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ORIGINAL PAPER

Implementation of a developmental model of teachers' and didacticians' learning through inquiry: design, operationalisation and outcomes

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Abstract
This paper addresses implementation with respect to the professional development (PD) of teachers of mathematics and the educators/didacticians who work with them, through an *inquiry-based* developmental model. In contrast with a PD model in which educators show, guide or instruct teachers in classroom approaches and mathematical tasks, we present a developmental model in which teachers and educators collaborate to inquire into and develop their own teaching practice. The project, *Learning Communities in Mathematics* (LCM: e.g., Goodchild, Fuglestad and Jaworski, 2013) exemplifies this developmental model. Here we focus on a project *Teaching Better Mathematics* (TBM) which extends LCM and implements its developmental model at larger scale. We trace the implementation process through analysis of data gathered during and after the extended project, including written reflections of key didacticians, minutes from leadership meetings and two versions of the project proposal. Particularly, we trace learning and development through an activity theory analysis of the issues, tensions and contradictions experienced by participation in TBM.

Keywords Implementation research · Developmental research · Inquiry-based teaching · Learning in mathematics

1 Introduction

The activity we discuss in this paper involves development of the teaching and learning of mathematics, using an inquiry model. Our *model* fronts a set of complex relationships in which groups of mathematics educators/didacticians/researchers work with groups of teachers to evolve *an inquiry-based* set of processes with the aim of developing students' understanding/skills related to mathematical concepts, teachers' understanding/skills related to teaching such concepts, and didacticians' understanding/skills related to supporting teachers in their evolution and providing research to facilitate and document its progress. This very long sentence tries to capture the 'three-layer' nature of our model, as we explain below.

In this model we engage in what we call 'Developmental Research' where practitioners are co-learning inquirers in the implementation processes (Jaworski, 2006; Wagner, 1997). Underpinning our model is the notion of 'activity'. We use the concept of *activity* in the sense of Activity Theory (Leovt'ev, 1979; Roth & Radford, 2011), as we explain in Sect. 3.3. We describe a developmental, collaborative process through which people engage with and reflect on their activity, through which knowledge and expertise evolve, and through which those involved grow and develop personal *awareness* and *being*. Important issues in this process of development are the emerging contradictions in the form of tensions the participants experience and how they impact the outcomes of the activity. Activity Theory provides tools that we use in bringing to light and analysing these tensions.

The developmental model and its implementation will be exemplified below through two projects in which the first author was involved: a medium-scale project LCM (Learning Communities in Mathematics) involving a university and schools in one district in Norway, and a larger scale project TBM (Teaching Better Mathematics) involving five districts

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and their schools. Building on the model and process, of developmental research though inquiry, established in LCM, we discuss the implementation of LCM's developmental model at a larger scale in TBM. We identify key factors in the implementation process and the associated developmental outcomes.

2 Implementation and implementation research

The call for papers for the special issue, *"Implementation and implementability of mathematics education research",* expressed *implementation* as:

> a change-oriented process of endorsing an action plan based on a relatively well-defined resource (such as a research finding, a digital tool, an innovation, a pedagogical framework, a curriculum, a textbook, or an institutional policy) that occurs in interaction of two communities, a community of the resource proponents and a community of the resource adapters.

In this definition, the critical notions of resource, proponents (CRP) and adapters (CRA) are related in different ways according to the adopted theoretical perspectives. For example, a "fidelity" model of implementation focuses on whether the resource (e.g. an innovation) is enacted (by the CRA) according to the intentions of the proposer (the CRP), while an "adaptation" model emphasises the process of implementation of an innovative idea and the conditions and contexts that affect it (Century & Cassata, 2016). According to Century and Cassata (2016) the fidelity perspective is an outsider's perspective where the proposer dictates how the innovation will be implemented while the adaptation perspective considers the users as active agents in a process in which contextual aspects and personal meanings guide their decisions. In this paper, we present an adaptation perspective: here the CRP are the didacticians/educators (sometimes also the school leaders), who define the basis of activity while the CRA include *both* the didacticians and the teachers who bring the activity to fruition.

Century and Cassata (2016) consider implementation research as the study of the several efforts of education research to bring successful and lasting change. They write (p. 172) "Contexts and conditions can affect innovation enactment in legitimate ways: … improving education requires processes for changing individuals, organisations and systems". The implementation of the developmental model focuses on the developmental processes as conducted in ordinary teaching practice and providing a critical review of outcomes relating to the developmental goals of the innovation.

Developmental research studies the developmental process and, in doing so, contributes to the development itself. In some ways it is similar to design research, but it goes beyond design research to be inclusive of practitioners who are co-learning inquirers in the developmental model (Jaworski, 2001; Wagner, 1997). Co-learning inquiry builds on the work of Wells (e.g., 1999) who writes about the idea of *Dialogic Inquiry* in which classroom activity and discourse can be seen as a *Community of Inquiry* in which teachers and students interact dialogically, and on the work of Cochran-Smith and Lytle (e.g., 1999) who write about *Inquiry as Stance*, in which the inquiry stance is a form of social positioning taken in a community of teachers in which inquiry has become one of the social norms in practice.

Forms of research that have similarities with implementing a developmental model are the implementation of design-based research (DBR) and design-based implementation research (DBIR). Gravemeijer and Eerde (2009), focusing on DBR, discuss the iterative character of design and implementation, the goals of the researchers to develop local instruction theories and of the teachers to adapt these theories in various instructional settings. They introduce the notion of dual design research to consider the development of the teachers: "We added that teacher learning might be an integral part of research on student learning" (p. 523). Extending DBR in large scale, Fishman et al. (2013) talk about DBIR emphasising the collaborative design and the capacity for sustaining change in systems. Implementation in DBR and its different forms is a more structured way of implementation than the developmental model. The goals of design in DBR are mainly defined by the academic researchers and focus on specific content areas.

The very complexity of a developmental project with its implicit tensions and contradictions contributes to problems in its processes of implementation at scale. The model is fundamentally 'adaptive': this means that there is no given set of procedures or tasks that can be conveyed, promoted or evaluated. The model has been implemented in various projects, mostly small in scale. Here our starting point is the medium-scale project "Learning Communities in Mathematics" (LCM), extensively reported elsewhere (e.g., Goodchild, Fuglestad & Jaworski, 2013; Jaworski, 2008). The activity and findings of LCM formed the basis of a second project TBM (Teaching Better Mathematics) in which the developmental model of LCM was implemented at a larger scale. Here we study the implementation of TBM to demonstrate the nature of and issues arising from implementation of a developmental research project.

This means that the *resource* for implementation in TBM is the *developmental model* developed in LCM; the proponents (CRP) are the didacticians who designed the developmental model for LCM (and hence as a basis for TBM) and the adapters (CRA) are the didacticians (a larger group) who

implemented TBM with teachers, and the teachers themselves; although, in this paper, we focus on didacticians in the CRA. Thus, we apply the methodology of implementation research to the TBM project in which we follow Century and Cassata in their five "Reasons to study implementation".

1. Inform innovation design and development;
2. Understand whether (and to what extent) the innovation achieves desired outcomes for the target population;
3. Understand relationships between influential factors, innovation enactment, and outcomes;
4. Improve innovation design, use, and support in practice settings;
5. Develop theory (p. 174).

We return to these five reasons at the end of the paper, when we have addressed implementation of TBM, and reflect on how this study informs us in the implementation process.

3 Developmental research and the LCM (learning communities in mathematics) Project

3.1 Inquiry leading to developmental research

The basis of our developmental model in LCM is the concept of *inquiry*. Inquiry involves exploration, questioning, investigation and problem solving. Inquiry may be seen, in its first manifestations (e.g., in inquiry-based mathematical tasks for use with students) as *a tool to aid development*; however, an aim of inquiry is that it becomes central to the *being* of the inquirer. Inquiry *as a way of being* is the mainstay of the developmental model (Jaworski, 2006). In activity theory terms, the inquirer's activity, most often seen within a *community of inquiry*, embeds all actions and their goals pervasive to cognition and emotion in the activity engaged.

The model is of *inquiry* in three layers: inquiry in *mathematics*, in teacher-student interactions; inquiry in *teaching*, by teachers, and by educators/didacticians, exploring, and hence developing their own practice; inquiry in *the research process*, in which researchers (including teacher-researchers) conduct research-inquiry within a systematic (rigorous) methodology. (Jaworski, 2019).

We focus on the idea of developmental research within a community of inquiry which involves collaboration between teachers and educators/didacticians, both active as researchers and both developing their own knowledge and practice. The two groups act in a mode of *co-learning partnership* in which.

> both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling (Wagner, 1997).

In the activity within this model, it is proposed to form *communities of inquiry* through which teachers and didacticians collaborate to explore inquiry-based practices and learn from outcomes. Theoretically, this leads to *critical alignment* in which practitioners align with the expectations and norms of practice while at the same time *looking critically at what they do and how they do it*, inquiring into their practice with a view to developing it for the better (Goodchild, Fuglestad and Jaworski, 2013; Jaworski, 2006, 2019).

3.2 Learning communities in mathematics (LCM)

The LCM Project, Learning Communities in Mathematics, was a four-year project in Norway. Its main aim was to use *inquiry-based* processes to gain insight towards improving students' mathematical learning outcomes and the developmental process(es) involved (within the developmental model). A team of didacticians (n = 12–15) worked with (~ 30) teachers in 8 schools ranging from lower primary to upper secondary. The didacticians designed the project and gained (substantial) funding. The project would be a collaboration between didacticians and teachers, all of whom would have their own sub-goals which could and would differ; particularly teachers' subgoals varied across the participating schools.

Project money paid for teachers to leave school and attend a workshop in the university for a half day every 2 months during 3 years of field-work. Teachers worked with their students, following up workshop activity and reporting back to subsequent workshops. Each workshop involved joint work on one or more mathematical problems offered by one or more participants, discussion in small groups and in plenary to provide insights to what might be done in classrooms (For further details see Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild & Grevholm, 2007).

Inquiry communities were implemented: teachers and/or didacticians worked together to explore relevant activity, to reflect on that activity and learn from it, and to feed back this learning to new activity, building on what had been learned. Project teachers worked with others in their school, hence involving a wider group of teachers with the project. The didacticians, each school team, and subgroups within schools formed their own communities of inquiry with differing characteristics and different goals to achieve. A research process, involving didacticians and (sometimes) teachers, analysed data from a range of sources to address

research questions related to inquiry in mathematics learning, teaching, activity in schools, collaborative relationships and so on. In all of these, the central outcome was recognisable development, in and beyond the project, which also included a recognition of issues and tensions that impeded learning goals (Jaworski, 2008).

3.3 LCM learning outcomes

Workshop activity was extremely successful in LCM with each group sharing views and making inputs related to their own specialisms. Particularly, presentations by teachers revealed classroom activity that explored inquiry-based processes with students, and resulted in insights for the nature of practice in both teaching and teaching development (Jaworski et al., 2011).

An important element of the learning from LCM was that ownership of the project needed to include all participants. Ownership of activity and developmental processes began with the didacticians but it was when teachers took the initiative to tell didacticians how activity should/could be different, that a more equitable relationship with agency on both sides developed. (Bjuland & Jaworski, 2009; Goodchild, Fuglestad and Jaworski, 2013).

Teachers and didacticians had initial views and expectations of each other that stood in the way of reaching an equitable balance: teachers saw didacticians as knowledgeable experts who would show them how to teach better. Didacticians tried hard to dispel this perspective by words and actions but remained unaware of the depth of feeling and disappointment when expectations were not fulfilled. It was felt that inclusion of schools and teachers at the design stage would be important for future projects (Goodchild, 2007; Goodchild & Jaworski, 2005).

Analysis of issues and tensions using activity theory provided further insights into key elements of the developmental model (Goodchild, Fuglestad and Jaworski, 2013; Jaworski & Goodchild, 2006).

3.4 Activity theory and developmental research

In order to address tensions and contradictions that arose in implementing the developmental model of LCM in the TBM project, we use Activity Theory which we have adopted from Vygotskian scholars, Leont'ev, Cole, Engeström, Roth and Radford (e.g., Jaworski & Potari, 2009). Activity theory allows us to recognise issues and tensions in the developmental process and the ways in which human participants cognise and emotionalise their actions and goals in activity.

Particularly we draw on Leont'ev (e.g., 1979) to focus on *actions* and *goals* in classroom practice and teaching development. According to Leont'ev, "human *activity* is the non-additive, molar unit of life … a system with its own structure, its own internal transformations, and its own development … motivated within the sociocultural and historical processes of human life" (Leont'ev, 1979, p. 46). The actions and goals are central to the activity which is engaged and depend on *operations* and *conditions* in the environment in which activity takes place. As we study processes in our developmental work with didacticians and teachers, it is important to locate the goals and their associated actions and to consider where actions lead to the achievement of goals, or not. Where not, we seek out the challenges in human relations and environmental conditions that lead to contradictions or tensions in the developmental activity. These challenges are often embedded in the activity as the participants (subjects) create their own image of the object of the activity (in our case teaching that achieves students' mathematical learning through inquiry). This dialectical relationship between the object as it is in the reality and as it is viewed by the subject is an example of what Roth and Radford (2011) based on the work of Leont'ev and Marx call "inner contradiction" (p. 23). Similar contradictions exist in the way that the subject experiences, through division of labour, the universal (e.g. the interest of society) and particular (e.g. the particular interests of the individuals). In our case didacticians and teachers interpret differently the object of the developmental activity so the individual and the collective goals may bring into light these contradictions. This dynamic realisation of the developmental model allows us to address issues in implementation and to consider ways of addressing or circumventing the challenges. In our study, we consider Activity as 'the whole' (Leonte'ev, 1979) in contrast with activity which includes aspects of practice in the project (e.g., use of tasks in classrooms, small group interactions in workshops); we rely on context to make the usage clear. We will expand on these ideas to show our use of activity theory later in this paper.

4 Implementation of the LCM-based developmental model in TBM

We focus centrally on the project TBM, as an implementation initiative based on the lessons learned from developmental research in LCM, with the following questions for its implementation:

1. Design: How was the developmental model (of LCM) implemented in the larger scale project and with what implications?
2. Operationalisation: In what ways did scaling and extension from LCM affect the operationalisation of TBM?
3. Outcomes: What issues, tensions and contradictions emerged related to scaling and extension?

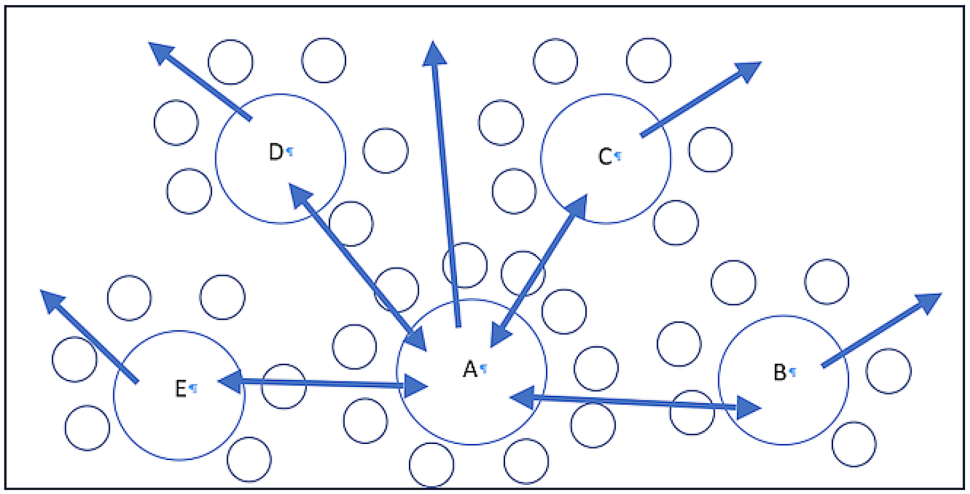

Fig. 1 Schematic diagram of the proposed TBM consortium—arrows indicating development between and beyond the particular centres

We focus on activity and development within the community of didacticians, drawing on data from a number of sources. For identification of issues and tensions we use activity theory to analyse their (complex) contribution to implementation of TBM. We address in 4.1 the design of TBM; in 4.2 the methodology of the empirical study; in 4.3 the operationalisation of, and development in TBM; and in 4.4 the implementation of the developmental model and its outcomes.

4.1 The design of TBM

TBM, a 4-year project, was initiated to build on and expand LCM in several directions. Implementation, in TBM, of the developmental model of LCM involved the following goals:

a. To institute or maintain a developmental focus based theoretically in inquiry-based learning and teaching of mathematics in schools through the formation of communities of inquiry between teachers and didacticians;
b. To build on developmental research from LCM:
    i. with an expanded population in the local site (Centre A);
    ii. with four more sites in Norway (Centres B, C, D, E) with related theoretical and operational focuses;
c. To elevate the participation of schools to joint decision-making and their own funding.

The proposal for funding for TBM came in response to a call, from the research council, for research into a range of educational interests including, *Subject didactical challenges in light of the education reforms* and *New forms of school practice in teacher education.* Participants in LCM wished to extend the developmental model of LCM in a more overt partnership with their district schools (i.e., involving schools in design and decision-making from the beginning) and with new schools and didacticians (henceforth Centre A). Due to the interest of colleagues in several other cities, the extension to four other parts of Norway was considered (henceforth Centres B, C, D, & E). At the end of a nation-wide conference to disseminate LCM, representatives of each proposed new centre met with didacticians at Centre A to initiate TBM. Several further planning meetings followed in which centre leaders discussed theoretical and operational principles considering factors in a, b and c above. New elements in TBM (beyond LCM) involved extending schools to include 'barnehager' (infant schools), the inclusion of teacher education, and new theoretical perspectives. A proposal was sent to the funders. The result was acceptance with several conditions including a serious reduction in the requested funding resulting in trimming of personnel and resources. A major requirement was that schools sought their own funding from relevant sources.

A consortium was set up with the five centres including a university and schools in each of the five districts. Each centre had to plan with local schools to achieve what was proposed. The schematic diagram in Fig. 1 represents the consortium, circles representing centres with university and schools, arrows drawing attention to developmental relationships with some extending beyond the centres to indicate further spread and influence of the project.

A, B, C, D E are regional centres, in each case a university linked with surrounding schools in the local district. Centre A, had been centre of the LCM project: in the expanded project, some schools from LCM continued to TBM and some new schools (including barnehager) were recruited; teacher education became a new focus. Teachers continuing from LCM acted as mentors for new recruits joining TBM. For the consortium, Centre A then linked with Centres B, C, D and

E, each with their own schools. Didacticians from Centre A led the way in guiding those in the other centres. Thus, the TBM project had a complex linkage in which the university teams planned to achieve goals based on the developmental research from LCM with new elements. Each new centre worked with their own schools in establishing their own developmental project based on the theoretical origins of LCM in Centre A, but taking directions that were relevant to their own and their schools' interests.

The adaptive nature of TBM, evolving from LCM, raised issues of structure and control. Primarily, this meant control of the developmental model in relation to its aims, and the associated structuring. However, structuring within Centres B to E was largely in the hands of the centre, without direct experience of LCM goals and outcomes, an important issue in their development. As we evidence below, schools' autonomy in the interpretation of classroom mathematics teaching sometimes conflicted with achieving the inquiry-based goals of the project.

To build on varying knowledge and experience in the consortium, didacticians at the five centres formed a leader group which met frequently in the first year of the project and less frequently after this. These meetings allowed a developing discourse in which new elements of theory and new ways of working were discussed. For example, two centres (TBM-B and TBM-E) were particularly interested in the importance of language in mathematics and its contribution to classroom activity. Another (TBM-C) wanted to explore mathematical knowledge in teaching (MKT). This brought new possibilities for inquiry-based processes with a focus on language and on MKT. In TBM-D, there were only two didacticians, both experienced teacher educators but inexperienced in running a research project, so they built their research activity of what they learned within the consortium.

4.2 Data from the TBM project and its analysis

In this section we discuss our data and analysis in addressing our research questions. Here we identify the elements of data we used, their nature and origins and associated analysis. TBM involved a consortium of schools, teachers and didacticians in five centres (A, B, C, D, E): we described their inter-relationships in Sect. 4.1.

The main data that we use in this paper to develop understanding about the design and the implementation of the developmental model in TBM are (i) the two versions of the proposal (the second following the reduction in requested funding) written to the research council with details of what was proposed in TBM; (ii) minutes from centre or consortium leadership meetings at different stages of the project, (iii) end of the project written reflections of nine didacticians in Centre A, and (iv) the authors' requested written reflections of the didacticians leading TBM in the centres B, C and D. It was not possible to access data from Centre E. Sources (ii) from leader meetings enabled us to report on the design of TBM and reveal activity and issues pertinent to the development of the project. Sources (iii), had been obtained by a leading member of Centre A and consist of reflective remarks related to didacticians' experiences in TBM. Sources (iv) were sought as part of our work for this paper from key didacticians in centres B to E – members of the consortium leadership group. We were interested in developmental outcomes, as recognised by the individual partners of TBM, that have had influence more broadly, with a focus on the addressing of tensions and contradictions indicative of growth. Didacticians in Centres B, C and D were given a set of questions related to authors' research questions to report on their experiences. They were asked to write reflections about the participants in the collaborative groups (teachers and didacticians), the development of the goals of the didacticians and the teachers in implementing the developmental model, the conditions that facilitated or prohibited the process and the outcomes of TBM for all the participants.

In the analysis of the data an open coding method was used initially, independently by each author, first extracting key statements relating to development in TBM and tabulating these for a following round of categorisation (axial coding – Strauss & Corbin, 1997). Emerging themes were identified through discussion between the authors. The main themes concerned: the collaboration among teachers and didacticians in the different sites and among the didacticians of the five centres; the outcomes of the implementation for the teachers and the didacticians; the different goals of the teams and participants; the conditions and their impact on the process and outcomes of the implementation; and tensions emerging in the process of implementation. These issues were further analysed under the perspective of activity theory in the attempt to address our research questions related to the design, operationalisation and outcomes of the developmental process addressing the conditions and the implied contradictions involved in its implementation.

4.3 Operationalisation and development in TBM

Early meetings in TBM focused on principles and goals and their operationalisation: for example, "How do we build a community in each place?", some colleagues suggesting this would be a local decision albeit learning from LCM. "Useful things" were seen to be discussion of theoretical and methodological issues, since didacticians in mathematics came with different educational backgrounds, some not familiar with theory and the rigour of educational research.

Different centres worked with different numbers of schools; this had an influence on the structure of activity,

and on internal relationships. An aim for one early leaders' meeting was as follows:

> To start to focus on research from the point of view of teachers. What is the sociocultural ambience/milieu in which teachers function and have their knowledge? What kinds of constraints do teachers face in trying to develop/improve the learning opportunities in mathematics for their pupils?
> When we think about research and development from didacticians' perspectives, do we really have any deep understanding of where the teachers are coming from? Can we really seek "equality" in our partnerships with teachers?
> [Item in consortium leader meeting agenda early in TBM]

LCM had revealed issues and tensions between teachers and didacticians so, in TBM, there was an initial awareness and expectation of differences. A requirement (from the research council) was for schools to attract their own funding for their participation in the project. School focuses (goals) for this funding were not always consistent with the inquiry-based philosophy of TBM: school leaders or district officials often had a differing view, from project leaders, of what their funding should achieve. This led to a fragmentation of focuses, goals and research questions from these differing perspectives affecting the ways in which teachers saw their participation in TBM.

During the first year of TBM, a two-day consortium leader meeting was planned to address key questions arising for didacticians. It was held in a neutral environment, with major focuses on inquiry in mathematics and on research methodology, for example:

> "What characterises mathematical thinking? How can our student-teachers and pupils be supported to develop an inquiry attitude and approach in their learning and work on mathematics?
> "Some of us are sitting on substantial amounts of data, and the question arises, how to get this into a form that can be presented e.g. in the shape of a paper."
> [Questions for the two-day meeting]

The developmental model initiated in LCM was implemented in a range of directions in TBM leading to learning for both didacticians and teachers. The nature of this learning was not pre-given (as in a fidelity model), and indeed it varied between the centres as each developed their activity with teachers in their own ways. This learning was shared at local and national conferences in Norway, as well as elsewhere, leading to a greater awareness of professional growth for both teachers and didacticians in relation to students' learning of mathematics in Norway. A special issue of a (Norwegian) professional development journal, consisting of papers from the five TBM partners and the consortium, communicated the TBM study to date (*Tidsskriftet FoU i praksis* 4(3)).[1] In addition, each partner published from their own project (mainly conference papers, including local, PME and CERME papers).

An issue that affected the continuing work of the consortium was that the consortium convener left the project to take up a new job. An experienced colleague took over the role of coordinator. The didactician concerned was also leader in Centre A. The demands of this dual role had implications for the ongoing work of the consortium.

4.4 Implementation of the LCM-based developmental model in TBM

4.4.1 Participants' goals and collaboration

Didacticians goals were expressed mainly implicitly in their written reflections. They talked about the development of inquiry communities, "in and across the schools" (TBM-B); they wanted to consider teachers' needs in the TBM project, "We emphasised that it was important for us as didacticians to take the teachers' needs and wishes as our starting point" (TBM-C); to incorporate inquiry to the local research projects, "We as researchers/educators took an inquiring attitude to the project and to the teachers" (TBM-C), "These groups worked mainly on research on mathematics teaching, with a focus of research on "inquiry" in mathematics teaching" (TBM-D); to engage in research and development in the context of TBM, "It was really a challenge to do both developmental and research activities at the same time" (TBM-A). These goals were aligned to the aims of the TBM proposal but, as we indicate below, they were sometimes in tension with the goals of the teachers and especially of the school leaders. For didacticians, the development of an 'inquiry-way-of-being was evident in the discourse, although perhaps less so for teachers.

Didacticians found it difficult at some instances to integrate the overall aims of the developmental model with their local research goals and with teachers' goals.

> It seems that we and the schools have different ideas about what it means to do a developmental project based in one subject [mathematics]. These problems were not resolved, and although the teachers were interested in collaborating with us, and we had no problems being admitted to their classrooms, it was very difficult to find time to work with them outside

[1] *Tidsskriftet FoU i praksis* 4(3) can be obtained from Frode Rønning frode.ronning@ntnu.no. Only one of the included papers is in English.

of the school hours, in an attempt to develop inquiry communities. (TBM-B)
As I recall, the school leaders were more into developing organisational structures at and between the schools, whereas the didacticians, and also the teachers were more into development in the subject and with the pupils. (TBM-B)

However, they mentioned positive experiences of the collaboration and they developed further understanding of important aspects of the collaboration such as teacher ownership.

In the workshops the groups were relatively stable and my impression is that teachers, and I also include myself, developed confidence in the group and dared to be more open and to express opinions and suggestions when working with tasks or when discussing didactical matters during the time of the project. I will propose that we developed a learning community within the group. Ideas were shared and school visits showed that teachers tried to adapt and implement some of those ideas in their teaching. (iii: TBM-A)
Ownership to developmental projects was important to implement. We learned that the most important thing for a school development project to be successful is that it had roots in the staff involved. Furthermore, we discussed our roles as educators and researchers (didacticians) in relation to the teachers, and that it is of crucial importance for a school development project to be based on the teachers' needs. (iv: TBM-C)

However, many didacticians reported that to adopt inquiry in different levels was interpreted in different ways by the participants on several occasions:

"All actors in the collaboration did not seem to share the same goals of the project" (ii: TBM-B);
"When it comes to Inquiry - as a way of being - we can say that we obtained that to a certain extent in the didactician/teacher group. However, we have not seen "inquiry" to the extent we could wish in the teachers' mathematics teaching". (iv: TBM-C)

A subtler reasoning, supported by the feedback, suggests that the inquiry philosophy was not well enough understood. At a simplistic level, it seemed that inquiry was possible alongside all the other focuses: e.g. a focus on MKT can have inquiry-based goals. However, one response can serve as an example of dissonance here: one respondent commented on the difficulty of providing tasks and "getting teachers to use them" [iii: TBM-A]. This suggests a fidelity mode of thinking on the part of the didactician, but we might see it as a perception of too many directions leaving teachers and didacticians unsure of their goals.

4.4.2 Tensions and contradictions

While inquiry was the overall innovation at research and practice level, each research team had other specific goals (e.g. focus on MKT, mathematical language). This created tensions between personal research perspectives versus project research perspectives, suggesting too many directions to make sense together and leading to a perceived fragmentation. Some respondents pointed to too many choices and too much openness. As we have said above, there was a requirement (from the research council) for schools to attract their own funding for their participation in the project. School focuses (goals) for this funding were not always consistent with the inquiry-based philosophy of TBM: school leaders often had a differing view, from project leaders, of what their funding should achieve. This led to a further fragmentation of focuses, goals and research questions from these differing perspectives affecting the ways in which teachers saw their participation in TBM. We provide below some examples from the didacticians' comments to illustrate the above tensions:

Many of the members [didacticians] struggled with the concepts throughout the project period. We also felt "locked into" the individual research questions we had formulated at the start of the project, even when we began to understand that the common goal was suffering from the fragmentation. (iii: TBM-D)
We experienced a tension between the goals of the school leadership, both locally at the schools and in the leadership at "kommune"-level [district level], and the goals of the didacticians. The teachers were in a way squeezed between these goals. (iv: TBM-B)

The coordinator of TBM reported a personal tension: while she found the consortium leader group a valuable forum for sharing and support, coordination of TBM together with leadership in Centre A in the original LCM environment proved too demanding – there were just too many threads to attend to at any time. On reflection it would have been better to split these responsibilities. Overseeing and coordinating the differing strands of TBM was a demanding job in itself.

A consequence of issues above was that a main force within TBM was towards five different projects each with its own focus (alongside inquiry) and each with teachers' own perceptions and interests within its schools. This led to a number of tensions for project philosophy – one respondent suggested there were too many inbuilt risks for a cohesive response to the project. Another didactician pointed out the difficulties to move from small scale projects to large scale projects when the didacticians did not have prior experiences "It is certainly true for me that in earlier research projects that I have led or taken part in there has been only one to

five researchers in the collaboration. So, it makes a difference to be so many as we have been in the TBM-project" (iii-TBM-A).

Almost all didacticians experienced it as a tension to balance development and research. In a demanding project, much time and effort were needed for the developmental activity, leaving too little time for addressing research questions, analysing data and writing papers, a requirement of their job. Except for the one journal special issue, there is little else published about TBM *as a whole*, although individual consortium members published from their project. Indeed, one partner (Centre E) received further funding from the research council to explore their own particular focus, from which two books (in Norwegian) emerged – a further element of scaling.

The project leader in Centre A, and TBM coordinator wrote:

> I would like to have the capacity and inspirations to work on research and publications alongside the developmental work. In a new project we have to be careful in planning this into the project plan. Be more specific what to do. I have tried to put research on our agenda, but very often we had not much time. (iii: TBM-A)

4.4.3 Outcomes

Alongside the emerging tensions most didacticians reported also positive outcomes of the TBM project. These concerned both the development of research and of mathematics teaching at school.

In terms of research, young researchers felt that they gained a lot by collaborating with experienced researchers and with the teachers in schools.

> To work together with researchers in a group as we did, was for me a new experience, and I really appreciate that we were so different: different when it comes to age, experience with developmental work and also we had very different backgrounds. I had the pleasure to prepare a workshop and have a presentation together with [an experienced researcher], and to present the project and some results together with [other researchers] near the end of the project. The opportunity to work with more experienced persons has given me valuable insights into such work. (iii: TBM-A)

In some centres, didacticians felt that research outcomes were not as they had expected mainly because balancing research and development was demanding: "the focus on research was difficult to keep and even if we mentioned this issue several times it was really a challenge to do both developmental and research activities at the same time" (iii: TBM-A). However, they also recognised long term outcomes in their research career offering as reasons mainly that experiences developed in the context of TBM contributed to their future research activities: "Results from TBM have had an impact on the research in [a subsequent project], e.g. the analysis of a multiplication situation [for a publication] was important for understanding certain episodes on multiplication encountered in [the subsequent project]" (iv: TBM-B).

In terms of the development of mathematics teaching and teacher development, the didacticians felt that there was not a lot of evidence about the use of inquiry approaches in mathematics teaching. They talked, nevertheless, about gains for the teachers:

> "teachers, and I also include myself, developed confidence in the group and dared to be more open and to express opinions and suggestions when working with tasks or when discussing didactical matters during the time of the project…ideas were shared and school visits showed that teachers tried to adapt and implement some of those ideas in their teaching" (iii TBM-A)

Several respondents suggested that the people who gained most from the project were the didacticians. Having deeper understandings of the philosophy of inquiry than teachers and, simultaneously, through their visits to schools, recognising the various issues experienced by teachers', they were able to see more clearly where the tensions lay, where the goals were not clear, or were contradictory. This was certainly a valuable learning experience for the didacticians which enriched their own knowledge and awareness.

There was evidence that other pressures on schools affected classroom practice in mathematics (Here LBM, *Learning Better Mathematics,* was an acronym adopted by schools who gained their own funding for TBM in Centre A):

> Our partners in LBM (the local district) have asked us to consider if it is possible to find out, or to test whether we (LBM/TBM) achieve worthwhile results or not in the project? Do we want to design a tool that can measure the activity in the project in order to document the development and students' improved learning? Is it possible and how do we manage to do this? What will be the quality criteria?
> We had a fruitful discussion where ideas came on the table. We all agreed that we do not want to test the students in school in order to try to document any improved learning. This will be too complicated and difficult to do. On the other side, we understand that the institutions that support the project want evidence that their money leads to some results. [Centre A: Didacticians' TBM meeting]

Looking back to LCM, it was recalled that evidence of classroom activity and students' mathematical achievements

had been visible in video recordings teachers had made from their lessons – a compilation of video extracts had offered visible high quality evidence: this was followed up subsequently in TBM.

4.4.4 Addressing the research questions

1. Design: How was the developmental model (of LCM) implemented in the larger scale project and with what implications?

   We have reported on complexity and fragmentation. The complexity was undeniable, with a clear issue of overall monitoring and control. For future projects we would emphasise necessity of an experienced developmental researcher to hold and exercise this responsibility without other significant demands. Fragmentation was seen in some cases, where a perception of too many focuses and demands got in the way of the big picture. However, there is considerable evidence of the developmental model with the associated learning of didacticians developing an inquiry way of being and in their experience of critical alignment. This points to recognisable potential for future teacher education. The recognised limitations in research outputs points to the original work plan in the consortium related to the financing of TBM. Due to the cuts in funding required by the funding council, a number of research positions had to be cut, which shifted research responsibility onto the experienced developmental practitioners. A timeline and guidelines for joint research across the centres, with dedicated responsibility, should have been instituted and followed. Research in the individual centres would have complemented TBM as a whole, rather than emphasising fragmentation.
2. Operationalisation: In what ways did scaling and extension from LCM affect the operationalisation of TBM?

   There was no doubt that scaling and extension from LCM affected the operationalisation of TBM in significant ways. In particular, the collaboration between the participants involved a more demanding handling of the different individual goals (e.g. goals of didacticians, teachers, schools and districts) in developing collective goals and actions to achieve the object of the activity and avoid perceptions of fragmentation.

   The tensions and contradictions that emerged in the scaling up process were mainly about balancing research and development. The complexity of the project involved development of a large number of tasks, interaction with a large number of schools, supporting them, for example, to develop their own proposals for funding as well as demands on them from external authority. Thus, the development of research collaborations among the didacticians were not facilitated due to time constraints linked to lack of adequate funding.

   Despite these constraints, the central developmental activity of TBM in collaboration between didacticians and teachers was initiated and sustained over the time of TBM, albeit in different forms. All reports indicate activity related to the developmental goals of the project with associated learning for both teachers and didacticians, and especially for the didacticians. The implications are for positive influences on teaching, learning and teacher education in the future.
3. Outcomes: What issues, tensions and contradictions emerged related to scaling and extension?

We list the main issues, tensions and contradictions as indicated above: the coordination of activity across TBM centres; the complexity of TBM and perceptions of fragmentation impeding its smooth progress; the issue of research time and the pressures of the developmental activity; the tension between the aims of TBM and external pressures on schools; the meaning of *inquiry* and how this was interpretated across TBM and between experienced practitioners and newcomers.

However, in relation to an inquiry approach to teaching and learning, the development in schools, learning of didacticians, spawning of further projects, and publications in Norwegian journals and international conference proceedings testify widely to the impact of these projects on the development of learning and teaching mathematics in Norway and hence to the implementation of the TBM project.

Thus, in TBM, we acknowledge both the many tensions and contradictions arising from the scaling of the project and the outcomes that address areas of impact. We address these from an activity theory perspective in our concluding section below.

5 Looking critically at implementation through the lens of activity

The developmental activity in TBM involved different conceptualisations of its object to improve mathematics learning and teaching in schools. The object develops throughout the period of collaboration as teachers and didacticians become more conscious of it and its implications, while tensions and their underlying contradictions are related to the different meanings of the object of the activity. The scale of implementation in TBM and the openness of the project made the contradictions more apparent. Contradictions also emerged in the division of labour and were related mainly to the individual versus collective goals and actions. The multiple perspectives of the participants often led to different outcomes

and made the developmental model more complex (some said 'fragmented').

Teachers and didacticians have different standpoints and meanings of the inquiry process in mathematics learning, in teaching and in research. Key elements of inquiry-based development involved collaboration in *being* and *becoming* inquiry practitioners in inquiry communities. As the project progressed, inquiry activity evolved new ways of seeing and doing, new ways of being and becoming. These new insights and associated actions are indicative of *change*. According to Roth and Radford (2011), writing of cultural-historical activity theory, "change is the fundamental unit, which means that this unit contains an internal contradiction (understood in a dialectical sense). It is *difference in itself* rather than difference *between* two identifiable states, one of which is transformed into the other" (p. 7: Italics in original).

These ideas of change and contradictions are complex, needing more space to address them than we have here. However, we see *development* in these projects to embody, for participants, internal contradictions seen in practice. Everyone in the TBM project is engaging with practice within an interpretation of *inquiry*, the *object* or *motive* of Activity. We engage and act according to a multiplicity of goals and conditions. As we interact teacher-teacher, didactician–didactician, teacher-didactician, teacher-student, our inner sense of inquiry is challenged through actions and inter-actions within the histories and cultures in which we have being.

The meanings develop in the continuous process as they emerge in the activity and they encounter "diverse and often conflicting cultural forms of being, knowing, and feeling and the unfolding subjectivity that is continuously produced and updated in the course of those encounters" (Roth & Radford, 2011, p. 47). This continuous process of being and becoming helps us to understand the complexity of implementation in collaborative settings between teachers and didacticians.

The process of becoming involves tensions in the participants' attempts to address each other's meanings. In the TBM project, tensions emerged in the attempts of the didacticians to promote their personal research interests and at the same time to implement the inquiry process. Moreover, the attempts of didacticians to balance research and development outcomes involved tensions. The emerging tensions and their role in the implementation indicate that implementation is not a smooth process where the researchers/proposers introduce innovation and the teachers/adapters enact it. Becoming aware of the tensions and trying to handle them leads to new understandings of all participants that drive the outcomes of the implementation. The teachers and the didacticians themselves become different persons throughout the years and they develop continuously new meanings of the enactment of the innovation. Implementation is not only an adaptation of specific subjectivities to an innovation but in the context of developmental research, the subjects themselves (here, teachers and didacticians) change and contribute to the transformation of the object of the implementation activity. This becomes more demanding for all participants when the implementation activity involves multiple centres and participants as in the TBM project.

Thus, we can see development as *states of being* in which *changes*, as they relate to the growth of inquiry, are manifested in the observed issues and tensions and the associated actions and goals of socio-historical-cultural consciousness in the communities of participants. Engagement and consciousness are dialectical within complexity. Awareness of tensions, in realisation of goals, challenges participants' consciousness to reveal its developmental frames. In research terms, these are the elements essential to research reporting to allow others to gain access to the subtleties of action and activity (Leont'ev) which give substance to reflective experience.

Implementation, therefore, is not a difference between initial states and final states in the progression of a project but is constituted in the Activity as manifested complexly in the project. The implementation research in which we have engaged, revealing issues and tensions and their relationships with the cultural-historical forces through which we are constituted, reveals key elements of the developmental model that contribute to what we call 'implementation'.

For ourselves, reconsidering *development* in connection with *implementation* has enabled us to describe new depths in the developmental processes employed. In TBM, the undoubted changes that took place in the activity described were premised on participants addressing the challenges they faced. Recognising these challenges as tensions or contradictions through an Activity Theory analysis, allowed us to see more clearly the *ways of being* of our collaborators in the project that led to the changes observed. Here, we can understand better how the social conditions in the different centres (e.g. funding, partnerships between schools and universities, collaboration between the universities) influenced the implementation of the developmental model, and its outcomes. We understood also the need for certain adaptations of the research goals and perspectives of the sites to fit the overall perspective of the implementation (of the inquiry process).

Finally, in response to the five reasons for studying implementation (Century & Cassata, 2016, p. 174), we believe that, in this paper, we have informed design and development with relation to an adaptive model based on developmental research. We have addressed the extent to which the innovation achieves its desired outcomes, particularly with respect to the issues and tensions it creates for participants. In doing this we have dealt extensively with influential factors, innovation enactment and outcomes. Regarding innovation design, use and support in practice settings we have communicated the close-to-practice nature of our study in

which we learn from our activity as we engage in and with it. Finally, our theory of inquiry-based activity and its contribution to learning in practice settings develops with every such project as we explore its developmental contributions to didacticians and teachers in their irreplaceable work with students.

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