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Resources in translation: towards a conceptual and technical apparatus

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Abstract

In mathematics education, translation of resources from one language to another occurs in a wide range of situations. This paper explores how conceptual and technical apparatus from contemporary translation studies may be of use in guiding and analysing such translation. Key concepts—including those of source, target and intermediate text, of paradigms of equivalence, purpose, uncertainty and localisation, and of semantic, syntactic and epistemological equivalence—are introduced and illustrated, primarily with reference to translation of a series of school mathematics workbooks. Significant types of tool to assist translation—translation protocols and machine translation—are examined. A more detailed case study illustrates techniques (and associated tools) for analysing translation shifts and terminological translatability, applying them to examine translation of documentation of a framework for researching teachers' interaction with resources—the Documentational Approach to Didactics. Specific insights that emerge are how translation is shaped by attention not just to fidelity to the source text but to the audience for, and function of, the target text; and how challenging it can be to formulate terms translatable across languages in such a way as to consistently anchor meaning in existing wordforms.

Keywords Mathematics education · Textual resources · Translation equivalence · Translation paradigms · Translation shifts · Translatability of terminology

1 Introduction

Translation of resources in mathematics education from one language to another occurs in a wide range of situations. Within an educational system, curricular resources composed in the official language may be translated into versions more accessible to other language communities. Borrowing curricular resources from one educational system to another often involves translation between their respective languages. Where the target audience for an assessment instrument is linguistically diverse, translation plays a key part in developing parallel versions. In communicating research, translation makes ideas accessible and tools usable in languages other than the original. Although many resources are multimodal, the focus here is on translation of written text, usually central to realising the communicative function of resources.

This article aims to introduce concepts and techniques from translation studies, illustrating their use in guiding and analysing translation of various types of resource for mathematics education.[1] It provides an overview of key translation concepts (Sect. 2), an outline of significant tools to assist translation (Sect. 3), and a detailed case study illustrating particular techniques (and associated tools) for analysing translation (Sect. 4).

2 Translation concepts

This section introduces key translation concepts, illustrating them in relation to mathematics education.

2.1 From source text to target text

A translator is constrained not just by the form and content of the existing *source* text—the text to be translated—but by

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[1] The examples have been chosen because the form that they take and the way in which they have been documented enables them to serve their illustrative function well. While they are varied, no claim is made that they are representative.

the intended audience and function of the *target* text—the text to be created. Consider the example of translation into English of a series of Chinese-language school mathematics workbooks to create two differing English-language editions: one published in Britain (Fan, 2017) under the title *The Shanghai Maths Project: For the English National Curriculum;* another published in China (Fan, 2018) under the title *Shanghai Maths: One Lesson One Exercise*.

The preface to the English-language edition published in China acknowledges different audiences, and correspondingly different functions:

> [W]e have two main aims in publishing this English language edition. The first is for it to serve as a useful resource for school students, teachers and parents, not only in China but also in other countries, in their learning, teaching and tuition of mathematics using English as a medium. The second is to provide an important window for international readers, in particular mathematics teachers, mathematics education researchers and policy makers, to look through and thereby to understand better the content, pedagogy and assessment of Chinese mathematics teaching and learning in schools, as reflected in this series of learning resources (Fan, 2018, Preface)

Thus the first function of this target text corresponds to that of the source text (to serve as a mathematics practice workbook) and the intended audience is comparable (students, their teachers and parents, but using English as the medium rather than Chinese). However, the second function and intended audience (to provide a window for specialists outside China into practices of mathematics teaching and learning there) are additional to, and different from, those of the source text. Accordingly, the translation process deliberately sought to retain characteristics of the source text:

> To keep the originality and authenticity of the Chinese edition, the English language edition is essentially a direct translation of the Chinese edition, so the loss of meaning in translation is kept to a minimum (Fan, 2018)

The English-language edition published in Britain shares the (first) function of serving as a mathematics practice workbook. However, it differs from the edition published in China in ways which are clearly intended to adapt it for an English audience (as its subtitle, *For the English National Curriculum,* highlights).[2] For example, the positioning of chapters changes (in line with English curriculum sequencing); at the start of each chapter an explicit learning objective is added (in line with a pedagogical norm in England);

some tasks are dropped and others added (to better match English curriculum coverage); and, on occasion, answers are prompted in a way not present in the original (reflecting differences in pedagogical norms between the two systems). Fan et al. (2018) report a detailed analysis of the adaptation of cultural references between versions, finding changes in elements such as people and places, everyday objects and daily life, food and drink, plants and animals, and currency.

The example of these two translations of the same original illustrates, then, how translation takes account of the intended function(s) of, and audience(s) for, the target text, as well as the substance of the source text.

2.2 Translation paradigms

Digesting the extensive literature on translation, Pym (2014) identifies a small number of overarching paradigms, each of which captures some major strand within the field. The example of the doubly translated mathematics workbooks from the previous section can be used to illustrate these. Inasmuch as the aim of the translation process was to achieve fidelity of the target text to the source text, the influence of an *equivalence* paradigm (Pym, 2014, ch. 2 & 3) was in play. Inasmuch as the aim was to repurpose the source text to create a target text adapted to a new audience and/or differing function, the influence of a *purpose* paradigm (Pym, 2014, ch. 4) was in play. (Although, in one case, that aim was—uncharacteristically—judged to be best served by maximising the direct equivalence of source and target texts.)

However, Fan (2018) reports encountering some difficulties in translating the workbooks. At certain points, where there appeared to be no adequate equivalent between the two languages, it proved necessary to leave out of the English-language translation some items included in the Chinese-language version. As my use of the qualifier 'adequate' signals, an equivalent is simply a translation judged (on one basis or another) to be close enough in meaning to the original and clear enough in conveying that meaning. The difficulties that translators meet in finding such matches, which become ever greater as the matching criteria are set stronger, underpin an *uncertainty* paradigm (Pym, 2014, ch. 6), not only sceptical about the possibility of establishing equivalence between source and target texts, but also emphasising indeterminacy and ambiguity in texts.

Finally, in this case, two translated versions of the workbooks were produced, intended for different audiences. Almost certainly, jointly producing these versions in the same language gave rise to some economy of translation

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[2] Assured that the English-language edition published in China corresponds closely to the Chinese-language edition, I have compared the former with the English-language edition published in Britain.

effort compared to creating two versions independently. On a larger scale, in an increasingly 'globalised' society, organisations create essentially the same product for use around the world, but in multiple versions, each 'localised' to some degree to a particular language and culture. Against this background, a *localisation* paradigm (Pym, 2014, ch. 7) is emerging, supported by the use of digital translation tools (such as translation-memory tools which access previous translations of similar segments of text), to increase the ease and efficiency of this process. Often, translation involves creating an *intermediate* text—between an original source text and multiple target texts—in an 'internationalised' form designed to facilitate translation (through techniques such as the use of a controlled language register with standardised equivalents in the target languages) and customisation (through techniques such as replacing localised conventions in the source text by generic markers in the intermediate text, which can then be (re)localised—via a relatively automatic process—in each target text).

2.3 Equivalence

In the equivalence paradigm, translation is viewed as the search for a target text which, as far as possible, preserves the meaning and form of the source text. However, there are multiple levels at which equivalence of meaning and form may be sought, with differences between languages often making it difficult to achieve equivalence at all levels. Even within the equivalence paradigm, perfect equivalence is recognised as a strategic fiction. Nevertheless, this paradigm is fundamental in the sense that any translation process seeks to reproduce, in the target text, at least some features of the source text. Even where other paradigms highlight the unattainability of perfect equivalence (as does the uncertainty paradigm) or question its desirability (as does the purpose paradigm), they assume that the translator does seek some kind of equivalence between source and target text.

As already noted, equivalence can be of many types, which may not be mutually compatible, obliging a translator to choose one type over others. Take a simple example, again relating to the workbook series presented earlier. The English-language translation published in China has an exercise with the title "Gougu theorem", where 'Gougu' is a phonetic rendering of the original Chinese. Recalling that the aim was for this translation to keep as close as possible to the Chinese-language source text, the choice has been made to treat 'Gougu' as a proper name to be conserved phonetically in the English-language target text. However, another option, in some ways more consistent with this aim, would be to acknowledge 'Gou-Gu' as a compound of two words, and so to translate these components as 'Base-Leg' (c.f. Qu, 1997) or, more conventionally in English, as 'Base-Altitude'. Thus the choice made to translate the title 'Gougu'

phonetically rather than in terms of its components means that the English-language target text loses the mathematical reference that the Chinese-language source text offers to the names used for two of the sides of a right-angled triangle.

At the same time, the text includes a footnote: "Internationally, this theorem is also called Pythagoras' theorem or Pythagorean theorem". This indicates that a further option for translating the title would be to adopt the Pythagorean name conventional for an English-speaking audience. On the other hand, an internationalised intermediate text might choose a more culturally neutral and mathematically descriptive translation of the title, such as 'Right-angled triangle theorem'. In summary then, the many alternatives available in this example illustrate how translation, even of a simple phrase, may involve choosing between very different options, each informed by a different notion of equivalence, guided by the intended purpose of, and audience for, the text.

2.3.1 Semantic equivalence

More fundamental problems of equivalence arise where two language communities do not make conceptual distinctions in the same way. For example, discussing translation of terms occurring frequently in the workbooks, Fan (2018) comments that in Chinese a 'deng shi' (such as $2+3=5$) is different from a 'fang cheng' (such as $2x-3=7$), whereas in English both can be referred to as an 'equation'. In order to translate a task depending on this distinction, it is necessary to find equivalent terms in English. For example 'equality' could be introduced to designate 'deng shi' and 'equation' reserved for 'fang cheng', but this would conflict with a customary use of 'equation' to refer to both. Alternatively, distinguishing adjectives could be placed in front of 'equation', such as 'numeric' or 'arithmetic' for 'deng shi' and 'algebraic' or 'symbolic' for 'fang cheng'. However, if these terms are not in customary use, they cannot be expected to be familiar to the intended users of the workbooks. Fan (2018) reports that lack of adequate equivalents between the two languages occasionally made it necessary to modify or drop items in the English-language translation.

Moreover, the alternative English terms identified in this example illustrate how, within any language, the same idea is open to alternative textual formulations which employ different terms (albeit ones signifying cognate concepts). This points to the way in which expressive use of language, including the authoring of a text, depends on (largely subliminal) selections from the *lexical/semantic fields* of related words and phrases which provide means of articulating concepts (Lyons, 1981). In the English semantic field relating to the example above, 'numeric' is close to 'arithmetic', and 'algebraic' to 'symbolic'. All four adjectives have potential to increase the precision of 'equation'. However, a word within a semantic field may have several senses, and

these senses may, in turn, overlap with those associated with neighbouring words. For example, 'symbolic' can be used in a narrower sense which differentiates 'algebraic' expressions from 'arithmetic', but it can also be used in a broader sense which includes both 'arithmetic' and 'algebraic' expressions: for that reason, 'algebraic' is a better choice than 'symbolic' in the situation in question. Conversely, the interpretive use of language, including when reading a text, depends on (again largely subliminal) identification of contextually appropriate senses for terms in order to establish what idea may be being conveyed. As shown by the English/Chinese comparison in the example above, cognate semantic fields in different languages are rarely isomorphic.

2.3.2 Syntactic equivalence

Equally, there may be important syntactic differences between languages, limiting the degree of equivalence which can be achieved between texts in different languages. In particular, Galligan ([2001](#)) argues that such syntactic differences may have important implications for the cognitive processing of mathematical texts. In providing examples, Galligan[3] first presents a text in Chinese, followed by a text (referred to here as a Chinglish[4] text) which seeks to reproduce in English the syntactic structure of the Chinese text, and finally a text (referred to here as an English text) which employs a conventional English syntactic structure to convey the meaning of the Chinese text.

One aspect of syntactic difference is complexity of sentence structure. Based on a review of relevant research, Galligan reports that use of subordinate clauses is more prevalent in English, whereas Chinese tends to use shorter sentences, or to combine these using linking attributes. The example that Galligan provides compares the Chinglish text, "A square's diagonal length 10 cm. Find this square's side length." to the English text, "Find the side length of a square whose diagonal is of length 10 cm." Of course, as the syntactically literal Chinglish translation shows, English could reproduce the Chinese structure (subject to the insertion of 'is' in the first sentence). Indeed, one can imagine that, by making simpler syntactic structures available to the translator, a Chinese source text might prompt use of these simpler structures in an English target text. On translation in the other direction, however, one could argue that the incongruence with Chinese of a complex syntactic structure in the English source text might necessitate the use of simpler structures in the Chinese target text, so reducing syntactic

equivalence. Hence there seems to be a potential for directional asymmetry in the translation process between the two languages as regards maintaining syntactic equivalence.

Another contrast between structures in the two languages can be illustrated by reference to this same example. Galligan reports the tendency of Chinese to use left-embedded noun phrases which impose greater cognitive load on the reader. The left-embedded structure of 'this square's side length' in Chinese contrasts with the structure of 'the side length of a square' in English. Again, as the syntactically literal Chinglish translation shows, English too can employ left embedding, but as the more colloquial version illustrates, English affords expression of the same idea in a form which reduces cognitive load; indeed, arguably, 'the length of the side of a square' would be even less burdensome. Again there seems to be a potential for directional asymmetry in the translation process, although here the relative simplicity and complexity between languages is reversed.

2.3.3 Epistemological equivalence

The internationalisation of scientific fields such as mathematics and the accompanying harmonisation of the specialised linguistic registers that they employ has created substantial alignment across languages with a long history of cultural exchange and exposure to globalisation. But few texts can escape some use of an ordinary register. Here, certain differences in the ways in which languages grammatically encode logical, temporal and spatial relations might be considered sufficiently profound to threaten the achievability of epistemological equivalence in translation (Hewson, [2012](#); Svorou, [1994](#)). Nevertheless, given the long interaction between major world languages, while there may be differences in their customary framing of certain issues, in ease of expressing them, in associated metaphors and images, these rarely stand in the way of finding a translation to convey the gist of the core idea from one language to another, even if some nuance is lost. But nuance can be vital. As English has become an international *lingua franca*, not least in this and other research journals, members of other language communities face challenges in conveying their ideas to an international audience, particularly to the degree that these ideas are distinctive to their community and dependent on its particular forms of expression (Geiger et al., [2018](#)).

A more fundamental impediment is where a language lacks a particular specialised register. Especially in countries with a history of colonisation, academic mathematics was often taught and practised solely in the language of the colonial power. The post-colonial impetus has been to pursue mathematics through the medium of vernacular languages. Typically, however, this necessitates extending these languages by developing a suitable specialised register. In Aotearoa New Zealand, for example, experts in

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[3] For illustration I have selected only two of the differences itemised by Galligan.

[4] Used without the pejorative connotation which sometimes accompanies it.

mathematics education and the Māori language have developed a standardised Māori mathematics vocabulary for the complete school curriculum (Royal Society Te Apārangi Expert Advisory Panel, 2021). Barton et al. (1998) explain the complexity of this process. An earlier approach of simply incorporating phonetic transliterations of English terms into Māori was rejected in favour of an approach which sought to express these terms in ways which drew wherever possible on Māori forms. The process was guided by four basic principles: the terms should be consistent with each other; they should be as short as possible without over-simplification; they should sound correct to a native speaker of Māori; and the usage should be grammatically correct. Guided by these principles, more detailed techniques were developed and applied in a systematic way. For example, the term chosen for mathematics, *pāngarau*, refers to multiple (*rau*) relationships (*pānga*) (Royal Society Te Apārangi Expert Advisory Panel, 2021).

3 Translation tools

This section examines two types of tool to assist translation, identifying their particular interest for mathematics education.

3.1 Translation protocols

Issues of equivalence in translation become particularly prominent in developing 'internationalised' resources in parallel versions in several languages. In particular, under the localisation paradigm, when multiple persons contribute to a translation process, developing some kind of protocol supports consistency of approach. In mathematics education, one important area where this takes place is in international studies involving standardised comparisons of student outcomes and learning contexts, based on administration of common test and questionnaire instruments across many educational systems and official languages. For the multiple versions of each instrument to be capable of producing comparable data across systems and languages, they must be equivalent to a high degree. This has encouraged development of explicit protocols for translation.

The character of such a protocol is illustrated by the translation and adaptation guidelines for PISA 2022 (Organisation for Economic Cooperation & Development, 2019). For PISA test items, for example, the initial step in development is to select resource material, taking account of "plausible shared interests and concerns of 15-year-old worldwide" (p. 18), with experts in each participating national system evaluating such material for cultural suitability and familiarity. The intention is that "the majority of the PISA stimuli [should be] texts that the student would normally be likely to encounter in daily life when reading books, newspapers, magazines, web sites, etc." (p. 18).

In ensuing steps, parallel versions of an item are developed in English and in French. These are then revised in the light of assessment, by a team of linguists, of their equivalence in the two languages and their translatability into further languages. This first stage of the development process aims to create two standard versions of the item which will lend themselves to translation into other languages without compromising equivalence. At the same time, some tolerance is imparted to this equivalence by virtue of the English and French source texts also serving as an indicator of "the degree of translation freedom that might be deemed acceptable" (p. 8).

The next stage is the translation of the standard versions into the official language of each national system. For each target language, two translators work independently, one from the English version, the other from the French. A third translator, the 'reconciler', then has the task of merging the two translations "in such a way that the resulting national version is semantically equivalent to the source versions and that the wording is as fluent as possible" (p. 8). This process aims, then, to converge on a definitive text in the target language through refining the initial independent translations of the English and French source texts. Since the texts under translation often blend everyday and specialised language, the PISA protocol considers it essential for the translation team either to include members competent in the subject, or to have its work checked by subject specialists.

The protocol offers translators extensive general guidance, intended to "control frequently encountered issues" (p. 11). A prominent rationale underlying recommendations is to avoid altering the demand of an item by splitting a long sentence in two (or the reverse), substituting everyday words or phrases for technical terms (or the reverse), changing level of abstraction by replacing a verb form with a noun form (or the reverse), or modifying reference chains in which the same character or notion is repeatedly alluded to, often through pronouns or synonyms.

For each individual item, translators may also be offered more specific guidance on these same issues, warned against particular translation traps, offered alternative wording in case translation difficulties arise, and be directed on where national adaptation is required. Acknowledging that mathematical language and symbolism is "far less 'universal' than it is often said to be" (p. 19), the protocol emphasises the importance of checking the use of locally appropriate equivalent forms, and highlights areas of difference such as the written form of units, the multiplication symbol, the decimal separator, the formatting of numbers, and the presentation of times.

In this example, the equivalence and localisation paradigms condition a process engineered to create parallel

versions of a resource in multiple languages. The filtering of resource material and assessment of source texts for translatability shape choices of mathematical and contextual substance. While the translated versions acknowledge local idiosyncrasies of surface form, they seek to strictly maintain the core substance and structure of the original.

3.2 Machine translation

Machine translation tools are now widely available not just to professional translators but to laypersons. They are being used increasingly by educators and researchers in the mathematics education field to assist them in reading and writing in languages other than those in which they are fluent.[5]

Early machine translation employed a *linguistic rules-based* approach which sought to apply explicit lexical, syntactic and semantic rules to achieve corresponding types of equivalence. However, quite rapidly, a more indirect and implicit approach gained favour. This *statistical corpus-based* approach builds a probabilistic translation model from analysis of the patterning of basic textual units (word particle, word, word sequence) within some large body of existing translations. The data-driven equivalences and contingencies established by this model then guide translation of such units. Most recently, application of machine learning techniques has led to a more integrated and autonomous *artificial neural network* approach, which enables such tools to take account of the whole sentence in which a text unit is embedded, and to continuously refine their underlying translation model (Wu et al., 2016).

How do machine translations rate in terms of conventional criteria of equivalence? The development team for Google's neural system found that, depending on language and direction, its translations attracted mean ratings ranging from a little above a score described as "the sentence retains most of the meaning of the source sentence, but may have some grammar mistakes", to some way below a score described as "the meaning of the translation is completely consistent with the source, and the grammar is correct" (Wu et al., 2016). An illustration of current performance comes from using Google Translate to translate the opening sentence of the abstract for this article into Chinese and then back into English. "In mathematics education, translation of resources from one language to another occurs in a wide range of situations" becomes "In mathematics education, the translation of resources from one language to another is very broad", losing the sense of the final clause of the

sentence. However, a simplified version, "Translation from one language to another occurs in a wide range of situations", becomes "Translation from one language to another happens in a variety of situations", retaining the essential meaning.

This illustrates how current machine translation still has difficulties with longer or more complex sentences, even when they use mainstream modes of expression. Consequently, human 'post-editing' of machine output is the norm in professional translation (Viera, 2019). Comparisons have been made between bilingual post-editing (attention given to both source and target text by someone fluent in both languages) and monolingual post-editing (attention given only to target text by someone fluent in that language). Predictably, bilingual post-editing is superior in ensuring that the target text conveys the meaning of the source-text, although monolingual editing by domain specialists has some value (Viera, 2019). These results can be extrapolated speculatively to the situation of mathematics educators seeking assistance from machine translation, in effect acting as post-editors of its output. The implications are more encouraging for reading than for writing. In reading a machine translated text (in a target language in which the reader is fluent), domain expertise helps to make sense of it, although familiarity with the source language is needed to add confidence that this meaning reflects the source text. In writing, however, fluency in the target language as well as the preferred source language is essential to adequately assess the quality of machine translated output.

Machine translation tools reproduce any blindspots or biases in the corpus of translations on which they have been trained. A recent development, then, is of a system specifically adapted to "the domain of mathematical text, as produced by researchers in mathematics and related fields, and post-secondary teachers of these subjects" (Ohri & Schmah, 2021, p. 38,078). As a backup when it encounters linguistic features absent from the custom mathematical corpus on which it was trained, the system employs Google Translate with an added mathematical glossary. We can expect such specialised translation tools to become more readily available.

4 A case study: the DAD Multilingual project

This section illustrates, in detail, analysis of two types of translation issue. The first is translation, unit by unit, of a text; the second, translatability of special terminology.

DAD is an acronym for the Documentational Approach to Didactics, a framework of theory and method devised "to understand teachers' professional development by studying their interactions with the resources they use and design in/for their teaching" (DAD Multilingual Project,

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[5] As one reviewer wrote, requesting addition of a section on machine translation: "From my experience … with colleagues, who—in their scientific work—use also other than their dominant language… I know that… web-based automated translation can be of help…".

Table 1 Opening sentences of the *Introduction* to the DAD text

	English source text	French target text
A	1. Introduction	1. Introduction
B	Mathematics teachers interact with curriculum and other resources in their daily work.	Les professeurs de mathématiques interagissent quotidiennement avec des ressources conçues à des fins d'enseignement (que nous nommerons ressources curriculaires, par exemple un manuel) ou des ressources non conçues à des fins d'enseignement (par exemple un article de journal).
C	Their work with resources includes selecting, modifying, and creating new resources, in-class and out-of-class.	Leur travail avec ces ressources, en classe et en dehors de la classe, inclut la sélection, la modification et la création de nouvelles ressources.
D	This creative work is termed *teacher documentation work*, and its outcome/s *teacher documentation*.	Ce travail est appelé le *travail documentaire du professeur,* et ce qu'il produit est nommé la *documentation du professeur*.
E	Typical curricular resources include text resources (e.g. textbooks, curricular guidelines, student worksheets); or digital curriculum resources (e.g. digital interactive textbooks).	Les ressources curriculaires incluent typiquement des textes (par exemple des manuels scolaires, des programmes officiels, ou encore des productions d'élèves); des ressources numériques (qui peuvent aussi être des textes, trouvés par exemple sur Internet, mais aussi des logiciels).

2020). The DAD Multilingual project (DAD-ML) seeks to disseminate this framework internationally through translating a definitive English source text on DAD (Gueudet et al., 2020b) into many languages, and to develop the framework further by drawing on this experience. The translations have been carried out by researchers active in, or cognisant of, the research field, as well as fluent in the target language. For each language, there is at least one translator and one reviewer.

The project also has more dialogic intentions as a site for intercultural communication, seeing the process of translation as a means of exploring the central concepts of the DAD framework with a view to developing the framework further. To this end, the translator-reviewer team for each language produces a 'translation issues' report, guided by a common protocol. This protocol specifies two tasks. The first task is to describe the main issues that emerged during the translation (including review) process, distinguishing between those linked to the vocabulary, the concepts at stake, and the social, cultural, or curricular context. The second task is to report on the translation of key terms, reporting the possible translations considered, the reasoning behind the choice finally made, and scientific references using this term in the targeted language. Thus, while the initial translation phase is one of localisation, involving dissemination of an established system of ideas from a centre to multiple peripheral locales, this subsequent reporting phase introduces the possibility of further flows of ideas, not just back from periphery to centre but around the periphery.

4.1 Translation shifts

In describing the new texts that it produces as 'translations', DAD-ML implies that they are equivalent in one way or another to the original. This subsection examines forms of equivalence in the key translation within the project. This approach reflects a *descriptive* paradigm (Pym, 2014, ch. 5) in translation studies, which aims to describe what translations are actually like rather than to prescribe how they should be. To that end I present a close analysis of the translation between English and French of the definitive text. I write 'between' because, although this text originated as an entry in the *Encyclopedia of Mathematics Education*, the process of translating it into French led to revision, as explained by the authors:

> Launching the DAD-Multilingual project, we have, in the same time, updated the original English version, and started the process of French translation. Most issues were in fact lacks of clarity in the English text, that we noticed with the very precise kind of reading needed to translate. This led us to revise also the English text (Gueudet, Trouche & Aldon, 2020a)

In practice, too, it appears that some translators into further languages drew on the French as well as English version, so both have particular significance in the project.

To compare the English and French versions I will identify *translation shifts*—originally defined as "departures from formal correspondence" (Pym, 2014, p. 64), but which can be more constructively thought of as arrivals at informal correspondence. Underpinning the technique I will use is a taxonomy developed by Vinay and Darbelnet (Molina &

Hurtado Albir, 2002; Pym, 2014). The opening sentences of the two texts are shown in parallel in Table 1.

4.1.1 Literal translation

Entry A exemplifies correspondence at its simplest and most direct. The two texts are identical since both elements (the numeral "1" and the word "Introduction" and their associated meanings) are shared by source and target languages (SL and TL), and the ordering of these elements is the same in source and target texts (ST and TT). The start of entry B illustrates a slightly more indirect type of correspondence. While the texts are not identical in the signs employed and their sequencing, they correspond in syntactic structure and semantic meaning (with the noun phrase "mathematics teachers" corresponding to "les professeurs de mathématiques", and the verb form "interact" to "interagissent"). The reversal between ST and TT in the sequencing of adjectival form and noun form in the noun phrase respects the differing grammatical and idiomatic norms of SL and TL. Vinay and Darbelnet refer to both these types of translation as *literal*, considering that no shift of any substance has taken place between ST and TT. Literal translation, then, represents the 'formal correspondence' from which the various types of informal correspondence depart.

4.1.2 Calque (through translation) and loan

Where there is no adequate literal translation in the TL, one solution is simply to draw on the SL. The TT material of entry B employs the *calque* "ressources curriculaires", formed by *through translation*—literal translation, word by word—of the phrase "curriculum… resources" from the ST. Had the English phrase itself been retained in the TT, this would have been an example of *loan*—direct borrowing from the source.

4.1.3 Implicitation and explicitation

Other shifts involve subtle changes in meaning. In entry B, the "quotidiennement" of the TT (back-translatable simply to "daily") loses the reference to "work" of "in their daily work" in the ST, although this might be inferred from context. Vinay and Darbelnet refer to shifts of this type as *implicitation* (as, in this case, where something explicit in the ST is left implicit in the TT) or *explicitation* (in the reverse case, where something implicit in the ST is made explicit in the TT).

4.1.4 Reformulation and adaptation

Likewise in entry B, while "with… resources" in the ST corresponds to "avec des ressources" in the TT, the associated adjectival forms differ in substance, with "curriculum and other" becoming "conçues à des fins d'enseignement… ou… non conçues à des fins d'enseignement" (back-translatable to "devised for teaching purposes… or… not devised for teaching purposes"). This appears to be an example of *reformulation* (in which a rather different expression is used to render an idea from the ST in more culturally appropriate terms in the TT). A related type of translation shift is *adaptation* (in which a more culturally appropriate reference is used).

4.1.5 Addition and subtraction

These elaborations appear to be motivated primarily by the lack of an established equivalent for 'curriculum… resources' in the TL. This is supported by the way in which the TT for entry B concludes by inserting further material absent from the ST to exemplify types of resource. This action goes beyond Vinay and Darbelnet's categories, but is referred to by Nida (Molina & Hurtado Albir, 2002) as *addition* (as in this case, where information that is not present in the ST is inserted into the TT).

A modest example of *subtraction* (the reverse case, where material present in the ST is omitted from the TT) is provided by the opening phrase of entry D, where "This creative work" in the ST becomes simply "Ce travail" ("This work") in the TT.

4.1.6 Generalisation and particularisation

Entry D also illustrates another type of shift. The ST employs the word "termed", translated both as "appelé" and "nommé" in the TT. Semantically, "nommé" in the TL is closest to "named" in the SL, while "appelé" is closest to "called" (although these respective meanings are also close within SL and TL). However, the SL has a further cognate, "termed", as used here in the ST. What differentiates "termed" from "named" or "called" is its particular connotation that this is a name subject to precise definition and/or specialised use. While the TL has a noun with this sense, "terme", it lacks a corresponding verb or verb-derived adjective, so that a more extended phrase would be necessary in order to convey this particular connotation. Here the translation opts for brevity at the cost of some loss of specificity. Vinay and Darbelnet refer to shifts of this type as *generalisation* (as in this case, where a more general or imprecise term is used in the TT than in the ST) or *particularisation* (in the reverse case, where a more precise or specialised term is used in the TT than in the ST).

4.1.7 Modulation and transposition

The opening phrase of Entry E illustrates a further type of shift, where "Typical curricular resources include" becomes

Table 2 Concluding sentences of the *Introduction* to the DAD text

	English source text	French target text
H	In theoretical terms the work of teachers with curriculum resources has been studied from many angles and theoretical perspectives for example in the Anglo/American research literature through the notion of 'enacted curriculum'.	Du point de vue théorique, le travail des professeurs avec des ressources curriculaires a été étudié internationalement en mobilisant une multitude de concepts. Dans la littérature anglo-américaine, on note les distinctions entre *intended curriculum* et *enacted curriculum*, que l'on peut traduire respectivement par *curriculum visé* et *curriculum en actes.*
I		Le passage du curriculum visé au curriculum en actes se rapproche de l'une des étapes du processus de transposition didactique, le passage du *savoir à enseigner* au *savoir enseigné* (étape nommée *transposition interne* par Ravel 2005).
J	In continental Europe the notion of '*Didaktik*' is a common concept.	
K	This entry describes, explains and illustrates the *Documentational Approach to Didactics* (DAD), which has its roots in French European Didactics.	Dans ce texte nous présentons l'approche documentaire du didactique (ADD dans ce qui suit), issue de la tradition française de recherche en didactique.

"Les ressources curriculaires incluent typiquement". Whereas in the ST, "typical" is an adjective applying to the noun "resources", in the TT, "typiquement" is an adverb applying to the verb "incluent". Thus there have been shifts both in the terms in which typicality is framed and in the grammatical category conveying the idea. Vinay and Darbelnet refer to shifts of this type as *modulation* (as in this case, where a shift in frame of reference takes place between ST and TT) and *transposition* (as in this case, where a shift in grammatical category takes place between ST and TT).

4.1.8 Uncertainty in classification

Nevertheless, there may be uncertainty as to how to classify a unit of translation in terms of type(s) of shift. In entry E, for example, "student worksheets" is translated as "productions d'élèves." A more conventional and direct translation of "student worksheets" would be "fiches d'exercices" or "feuilles de travail" (or some permutation of these); equally, "productions d'élèves" would conventionally be (back-)translated as "student productions" (for a specialist audience familiar with that turn of phrase) or as something like "students' work" (for a more general audience). From this perspective, this translation combines subtraction—omission from the TT of information in the ST—with addition—insertion into the TT of information absent from the ST. But perhaps this is adaptation—the substitution of a more culturally appropriate reference—although a basis for that is unclear. Likewise, at the end of entry E, the substitution of "e.g. digital interactive textbooks" in the ST by material in the TT back-translatable as "which can also be texts, found, for example, on the Internet, but also software" might be adaptation, but in the absence of any clear cultural nuance, this is more plausibly a combination of addition and subtraction.

4.1.9 The limits of informal correspondence

In analysing these opening sentences, this taxonomy has proved useful in identifying and characterizing translation shifts. It has made it possible to capture nuances of informal correspondence in a systematic way. As such the taxonomy has demonstrated its capacity to contribute to fine-grained analysis of the translation of resources in mathematics education. In particular, it can be a useful tool for translators, providing an inventory of potential shifts to achieve informal correspondence, and a mode of analysis highlighting issues of consistency and clarity.

However, the concluding sentences of the Introduction (as shown in Table 2) illustrate how the nuanced insights offered by the taxonomy are relevant only when translation seeks to achieve close correspondence.

In these concluding sentences we find more radical divergences between the ST and the TT, at the extremes of addition and subtraction. Entry I is found only in the French TT, entry J only in the English ST. Both entries serve a particular purpose: that of providing further information adapted to the assumed knowledge of readers in the language concerned. Thus, in the French TT, entry I continues the elaboration of the idea of "intended curriculum" begun in entry H, by relating it specifically to ideas from French didactique. Likewise, in the English ST, entry J introduces the idea of European didactics, as a preamble to the reference to "French European Didactics" in entry K. In their report on the translation from English to French, the authors and self-translators write:

> In French we felt the need to explain the link between DAD and didactic transposition… In the English version, we only cited "intended curriculum" and "enacted curriculum", but it is not possible in France

to present these notions without referring to didactic transposition. (Gueudet, Trouche & Aldon, 2020a)

One might summarise these observations by saying that the freer translation of the concluding sentences looks beyond the formal (or close informal) equivalence of each of the smaller units making up source and target texts, to a *functional* (or *dynamic*) *equivalence* (Pym, 2014) of the larger ideas conveyed by the texts. From such a perspective, translation needs to take account of cultural differences in readers' knowledge in order to appropriately scaffold understanding within their respective language communities. In effect, translating across languages and cultures may—in some respects—require the sacrifice of a degree of close formal or informal equivalence between source and target texts.

At the same time, another phenomenon may also be in play here. As noted earlier, the translation into French was undertaken by two of the authors of the English original. It has been argued that because "self-translators can access their original intention and the original cultural context or literary intertext of their original work better than ordinary translators", they "will feel justified in introducing changes into the text where an 'ordinary' translator might hesitate to do so" (Montini, 2010, p. 306). The influence of intertextuality may have been particularly strong here, since the DAD framework was originally formulated by the self-translators, not in English but in French, under the influence of theories expressed in French.

4.2 Terminological translatability

The study of *language for special* or *specific purposes* (LSP) acknowledges that the linguistic variants used in professional and other specialist settings have distinguishing features (Gunnarsson, 1997). In particular, such language not only employs specialised terminology and symbolism but deploys distinctive structures and genres (Schubert, 2010). An important feature of scientific language, then, is its creation of terminological systems to express key concepts. Terms are words or phrases intended to convey a precise meaning when used by experts in a scientific field. Accordingly, they play an essential part in representing and communicating knowledge within the field. Linguistically each term represents a specific concept and cognitively it evokes that concept.

In DAD-ML the crucial part played by terminology receives particular attention in the protocol for reporting on the translation process. This specifies that the report should identify:

> Certain concepts, or processes [which] raised difficulties, or discussions between the translator and the reviewer. We suggest that you explain these difficulties, and the choices you have made, for the notions

of *resource*, *document* and for… other notions which seemed… complex (e.g., scheme, operational invariant…) (DAD-ML, 2020)

This protocol, then, raises the issue of the translatability of key terms of DAD. I will use lexicological techniques to examine this.

4.2.1 Recontextualisation of terms

Systems of terms and concepts are often recontextualised from one setting to another. Such recontextualisation is apparent in DAD. For example, as the originators of DAD note, they borrowed concepts such as *scheme* and *operational invariant* from Vergnaud's theory of *conceptual fields*. Indeed, the situation is more complex. As Vergnaud (2011, p. 45) makes clear, he had already recontextualised the system of concepts cited above (and also that of *genesis*) from Piaget's theory of *genetic epistemology*:

> Although the analyses presented above are rather different from those of Piaget… and show a greater concern for definition and generality concerning the concept of scheme, the borrowing from Piaget of the terms 'scheme' and 'operational invariant' indicates clearly that the primary inspiration for this theory came from him.[6]

Where these Piagetian terms are concerned, then, there will already be accepted equivalents in other languages, established through the translation of this prior body of work. What is perhaps more significant, however, is the point which Vergnaud makes that such borrowing of terms risks masking conceptual and methodological shifts. It highlights a potentially deceptive form of 'translation' which takes place between settings—in which an established term is appropriated to convey a closely related but not identical sense—which needs to be taken into account in translation. Thus, just as Piaget's conceptual and terminological apparatus are recontextualised in Vergaud's theory of *conceptual fields,* and apparatus from both these prior theories in Rabardel's theory of *cognitive ergonomics*, so the apparatus of Vergnaud and Rabardel is recontextualised in DAD.

Because the work of Vergnaud and Rabardel has already been translated into English and other major languages, clear precedents exist for translating key terms into these languages. However, translating them into some further

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[6] Translation of "Bien que les analyses présentées ci-dessus soient relativement différentes de celles de Piaget… et témoignent d'un plus grand souci de définition et de généralité concernant le concept de schème, l'emprunt à Piaget des termes de «schème» et «d'invariant opératoire» indique clairement que c'est lui le premier inspirateur de cette théorie.".

language involves finding or creating appropriate equivalents in that language. It is also important to give attention to precedents in other languages so as to ensure consistency and facilitate translatability between multiple languages. For example, in the absence of translation of many of Piaget's and Rabardel's terms into Ukrainian, the DAD-ML translators took account of precedents in the related Russian language.

4.2.2 Formulation of terms

Developments in science call, not just for the adaptation seen in recontextualisation, but for innovation in language and concepts. The following extract from the definitive text captures the key conceptual and terminological contribution of DAD:

> [D]uring the interaction with a particular resource or sets of resources, teachers develop their particular *schemes of usage* with these resources... These are likely to be different for different teachers, although they may use the same resource, depending on their dispositions and knowledge, for example. The outcome is the *document*, hence: *Resources + scheme of usage = document.* The process of developing the document (including the teacher learning involved) has been coined *documentational genesis*. (Gueudet, Trouche & Pepin, 2020b)

Here, then, a (re)new(ed) scientific concept—expressed in the linked terms *document*, *documentation* and *documentational*—is introduced and defined. The DAD text explains its source as follows:

> Considering resources as the matter feeding teachers' work, a word was needed for naming what a teacher develops for a precise aim through his/her work with these resources. The word *document...* had already been used in the field of information architecture for designing 'something bearing an intention', and dedicated to a given usage in a given context. This choice is the origin for the name of the approach, "documentational approach to didactics." (Gueudet, Trouche & Pepin, 2020b)

However, a lexicological perspective reminds us that the word 'document' has established meanings which may support or subvert the new and specialised sense. This raises the issue of how new scientific terms should be formed, in the interests both of clarity of meaning in the original language and translatability into other target languages. The text cited above as the source for 'document' is written in French, and the DAD-ML source text uses its English cognate. The established traditional meaning of the noun form is similar in the two languages: the Académie Française offers "Écrit

ou, par ext., tout objet pouvant apporter un renseignement, établir ou infirmer un fait"[7] ("Written item, or, by extension, any item able to provide a piece of information, establish or negate a fact"), while the Oxford English Dictionary offers "Something written, inscribed, etc., which furnishes evidence or information upon any subject".[8] The now established modern usage of 'document' to also refer specifically to a digital file (for) holding information extends the traditional usage to a new medium. Design of digital systems reinforces this continuity through correspondences between 'documents' displayed on screen and printed on paper. Such correlations play an important part in the uptake and acceptance of new meanings.

In DAD, however, the psychologisation of 'document' as 'resources + scheme of usage' disrupts such continuity: what is a 'document' for DAD is not a 'document' as commonly understood; and what is commonly understood to be a 'document' is, for DAD, not a 'document' (although it might form part of the 'resources' component of one). Indeed, conveying the psychologised meaning while maintaining continuity with established usages might be better achieved by reversing the roles of the terms 'document' and 'resource' within DAD. The established meanings of 'resource' and 'resources' include not just "a source of help, information…" and "stocks or reserves of… materials… [to] be drawn on" but also "an action or strategy which may be resorted to…" and "personal attributes and capabilities…".[9] Arguably, then, marrying these senses by defining a 'resource' as 'document(s) + scheme of usage' would offer greater continuity with established meanings.

The main divergence of meaning between English and French arises in the verb forms of 'document'. Both languages have the form in which one 'documents' something (in the sense of creating or providing a documentary record of it, or, more loosely, of gathering or supplying information about it). But, in French, there is also a form in which one 'documents' someone (in the sense of supplying them with information),[10] whereas this form is long obsolete in English.[11] Likewise, French offers a reflexive form of the verb ('se documenter') in which one 'documents' oneself (in the sense of gathering information for oneself). In these French locutions, then, persons can be, in grammatical terms, both subject and direct object of the verb, and, in semantic terms, recipient as well as agent of the act of documenting, taking the meaning of 'document' closer to the psychologised sense in which it is used within DAD.

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[7] https://www.cnrtl.fr/definition/academie9/document

[8] https://www.oed.com/view/Entry/56328

[9] https://www.oed.com/view/Entry/163768

[10] https://www.cnrtl.fr/definition/academie9/documenter

[11] https://www.oed.com/view/Entry/56329

The various DAD-ML translation reports indicate that many languages, particularly those culturally close to English and French, possess the word 'document'—give or take differences in grammatical and orthographic conventions. Likewise, those reports that refer to the established meanings of the word indicate that these are similar to the ordinary meanings common to English and French. At the same time, many of these reports comment on the considerable gap between these established meanings of the term and the meaning intended by DAD. Nevertheless, in general, the translators simply choose 'document': the exception is the Greek translation which modifies the term through particularisation, to a phrase expressing 'document for teaching'. The reports from more distant languages, Chinese and Japanese, indicate that while they lack the word 'document' (although it could be borrowed and rendered phonetically), there are already words within these languages which have similar meanings. The Chinese team choose to render 'document' in terms of a word meaning 'any carrier that records knowledge', and the Japanese team in terms of a word meaning 'idea expressed in text', again somewhat distant from the meaning intended in DAD but close to the established meanings of 'document' in other languages. Finally, the translation reports indicate that reproducing the 'document–documentation–documentational' system with its associated noun, verb and adjective forms proved difficult in languages with orthographic and grammatical systems which did not afford devices for making all of the necessary distinctions.

This example, then, shows how challenging it can be to formulate terms which will be translatable across languages in such a way as to consistently anchor meaning in existing wordforms.

5 Conclusion

This paper has demonstrated how conceptual and technical apparatus from contemporary translation studies may be useful in guiding and analysing the translation of resources in mathematics education. While the uncertainty paradigm highlights reservations about the degree to which translation can achieve full equivalence—semantic, syntactic and epistemological—between texts, the equivalence paradigm provides tools—such as Vinay and Darbelnet's taxonomy and the PISA protocol—to assist translators in achieving close correspondence. The descriptive paradigm repurposes such tools in order to analyse—in terms of translation shifts—the types and degree of correspondence achieved in translation. The purpose paradigm directs attention beyond textual equivalence to intended function and audience. An emerging localisation paradigm is establishing techniques—increasingly assisted by digital tools—for efficient translation into (and between) multiple languages.

Nevertheless, it is important to remember that this is the apparatus of a particular time and place. For example, might notions such as those of source, intermediate and target text need to be reformulated in a world of the continuous updating of online resources and their repurposing by others, a world of resources translated by teams from around the globe into multiple parallel versions? Equally, in a world that is both culturally and linguistically diverse, how might translation techniques take better account of the cultural aspect beyond the linguistic one?

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