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ORIGINAL PAPER

Characteristics of Chinese high-quality mathematics lessons from a lesson structure perspective

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Abstract

In this study we aimed to capture the characteristics of Chinese high-quality lessons from a lesson structure perspective, specifically addressing the elements of teaching activity and their sequence. Sixteen videotaped middle school mathematics lessons were selected from 224 daily lessons collected from urban districts in five cities located in the east, north, west, south, and center of China, approved by three independent parties (based on the scores of an instruction quality assessment and ratings by two experts). We found the following five common features among most mathematics lessons: (a) most lessons began with a review; (b) pure learning of mathematical knowledge was emphasized; (c) recurrent teaching activities related to mathematics problems ran throughout the lesson; (d) highlighting and summarizing important points during the lesson and summarizing the entire lesson at the end of the class were frequently used; and (e) teaching activities were dense and large in volume. The research findings paint a broad picture of high quality instruction (deepening our understanding of high-quality teaching in different cultures) and provide multiple choices for teachers.

Keywords Classroom teaching · High-quality mathematics lessons · Instructional quality · Lesson structure

1 Introduction

Many studies have shown that classroom instruction is an essential factor that influences students' learning (An et al., 2006; Klein et al., 2000; Kyriakides et al., 2018; Muijs et al., 2018; Stigler & Hiebert, 1999). To better understand and improve the effectiveness of students' mathematics learning, it is necessary to conduct in-depth research in the place where mathematics education occurs—the classroom. Teachers aim for high-quality lessons, and other stakeholders are concerned about them. Improving teaching quality has attracted considerable research interest in many countries (Charalambous & Praetorius, 2018; Huang & Li, 2009; Lin & Li, 2009; Pang, 2009; Shimizu, 2009). For example, researchers from twelve countries participated in the Learner's Perspective Study (LPS)—a study of the practices and associated meanings in 'well-taught' eighth-grade mathematics classrooms (Clarke et al., 2006a, 2006b).

The past years have seen an accumulated body of studies aiming better to conceptualize, operationalize, and measure instructional quality through different lenses (Charalambous & Praetorius, 2018). To deepen our understanding of high-quality instruction, the features and instructional aspects identified in different countries with different cultural values and traditions warrant study (Praetorius & Charalambous, 2018). China is indispensable in this regard, with specific teaching traditions and circumstances such as large class sizes, fierce competition and unified curriculum standards. Chinese students have excelled in school mathematics in certain cross-national studies (Leung, 2018; Lapointe et al., 1992; Mullis et al., 2004; Organisation for Economic Co-operation and Development, 2010). In addition, since the beginning of the twenty-first century, China has embarked on reforming its mathematics curriculum on an unprecedented scale, and in the past two decades, classroom activities have undergone a substantial change to promote students' learning opportunities.

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Due to the complexity of classroom teaching, which involves various factors including the teacher, students, mathematics content, and organization, it is challenging to study classroom teaching thoroughly in one study. There are many factors a teacher must consider when designing a class, especially how to organize and structure the class. The lesson structure, including the elements of teaching activities and their sequence, is imperative for capturing classroom teaching. Lesson structure is an important indicator when observing the instructional quality of a class (Kyriakides et al., 2018; Walkington & Marder, 2018). In the 1999 Third International Mathematics and Science Study (TIMSS) video study, the lessons were analyzed based on division into three elements, namely, reviewing, introducing new content, and practicing new content (Hiebert et al., 2003). Shimizu (2002) developed a framework with thirteen elements to study lesson structure. In this study, we aimed to capture characteristics of Chinese high-quality lessons from a lesson structure perspective. This study was aimed to facilitate a deeper understanding of instructional quality, paint a broad picture on high quality instruction in a unique cultural setting, provide multiple choices for teachers worldwide to improve their teaching as well as understand Chinese students' mathematical learning better.

2 Literature review

2.1 Research on high-quality classroom instruction

The quality of mathematics classroom instruction is key to improving students' performance (Martin, 2007; Muijs et al., 2018). Several attempts have been made to improve students' learning by exploring and understanding mathematics classroom instruction practice in high-achieving education systems (Clarke et al., 2006a, 2006b). Specifically, researchers worldwide have shown increased interest in addressing the characteristics of high-quality instruction in their respective countries. For example, Pang (2009) examined the characteristics of high-quality teaching and learning in Korea by describing a sixth-grade teacher's mathematics instruction in depth. Shimizu (2009) examined the key characteristics of 'good' mathematics lessons in Japan from the learners' perspective. Clarke et al. (2006a, 2006b) analyzed the lessons taught by 'competent teachers' in different countries, where the meaning of 'competent' was determined according to local criteria.

Globally, different research perspectives have been adopted to study high-quality mathematics instruction. Some studies have captured the characteristics of high-quality lessons by considering the lessons delivered by experienced teachers (Kaur, 2009; Wang & Cai, 2007). The most widely accepted method is to first identify excellent teachers and then develop methods to confirm that they deliver high-quality lessons. For example, Kaur (2009) captured the characteristics of good mathematics teaching in eight-grade classrooms in Singapore based on a juxtaposition of teachers' practice and students' perception. She analyzed the teachers' practices by considering activities such as whole-class demonstration, seatwork, and whole-class review of student work.

Although significant work has been done over the last decades in studying instructional quality, the research findings are not very consistent, and their meaningful relations to student learning are not entirely convincing. The features and instructional aspects identified in different countries and regions with different cultures and traditions needed to be studied further (Praetorius & Charalambous, 2018). In the worldwide perspective, superficially, mathematics teaching in the Chinese systems looked very traditional and backward, but a more fine-grained analysis of the data in some international studies of classroom teaching showed that the quality of teaching was high (Leung, 2018). Some prior studies have been conducted to examine how Chinese teachers teach mathematics and develop their knowledge and expertise through practice (An et al., 2004; Huang et al., 2010; Huang & Bao, 2006; Huang & Leung, 2004; Huang & Wong, 2007; Li & Huang, 2008a, 2008b; Ma, 1999; Watkins & Biggs, 2001). However, there are only a few studies on high-quality lesson characteristics (e.g. Ding, 2021; Ding et al., 2019; Huang & Li, 2009; Lin & Li, 2009). Although some studies investigated effective teaching in China from the experienced teachers' perspective (Cai & Wang, 2010; Huang et al., 2005; Wang & Cai, 2007; Yang, 2012), the teachers' beliefs and their practices were not consistent (Cai & Wang, 2010).

To identify the features of mathematics classroom instruction excellence valued in China, the selection of sample lessons was most important. Huang and Li (2009) examined a particular exemplary lesson in depth. Lin and Li (2009) explored the general overview of mathematics classroom instruction valued in Taiwan by analyzing 92 lessons from six experienced teachers according to the following three themes: features of problems and their uses in classroom instruction, aspects of problem–solution discussion and reporting, and the discussion of solution methods. In the above-mentioned studies, the available methods for examining the characteristics of high-quality instruction were presented, and readers were provided with information to understand classroom instruction identified and valued in China. However, the following concerns needed to be addressed. Not every lesson delivered by an experienced teacher can qualify to be an excellent lesson. Some selected lessons were well prepared and were very different from daily lessons. In daily teaching, most teachers may not be able to spend extensive time and energy preparing a normal lesson. This is why, although most teachers value the characteristics of example lessons or prize-winning lessons in the teaching contest, they cannot design and deliver such lessons in their regular classes. The sample lessons should be selected from the daily lessons and be recognized as high quality. Thus, using Complementary Accounts Methodology (Clarke, 1998), high-quality lessons from daily lessons were selected for this study to serve as example lessons, and, concurrently, were closely related to teachers' normal teaching practice.

2.2 Analyzing high-quality mathematics lessons from a lesson structure perspective

Teaching is a system of interacting elements embedded in a cultural context (Stigler & Hiebert, 1999). Classroom teaching is complex with the interaction of various factors, including the teacher, students, mathematics content, and organization. Researchers from different fields have developed different observational instruments to capture instructional quality with a focus on generic versus content-specific dimensions or a combination of both (Charalambous & Praetorius, 2018). However, the recurrence of particular lesson components in the practice of teachers participating in the same or similar school systems suggests that some form of typification is possible. There could be a high degree of regularity in the lessons or in the sequencing of particular types of instructional activity in the delivery of a topic (Clarke et al., 2006a, 2006b). For teachers, especially new teachers, when preparing a lesson, how to structure a lesson is among the predominant problems that must be considered seriously. Lesson structure is an important indicator when observing the instructional quality of a class (Kyriakides et al., 2018; Walkington & Marder, 2018). It refers to the elements of teaching activities that constitute a class, and their sequence.

Stigler and Hiebert (1999) reported that U.S. lessons can be characterized by the recurrence of the following four distinct classroom activities: reviewing previous material, demonstrating how to solve problems for the day, practicing and correcting seatwork, and assigning homework. Meanwhile, German classroom activity types were described as follows: reviewing previous material, presenting the topic and problems for the day, developing the procedures to solve the problems, and practicing. Japanese activities were described as follows: reviewing previous lessons, presenting the problem for the day, students working individually or in groups, discussing solution methods, and highlighting and summarizing the major points. In the TIMSS 1999 Video Study, lesson structure was inclusively analyzed using the categories of reviewing, introducing new content, and practicing new content (Hiebert et al., 2003).

In a series of LPS project studies that aimed at better representing the richness of the teaching activities in Japanese classrooms, Shimizu (2002) reclassified the teaching activity elements and developed a new schema that included thirteen elements for coding Japanese lessons. In a symposium at the 10th Biennial Conference of the European Association for Research on Learning and Instruction, which compared mathematics classrooms' lesson structures in different countries from various insiders' perspectives, Jablonka (2003) revealed that German mathematics lessons exhibit a much higher variety of lesson patterns in interaction, talk, and teachers' shape and perceptions of the lessons. In addition, Clarke et al. (2006a, 2006b) reported the nature of lesson structures from each of the USA, Germany and Japan as results of a study in which they analyzed sequences of ten lessons, interpreted through the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews. These findings, obtained from different insiders' perspectives, can be accumulated and serve as useful information for teachers and educators to learn from each other and reflect on local educational practices.

It is also easy to find that some elements of teaching activities in the above studies cut across cultural, national, and country boundaries, such as reviewing, problem solving, and practicing. Meanwhile, the sequencing of the elements of teaching activities as well as some specific elements of teaching activities differ among countries. China has a particular educational condition and cultural background. Chinese lessons, as indicated by prior research (e.g. Mok, 2006; Huang & Li, 2009), seemed to demonstrate unique features. However, the details regarding what exactly the unique features were, still called for further study. The above studies provided a good basis for developing a suitable framework to analyze the structure of Chinese high-quality lessons. In this study, we aimed to capture the characteristics of Chinese high-quality lessons from the perspective of lesson structure, for the purpose of painting a broader picture of instructional quality. Specifically, in this study we aimed to address the following two questions:

1. What elements of teaching activity do Chinese teachers use to organize a lesson?
2. How are the elements of teaching activity sequenced in high-quality lessons?

3 Methodology

3.1 Selection of high-quality lessons

To select high-quality mathematics lessons recognized in the Chinese educational environment, which closely related to teachers' daily teaching practice, a three-stage procedure was adopted. First, five to ten junior high schools were selected from five regions of China (east, north, west, south, and center), from which five to seven Grade-7 mathematics teachers were randomly selected. In order to ensure the quality of research data, each selected teacher was asked to record two lessons; thus, 448 lessons from 224 teachers were recorded, videotaped and converted into digital format. These videotaped lessons were collected during the 2011–2013 school year. Standardized procedures for camera use were developed, tested, and revised, and videographers were trained according to these procedures. Next, an instrument for instructional quality assessment, containing ten criteria classified into three categories (Instructional Content, Instructional Procedures, Instructional Strategies), was developed by our research group. Then, one relatively good quality recorded lesson was selected from each teacher to constitute the studied data for this study, namely a total of 224 recorded lessons were evaluated using this instrument. In addition, 12 geometry and 12 algebra lessons with the highest ratings were preliminarily selected. Then, a university professor who specializes in the field of mathematics teaching theory (Expert 1) and a well-known master teacher with over 30 years of teaching experience (Expert 2) were invited to screen and verify the 24 selected lessons separately, following their particular experience and criteria. After screening and further evaluation, 16 lessons (eight algebra and eight geometry lessons), all approved by three independent parties, were selected as the final research sample. The selected lessons could largely represent high-quality mathematics lessons from Chinese daily teaching. Almost all teachers of the selected lessons had over 10 years of teaching experience. In addition, all teachers graduated from four-year normal universities with majors in mathematics; five of them were male, and the rest were female.

3.2 Lesson coding schema and coding reliability

For the coding process, preliminary codes were first established by referring to the literature (e.g. Hiebert et al., 2003; Lindorff & Sammons, 2018; Shimizu, 2002; Stigler & Hiebert, 1999). Stigler and Hiebert (1999) developed different teaching activity types to code German, American, and Japanese videotaped data. For the Japanese data, the following five activities were used: reviewing the previous lesson, presenting the problem for the day, students working individually or in groups, discussing solution methods, and highlighting and summarizing the major points. Considering that the teaching activity categories adopted by Stigler and Hiebert (1999) were too inclusive to be useful in representing the richness of the activities, Shimizu (2002) developed thirteen activity elements to code the Japanese lessons, as follows: reviewing the previous lesson (RP), checking homework (CH), presenting the topic (PT), formulating the problem for the day (FP), presenting the problems for the day (PP), working on the sub-problem (WS), students working individually or in groups (WP), presentation by students (PS), discussing solution methods (DS), practicing (P), highlighting and summarizing the main point (HS), assigning homework (AH), and announcement of the next topic (AN). Five of these were derived from the Japanese lesson pattern reported by Stigler and Hiebert (1999), whereas the remaining elements were newly developed or modified. In this study, six of these elements were maintained, and four elements were newly developed or modified after observing the first three sample lessons. For example, FP and WS were excluded because they were not typically observed in Chinese sample lessons. In addition, PS was merged with WP. Practicing was described in more depth as presenting a problem (PP), working on the problem individually or in a group (WP), and teacher-guided solution discussion (TG). The teaching of pure mathematics knowledge was added because it is an important regular teaching activity in Chinese mathematics lessons. Thus, the final coding schema included ten elements of teaching activities, which are described in detail in Table 1. The recorded lessons were coded with the help of a software package called NVivo. First, the videotaped lessons were imported through the *data* button, and *nodes* were built following the coding schema. Cutting the beginning and end time of each particular element of teaching activity followed. After that, specific codes were assigned. By repeating these steps, the software helped to ensure that the elements of teaching activities, timelines, and coding bars were aligned such that the data were suitable for further analysis. All of the sample lessons were coded by one of the authors, and 20% of the sample lessons were double-coded independently by a doctoral student majoring in mathematics education. She read the coding framework, watched the lesson videos and coded them according to the framework. The inter-rater agreement (Cohen's Kappa) for coding was 0.91.

4 Lesson structure-based analysis of the characteristics of high-quality lessons in China

Figure 1 displays the 16-sample mathematics lessons' structure regarding the elements included and their appearance. The characteristics regarding how single elements and sequences of elements were arranged in the lessons' instruction process are presented in the following subsections.

4.1 Analysis of single elements of teaching activity in the lesson structure

4.1.1 Most sample lessons began with a review

Reviewing previous lessons (RP) was a popular teaching activity in most sample lessons. Reviewing what was learned aims to activate students' existing cognitive basis and helps them to be well-prepared for the new class. Two types of

Table 1 Ten elements for analyzing the structure of sample lessons

Elements of lesson activity	Description
Reviewing previous lessons (RP)	The content of previous lessons and the relevant mathematics knowledge that students have learned in the former class are reviewed
Introducing the topic (IT)	The teacher uses various methods to introduce the lesson's topic. Usually, the topic is written by the teacher on the blackboard or whiteboard or exhibited through a premade presentation
Teaching pure mathematics knowledge (TM)	The teacher introduces a specific mathematics concept, formula, rule, theorem, or algorithm that has been planned for the lesson; provides corresponding explanations and deductions, and emphasizes the key points
Presenting a problem (PP)	The teacher assigns a mathematics problem to the students, explains the context, asks the students to think about and explore solutions
Working on the problem individually or in a group (WP)	The students are given some time to think, explore solutions, and provide answers, either individually or in groups rather than as a whole class
Teacher-guided solution discussion (TG)	The teacher guides the whole class to work together to discuss the solutions to a problem and then reviews the correct procedures to solve the problem and provides the answer
Highlighting and summarizing the main points (HS)	The teacher reviews and summarizes the most recently taught key knowledge points, problem-solving strategies, or any other point that they think requires emphasis
Summarizing the whole lesson (SW)	The teacher highlights and summarizes the main points of the entire content covered in the lesson or expands upon the taught knowledge
Assigning homework (AH)	The teacher assigns problems to the students to be completed after the lesson
Checking homework (CH)	The teacher discusses the submitted homework, such as indicating common errors and introducing good examples or reviewing the correct answers with the students

a

Lessons	Lesson-structure Sequence with Different Teaching Activities																						
1	RP	IT	WP	TG	TG	WP	TG	TG	TM	TG	PP	WP	TG	SW									
2	RP	TG	WP	TG	TG	TG	WP	TG	WP	TG	WP	TG	TG	TG	SW	WP	TG	TG	CH				
3	RP	PP	TG	IT	TM	TM	TM	PP	WP	TG	PP	WP	TG	SW									
4	IT	TM	TG	TG	TG	TG	TM	TG	WP	TG	TG	WP	TG	SW	PP	TG	TG						
5	IT	TM	TG	TM	TG	HS	TG	TG	HS	WP	TG	SW											
6	RP	IT	TM	HS	TG	TG	TG	TG	TG	WP	TG	AH											
7	IT	RPL	PP	WP	TG	TM	PP	WP	TG	CH	PP	PP	SW	AH									
8	TG	TG	TG	HS	PP	WP	TG	HS	PP	WP	TG	SW											
9	IT	PP	TG	TG	PP	WP	TG	TG	HS	TG	TG	TG	TG	TG	HS	TG	PP	WP	TG	SW			
10	RP	IT	TM	TG	TM	TG	PP	WP	TG	TG	HS	PP	TG	WP	TG	SW							
11	RP	IT	TM	TG	PP	WP	TG	TG	WP	TG	WP	TG	TG										
12	RP	IT	TM	TG	TG	TG	WP	TG	WP	TG	HS	TG	SW	WP									
13	RP	IT	TM	TG	TG	WP	TG	TG	HS	TG	TG	TG	TG	TG	TG	TM	SW	TG	AH				
14	RP	IT	TM	TG	WP	WP	TG	WP	TG	TG	WP	TG	TG	TG	WP	TG	WP	TG	HS	WP	TG	SW	AH
15	RP	IT	TM	PP	WP	TG	PP	TG	WP	TG	HS	PP	WP	TG	PP	WP	TG	TG	PP	WP	TG	TG	SW
16	RP	IT	TM	TG	HS	TG	PP	WP	TG	WP	TG	PP	WP	TG	SW	AH							

b

RP	IT	TM	PP	WP	TG	HS	SW	AH	CH

Fig. 1 Lesson–structure sequence with different elements of teaching activities of sample lessons. *RP* reviewing previous lessons, *IT* introducing the topic, *TM* teaching pure mathematics knowledge, *PP* presenting a problem, *WP* working on problem individually or in a group, *TG* teacher-guided solution discussion, *HS* highlighting and summarizing the main points, *SW* summarizing the whole lesson, *AH* assigning homework, *CH* checking homework

strategies were used by the teachers to review mathematics knowledge, as follows: (1) directly reviewing the key mathematics knowledge, such as asking students the meaning of a concept, content of a formula, or theorems, and (2) solving some practical problems requiring the application of the learning of the former class. The teachers could also make these problems produce a double effect. A typical example is exhibited in a screenshot of the video of Lesson 11, as shown in Fig. 2. Here, the teacher launched four problems to be solved independently by the students to review *multiplication between polynomials*. The students could solve these problems by applying the rules learned in previous lessons. These problems were unique, as the formulas of all four of them could be abstracted to a common form (a + b)(a − b). Through multiplication, some students found that the results followed a fixed rule. This reviewing process helped students strengthen what they had learned in previous lessons and laid a strong foundation for the following topic of this lesson: *square difference formula* at the same time. In short, Reviewing Previous Lessons (RP) proved effective in helping students prepare for a new class.

4.1.2 Pure mathematics knowledge learning was emphasized

Evidence for the emphasis on the learning of pure mathematics knowledge was obtained by analyzing the activity of Introducing the Topic (IT) and Teaching Pure Mathematics Knowledge (TM). On the one hand, all new lessons had a particular lesson title that was a concrete mathematics knowledge topic, such as a trapezoid or a quadratic equation throughout the class. At the beginning of the class, the teacher usually wrote a specific lesson title on the blackboard, which was a key learning goal and laid the foundation for the learning that followed. On the other hand, most sample lessons (81%) contained the activity of TM, including mathematics concepts, formulas, rules, or theorems. Before teaching a specific mathematical knowledge topic, the teachers usually spent time foreshadowing it and then tried to demonstrate the topic from different angles, such as providing some examples and non-examples to help students understand the essence of the topic. Then, the teachers launched some example problems and practical problems to help students further master and apply what they had learned.

Evidence for the emphasis on pure knowledge learning was further obtained by analyzing the position at which the two related teaching activities intersected in the sequence of teaching activities. As shown in Fig. 1, these teaching activities occurred in the first half of most lessons, where the students paid maximum attention. For many students, learning mathematics concepts and formulas can be boring and abstract. The teachers tried to help student build meaningful understanding of abstract concepts or formulas by variation, which is a popular way among Chinese teachers to promote effective mathematics learning (Gu et al., 2004). Consider, for example, Lesson 11. First, the teacher launched four concrete mathematics expressions and allowed the students to find the square difference formula; then, she abstracted and presented the formula on the blackboard and analyzed the formula's characteristics (the sum of two parts multiplied by the difference of two parts is equal to the difference of the two parts' squares) to help students consolidate their memory of the formula and use the formula skillfully. The teacher also emphasized the holistic thinking involved in this formula, where *a* and *b* could represent both numbers and algebraic expressions.

4.1.3 Recurrent teaching activities related to mathematics problems ran throughout the lesson

Figure 1 clearly shows that problem-related teaching activities constituted the main part of each lesson. The average number of problems launched in each sample lesson was

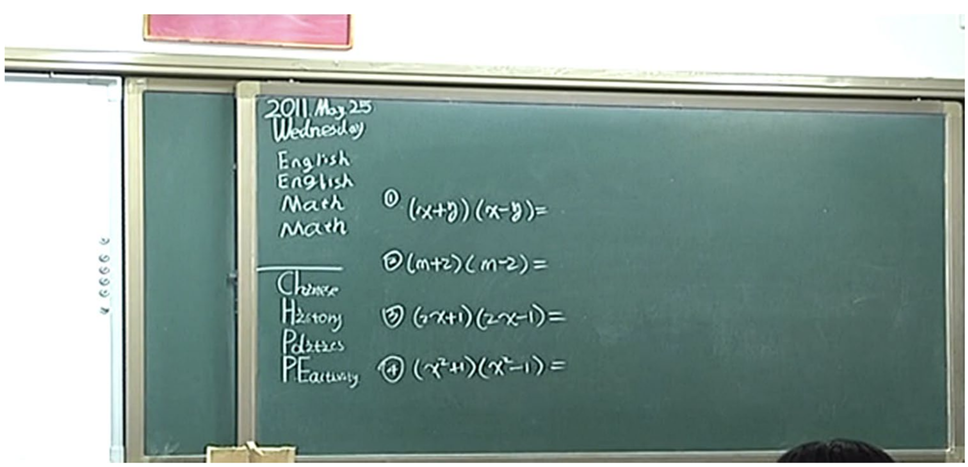

Fig. 2 Problems used for reviewing in Lesson 11

seven. Four patterns of problem-related teaching activities were found in the sample lessons.

The first category was PP-TG (Presenting a Problem followed by Teacher-Guided Solution Discussion, which was a minority category—5%). This category's characteristics included high cognitive demand geometry problems or algebraic problems in a real-life context. The teachers normally guided the students to read the problem and explained some difficult parts to help them understand the problem more clearly.

The second category was WP-TG (Working on the Problem Individually or in a Group, Teacher-Guided Solution Discussion), in which the presenting part was not coded because the problem was usually displayed on the slides and the teacher did not spend any time reading or explaining the problem but rather asked the students to explore the solutions directly. The percentage occupied by this category was 22%, and it was usually implemented in the lessons' second part.

The third category was PP-WP-TG (Presenting a Problem, Working on the Problem Individually or in a Group, Teacher-Guided Solution Discussion) with an occupied percentage of 15%. The characteristic of the problems in this category was high cognitive demand. Here, the teacher left some time for students to think, inquire, or communicate and, finally, guided the whole class to check the answers, share ideas, summarize the solutions, and emphasize the essential points to solve the problems. This category provided the students with more learning opportunities. Obviously, the number of mathematical problems decreased if the teacher preferred to implement the problem according to this category.

The last category was TG (Teacher-Guided Solution Discussion) with an occupied percentage of 58%. In this category, neither problems presented nor explored were coded because of the short time allocated to them. This finding indicated that in the sample lessons, the teacher preferred to guide the whole class to solve the problems together by asking the students a series of heuristic questions until they determined the solutions rather than providing them enough time to think individually or work in groups. Moreover, they were not taught the solutions directly. The teachers engaged the students in the class by questioning and interpreting the problems heuristically. In this way, the students obtained opportunities to think and practice. For example, in Lesson 15, the teacher introduced a series of variant problems, as shown below, for the students once they had learned the square difference formula. Some of these problems were launched to understand and apply the formula easily (see Example 1), whereas some were launched to apply the formula in more complicated situations (see Examples 2 and 3).

1. Please judge which of the following problems can be calculated using the square difference formula, and then calculate:

   (1) $(a+b)(a-b)$;

   (2) $(-a+b)(-a-b)$;

   (3)$(a-b)(-a+b)$;

   (4) $(a+b)(-a-b)$;

   (5) $(b-a)(-a-b)$;

   (6) $(-3x+5y)(3x-5y)$;

2. Calculate using the square difference formula:

   (1) 101×99;

   (2) $1007\times(-993)$;

3. Simplify and calculate:

   $(2x-y)(2x+y)-\left(2y+x^2\right)\left(2y-x^2\right),\ x-1,\ y=2$

4.1.4 Highlighting and summarizing important points during the lessons and summarizing the whole lesson at the end of the class were frequently used

Most sample lessons (88%) contained the teaching activity of Summarizing the Whole Lesson (SW). In this activity, some teachers provided the students with a framework to guide them to consolidate and extend the whole lesson's key knowledge points, some preferred to summarize the entire class through questioning, and some tended to select particular students to answer particular questions. When the class time was insufficient, the teachers tended to summarize by lecturing, during which the students just needed to follow and understand the teacher. Regarding the content of summarization, it was generally not a specific mathematics problem but a review of general and inductive content, such as indicating the key points when solving various problems, or the essence of concepts. Some teachers even categorized the problems so that the students could apply the methods to the corresponding category. The entire lesson's review was aimed to emphasize its core content, such as mathematical thinking and ideas, strengthen students' mastery of the new knowledge and skills, and enhance their learning effectiveness.

Consider, for example, Lesson 10. Here, the teacher guided the students proficiently to master several key concepts and the properties derived from them.

> *T: Let us talk about what you have learned in this class.*
> *Ss: Definition of a trapezoid.*
> *T: OK. What is a trapezoid?*

Ss: A quadrilateral with a pair of parallel opposite sides and another pair of nonparallel opposite sides.
T: What about an isosceles trapezoid and a right trapezoid?
Ss: An isosceles trapezoid is a trapezoid with two equal waists, and a right trapezoid is a trapezoid with a right angle.
T: Good. So, what are the properties of an isosceles trapezoid?
TS: Two base angles on the same base are equal, and two diagonal lines are equal; it is an axisymmetric figure, and the line activity with two middle points of the base is the axis of symmetry.

The teaching activity of Highlighting and Summarizing the Main Points (HS) could be conducted anytime during the lessons, whenever the teachers considered it as necessary. Take Lesson 5 as an example, where the mathematics topic studied was *congruent triangle*, and the teacher highlighted and summarized the main points twice. The first time was after the students solved a problem that asked them to determine the corresponding side and angle of the triangle. The teacher highlighted a strategy according to which if there was a common arm, then they were the corresponding sides. The teacher highlighted them in red for emphasis and asked the students to take note. The other highlighting and summarizing activity involved the strategy of determining the corresponding angle after the students had solved the other two problems. The teacher summarized this using two congruent triangles, where both the equal and opposite angles were the corresponding angles.

4.2 Analysis of the sequence of lesson-structure elements

4.2.1 Elements of teaching activities were of high density and large volume

Classroom instruction was composed of different elements of teaching activities, the number and sequence of which constituted the characteristics of different classes. The largest number of whole elements of teaching activities was 23, the smallest number was 12, and the average number was 16. The number of whole teaching activities was always larger than that of different teaching activities, meaning that some teaching activities appeared more than once. The largest, smallest, and average numbers for the different teaching activities were 9, 5, and 7, respectively. Some teaching activities were repeated many times, especially problem-related teaching activities. In most lessons, the ratio of the total number of whole teaching activities to that of different teaching activities ranged from 2 to 3. This indicates that the Chinese high-quality lessons were highly dense, with different teaching activities being switched or repeated frequently in a 'spiral' rather than a 'linear' structure. The teachers attempted to fulfill the teaching goal and provide students with as many opportunities as possible.

4.2.2 Most lessons exhibited a similar structure pattern

Most lessons were structured according to the following pattern: starting with reviewing the previous lesson, then introducing the lesson topic through different strategies, followed by teaching pure mathematics knowledge, and continuing to launch and solve various problems. During these teaching activities, the teachers highlighted and summarized the main points. Usually, whole-class summarizing and assigning homework were used to end a lesson. Although the number of problems solved differed among different lessons, they were designed for understanding pure mathematics knowledge in the class or helping students improve their problem-solving ability by applying mathematics knowledge.

5 Conclusion and discussion

In this study we aimed to capture the characteristics of high-quality lessons in China by analyzing single elements of teaching activities and the elements' sequences that were important when organizing classes. This research offers an important complement to the research on high-quality instruction worldwide, as well as an in-depth understanding of Chinese students' mathematical learning, and provides multiple choices for mathematical teachers worldwide when considering how to improve their teaching efficiency.

Mathematics teaching is considered as a cultural activity. Only through better understanding of instruction can we really improve it and consequently have an impact on student learning (Charalambous & Praetorius, 2018). Due to the unified mathematics curriculum standards and similar social and cultural backgrounds, the sampled Chinese mathematics teachers exhibited a particular teaching pattern that included the following: reviewing previous lessons; introducing new topics; learning new concepts, rules, formulas, or propositions; working on typical examples together with teachers and students; practicing variant problems; and highlighting and summarizing the main points. This sequence aligns with that of Rosenshine and Stevens(1986), who found that student learning is positively influenced when teachers actively present materials and structure them as follows: (a) beginning with overviews and/or review of objectives, (b) outlining the content to be covered and signaling transitions between lesson parts, (c) calling attention to main ideas, and (d) reviewing main ideas at the end. This clear and well-organized lesson structure also echoes that advocated in the theoretical underpinnings of the UTeach Observation

Protocol (UTOP) (Walkington & Marder, 2018). Structuring activities assists students to develop links between the different parts of a lesson instead of dealing with them in an isolated way (Kyriakides et al., 2018).

5.1 Instructional coherence was ensured by adopting specific elements of teaching activities

At the beginning of each lesson, the RP activity was widely used to help students consolidate what they had learned and build new concepts on prior knowledge, which is an effective teaching strategy according to prior research (e.g., Walkington & Marder, 2018). According to Cai et al. (2014), in the Chinese teachers' views of instructional coherence, they emphasized the interconnected nature of mathematical knowledge beyond the flow of teaching. During the lessons, the teachers spent several minutes highlighting and summarizing the important points, some of which were related to the essence of a concept, others of which were key points required to solve a category of problems, and still others of which were prevalent mathematical thinking and methods. At the end of each lesson, the whole lesson was summarized to deepen students' understanding of the lesson's important concepts, which echoes the Japanese perspective that highlighting and summarizing the major points seemed to be an indispensable element in any successful lesson (Shimizu, 2006). Through the three stages of reviewing, highlighting, and summarizing, students can effectively achieve their learning goals. Moreover, they can master new concepts, theorems, rules, or formulas—understand their meaning and value, memorize them, and apply them to solve problems.

Both memorizing and understanding were emphasized by the teachers. They not only required their students to understand what they had learned but also to memorize further, followed by proficiently applying and practicing. The effect of practicing in supporting students to solidify what have they learned has been emphasized in some observation frameworks of instructional quality (e.g., Kyriakides et al., 2018; Lindorff & Sammons, 2018; Schaffer et al., 1998; Walkowiak et al., 2018). When teaching concepts, formulas, theorems, or key problem-solving strategies, the students are often required to take notes so that they can consolidate and review them after the class. This parallels a previous study (Zhang et al., 2019) in which fluent memorization of key points was among the main teaching goals and memorization and understanding were intertwined. A popular ancient Chinese educational proverb is *looking back to the old if you want to learn the new*. The academic selection function of mathematics significantly influences high-stakes examinations in recruitment for universities and at other levels and even at compulsory levels (Wang & Guo, 2018). Thus, teachers wish to help students obtain higher achievement through high classroom expectations—not only to understand the meaning but also to obtain fluency through memorization.

5.2 The lesson was mainly structured by content-related teaching activities

Mathematical content is valued in many instructional frameworks, such as TRU (Schoenfeld, 2018), MQI (Charalambous & Litke, 2018), and M-Scan (Walkowiak et al., 2018). Content selection and presentation was emphasized in the comprehensive framework of observing instructional quality developed by Praetorius and Charalambous (2018), including elements of selecting mathematically worthwhile and developmentally appropriate content, motivating the content, and presenting the content in structured, mathematically accurate, and correct ways. The utilization rate of the activity TM (Teaching Pure Mathematics Knowledge) in the sample lessons aligns with the findings of the above research. In the lesson sequence of teaching activities, this activity often appeared in the first half of the class, after the teaching activity of introducing the topic and before the example and exercise problems were launched. Irrespective of the type of introduction strategy adopted, the teacher's goal is always to lay a strong foundation for students to learn new mathematics knowledge, with the follow-up example problems and exercise problems aimed at promoting and consolidating the understanding and mastery of new content. Although the time of this teaching activity was short, it played a central role in the classes. The meanings of concepts, rules, formulas and mathematical ideas were overarchingly emphasized. As Schoenfeld (2018) advocated, the powerful classroom provides students with opportunities to learn disciplinary ideas, techniques, and perspectives, make connections and develop productive disciplinary habit of mind.

The findings of the TIMSS 1995 video study showed that the Japanese lessons' structure was characterized as "structured problem solving" (Stigler & Hiebert, 1999; Stigler et al., 1999). In the TIMSS 1999 video study, the findings showed that a considerable portion of lesson time in every participating country was spent on solving mathematics problems, which is consistent with our findings. When considering the sequence of each element of teaching activities, it is easy to see that the teaching activities related to mathematical problems emerged after or before the TM activity (Teaching Pure Mathematics Knowledge). The purpose of problem-related activities was to explore or deepen understanding of the new mathematical knowledge (including concepts, formula, rules, theorems, and so on) or to cultivate students' abilities to apply the newly learned knowledge to solve problems.

A problem-based teaching approach was seen as an effective approach to delivering high-quality lessons (Huang &

Li, 2009). The teachers in our sample lessons provided students with opportunities to solve plenty of varied problems. When both generalized and concrete strategies are available for problem-solving, Chinese teachers usually expect the students to choose the generalized strategy as the better one (Cai, 2004). However, note that as most of the selected mathematics problems are routine problems, students lack experience in improving their ability to solve problems without clear solutions. Following previous studies, Chinese students outperformed American students in solving simple computational problems and processing closed problems; however, their performance in processing open problems was weaker (Cai, 1995, 2000). The experience of solving unconventional problems is closely related to the cultivation of innovation consciousness and ability, which is also one of the goals advocated by the new curriculum standards and which should be strengthened in the future.

5.3 Balancing the dominant position of the teacher and stressing student subjectivity; trying to build a peer-learning community

Some researchers identify the nature and quality of classroom instruction by considering students' opportunities to engage in cognitively demanding mathematical work and discussions (e.g.Boston & Candela, 2018; Schoenfeld, 2018) whereas some highlight the work of teachers (e.g., Kyriakides et al., 2018; Muijs et al., 2018; Schaffer et al., 1998). Based on the results of this study, we advocate a 'double center' idea, wherein both teacher-guided and students-engaged aspects are important features of high-quality classroom instruction, especially with large class sizes and limited class time. The teacher should take charge of classroom management and selecting and implementing tasks while also paying attention to maximize student's engagement, ownership and autonomy.

Mathematics learning is both an individual and a social process (Cobb, 1994). In our sample lessons, learning through a social process, especially through teacher-student interactions, was very common. Among all four categories of problem-based teaching activities, the teacher-guided solution discussion occupied the maximum part. When tackling mathematics problems, compared to providing enough time for students to think independently or explore in groups, the teachers preferred to guide the whole class together to solve problems by asking questions or through heuristic tutoring. It is highly challenging for these teachers to address specific individuals given the large class sizes. Instead, they try to consider students' general needs, as indicated by Wang and Murphy (2004). The teachers are responsible for introducing a topic, teaching pure mathematics knowledge, highlighting and summarizing the main points, and summarizing the whole lesson. Concurrently, they also care about students' engagement in the class, including their trying, struggling, thinking, comprehending and answering. Therefore, even in the teacher-centered model, students can develop abstract cognitive understanding after 'passively' accepting and memorizing basic knowledge (Cai & Wang, 2010).

Meanwhile, we must also recognize that such a fast-paced situation lacking enough independent thinking might induce negative learning attitudes among students, especially those with learning difficulties. In the new era, society needs more talent with the ability to cooperate and innovate. As Chi et al. (2018) noted, collaborative learning has the highest level of cognitive engagement. Thus, some possible suggestions are to select mathematics problems that can provide more learning opportunities, such as problems with higher cognitive demand without clear procedural solutions, to moderately slow down the teaching pace, to build a positive and cooperative learning environment, and to provide appropriate time and space for students to think independently and work in groups.

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