; Price et al., 2020). Typically, such studies explore the opportunities virtual reality technology offers for the learning of geometry, and in particular move from 2D to 3D activities.

New DTs may also invite new design paradigms. For example, Soldano et al. (2019) highlight the opportunities to include game elements in design. A very promising design paradigm is embodied design. As an example, Shvarts et al. (2021) designed embodied learning activities that aim to develop students' body-artifact functional systems, in which an intentionality, a bodily experience, a practice of using artefacts, and a mathematical conceptualization come together. This approach builds on earlier work on embodied design, such as the study by Palatnik and Abrahamson (2018), in which the bodily experience of rhythmic movements was used to develop the notion of proportion.

To summarize, new digital tools invite the design of new types of student activity, and in this way come with new design paradigms, expressing new views on mathematics. An issue here, however, may be the limited access to these DTs, which we will discuss now.

(iv) To ensure equitable access to learning

Access to digital technology is an important factor when considering equity in mathematics education. Mathematics education research on the use of DTs has not traditionally focused on designing DTs for particular groups of students who may be minoritized for different reasons. An exception to this can be found in the work of Healy and colleagues (for example, Healy & Fernandez, 2011), who have experimented with the design of technologies particularly suited for deaf and blind learners. Emprin and Petitfour (2021) study the use of simulators to help children with dyspraxia learn geometry. The recent emergence of universal design principles has encouraged some researchers to consider how their DTs can be used by students with behavioural and physical disabilities, as discussed in Abrahamson et al. (2018).

Other equity-deserving groups of learners might include racially and socioeconomically minoritized students. Given the growing research in various forms of critical studies, it is becoming clear that taking into account the particular histories and circumstances of certain learners can have an important effect on the effectiveness of DTs for mathematics learning (e.g., see Sandoval & Trigueros, 2022). Therefore, one purpose of research on the use of DTs in mathematics education might be to study the ways in which DTs can be designed or used to support equitable access to technology and/or mathematics. A recent example of researchers facing the socio-political dynamics of mathematics education in relation to DTs is the study by Leonard et al. (2019), which focusses on designing DTs-enhanced learning opportunities that target equity-deserving students. In this case, the purpose of the research was to broaden urban students' opportunity to participate in STEM using culturally responsive instruction. The authors are less concerned with assessing student learning than with studying how the teachers were able to deploy culturally responsive instruction in the digital environment and how students experienced the use of these environments.

The issue of equitable access to DTs became acute during the COVID-19 pandemic. As many schools were closed, mathematics teachers moved en masse to Emergency Remote Teaching. The Special Issues published by *Educational Studies in Mathematics* (Chan et al., 2021) and *ZDM – Mathematics Education* (Engelbrecht et al., 2023) clearly raise the issue of equity in terms of access to the digital technology needed to attend online teaching, and the home conditions such as a quiet room, a desk, and parental support. Even if the long-term picture is not yet clear, it seems reasonable to expect learning delays for less privileged students, which is a concern, also for potential future disruptions.

(v) To change curricula, and teaching and assessment practices through the use of DT

A fifth and final purpose of using DTs in mathematics education is to foster educational change. Many current digital tools challenge the content of traditional curricula and invite considering curriculum reform. They also allow for different teaching practices, whose implementation may require teacher professional development. Finally, both summative and formative assessment practices are subject to debate, thanks to the new opportunities provided by DT. The underlying perspective of this reform purpose is that education tends to be rather static, reluctant to change, and lagging behind developments in society and work, and DTs may induce change.

Concerning *curricula*, many of them focus on "old" mathematical skills, the relevance of which can be questioned in the light of today's tools. This question is not new. Already in the early ages of computer algebra, Buchberger (1988) wondered whether it was still important for students to learn integration rules. In the same year, Heid (1988) provided a "proof of concept" for a technology-rich calculus course resequencing concepts and skills. Today, literature shows much attention to higher-order thinking skills and competencies, such as problem solving, abstraction, and modelling. For example, the PISA 2022 framework is "based on the fundamental concept of mathematical literacy, relating mathematical reasoning and three processes of the problem-solving (mathematical modelling) cycle.[1]" This problem-solving cycle includes formulating, employing and interpreting mathematics to solve problems in a variety of real-world contexts.

In line with this, a plea has been made to better connect the mathematics curriculum to computer science through the focus on computational thinking, algorithmic thinking, programming and coding. For example, in a literature-informed Delphi study Kallia et al. (2021) found that mathematical and computational thinking share common ground: they both focus on problem solving, abstraction, generalization, modelling, and algorithms. Research suggests that a further integration of computational thinking in the mathematics curriculum is desirable, like the mathematics curriculum was extended in the past to include statistics and probability.

Concerning *teaching practices*, regular whole-class teacher-centred lectures can now be replaced by other working formats, in which students are exploring and developing mathematics through DT. Clearly, this requires new teaching skills, as expressed in the TPACK framework mentioned above. In recent literature, increasing attention is paid to teacher professional development. In line with the notion of instrumental distance (Haspekian et al., 2023), teachers have to become aware of the qualitative gap between using the paper-and-pencil environment and digital environments, and find ways to bridge it. This bridging (or transition) was precisely the focus of two special issues in *Digital Experiences in Mathematics Education* published in 2023. Also, teachers nowadays have a myriad of digital and non-digital resources available to design their teaching, which may be confusing and challenging to integrate in a coherent way (Trouche et al., 2020). As an example, Sinclair et al. (2020) show how teachers may need to reconsider both their practices and their views on the mathematical notions addressed, when DTs offer new lenses or invites new practices, as was the case for the *TouchTimes* application.

A final point on teaching practices concerns the disruption that took place during the aforementioned pandemic, which in many cases involved online teaching. Many studies investigated the newly emerging teaching practices, and documented teachers' struggle to do so. For example, Huang et al. (2023) describe a case study of two teachers' emerging ERT practices in Shanghai, based on the documental approach to didactics and a resources view.

Assessment has always been a "hot topic" in the discussion on DTs in mathematics education. On the one hand, changing curricula and teaching practices (should) necessarily involve changing assessment. On the other hand, DTs come with affordances for assessment, such as automated scoring, feedback and adaptivity (Hoogland & Tout, 2018). For the case of high-stakes summative assessment using computer algebra, Leigh-Lancaster and Stacey (2022) offer an overview of the 20-year long experience in Victoria (Australia). In their examples, the authors show that having sophisticated DTs at hand does not necessarily make the assessment easy. For example, while using CAS, students needed to be versatile in recognizing equivalent

[1] https://pisa2022-maths.oecd.org/#Overview.

mathematical expressions. Also, Leigh-Lancaster and Stacey show that changing assessment is not just a technical, but also a tactical affair, which should involve stakeholders and needs time. Olsher et al. (2023) highlight the subtleties of design decisions, administration and scoring, and reporting of digital tests in mathematics, and describe the paradox of automated assessment in mathematics: easy calculation skills are the easiest ones to score automatically in a digital environment, but in the meantime are the less interesting ones now that they can be carried out so perfectly through DT. To mirror higher-order mathematical activity in a digital assessment puts high demands on the digital environment and is a challenge for the designers. To address this challenge, digital tools have been designed for assessing more specific and subtle student skills, e.g., for the case of reasoning (Yerushalmy & Olsher, 2020), quadrilaterals (Popper & Yerushalmy, 2022), or function (Ruchniewicz & Barzel, 2019), some of them focusing more on formative than on summative assessment.

5 Conclusion and discussion

In this narrative overview, we identified some of the significant purposes motivating the use of DTs in mathematics education research and their underlying perspectives, as presented in the research literature since 2017. As an organizing framework, we identified five different purposes that researchers address. These purposes entail quite different perspectives, theoretical assumptions and methodological choices. Together, this purpose framework provides a lens to describe the research area and to distinguish the different types of results and insights that are available.

Rather than closing in on specific answers, the breadth of papers cited in this article shows how new knowledge is leading to more questions and to more concerns that deserve attention. Such concerns will likely grow as more and more intelligent DTs fundamentally challenge the goals of mathematics education, as well as the place of mathematics as a school subject. Will there be specific mathematics courses in future school curricula, or will mathematics be integrated with computational thinking courses, or within multidisciplinary projects? Or will the future show that higher-order mathematical skills are the only skills that really matter, if one lives in a technology-driven society?

Our impression is that research on DTs in mathematics education was initially somewhat isolated from research on mathematics teaching and learning in general, with its own journals and conferences. Nowadays, it seems that the two worlds are finding increasing overlaps, as we can tell from the references we identified. This suggests that the field has become more mature and integrated, focusing not only on the DTs themselves, but also on how they relate to teaching practices, to assessment, to affect and to equity.

Despite this growing maturity, it goes without saying that there are still many topics to investigate, not only because of new technical developments, but also because of the growing tension between what digital technologies can do—and how they change mathematics—and the traditional nature of most curricula. As a future research agenda, we recommend further study of (1) how the use of DTs in mathematics education impacts the time required for learning—as well as the temporality of teaching and learning, (2) how it changes the nature of doing mathematics, and the relation to basic skills and higher-order skills in particular, (3) how curricula, teaching practices, and assessment may change due to the availability of sophisticated mathematical tools, (4) how DTs and other resources may be combined in teaching and learning, and (5) how the design and/or use of DTs may help address equity issues in education. These questions prompt the development of new theoretical constructs and approaches. Much work is waiting to be done, and we are curious to see what a future review of this field will look like in, say, 2050.

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**Ferrara, F., & Ferrari, G. (2022). Kindergarten children and early learning of number: embodied and material encounters within the classroom. *Digital Experiences in Mathematics Education*. https://doi.org/10.1007/s40751-022-00117-y. (This study explores the opportunities of multi-touch technology, in combination with physical paper strips, for 5-year-old childrens' understanding of number. Drawing on assemblage theory, the design invites students to explore ordinal and relational aspects of number. After using a number strip, the children work with the multi-touch <Emphasis Type="Italic">TouchCount</Emphasis> app on an iPad. The authors conclude that the notion of assemblage was helpful in noticing how movement animates the mathematical activity, and as such enriches their understanding of how the notion of number emerges from the activities).

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**Kallia, M., Van Borkulo, S. P., Drijvers, P., Barendsen, E., & Tolboom, J. (2021). Characterising computational thinking in mathematics education: A literature informed Delphi study. *Research in Mathematics Education, 23*(2), 159–187. https://doi.org/10.1080/14794802.2020.1852104. (This article investigates the relationship between computational and mathematical thinking, that, according to a literature study, seem to have common ground. Results from a literature-based Delphi study involving 25 mathematics and computer science experts highlight three important aspects of CT to be addressed in mathematics education: problem solving, cognitive processes, and transposition).

*Karavakou, M., & Kynigos, C. (2022). Mathematical creativity in coding dancing animations: a fallibilistic view. In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*. ERME / Free University of Bozen-Bolzano. https://hal.science/hal-03748407/. Accessed 17 Oct 2023.

**Leigh-Lancaster, D., & Stacey, K. (2022). Evolution over two decades of CAS-active senior secondary mathematics curriculum and assessment. *Mathematics, 10*(13), 2333. https://doi.org/10.3390/math10132333. (This contribution summarizes 20 years of experience with using computer algebra in high-stakes summative assessment in Victoria (Australia). The authors describe the changes over time, also caused by device capabilities and increasing teachers expertise. In their examples, the authors show that having sophisticated DTs at hand does not necessarily make the assessment easy. The authors conclude that changing assessment is not just a technical, but also a tactical affair, which should involve stakeholders and needs time).

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**Shvarts, A., Alberto, R., Bakker, A., Doorman, M., & Drijvers, P. (2021). Embodied instrumentation in mathematics learning as the genesis of a body-artifact functional system. *Educational Studies in Mathematics, 107*, 447–469. https://doi.org/10.1007/s10649-021-10053-0. (This article reconsiders the instrumental approach from a radical embodiment perspective. Rather than focusing on mental schemes, the authors see a key role for body-artifact functional systems, in which an intentionality, a bodily experience, a practice of using artefacts, and a mathematical conceptualization come together. This notion of artifacts being included in the perception-action loops of instrumented actions is illustrated by an empirical example).

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**Trouche, L., Rocha, K., Gueudet, G., & Pepin, B. (2020). Transition to digital resources as a critical process in teachers' trajectories: the case of Anna's documentation work. *ZDM - Mathematics Education*. https://doi.org/10.1007/s11858-020-01164-8. This paper addresses the impact of digital resources on mathematics teachers' work and professional development. Based on the Documentational Approach to Didactics, the authors describe a case study of a teacher using digital resources. The paper shows the development of the teacher's resource system, and, as such, her professional development over time.

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