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ORIGINAL ARTICLE

Can digital technology change the way mathematics skills are assessed?

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Abstract
Formative assessment strategies have been studied for a long time. Drawing on data from the FaSMEd (Formative Assessment in Science and Mathematics Education) project, this paper has the aim of contributing to research about formative assessment and the use of technology, in the field of mathematics education, by claiming that digital technology does modify classroom assessment processes when mastered by teachers, especially regarding the implementation of formative assessment strategies, but also by discussing how and to what extent this occurs, taking into account the different perspectives of the actors involved. The methodology of this research is founded in the design-based research paradigm, and the work with teachers is detailed in order to show the contributions of the project both in providing research results and in examples of practical use in the mathematics classroom.

Keywords Formative assessment · Technology · Feedback · Assessment situation · Instrumental approach

1 Introduction

The context of the research presented in this paper is the European project FaSMEd (Formative Assessment in Science and Mathematics Education[1]) which ran for 3 years and concluded at the end of 2016. This project involved a set of international partners, all of whom have recognized expertise in the analysis and implementation of pedagogies based on scientific investigation integrating the use of technologies. The purpose of this project was to consider the role of technologies in formatively assessing student achievement in science and mathematics.

The objectives of the project were stated as follows:

– to produce a set of resources and methods (known as a 'toolkit') to support the development of practices from a teacher professional development perspective,
– to build formative assessment approaches using technologies,
– to disseminate research results in the form of online resources and professional and research publications.

Each of the project partners worked with a set of schools in a Design-Based Research perspective (Wang and Hannafin 2005; Swan 2014); i.e., research involving a sequence of lessons designed jointly by teachers and researchers, implemented in classes and analysed jointly, according to both empirical data and theoretical perspectives, for reformulation and new implementation. In the case of France, the levels of the classes involved varied from primary school (grades 4 and 5) to lower secondary school (grades 6–9) and upper secondary school (grade 10). In this paper, we chose episodes which can significantly help in understanding if and how digital technology can change the way mathematics skills are assessed. Such episodes were selected especially from a grade 4 class (equipped with clickers) and a grade 9 class (equipped with tablets). In these classrooms, teachers volunteered to test a pedagogy based on both Formative Assessment (hereafter, FA) and the use of technology. The two classes were chosen, in this paper, as sources of episodes

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to show that the model can be applied at any school level and to various themes and situations. Operational invariants were found at different levels in the teachers' didactic choices and strategies for the purpose of implementing formative assessment with technology. The research question addressed in this paper directly reflecting its title—*Can digital technology change the way mathematics skills are assessed?*—is not new in research in education. We present the state of the art in the first section, then the theoretical framework of this research study, and we network the theoretical lenses through which we analysed our data. In Sect. 4, we present the way we worked with the teachers involved, following a Design-Based Research approach, and the data on which our analyses in Sect. 5 are grounded. In the conclusion, we present elements in answer to the title question, drawing on the analysis of examples selected from our data, and detecting operational invariants in the use of technology for implementing FA strategies. These elements draw mainly on the teacher's level, although we also consider the students' level in responding to teacher action.

2 Literature survey

In order to answer the question we network different concepts coming from FA frameworks, namely, Brousseau's Theory of Didactical Situations (TDS, Brousseau 1997) and the instrumental approach.

Several studies have analysed technology-enhanced learning environments such as classroom connected technologies in general (e.g., Roschelle and Pea 2002), and in particular in the mathematics classroom (Stroup et al. 2002; Pape et al. 2013). A precise focus on technology enhanced formative assessment was made by Beatty and Gerace (2009) in the context of a classroom response system (clickers) and by Spector et al. (2016) who conducted a survey of all existing teaching and learning environments of this kind, allowing teachers to support and enhance student learning through formative assessment and feedback. Burns (2017) shows how technology tools can energize teacher practice by providing easy ways to implement formative assessment every day. Increasing interest is also arising concerning the professional development of teachers integrating technology into their practices in order to enhance formative assessment (Lee et al. 2012; Panero and Aldon 2016). Drawing on the existing frameworks of formative assessment, in particular Black and Wiliam's (2009) key strategies, we consider the FaSMEd model (Cusi et al. 2017; Ruchniewicz and Barzel 2019), created in the project to analyse the role of technologies in the implementation of FA strategies. In this paper we aim to contribute to such research, in the field of mathematics education, by claiming that yes, digital technology does modify classroom assessment processes, especially the implementation of formative assessment strategies, but also by discussing *how* and *to what extent* this occurs, taking into account the different perspectives of the actors involved.

The TDS is concerned with the conditions required to produce, communicate and learn knowledge that is recognised as mathematics, modelling them as situations having a particular *milieu* that consists of the objects with which the student interacts in a situation. It has been widely discussed, commented on, and expanded since its origin (Brousseau 1986, 1997, 2006, 2010, Perrin-Glorian 2008, Artigue et al. 2014). More particularly, the concept of *milieu* has been addressed in mathematics education (Margolinas et al. 2005; Artigue 2009; Kidron et al. 2014) but also outside mathematics education (Achiam et al. 2013); we refer to Margolinas and Bloch's refinements of the *milieu* structuring (discussed below) (Margolinas 2004; Bloch and Gibel 2011; Ainley and Margolinas 2015), which describe and organise the mutual roles of the student, the teacher and the *milieu* at different levels of interaction.

The instrumental approach, born in the field of cognitive psychology (Rabardel 1995), has been adopted and adapted in mathematics education (Rabardel 1999; Artigue 2002); numerous researchers in their studies have developed this theoretical approach by highlighting and clarifying the concept of scheme, used in the psychologists' sense (Vergnaud 2009; Määttänen 2016; Roorda et al. 2016) or as technical and conceptual components (Ceratto Pargman et al. 2018; Buteau et al. 2019). We adopt the definition of Vergnaud (2016) where schemes are "invariant organizations of the activity for a defined class of situations" (p. 290).

The originality of our approach lies in the local integration (Bikner-Ahsbahs and Prediger 2010) of these theoretical frameworks, allowing us to combine the concepts of *milieu* with the instrumental approach in a context of formative assessment with technology.

3 Theoretical framework

In this section, we present the construction of the theoretical frame underlying our work. Our aim is to detect schemes of use that are developed by the teacher and the students when working with FA-oriented technology, depending on the *milieu* structure level in which they are operating.

We present successively the FA approaches then the Theory of Didactical Situations (TDS) with the concept of *milieu*, the instrumental approach and the notion of scheme. The result of the networking of these frames appears as the model of analysis.

Table 1 Key-strategies of formative assessment (Black and Wiliam 2009, p. 8)

	Where the learner is going	Where the learner is right now	How to get there
Teacher	(A) Clarifying learning intentions and criteria for success	(B) Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding	(C) Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	(D) Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	(E) Activating students as the owners of their own learning	

3.1 Formative assessment

We consider formative assessment or assessment for learning as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black and Wiliam 2009, p. 9). The consequence of this interpretation is a deep modification of the teaching process leading to the construction of situations based on FA strategies. Formative assessment extends the intervention times that occur at different points in learning, and is further enriched by a communication perspective in the intertwined processes of teaching and learning. Even if different definitions of formative assessment still coexist (Allal and Lopez 2005; Dunn and Mulvenon 2009), the perspectives developed in the Anglo-Saxon world and organized around 'feedback' contribute to giving to formative assessment the status of a tool for the benefit of learning by shifting from 'assessment of learning' to 'assessment for learning' and even 'assessment as learning'. From this perspective, Black and Wiliam's (2009), based on their work, propose a model in which regulatory perspectives cross communication perspectives, leading to FA strategies that are integrated into the teaching/learning process. So, FA focuses on the teacher, the single student and groups of students. The three main questions of FA—*Where is the learner right now?, Where is the learner going? and How to get there?*—lead to a set of FA strategies that are adapted to the different agents and are represented in Table 1.

3.2 Theory of Didactic Situation

We speak of 'assessment situations' in order to insist on the idea that FA is the core of the teaching–learning process. This is also in reference to the Theory of Didactical Situations (TDS; Brousseau 1997) where the learning process is designed by the teacher in order to confront the learner with a *milieu* sufficiently responsive and challenging to augment his/her knowledge and meanings. The *milieu* of a situation is a central concept of the TDS and it is presented by Brousseau (2010) as follows:

> A situation is characterized in an institution by a set of relations and reciprocal roles of one or more subjects (pupil, teacher, etc.) with a *milieu*, aimed at transforming that milieu according to a project. The milieu consists of objects (physical, social or human) with which the subject interacts in a situation. The subject determines a certain evolution amongst the possible, authorized states of this milieu which he judges to conform to his project. (p. 2)

The *milieu* is both the tangible environment that the teacher designs for his/her students, including pedagogical methods (e.g., social organization, mode of communication) and all the previous knowledge or beliefs that the student, considered here as a generic student, can mobilize. Looking more deeply into the concept of situation, Brousseau (1997), then Margolinas (2004), Bloch and Gibel (2011), defined three levels of construction and analysis of situations: (a) a theoretical level where specific knowledge is linked to a didactical situation, which produces the theoretical game of a situation and the mathematical knowledge at stake; (b) the experimental level which considers the adaptation of the situation to a given level of education, modifies the didactical variables and pinpoints the entry of students into the game, the so-called devolution; and (c) the contingent level that corresponds to the effective implementation of the situation in a given classroom. These three levels of a situation lead to structuring it from the design stage to the implementation stage. The structuring of the *milieu*, from the pupil's or the teacher's point of view, highlights the different kinds of situations that correspond to distinct projects evolving from the teacher's design and devolution to the pupils, to their acceptance of the responsibility of engaging in them.

At each level S_n the *milieu* M_n of the situation is constituted by the entire situation S_{n-1} at the lower level (see Table 2). For instance, in the phase of constructing situations for the classroom, the teacher prepares, organizes and forecasts the students' material *milieu* and detects elements that allow him/her to grasp the dynamics between

Table 2 The structure of a didactic situation

Level	Student	Teacher	Situation	Milieu
M_{+3}: Design	–	T_{+3}: Noospherian	S_{+3}: Noospherian situation	Superdidactic levels
M_{+2}: Project	–	T_{+2}: Designer	S_{+2}: Design situation	
M_{+1}: Didactic	St_1: Reflexive	T_{+1}: Projector	S_{+1}: Project situation	
M_0: Learning	St_0: Student	T_0: Teacher	S_0: Didactic situation	
M_{-1}: Reference	St_{-1}: Learner	T_{-1}: Regulator	S_{-1}: Learning situation	A-didactic levels
M_{-2}: Objective	St_{-2}: Acting	T_{-2}: Observer–Devolver	S_{-2}: Reference situation	
M_{-3}: Material	St_{-3}: Objective	–	S_{-3}: Objective situation	

the material *milieu* and the objective *milieu* of the students. Several approaches to this model can then be considered:

- a top-down analysis (Margolinas 1995) which corresponds to an a posteriori analysis; we used this approach in the analysis of the lesson design;
- a bottom-up analysis (Bloch 1999) that analyses the realization of the situation, included in an a-didactic situation (or in a-didactic phases of a situation); we used this approach in class observations.

In this frame, the three levels of construction and analysis of an 'assessment situation' mentioned above are taken into account:

- in the lesson design, when the FA theoretical principles lead the design of the *milieu* and the use of technology is supposed to enhance a particular FA strategy;
- in the lesson implementation, when the main ideas guide the organization of the class and its orchestration;
- at the contingent level when decisions are made by the teacher to manage didactic time and when students take advantage of the confrontation with the designed *milieu* to build new knowledge.

From this perspective, we highlight what a technology-enriched *milieu*, at different structural levels, brings to the assessment dimension of the mathematics teaching–learning process; that is to say how teachers and students benefit from the functionalities of technology in detecting and assessing mathematical skills.

3.3 Instrumental approach and 'FA instruments'

The second, more functional aspect of this study focuses on the role of technological tools for implementing FA strategies.

In the FaSMEd project, we distinguished three specific functionalities of technology, namely, sending and displaying, processing and analysing, and providing an interactive environment. Then we crossed them with FA strategies and with the agents (teacher, individual students, peers) to build a three-dimensional model (Fig. 1). This grid can be used to describe and analyse the dynamics created by the

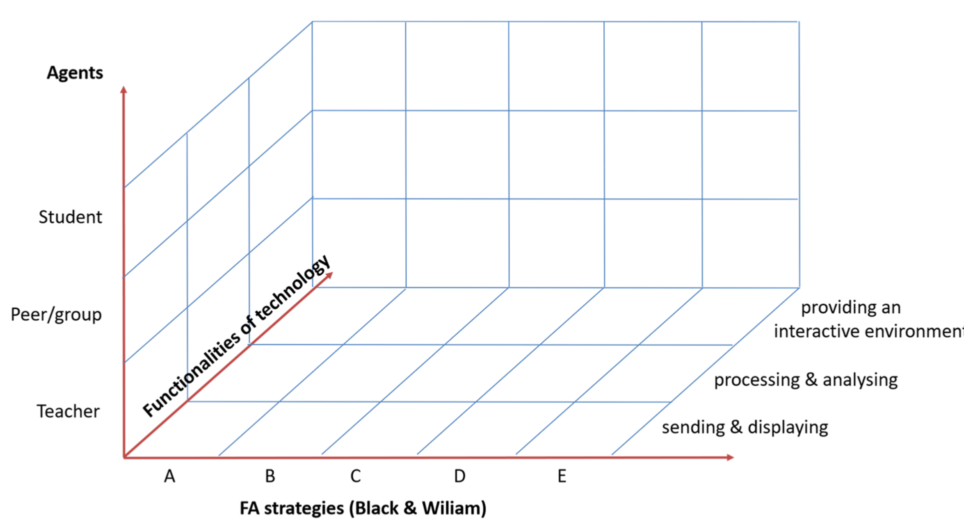


Fig. 1 FaSMEd three-dimensional model, where A, B, C, D and E are the FA key-strategies identified by Black and Wiliam (2009; see Table 1)

introduction of technology-assisted or technology-oriented FA strategies (Aldon et al. 2017; Ruchniewicz and Barzel 2019).

In a technology-enhanced environment, all the agents involved need to appropriate the tools in order to exploit fully and efficiently their different functionalities for FA purposes. Such tools are initially nude artifacts that, according to the instrumental approach, become instruments when combined with specific schemes of use, including an actor, rules of action, operational invariants and adaptation to the situation. This process, called instrumental genesis, is carried out through two dialectical phenomena, the instrumentation which allows the actor to shape the artifact by modifying it for his/her personal use and the instrumentalization which modifies the way the actor will act with the artifact, in this case the way of teaching and learning with technology. In the FaSMEd project, teachers were used to implementing FA strategies in their classrooms, but usually in a poorly structured way and often without technological tools or without an intentional use of a specific functionality of the technology. When the artifact-technology is used by an agent with a specific functionality to implement a particular FA strategy, it becomes a specific instrument for that agent. We thus define a *formative assessment instrument* (hereafter, FA instrument) as a given artifact-technology to which a specific scheme of use (agent, functionality, strategy) is associated. Let us clarify this idea and show it in the FaSMEd model (Fig. 2), with an example that is discussed further in this paper. Imagine that the teacher uses a connected classroom technology (e.g., NetSupport School) to display some students' solutions to a given problem with the aim of *engineering an effective classroom discussion eliciting student understanding*. We can identify this classroom connected technology along with the scheme of use (teacher, sending and displaying, FA strategy B) as a FA instrument. At the same moment, for another agent the same technology can become a different FA instrument. For example, individual students who *send* via tablet their own solution to the teacher and then see it displayed on the screen, in order to be discussed and validated by their classmates, are activating themselves as resources for their peers. Therefore, the tablet, as a part of the classroom connected technology along with the scheme of use (student, sending and displaying, FA strategy D), can be considered a different FA instrument. It appears clearly, from this example, that it is not just a matter of changing point of view (that of the teacher or that of the student) but it is an actual change of the entire scheme of use.

Clearly there is a dialectic between these two schemes of use as the agents interact in the FA process. With a specific focus on the teacher, he/she has to orchestrate, in dialogue with his/her scheme of use, all the students' instrumental geneses, which develop according to different rhythms in the classroom, and which can be different from time to time in the mathematical activity. Hence, advanced orchestration skills (Trouche 2004) have to be developed by the teacher to set a suitable technology-enriched *milieu* for the students and to implement FA strategies (Panero and Aldon 2016).

In this context, FA instruments can be different according to the different levels of *milieu* in which they are developed. For instance, in the example described above, teacher and students are in a didactic situation (S_0). The learning *milieu* (M_0) is enriched with connected classroom technology (tablets + NetSupport school) used as a teacher's FA instrument and a student's FA instrument. The student is *active as an instructional resource for his/her classmates* and the teacher *engineers a classroom discussion that elicits student understanding*. The question is: what is the additional value from the fact that both are using technology as a FA instrument?

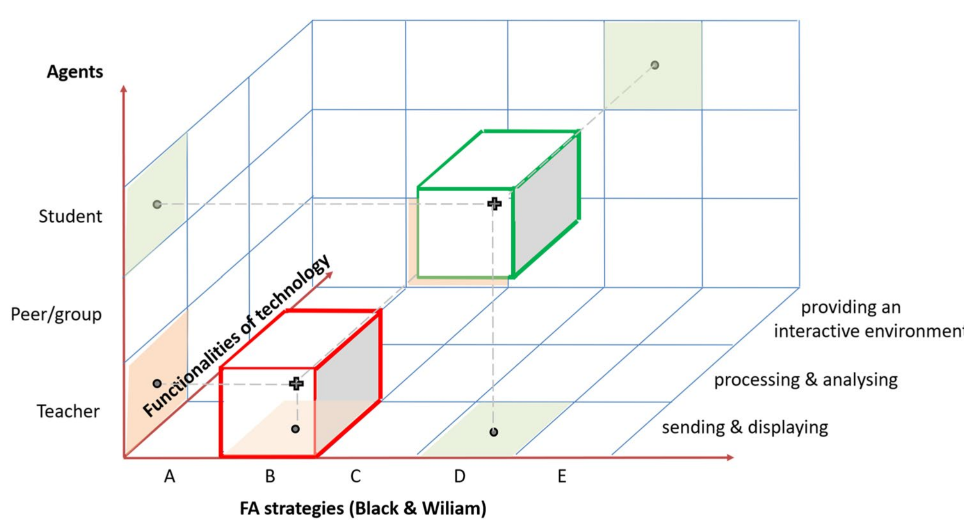


Fig. 2 Schemes of use activated by the teacher (red cube) and by the student (green cube) in the example of a classroom connected technology

4 Methodology

The methodological principles of the FaSMEd project included a collaborative methodology, grounded on the Design-Based Research, which means a methodology engaging teachers as practitioner researchers using a 'lesson study' method for professional development. In that sense, teachers and researchers involved in the project collaborated to construct, observe and refine one or a small number of lessons, and spent a long time working to reflect on lessons that were designed under the teachers' responsibility. The collaborative work, also described and analysed as reported by Aldon et al. (2017), led to organizing the data collection through interviews, class observations and common feedback and analysis, and personal journals during the two years of the project's intervention cases. Furthermore, in order to understand and discuss the FA strategies implemented in the classroom, their effects on learning as well as their evolution throughout the process by which the different agents appropriated the tools, it was fundamental to build the method of data collection over the long term, by involving the actors in reflection as well as observing their practices.

As already mentioned, in this paper we draw on data coming from two classes. This selection is motivated, on the one hand, by the diversity of school levels (primary and secondary), which makes it possible to highlight operational invariants at different levels within different teaching contexts, and, on the other hand, by the different technologies used.

The first is a grade 9 class in a lower secondary school in a small town in the south-east of France. The class was equipped with one tablet for each student, an interactive whiteboard (IWB), and classroom connected technology (e.g., NetSupport School and Maple TA). Due to the distance from Lyon, we proposed that the teachers document their work over the course of the year, and also we organized some meeting windows of about two or three days during which researchers became part of the normal functioning of the classroom observed, by attending all courses and working outside the classroom with the teachers from the perspective of feedback and design review. Thus, in the two years of joint work, the data collected were of a different nature: lesson plans; classroom observations that went beyond the mathematics course alone and resulted in a report for the classroom teachers' team and the college management team (Aldon and Panero 2016); notes from meetings with all partner schools; interviews with teachers, sometimes organized as a joint preparation of a formative assessment lesson as described below; and finally interviews with students. Regarding the focus of this paper, we draw on data collected during three consecutive days of observation in the school, a work meeting between with the mathematics teacher, the teacher's lesson plan, and notes from interviews and informal discussions with the teacher.

The chosen observation represented a key moment of the project for two main reasons. First, the observed lessons were published as the French mathematics case study in the report of theFaSMEd project.[2] Second, it was carried out in October 2015, which means after two years of consolidated, joint work with the teacher, but just at the beginning of the school year. Hence, this was the last iteration of a design-based research cycle in which FA artefacts were becoming FA instruments for the teacher. However, the students involved in this observation were 'beginners' since in the previous year they had not participated in the project. So they had just one month and a half of experience in learning through FA lessons, namely in a technological environment. In this example, the FA lessons focused on the interpretation of a graphic representation, which was shared by all the FaSMEd partners as the common theme to be implemented in the project. Drawing on some tasks and ideas proposed by the FaSMEd partners, the teacher articulated the lessons into several steps over three days (1 h class per day) that are briefly described in Sect. 5.1. Our analysis focused on three specific and significant episodes of these lessons.

The second example comes from a 4th-grade class in a primary school in the suburb of Lyon, equipped with an IWB, clickers and personal calculators. This example is less developed; however, it seemed important to us to test the theoretical construction in a different context, with different tools and at a different school level. Thus, the purpose of this brief analysis is to show that the theoretical construction is well-decontextualized and also usable in other contexts. Our analysis is based on FA lessons about the concept of fractions in grade 4, which were designed by the teacher and analysed jointly.

5 First example

We present the activities planned for the lessons in the grade 9 classroom, then we describe and discuss three episodes taken from the observation of these lessons, and finally we provide an analysis of the teachers' schemes through the FaSMEd 3D model.

5.1 Lesson plan

During the first activity (day 1), students worked in groups and were invited to share their stories through a digital notebook available on their tablet, namely OneNote.

[2] https://research.ncl.ac.uk/fasmed/deliverables/.

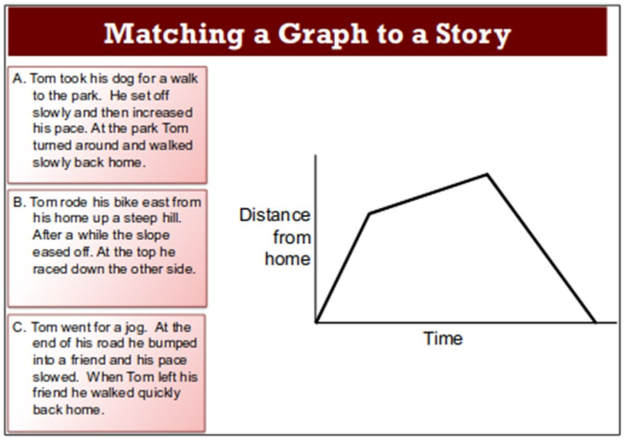


Fig. 3 Activity 2: matching a graph to a story

The second activity (day 1) consisted of an individual question that was posed on the platform Maple TA. The picture in Fig. 3 had been copied and pasted to create a Maple TA question and the students had to enter A, B or C as an answer. Students worked individually: they accessed the platform from their tablets using their personal accounts, they opened the multiple-choice question, reflected a while on the problem and then submitted their answers.

In a third step (day 2), the teacher commented on the students' scores regarding the second activity. While doing so, he displayed the scores and the class' rate of success on the IWB.

A fourth step (day 2) followed and the teacher displayed the graph in Fig. 4 and asked two students to come and tell their story to their classmates, showing the different

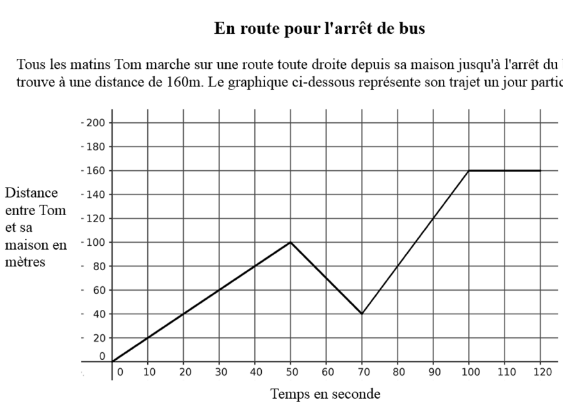


Fig. 4 Activity 1: "Journey to the bus stop. *Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 m. The graph shows his journey on one particular day. Describe what may have happened*"

moments. The story telling brought out two students' difficulties/misconceptions: conceiving the graph as a map, and misinterpreting the constant part of the curve. The teacher did not validate the stories and invited the classmates to ask questions.

The fifth step (day 2) was devoted to another activity: the teacher asked the students to match ten stories to ten graphs, still using Maple TA.

After these five steps, between day 2 and 3, teacher and researchers met in order to analyse the first proposed activities and to organize the next lesson that took place the next day.

5.2 Three episodes and a (long) discussion

5.2.1 Presentation

The teacher (hereafter called Thierry) agreed to design and implement this situation in his class. As he said in the discussion meeting between day 2 and day 3:

> "It's a notion that I had a lot of trouble getting through, I was bored doing it, I bored the students... when you proposed it to me, if it wasn't you who proposed it I wouldn't have thought of it myself, so you didn't create a constraint you gave me the opportunity to do it!"

The subject was not familiar to the teacher and, in the noospherian situation S_{+3}, he agreed to use it in the classroom because of his confidence in the researchers. However, after having studied the different proposed activities, Thierry as a Teacher–Designer T_{+2} planned his lessons drawing on his personal confidence with technology (Netsupport School and Maple TA) and his personal schemes of use with it, namely, teacher, sending and displaying, FA strategy B. These schemes of use were developed in parallel with his increasing mastery of the technological tools—NetSupport School allowing him to connect all the tablets of his students with the IWB and Maple TA allowing him to question them—and were the result of an instrumental genesis that occurred gradually through the cycles of FA lessons that were the focus of experiments in the previous years of the project.

5.2.2 First part of the process

The first episode corresponds to the first activities proposed in day 1. Working in groups of three or four, the students had to write a story that corresponded to the graph in Fig. 4. Everyone had to note down the proposal shared in his/her group; one student per group had to write the story on his/her tablet using OneNote.

Following a student's request for clarification, Thierry explained to the whole class that every remark, comment, or inconsistency noted by students must be written down. He wrote on the board: "if the graph shows any problem, write it!".

The first observed group began the investigation by interpreting the graph in terms of distance and time. A student wrote on OneNote:

160 m, 2 min, 100 m, 50 s.

which corresponded to the distances and times represented on the two first segments. After they suggested another interpretation, the following exchange occurred:

Kevin: We can say that he went down and went faster (indicating the declining segment).
Jean: Here he is following a river.
Michel: Yes! he couldn't cross here (indicating the declining segment).
Thomas: Tom walks two minutes to reach his bus stop.

They finally wrote: "Tom is living in Lyon. He walks 100 m and arrives in front of the river Rhône. He walks 60 m following the river to take the bridge which is 60 m long. He's going over it. Finally he walks 20 s to his bus stop" (Fig. 5).

This group of students in the objective situation S_{-3} is confronted with the material *milieu* M_{-3} which gave them indications to be interpreted (distance, time) but they confused a mathematical representation with a map, that is to say a graph (distance, distance), when they said: "following the river", "crossing the bridge", etc. In the reference situation S_{-2}, they took into account their understanding of the mathematical situation to interpret and answer the question. At that stage, they did not have any reflection on their own work. However, they used the device provided to them to accomplish their task, that is to submit a proposal previously discussed in the group. In that sense, the teacher proposed and orchestrated NetSupport School as a FA instrument with the scheme of use (teacher, sending and displaying, FA strategy B) to, more specifically, *engineer tasks eliciting students understanding*. The students accepted the proposed artifact which became for them a FA instrument with the scheme of use (peers, sending and displaying, FA strategy E) since they were *activated*, working in a group, *as the owners of their own learning*.

The second observed group also began the investigation with the interpretation of distance and time through reading the graph and the projections on the coordinate axes. They began to write: "Tom takes 30 s to walk 100 m, then he takes 20 s to walk 80 m, then he takes 30 s to walk 120 m". Then the following dialogue follows.

Émilie: It doesn't make any sense.
Aurore: He went walking, trotting, galloping; he is riding a horse!.
Émilie: There is something that doesn't make sense. He did 100m, 60m and 120m, and it is said that the bus stop is 160m from his house; something is wrong.
Thierry: So write it! (on hearing the remark).

They wrote: "Once upon a time there was a false calculation: $100+60+120 \neq 160$" (Fig. 6).

We interpret in the same way the confrontation of this group of students with the material *milieu* M_{-3}, but in the reference situation S_{-2} and starting from their first interpretations, they forgot the meaning of the axes (particularly the y-axis) and did not interpret correctly the decreasing part of the graph. It is important to notice that the teacher, in this reference situation, acted as a Teacher–Observer leaving the responsibility of knowledge to students. Providing a rich mathematical situation, he was *activating students as the owners of their own learning,* which corresponds to a scheme of use of the technology tablets + NetSupport School as a FA instrument. As happened with the previous group, these students did not reflect, as reflective students, on their own work, but they took over the scheme of use (peers, sending and displaying, FA strategy E).

These two examples allow us to see how a FA-oriented technology, considered as an artifact, became an instrument through an instrumental genesis process constructed

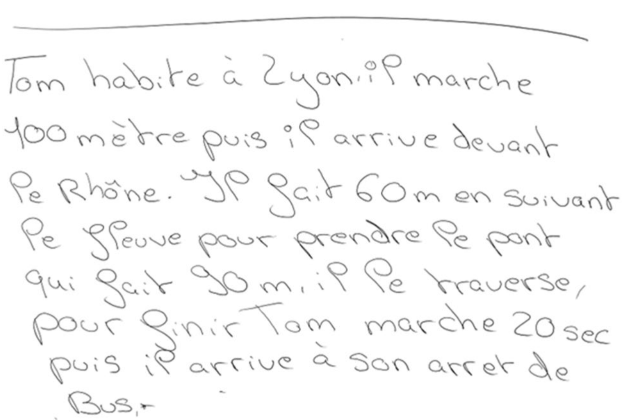


Fig. 5 Story sent by the first group

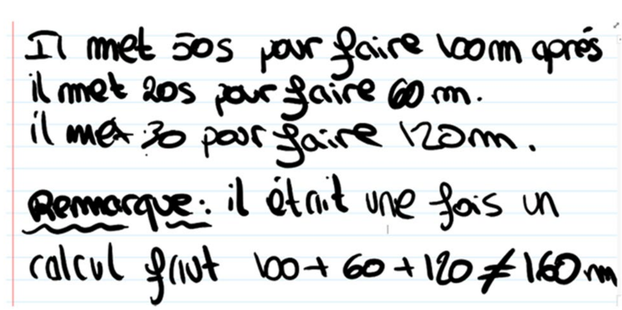


Fig. 6 Story sent by the second group

in parallel by teachers and students. Drawing on the material *milieu* provided by the teacher, students working in groups acted in a reference situation where the Teacher–Observer–Devolver organized feedback by asking students to make explicit their understanding of the situation. The joint action of students and teacher contributed to the process of the instrumental genesis, transforming artifacts made up of technology and a reference situation into an FA instrument, different for the teacher and for the students but effective for their respective objectives. It is also interesting to notice that the strategy *clarifying, understanding and sharing learning intentions and criteria for success* was not present; it may come from the "paradoxical injunction" (Brousseau 1986, p. 316) where the teacher, in order to let the student build his/her own knowledge, cannot express what he wants: "if the teacher says what he wants, he can no longer obtain it" (Ibid. p. 316). The didactic contract comes precisely at this point: "Learning implies, for him, that he accepts the didactic relationship but that he considers it as provisional and tries to reject it" (Ibid. p. 316). Thus we consider this strategy as a consequence of the didactic contract that had been negotiated since the beginning of the year, through the relationships established in the classroom by the teacher at a didactic level (S_0), concerning the way in which knowledge is shared.

5.2.3 Second episode

The second episode corresponds to the fourth step where students are invited to share their stories with the class.

The teacher displayed all the stories written by the students, then zoomed on each particular story while the representative of that authors' group was reading it. He invited the classmates to write down any remarks that came into their minds. In his FA schemes, he is using the functionality *sending and displaying* to *engineer effective classroom discussions and other learning tasks that elicit evidence of student understanding* for the whole class. Here is an excerpt of the discussion.

Paul (reading at the whiteboard): Tom was walking towards his bus stop when he realised that he lost his keys 60m behind him. After recovering them, he ran towards his bus stop. Finally, Tom missed the bus and walked to his school (showing the horizontal segment of the graph in Fig. 4 to reinforce his explanation).
Thierry: Where is the bus stop?
Paul: Here! (showing the endpoints (120,160) and (100,160) of the horizontal segment at the end of the graph).
Thierry: Can somebody answer more precisely the question? Where is the bus stop?
Jean, who was in the first observed group, came to the board and showed the endpoint of the graph.
Thierry: Why?
Jean: Because that's where it ends.
Thierry: Does anyone have any other opinions?
Nobody answered. Thierry stopped the discussion on this aspect.
Thierry (asking Jean): OK. Can you tell us your story?
Jean (after reading his group's story, Fig. 5): [...] Tom crosses the bridge which measures 90m.
Thierry: Where do you read 90m?
Jean: Here (showing the segment (70, 40)–(100, 160)).
Thierry: Precisely, how do you compute 90?
Jean: Well… Here it's 40 (showing the ordinate 40 on the y-axis) and to go to 160 (showing the ordinate 160 on the y-axis), it gives...
Laughter in the class.
Thierry: OK, we'll say 120.
Jean: Oh yes. Then, Tom walks 20s and he arrives at the bus stop.
[…]
Thierry (asking to the classmates): Do you have any comments or remarks?
Silence.
Thierry: They don't say the same thing. Who is right?
Marie: They are both right.
Paula (referring to Jean): He is right because he gives the units.
As a provocation, Thierry asked Jean to draw the river and then the cathedral of Lyon.
Thierry (asking the class): What do you think?
Marion: It's no longer a graph.
Luc: He drew the path.
Miriam: It's a plan.
Thierry: And are there seconds on a plan?
Students: No...

First, in terms of graphical interpretation, either Paul or Jean gave partially accurate and partially erroneous answers: the main mistake being the confusion between a graphical representation of the distance as a function of time and the actual path drawn on a plan. Both students knew how to locate a point in the plane, how to interpret the x-axis and the y-axis in terms of time and distance, but they got both confused in the interpretation of a point or a segment within the graph: "He walks to his school", Paul said; "He crosses the bridge", Jean said. The debate in the classroom showed also that other students were not yet confident regarding the interpretation of the graph and the teacher's provocation engendered contradictory opinions.

In terms of FA, the possibility of collecting all the answers and proposing them to the class informs the teacher about the state of knowledge and the difficulties encountered by his students, but it also allows students to share their own knowledge with their peers and be questioned on it, in this case their misunderstanding of a graphic interpretation. At a meta level, the teacher takes advantage of the *displaying* functionality of technology to provoke a debate about the mathematical contents at stake. So, in a didactic situation S_0, technology helps him better understand the reflection that students have built in the lower-level situations, and to use this understanding in order to confront students with their contradictions. On the one hand, the artifact becomes a FA instrument with the scheme of use (teacher, sending and displaying, FA strategy B). On the other hand, students develop also from these debates an habit to share their mathematical productions with peers and to provide feedback to each other using the technology as a FA instrument with the scheme of use (peers, sending and displaying, FA strategy D).

5.2.4 Meeting with the teacher

After these two first days, we met the teacher outside of school hours in order to take stock of what happened and how to go on for the next lessons. During this meeting, Thierry referred to a pragmatic concern after having observed difficulties among the students. A long discussion then took place in order to imagine the next steps in the work with the students. Pragmatic considerations about time had to be accommodated: a good balance between the willingness to put students back into a research learning situation S_{-1} that allows them to understand the meaning of the coordinate axes for the interpretation of a graph, and the teacher wanting to introduce the notion of speed.

> Thierry: I want it to be clear with them [the students]. What do we do in terms of formative assessment to know that they understand it? [...] I want to ask them, according to the graph, after they build, they'll build the graph, I want to ask them for the speed, to calculate the speed on each piece of the graph. And then I have a little time where I tell them this is how you calculate a speed, how you express a speed.
> [...]
> Researcher: But still, the question you're asking is not yet solved, what can we do at the end to see if it's the distance to home, or the distance traveled…? [...] [The problem] is really related to this issue which, in my opinion, is fundamental for them to detach themselves from the plan and enter into the interpretation of the graph.

In this short excerpt, researchers and teacher question the goal, 'where the learners go', starting from the students' behaviour in the previous activities. As stated above, in the FA instruments that the teacher is appropriating, FA strategy A did not appear. He relied on the students' work to adjust the didactical objective of the lesson and wondered how to make this learning intention readable to the students. Thierry was surprised by the interpretation his students gave and was not prepared to institutionalize this content. So he proposed a more familiar content (the introduction of the speed), while researchers suggested allowing students to overcome this difficulty before addressing a new concept. Looking more closely at the students' productions, which was made possible by the recorded traces of the IWB, a final agreement was built on a new *task to elicit student understanding*, more suitable for starting from 'where the learners are right now' and aimed at moving them forward or deeper, before introducing any new concept.

At a meta level, this meeting contributed in the instrumental genesis of the FA instruments and was later considered by the teacher, in his personal journal, as a fundamental moment of awareness of FA as a pedagogical method. Technology played an important role in making it possible to build on students' actual activity and not just on the results of their work. The noospherian T_{+3} was confronted with the idea of the FA lesson including his technological knowledge but addressing the mathematical objectives of a lesson that he was not used to planning before.

5.2.5 Last step

The day after this meeting, the teacher organized a debate around the proposals made by the students in the first step, aiming at institutionalising the information given by the axes in a graph reading. The Teacher–Projector organized his lesson around the FA strategy C: *providing feedback that moves the learner forward*. During the lesson, the Teacher–Regulator T_{-1} drew on students' main ideas, summarizing and relating them with the interpretations made previously. Technology appeared in this lesson as a memory of working in the classroom, allowing the teacher to go back to the history of the work and to analyse the different phases of the process along with the advancement of knowledge. To make an example, he proposed analysing the story proposed by the second observed group (Fig. 6).

> Thierry: What did they calculate, when they wrote 100+60+120?
> Luc: The distance.
> Aurore: How far away he is…
> Luc: The distance he has travelled.
> Michel: The extra distance he has travelled.
> Thierry (writing on the board): "Time-distance that Tom walks in total".

Then, Thierry drew an axis system on the board and wrote "distance walked in total" on the y-axis, and "time" on the x-axis. Finally he gave a new task: "With the same

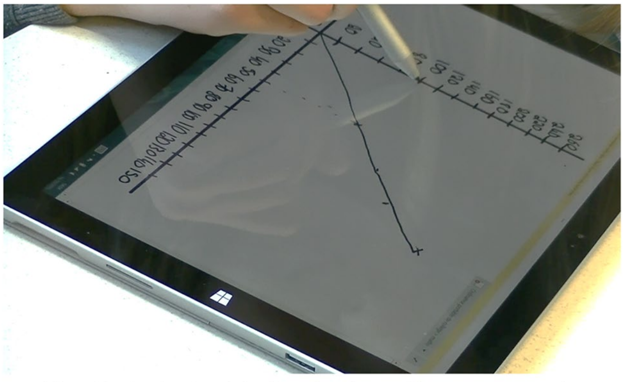


Fig. 7 A student trying to draw the new graph

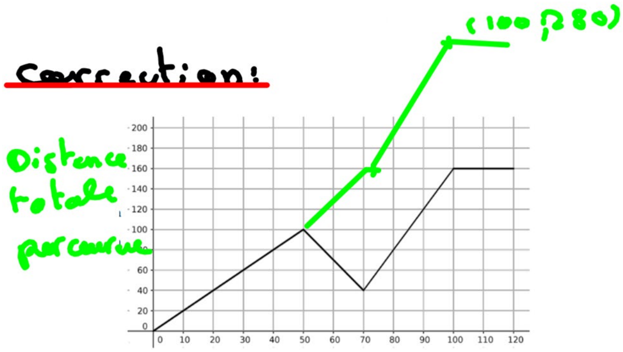


Fig. 8 The new graph corrected at the IWB

story, draw the graph. It is the same situation, but now you consider the distance walked by Tom".

Students came back individually on their tablets and tried to draw the graph (Fig. 7). The teacher walked from group to group to explain, verify the individual graphs and discuss possible mistakes. He finally used the IWB to give a correction to the whole class (Fig. 8).

With this new task, Thierry used the technology to *provide an interactive environment* where, starting from the information in the first graph, students had to draw a new one with another interpretation of the y-axis. The schemes of use (teacher, providing an interactive environment, FA strategy C) and (teacher, sending and displaying + processing and analysing, FA strategy C) emerged in the lesson. On the other hand, through dialogues with their peers and with the teacher, students developed FA schemes of use, namely (student, sending and displaying, FA strategy E) and (peers, sending and displaying, FA strategy D).

5.3 Analysis of the situation

The two analyses, bottom-up and top-down, of the situation allow us to describe a parallel instrumental genesis of the FA instruments in the technological environment.

The teacher in a noospherian situation, confronted with the design of a FA lesson, drew on his previous experiences built in the first iterations of design-based research process. The Teacher–Designer chose to activate students' FA strategies through the *sending and displaying* functionality allowed by the technology, by highlighting collaborative work between peers. The Teacher–Projector organised his classroom to implement his pedagogical construction which led to the effective organisation of the lesson. After back and forth between the learning situation and the reference situation, the Teacher–Observer and the Teacher–Regulator modified and adapted his lessons in order to take into account the students' actual knowledge and their stepwise learning achievement. The dynamics observed within the strategies and the tools used to implement such strategies testify to the construction of robust FA instruments, that is to say FA instruments at the teacher's disposal in his FA toolbox: FA instruments observed in this situation were also observed in other contexts.

On the other hand, students confronted with a material milieu, in an objective situation, were confronted with their ideas and conceptions of the graph, with their peers and the teacher. They took advantage of the technology at their disposal to engage themselves in FA strategies and they progressively transformed the artifacts into FA instruments dedicated to their own understanding of mathematical concepts at stake. Their positions as Student-Objective, Student-Acting and Student-Learner contributed to the appropriation of FA instruments designed to make them responsible for their own learning.

In order to support his didactic choices, the teacher developed schemes of use relative to the formative assessment process that he had implemented in different situations (see Fig. 9 for a representation in the 3D model):

1. starting from a rich mathematical situation, making the students work in groups to foster discussion and argumentation among students: (teacher, sending and displaying, FA strategy B);
2. collecting one production for each group, that is, one student writes on her tablet the proposal agreed upon in the group, while the others take notes either on their notebook or tablets: (peers, sending and displaying, FA strategies D and E);
3. displaying the different productions at the IWB and commenting on them with the students (peers, sending and displaying + processing and analysing, FA strategy B);

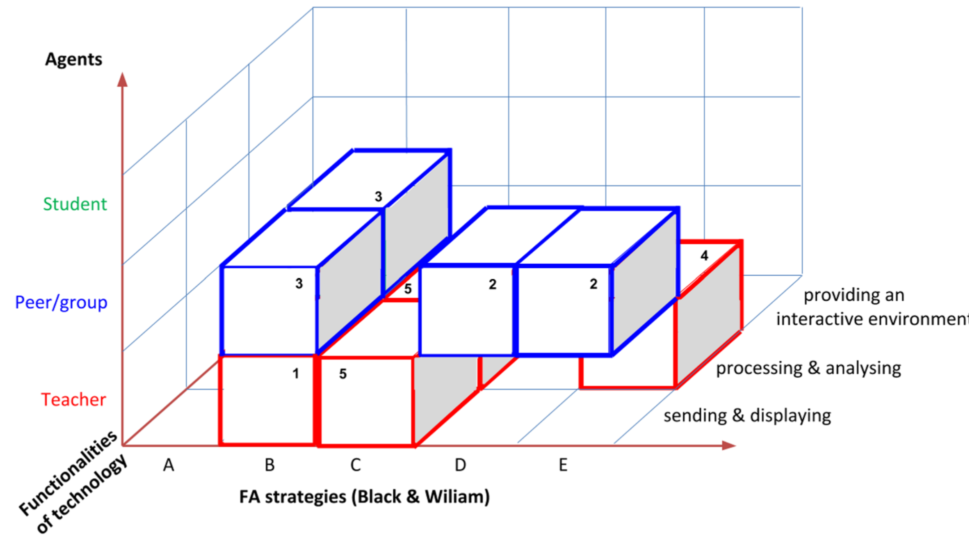


Fig. 9 Representation of the schemes of use of technology 1–5

4. using NetSupport School for managing and monitoring the groups' work and Maple TA for managing and monitoring individual work: (teacher, processing and analysing, FA strategy E);
5. providing feedback both individually and about the class' progression: (teacher, sending and displaying + processing and analysing, FA strategy C).

This observation came after a long period of time that had allowed an instrumental genesis to take place.

6 Second example

This second example was chosen in a different context to test the theoretical framework and highlight, as we did in the first example, possible invariants leading to the establishment of schemes of use of the available tools to implement formative assessment. The observed lessons deal with the concept of equivalent fractions in grade 4. Our analysis focuses on the 2 h that followed the introduction of the concept, thus in the phase of appropriation by the students. These lessons came after a first year of joint work, of teacher and researchers, on the implementation of technology-oriented formative assessment situations.

6.1 Description of the lessons

The teacher had introduced the concept of fraction with tangible manipulation of paper strips and coloring fractions of a disc or a rectangle (Fig. 10). Using this material, pupils had to imagine how many eighths are in two quarters or other similar examples.

The first observed lesson began with the teacher (hereafter called Vanessa) proposing three examples in an algebraic form, such as $\frac{3}{8} = \frac{27}{?}$, and asking pupils to explain the strategy they used to find the equivalent fraction. The objective of this lesson was to consolidate learning and to practice strategies to find fractions that are equivalent to a given one. Then Vanessa proposed a quiz using the clickers and organised the lesson as a game, whose goal was getting the greenest answers (the right answers) in the class. Students could see their answers projected on the IWB and position

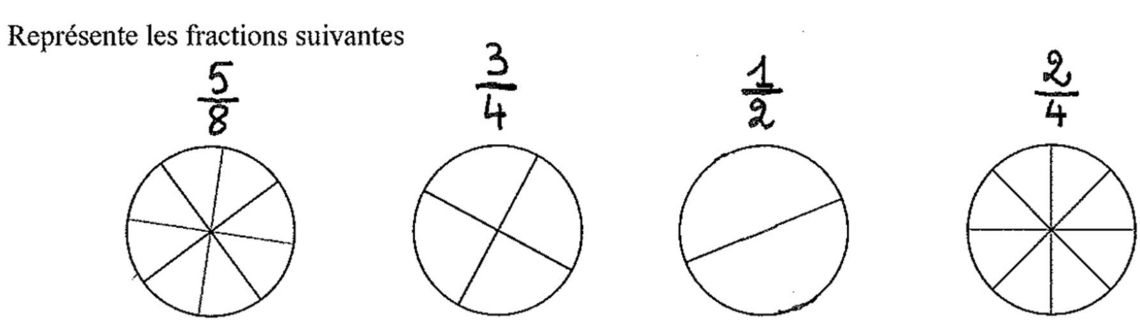


Fig. 10 Represent the following fractions

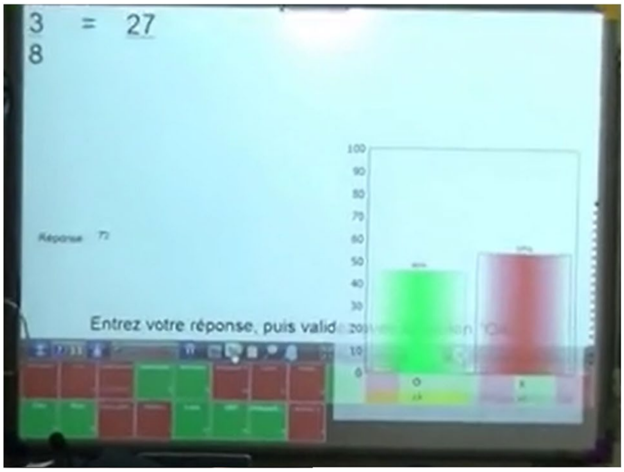


Fig. 11 The quiz game with the clickers

themselves in relation to the right answer, according to the colour (green or red) of their own answer (Fig. 11).

The teacher's analysis of a specific question helped them to be more successful in the following questions. Question after question, Vanessa took the time to comment on some answers and intervened individually with a few students. Also, she gave personalized feedback and explained again what was not clear for the whole class, asking for help from students who acted as resources for their peers: in the example $\frac{3}{8} = \frac{27}{?}$ pupils had to write on their clickers the denominator of the second fraction. Analysing the answers which were collected on the spot, processed and displayed in real time by the clickers software (Fig. 11), Vanessa commented on some of them, referring directly to each specific student: "Hussein, do you need more time? Samia, you made a mistake, you must not indicate 9, the factor, but the result of the multiplication of the denominator by this factor, 9 times 8. Do you understand? What is the result?".

Moreover, she suggested possible aids for lower achievers: "David, is there a problem with calculations? Take your multiplication tables with you".

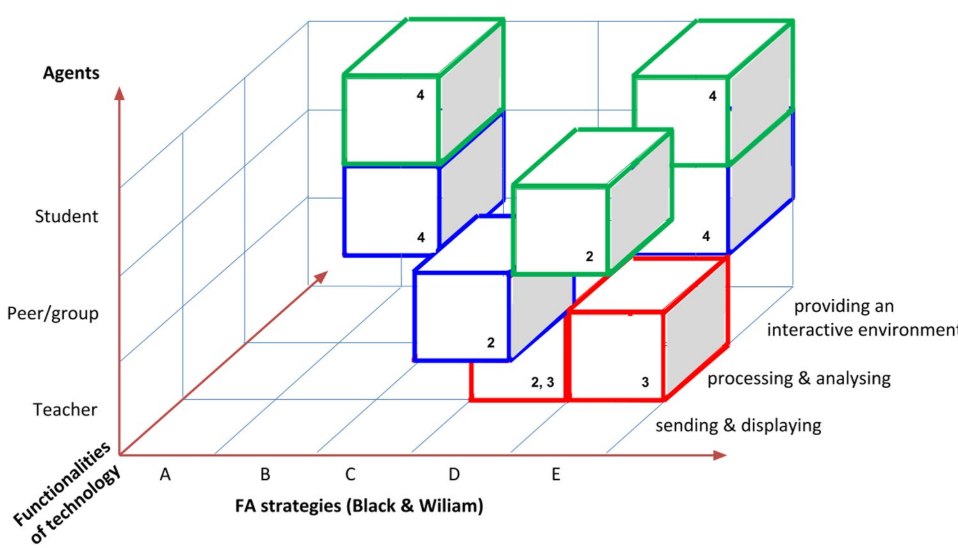
The next day, Vanessa differentiated the work for her pupils, by assigning them different activities with different degrees of autonomy in relation to the quiz answers that she could analyse after the lesson. Indeed, the software recorded all the answers and processed them in tables, making it easy to identify students who had no difficulties, those who had made easily interpretable mistakes, and those who did not seem to have a reliable strategy. She left the first group of students working alone with specific tasks to solve within the interactive environment of their personal calculators, and she paid more attention to the other groups. She often interrupted the work to ask students to come and explain their solution at the blackboard.

6.2 Analysis

In this example Vanessa built a FA instrument which is very different from that developed by Thierry. In any case, with a similar purpose, she aimed at integrating FA within her teaching methods. The developed FA instrument was often reproduced in Vanessa's lessons and was also shared and implemented by other primary school teachers working in the FaSMEd project. Vanessa co-developed with her colleagues specific schemes of use to implement the formative assessment process (see Fig. 12 for a representation in the 3D model):

1. introducing a mathematical concept with tangible manipulations and challenging learning tasks;
2. proposing a quiz with clickers to understand where the learners are in their learning at that moment: (teacher,

Fig. 12 Representation of the schemes of use of technology 2–4



processing and analysing, FA strategy B), (student, sending and displaying, FA strategy E) and (peers, sending and displaying, FA strategy D);

3. analysing deeply the individual results in order to create remediation groups: (teacher, processing and analysing, FA strategy D and E);
4. proposing differentiated activities for students: (peers and student, providing an interactive environment, FA strategies B and E);
5. give feedback to the whole class drawing on the students' work.

Starting from a mathematical situation, the Teacher–Designer organised the *milieu* in order to present the mathematical activity as a game where the goal for the students was to light the lower part of the screen in green (Fig. 11). She collected information about the individual knowledge of her students and reacted in real time to modify the students' set of knowledge. The teacher transformed a mathematical and technical situation into an assessment situation in which students became agents of their own learning. The Teacher-Regulator in the learning situation provided information using the *sending and displaying* functionality of technology. On the other hand, students in the learning situation, engaged themselves in FA-strategies, becoming alternatively owners of their own learning and instructional resources for one another.

7 Conclusion

The purpose of this paper was to propose a new angle on the analysis of assessment situations in a technological environment. The first conclusion that can be drawn from this study is to highlight the didactic nature of the formative assessment process. Indeed, it is clear both in the theoretical framework and in the examples developed that the mathematical content of the situations proposed guides the instrumental genesis. This process transforms the principles of formative assessment into instruments of formative assessment that include FA strategies and an operational system for implementing these strategies, for both the teacher and the students. Linking the framework of formative assessment with the Theory of Didactical Situation, and more precisely with the structure of the *milieu*, highlights the mathematical content at stake as well as the organisation of the lesson, reinforcing the consideration of FA as a teaching–learning method. The addition of the technological environment leads to analysis of the situation from the point of view of the instrumental orchestration.

The second important point that emerges from our study is that formative assessment with technology is not only a facilitation of the implementation of strategies but more profoundly a modification of the didactic contract, making students and the teacher co-responsible for teaching and learning. Through the dual instrumental genesis, this modification implies a reconsideration of formative assessment strategies from a perspective of joint action (Sensevy 2012).

To answer the question posed in the title of this article (Can digital technology change the way mathematics skills are assessed?) and the issues proposed in the introduction, we have highlighted the particular role of the actors involved in a formative assessment process that gives rise to a new posture, a posture of *formative assessor*.

It is interesting to notice that in the 3D model not all the cubes (Figs. 9, 12), that represent the schemes of use developed respectively by Thierry with NetSupport School and Maple TA and by Vanessa with clickers and calculators, are at the same level of *milieu* structure. This means that they are produced by the teacher or the students depending on the particular situation S_n in which they are acting. For example, in Fig. 9, the scheme of use (teacher, sending and displaying, FA strategy B) has the aim of devolving to groups of students a rich mathematical situation; thus it is an act of devolution, allowing a shift from S_0 to S_{-3}. When students are working on their tablets, the scheme (peers, sending and displaying, FA strategies D and E) is activated; this is an act of implication in which students work within S_{-1} and come back to S_0 when they send their written production to the teacher via NetSupport School. In future developments of this study, it could be interesting to analyse if these schemes of use are systematic: Is the scheme of use (teacher, sending and displaying, FA strategy B) activated in every act of devolution?

This could be a possible object for further research with the aim of identifying and characterising 'new' postures, namely those of formative assessors, of the agents in the *milieu* structure.

References

Achiam, M., Sølberg, J., & Evans, R. (2013). Dragons and dinosaurs: directing inquiry in biology using the notions of 'milieu' and 'validation'. *Journal of Biological Education*, *47*(1), 39–45.

Ainley, J., & Margolinas, C. (2015). Accounting for student perspectives in task design. *Task design in mathematics education,* (pp. 115–141). Cham: Springer.

Aldon, G., & Panero, M. (2016). Une classe tablette en mathématiques, *Mathematice*, 50. Available at: https://revue.sesamath.net/spip.php?article857 (consulted on 13.04.20).

Aldon, G., Cusi, A., Morselli, F., Panero, M., & Sabena, C. (2017). Formative assessment and technology: Reflections developed through the collaboration between teachers and researchers. In G. Aldon, F. Hitt, L. Bazzini, & U. Gellert (Eds.), *Mathematics and technology. Advances in mathematics education* (pp. 551–578). Cham: Springer.

Allal, L., & Lopez, L. M. (2005). Formative assessment of learning: A review of publications in French. In Centre for Educational

Research and Innovation (Ed.), *Formative assessment: Improving learning in secondary classrooms* (pp. 241–264). Paris: OECD Publishing

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245.

Artigue, M. (2009). Didactical design in mathematics education. In *Nordic research in mathematics education* (pp. 5–16). Leiden: Brill Sense.

Artigue, M., Haspekian, M., & Corblin-Lenfant, A. (2014). Introduction to the theory of didactical situations (TDS). In *Networking of theories as a research practice in mathematics education* (pp. 47–65). Springer, Cham.

Beatty, I. D., & Gerace, W. J. (2009). Technology-enhanced formative assessment: A research-based pedagogy for teaching science with classroom response technology. *Journal of Science Education and Technology, 18*(2), 146–162.

Bikner-Ahsbahs, A., & Prediger, S. (2010). Networking of theories—An approach for exploiting the diversity of theoretical approaches. In B. Sriraman & L. English (Eds.), *Theories of mathematics education. Advances in mathematics education*. Berlin: Springer.

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational assessment, Evaluation and Accountability, 21*(1), 5–31.

Bloch, I. (1999). L'articulation du travail mathématique du professeur et de l'élève dans l'enseignement de l'analyse en première scientifique. *Recherches en Didactique des Mathématiques, 19*(2), 135–194.

Bloch, I., & Gibel, P. (2011). Un modèle d'analyse des raisonnements dans les situations didactiques. Étude des niveaux de preuves dans une situation d'enseignement de la notion de limite. *Recherches en didactique des mathématiques, 31*(2), 191–228.

Brousseau, G. (1986). Théorisation des phénomènes d'enseignement des mathématiques. *Thèse de l'Université de Bordeaux 1*.

Brousseau, G. (1997) "Theory of Didactical situations in Mathematics". Recueil de textes de Didactique des mathématiques 1970–1990, traduction M. Cooper and N. Balacheff, R. Sutherland and V. Warfield. Dordrecht: Kluwer.

Brousseau, G. (2006). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990* (Vol. 19). Dordrecht: Springer Science and Business Media.

Brousseau, G. (2010). *Glossary of terms used in Didactique*, online: https://faculty.washington.edu/warfield/guy-brousseau.com/biographie/glossaires/GLOSSARY_Eng.doc

Burns, M. (2017). *Formative tech: Meaningful, sustainable, and scalable formative assessment with technology*. New York: Corwin Press.

Buteau, C., Muller, E., Mgombelo, J., & Sacristán, A. I. (2019, July). Stages of students' instrumental genesis of programming for mathematical investigations. In *43rd Annual Meeting of the International Group for the Psychology of Mathematics Education* (vol. 4). Oral Communications and Poster.

Cerratto Pargman, T., Nouri, J., & Milrad, M. (2018). Taking an instrumental genesis lens: New insights into collaborative mobile learning. *British Journal of Educational Technology, 49*(2), 219–234.

Cusi, A., Morselli, F., & Sabena, C. (2017). Promoting formative assessment in a connected classroom environment: Design and implementation of digital resources. *ZDM The International Journal on Mathematics Education, 49*, 755–767.

Dunn, K. E., & Mulvenon, S. W. (2009). A critical review of research on formative assessment: The limited scientific evidence of the impact of formative assessment in education. *Practical Assessment, Research and Evaluation, 14*(7), 1–11.

Kidron, I., Artigue, M., Bosch, M., Dreyfus, T., & Haspekian, M. (2014). Context, milieu, and media-milieus dialectic: A case study on networking of AiC, TDS, and ATD. *Networking of theories as a research practice in mathematics education* (pp. 153–177). Cham: Springer.

Lee, H., Feldman, A., & Beatty, I. D. (2012). Factors that affect science and mathematics teachers' initial implementation of technology-enhanced formative assessment using a classroom response system. *Journal of Science Education and Technology, 21*(5), 523–539.

Määttänen, P. (2016). The concept of the scheme in the activity theories of Ilyenkov and Piaget. *The Practical Essence of Man* (pp. 154–166). Brill: Leiden.

Margolinas, C. (1995). La structuration du milieu et ses apports dans l'analyse a posteriori des situations. *Les débats de didactique des mathématiques*, 89–102.

Margolinas, C., Coulange, L., & Bessot, A. (2005). What can the teacher learn in the classroom?. In *Beyond the Apparent Banality of the Mathematics Classroom* (pp. 205–234). Boston: Springer.

C Margolinas 2004 Points de vue de l'élève et du professeurHabilitation à diriger des recherches de l'Université de Provence-Aix-Marseille I Essai de développement de la théorie des situations didactiques

Panero, M., & Aldon, G. (2016). How teachers evolve their formative assessment practices when digital tools are involved in the classroom. *Digital Experiences in Mathematics Education, 2*(1), 70–86.

Pape, S., Irving, K., Owens, D., Boscardin, C., Sanalan, V., Abrahamson, L., et al. (2013). Classroom connectivity in Algebra I classrooms: Results of a randomized control trial. *Effective Education, 4*(2), 169–189.

Perrin-Glorian, M. J. (March). From producing optimal teaching to analysing usual classroom situations. Development of a fundamental concept in the theory of didactic situations: the notion of milieu. The first century of the International Commission on Mathematical Instruction (1908–2008), March 2008, Rome, Italy. Abstract page 308. hal-01660872

Rabardel, P. (1995). *Les hommes et les technologies; Approche cognitive des instruments contemporains*. Paris: Armand Colin.

Rabardel, P. (1999). Eléments pour une approche instrumentale en didactique des mathématiques. *Actes de l'école d'été de didactique des mathématiques, 18*(21), 203–213.

Roorda, G., Vos, P., Drijvers, P., & Goedhart, M. (2016). Solving rate of change tasks with a graphing calculator: A case study on instrumental genesis. *Digital Experiences in Mathematics Education, 2*(3), 228–252.

Roschelle, J., & Pea, R. (2002). A walk on the WILD side: How wireless handhelds may change computer-supported collaborative learning. *International Journal of Cognition and Technology, 1*(1), 145–168.

Ruchniewicz, H., & Barzel, B. (2019). Technology supporting students' self assessment in the field of functions: A design based research study. In G. Aldon & J. Trgalova (Eds.), *Technology in mathematics education* (pp. 49–74). Cham: Springer.

Sensevy, G. (2012). About the joint action theory in didactics. *Zeitschrift für Erziehungswissenschaft, 15*(3), 503–516.

Spector, J. M., Ifenthaler, D., Samspon, D., Yang, L., Mukama, E., Warusavitarana, A., et al. (2016). Technology enhanced formative assessment for 21st century learning. *Educational Technology & Society, 19*(3), 58–71.

Stroup, W., Kaput, J. J., Ares, N., Wilensky, U., Hegedus, S. Roschelle, J., Mack, A., Davis, S.M., & Hurford, A. (2002). The nature and future of classroom connectivity: The dialectics of mathematics in the social space (pp. 195–203). *Proceedings of the Annual Meeting of the International Group for the*

Psychology of Mathematics Education—North American Chapter, Athens, Georgia.
Swan, M. (2014). Design research in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 148–151). Dordrecht: Springer.
Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*(3), 281–307.
Vergnaud, G. (2009). The theory of conceptual fields. *Human Development*, *52*(2), 83–94.
Vergnaud, G. (2016). Forme opératoire et forme prédicative de la connaissance. *Investigações em Ensino de Ciências, 17*(2), 287–304.
Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development, 53*(4), 5–23.