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ORIGINAL ARTICLE

Ways of acting when using technology in the primary school classroom: contingencies and possibilities for learning

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Abstract
Digital technologies shape the processes of teaching and learning in the classroom. They create spaces for action, while at the same time they pose restrictions and can generate unexpected situations, both for teachers and students. In this paper, we show examples in which teachers respond to contingencies emerging from the use of interactive programs in the classrooms. By describing different ways in which teachers react to those contingencies, we show how the technology plays an important role, at times by creating unexpected situations, and at others in support of teachers' explanations. In both cases it can promote modifications in students' actions and shape teachers' actions and responses in relation to a particular mathematical situation or problem. Understanding how teachers react when they are faced with an unexpected situation is important in order to gain knowledge about those particular responses that result in effective behaviors that are related to mathematics learning, and also in terms of the construction of rich learning environments that promote those kinds of behaviors. Results show that the kind of interaction that technology has the potential for promoting plays an important role in making students and teachers more aware of students' doubts and misunderstandings, but that this potential needs to be accompanied by effective teachers' strategies through which they use contingencies as opportunities to promote both their own and their students' learning. This study contributes to deepening knowledge about teachers' effective strategies in primary school classrooms as well as providing examples to promote reflection regarding teachers' training programs.

Keywords Contingencies · Digital technology · Teachers' role · Enactivism · Mathematics learning

1 Introduction and literature review

Teaching mathematics in primary school is a demanding activity for teachers. The use of technology to accompany lessons can impose new pressures in this profession. Digital resources can be a source of rich activities, but can also present teachers with unexpected situations where they have to respond to students' questions or comments about what the whiteboard or screen shows, which might be different from what they usually encounter in textbooks or in their notes (Spiteri and Chang Rundgren 2020). The study of how teachers respond to such events needs further investigation in order to better understand those actions where unexpected situations might be used as opportunities for students' learning.

Several studies (e.g. Rowland and Zazkis 2013; Smit and Van Eerde 2011; Mason 2002; Arafeh et al. 2001) have highlighted the importance of teachers' mathematical knowledge and their openness to inquiry in order to foster students' mathematical learning. Researchers have been particularly interested in how teachers deal with contingent events or contingencies, understood as "unpredictable" moments; unpredictability "is witnessed in classroom events that were not envisaged in the teachers' planning" (Rowland, Twaites and Jared 2015, p. 75) or when a teacher is taken by surprise and needs to improvise (Rowland et al. 2015). Other studies regard these moments as scaffolding opportunities where the teacher can help students to develop their problem-solving abilities or to modify their previous knowledge (Makar, et al. 2015; Roll, et al. 2012). Stockero and van Zoest (2013, p. 127) describe them as "pivotal teaching moments" (PTM)

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where an interruption is an opportunity for the teacher to modify the lesson plan in order to extend, or change students' understanding of a mathematical idea. Finally, regarding unexpected situations Foster (2015) shows how they can be exploited to engage students in mathematical thinking.

Integrating technology in the classroom involves many difficulties (Lagrange and Monahan 2009). Managing knowledge, students and resources poses complex problems for teachers who have to develop skills in order to use the technology in an appropriate way at precise moments. Clark-Wilson (2014) developed the notion of "hiccup" to refer to perturbations occurring during lessons, triggered by the use of technology, in which teachers seem to hesitate, and which seem to make evident discontinuities in teachers' knowledge.

Contingencies, taken here as unpredictable unexpected events, can happen in any classroom, but when digital technologies are used, they can become an additional challenge for teachers to deal with when trying to create classroom environments that promote mathematics learning. The analysis of teachers' actions while they respond to unexpected situations can give useful information about how different teachers deal with these contingencies, and if their actions create rich contexts which promote students' mathematical learning.

In this paper we analyze mathematics lessons in which five different primary school teachers face contingencies that are closely related with the use of technological resources that can cause the course of the lesson to deviate, and that, sometimes, can be turned into learning opportunities by teachers. Our research questions were as follows:

- How does digital technology contribute to the emergence of contingencies that promote the modification of teachers' and students' actions in relation to mathematics?
- What characterizes responses to contingencies when teachers use them as opportunities for the promotion of mathematics learning

2 Theoretical framework

We work from an enactivist perspective, which we have used in the past and which illuminates our ideas about learning and acting in the classroom. Enactivism is a theory about learning stemming from the work of biologist Maturana and neuroscientist Varela (Maturana and Varela 1992). It considers knowing as effective action, which refers to actions which allow an individual to continue existing in a given context. Effective actions are not necessarily correct, or more efficient or better than other behaviors, but they are such that they allow individuals, in this case teachers and students, to continue participating in their particular context (Simmt and Kieren 2015). To act effectively in a given environment is to perform actions that are acceptable in that environment. Different criteria of acceptability are constructed and specified in different contexts; so, for example, in a given classroom talking to other students during an examination might not be acceptable, while in others it might be a common practice that reflects a classroom culture in which mathematics is taken as a collective construction. Another example would be a context in which the teacher is the one who provides correct answers, in contrast to a different environment in which students are used to determining the correctness of a response by using verification procedures, by asking other students or by using a computer program or the Internet. Behavior that is not effective will lead to the interruption of interactions and eventually will prevent the individual from continuing to participate in the particular context in which the actions are not acceptable (Lozano 2015). Therefore, it can be said that effective behavior is what allows students to carry on being students (and teachers being teachers) in the particular classroom in which they are located.

Effective behaviors account not only for the type of learning that requires deliberate thinking and deciding, but also to immediate coping, which refers to those actions which do not arise from analyzing or reasoning but by acting in the moment with what is being presented (Varela 1999, p. 5). Immediate coping, which according to Varela (1999) accounts for most of our mental and physical activity, is "transparent, stable, and grounded in our personal histories". We are mostly unaware of it, and it does not come to mind unless we reflect to it later on.

The idea of immediate coping, together with the enactivist idea of human beings being structured determined systems, means that in any given moment we act from the way in which we are constituted at the time, and we cannot act otherwise. Our structural state reflects our history of interactions and includes not only what we know in terms of the mathematics and its teaching, but also our ability to handle uncertainty, to question our own understanding and to modify our course of action according to what is happening. In regard to this modification in activity, enactivism considers that, even though in a given moment we act according to our structural state, our structures are highly flexible and they change as a result of interactions—often recurrent—that trigger and even demand different actions in us. Change in action, which in enactivism is identical to learning, happens when our systems are able to perceive certain features in the environment and then modify our behavior so that we respond differently.

When a contingency arises in the classroom, teachers will immediately cope by acting according to their current structural state. However, the contingency in itself can be a trigger for a change in behavior, whenever the commonly used actions are not 'good enough' to handle the emerging situation (Zack and Reid 2003). Sometimes that happens immediately, but it can also be the case that upon later reflection, when interacting with other people (students, colleagues, friends) and in conversation about the particular incident, new ways of acting can arise. In this regard and in the context of teacher training, Brown and her colleagues highlighted the importance of teachers being aware of opportunities that can arise when relating to other teachers, particularly because they consider that through immediate coping some processes can be reified; and therefore they point out that conversations can "open up to new awareness" (Brown et al. 2018).

In characterizing learning as effective actions in a particular domain, enactivism constitutes a 'middle way' between objectivist and subjectivist perspectives about learning. With constructivism, enactivist theories support the view that knowing is about organizing and re-organizing one's world of experience; however, individuals are not seen as isolated units but rather as part of a world that they actively participate in shaping. Cognition is seen as a collaborative enterprise and collective knowledge arises from shared action because individuals are considered to be part of complex systems with integrities of their own (Davis 1996, p. 192). A classroom is considered as a whole and participants construct their knowledge through their interactions. While they act together in a setting, individuals contribute in building it. Students and teachers participate in creating a culture that in return will influence individual learning. (Lozano 2004).

From an enactivist perspective, the use of technologies is part of human experiences, since they are part of human practices and their cultural experience (Davis et al. 2000). Technological resources mediate human activity and play an important role in learning because their use shapes the processes of knowledge construction and conceptualization. They generate a space for action and, at the same time, pose restrictions on users which make possible the emergence of new kinds of actions (Rabardel 1999). They thus open the space of possible actions for learners, although this opening is not immediate. Actions are shaped gradually through a complex process of interactions where activity shapes resources. Working with mathematical problems and ideas with technological resources is closely related to the possibilities offered by the tools.

Reformulating the purpose of our investigation from an enactivist standpoint, we are interested in exploring the way in which technology contributes to the emergence of unexpected situations and in shaping effective actions in the classroom. Additionally, we want to characterize those effective behaviors, arising from contingencies, which specifically promote mathematics learning.

3 Methodology

The choice of methods used in our investigation of mathematics learning is also inspired by the enactivist approach. "Enactivism, as a methodology [is] a theory for learning about learning" (Reid 1996, p. 205). From this standpoint, research is considered to be a way of learning, a flexible and dynamic recursive process of asking questions.

The work reported in this paper is part of a complex process of interaction which emerged as a result of our involvement in Enciclomedia, a large-scale Mexican project that was devised with the purpose of enriching primary school (years 5 and 6) teaching and learning of all subjects by working with computers in the classrooms. Our work has included the development of digital resources, and the investigation of their use in the classroom. So far we have interacted with about 50 teachers using Enciclomedia and we have investigated, in depth, the way in which 11 of those teachers use different mathematics interactive programs. These teachers differ in their background, professional training and experience in teaching mathematics and also in their training experiences on the use of technology. For each one of them, we have carried out classroom observations with video and audio recordings, together with follow up interviews in most cases.

When analyzing the data from the observations and video transcripts, one of the things we were interested in was looking at what difference the use of the digital programs made, especially regarding the teaching and learning of mathematics. In particular, we wanted to investigate responses to contingencies arising from the use of technology and how they relate to mathematics learning. For this research, we selected examples from five teachers. For each of them, there was agreement among the researchers that the cases selected contained instances which show the way technology shapes behavior in the classroom, either by creating a situation in which something unexpected happens—both from an observer's point of view and confirmed by teachers' comments during the interview—or by guiding actions in a specific way. These examples also show how teachers play a specific role in directing the course of actions by responding in a particular way to the situations arising in the classroom.

The researchers analyzed each case independently, looking in detail at the way in which the teacher handled the unexpected events and used the technology, paying particular attention to whether their actions promoted mathematics learning (or not). Later we met and discussed our observations thoroughly; we then selected those dialogues in which contrasting situations between teachers appeared. Quotes from teachers and students were selected by the three researchers as a result of the above-mentioned discussion and then translated from Spanish by one of the authors and checked by two other researchers.

4 Results

4.1 Cycle track: what is the difference between a motion's trajectory and its graph?

We show, in what follows, the analysis of results obtained when we analyzed the data of two teachers using the cycle track in their classroom. Both teachers faced similar contingencies arising from the use of technology, but their actions in dealing with them were different.

The purpose of the cycle track program is to introduce uniform movement as an example of a proportional relation. It dynamically simulates the movement of cyclists going from home to school at three different constant or constant average velocities. It shows one or two cyclists moving simultaneously with different speeds on straight tracks that can be shaped in terms of their incline or by dividing them in different straight segments by touching the screen. Graphs and/or tables that show the relationship between the cyclists' position and time can be displayed together with the cyclists' movement or by themselves (Fig. 1). In addition, an animation in which the proportional relation of the variables involved in constant velocity movement is explained is also available. In enactivist terms, the program is meant to trigger in students' mathematical effective actions such as identifying a proportional relation in uniform motion, comparing results in different representations, and comparing movement through different linear paths or paths consisting of linear segments.

4.1.1 Mrs. T: use of the animation

While using the cycle track, Mrs. T showed the students a cyclist moving along a horizontal path and discussed with them the variables involved in the description of the movement. Then she repeated the same scene but she added the distance-time graph which is constructed, simultaneously, as the cyclist moves along the track. Ana exclaimed: *That is wrong! The bike moves towards the right while the graph is inclined*. Mrs. T responded: *I don't think it's wrong*, but, as other pupils' comments coincided with Ana's, Mrs. T showed them the animation in which the variables involved are explained.

Mrs. T's immediate coping when the difference between trajectory and the movement-graph consisted of using the technology as a resource for giving an explanation, hoping that this action might trigger students' reflection (Mrs. T., 05/2012).

Fig. 1 Interactive program cycle track

When the explanation was finished, pupils were still confused, which meant that her actions were not good enough to resolve the situation. She opened a brief discussion space in which she reiterated: *The graph shows distance and time*. Most students still appeared confused, and started murmuring, but one of them, Luis, clarified: *In the graph the horizontal part is time and the vertical, the distance, that is why when you draw it is inclined*. Mrs. T continued: *Yes, the graph represents that distance is proportional to time, and the proportionality constant is the velocity.* She wrote $d = vt$ on the blackboard and asked students to do some textbook exercises.

Technology in this case made it possible to see what could not be seen either in the textbook or when teachers relate uniform movement with proportionality without looking simultaneously at the moving object and its representations. The possibilities offered by the program could have provided an opportunity for the teacher and students to reflect about both the relation between the cyclist's movement and its representations (on the coordinate plane or on a table) and between time and position.

Mrs. T's actions when the contingency was brought up by a student did not promoted students' mathematical actions to solve the conflict. She did not open a space for reflecting on the relation between the animation and the mathematics involved that could have promoted learning. During the interview she stressed that she liked to teach proportionality graphs but that: *movement is difficult for students at elementary level.* She added: *I trusted the animation explanation to solve students' conflict, since I have not done it before, but it didn't help, so when Luis explained the graph I better turned students' attention from movement to proportionality*. With this decision, Mrs. T missed the opportunity to support students. Her comments during the interview showed her insecurity on her possibility of explaining the relation of the graph with movement, thus she did not use the contingency for the construction of mathematics learning.

In this case, behaviors when facing contingencies included going back to the original objective of the lesson, which is a familiar topic for Mrs. T. In doing this, she disregarded the evidence presented by the different representations, missing an opportunity to help students learn.

4.1.2 Mrs. G: use of different representations

Mrs. G used two lessons to work with the Cycle Track. During the first one, she showed the movement of two cyclists traveling, with different speeds, along the same path, in order to guide students towards identifying the main variables involved in the mathematical situation: position, time and velocity. She then removed one of the cyclists to explore further the relation between those variables and displayed the distance-time graph at the same time. The following conversation took place:

> Ken: *Why is the graph slanted? I thought it should be horizontal as the path.*
> Mrs. G (to the group): *Do you all think as he does?* … (after some discussion with the group)
> Jim: *I don't know, but the graph shows distance vertically… then it should go up, but not sure…*
> Pau*: I don't get it…*
> Mrs. G: *We can use the movement table* (shows the table on the program and draws the axes of a graph on the blackboard). *Do you remember how we did it? We can take this point from the table and draw it on the graph, and then these other* (adds 5 points) … *What happens when we join them? We have done this before…*
> Pau: *Like the one [graph] for the cyclist! I see the difference; the graph tells the distance traveled for each point in time.*
> Mrs. G: *The cyclist´s movement is different from the graph that shows the position of the cyclist at each moment in time. How would the graph look like if the cyclist moved quicker?*
> Ken: *It is a slanted line.*
> Mrs. G: *And if the cyclist moves quicker?*
> Ken: *It would be more slanted.*
> Mrs. G: *Right! We can see it if we used two cyclists moving simultaneously. How would the graph of the faster cyclist be?*
> Lu: *Is it more slanted, … but is it possible that a horizontal graph describes a movement?*
> Mrs. G: *Let's see… Well, let's draw a horizontal graph here, for example, when equals 5 (*draws the line on the blackboard). *What do we see? What is the position of the cyclist at time 1 second?*
> All: *x is 5.*
> Mrs. G: *And if t is 2?*
> All: *It is also 5.*
> Mrs. G: *And when it is 15 seconds?*
> All: *He is still there. He doesn't move.*
> Mrs. G: *With everything we have done, what does the graph incline tell us about the cyclist's movement?*
> Ken: *The velocity.*
> Lu: *Is it proportional?*
> Mrs. G: *Yes, this is another example of proportionality, in this type of motion the distance travelled is proportional to the time elapsed, and the proportionality constant is the velocity.*

Mrs. G's immediate coping with the first contingency, triggered by a student, was to socialize the question with the group. Mrs. G took advantage of the dynamic possibilities offered by the technology, and used the data from the table shown on the screen to draw a graph by hand, instead of using the graph provided by the technology. This allowed students to follow the transformation from one representation to the other and to reflect, thus triggering the construction of relations between the variables involved, which strengthened the idea that the movement's graph is a slanted line. Students' answers confirmed the adequateness of Mrs. G's actions involving the technology and socializing with students to promote their reflection on the mathematics involved in the situation. Mrs. G's immediate coping with the contingency involved the use of different representations and opportunities for students to ask questions and validate their responses. By opening a collaborative reflection space, she helped them change their beliefs by noticing what makes the graph different from the trajectory, and so to learn.

When a new contingency arose, triggered by Lu, about the meaning of a horizontal graph, Mrs. G coped by pausing and then modifying her actions by posing a situation that stimulated students' reflection about the graph of a non-moving object. She later described that she did not expect this question, which cannot be illustrated with the program. At the end of this episode Mrs. G promoted again a shared space for discussion where students related the variables involved in movement, identified velocity as the incline of the line and associated uniform movement with proportional relations. To close the session, she let students use the program to compare different movements, tables and graphs. She created a context where students could experiment with the technology, reflect and predict through interaction. These actions fostered the emergence of mathematical ideas and relations between variables so that she could be sure they had understood.

During the following session Mrs. G changed the path as shown in Fig. 1. She chose a cyclist and asked: *What will happen?* After students describe the movement using their experience, she asked: *How will the graph of his movement appear?* She used the program to show the graph while the cyclist goes upwards. A contingency emerged when some students question the graph indicating a constant speed. This event was triggered by a limitation of the technology, which always uses constant velocity, in this case, average velocity, to keep the attention on uniform motion. Mrs. G explained the technology's limitations, gave an example from daily life and students accepted it. Mrs. G´s immediate coping with this contingency included directing attention to a restriction of the technology, introducing the concept of average speed and using it to give meaning to the graph. Through these actions she fostered the emergence of new mathematical ideas. The discussion continues:

Pau: *The same thing happens when it goes down.*
Mrs. G: *Yes, but the speed is negative because the distance decreases with time, the graph should go down, look.* (She shows the movement and the graph with the program.)
Ek: *But that doesn't show in the graph, the line goes up, not down, why?*
Mrs. G: *…I don't know, let's look at the table. Ah! I see, I was wrong. I can see here that as the cyclist goes down the distance to the starting point is not decreasing, it is still increasing. Can you see it? That is why, and its incline is greater because the average speed is greater. Do you understand it?*
All: *Yes.*
Nat: *When is the speed negative?*
Mrs. G: *The program cannot do this, but we can do it ourselves. Ek you'll move slowly until I tell you and we can measure the distance by counting the mosaics he covers, then come back a little bit quicker and I measure the time.* (Ek does it) *Now we calculate his speed, 12 mosaics in 15 seconds, 0.8 mosaics per second, and back … as he goes back, the distance is getting shorter, so the velocity will be negative, right? It is… − 1.8 mosaics per second. Is it clear? Let's draw the graph* (she draws it). *How is the line when he goes back?*
All: *Slanted, but it goes downwards.*

Mrs. G triggered a new contingency herself when her explanation did not match with what was shown by the technology. She handled it by showing the associated table from the program, where she noticed her mistake. She acted by recognizing it, giving a clear explanation and asking questions to make sure that students understood. Then when faced with the question about negative speed, as the program is limited, she coped by proposing that students actively explored a situation where position can be positive or negative in order to illustrate negative velocity. She created a rich context where students could learn.

Mrs. G's showed throughout the whole episode that she could use the technology to help students reflect, act and interact with each other. On all occasions she created purposeful activities, with and without technology, contextualizing her actions for the students to analyze the mathematical problems that appeared, thus guiding their actions and reflections. Even in the case where Mrs. G made a mistake or when the technology was not useful to deal with students' questions, her actions seemed to be effective in creating a context that promoted her and her students' learning.

In this case, behaviors when facing contingencies included providing opportunities for students to ask questions, taking their interventions into account and using them to explore, experiment and discuss with and without the technology. The program was both a source of contingencies and a means for guiding students' attention towards important mathematical concepts and relations. Mrs. G and the way she used the program promoted a reflective environment where students' curiosity emerged as manifested by their pertinent and important questions.

4.2 The balance: promoting mathematical actions using fractions

With the program the balance users can create balances with different numbers of weights and on different levels. On each weight, natural numbers, fractions and decimal numbers can be written. The program indicates, in real time, visually and with sounds, whether the balance is in equilibrium or not, according to the values which are assigned to the scales. It promotes mathematical effective actions such as comparing, adding, subtracting and dividing numbers. In the case of fractions, it also invites users to find equivalent quantities (Fig. 2).

We observed several teachers using the balance during a number of lessons, with different objectives and obtaining different results. Here we report on two of them, Mrs. S and Mrs. L, who show different ways of acting when facing a similar situation created by the use of the program.

4.2.1 Mrs. S: finding a need for dividing fractions

We recorded data from six sessions in which students and Mrs. S worked with the balance. The following description was reported in a previous study:

> At the beginning of the sequence of lessons, teachers conducted an exploratory discussion which included comparing fractions and decimal numbers. We observed that most students were not able to compare decimal numbers and fractions adequately. Some would think that, for example, .4 is equivalent to .04, or that the latter is greater than the former. Other students would also say that 1/3 is greater than 1/2 because 3 is greater than 2. During the following sessions, students were asked, in whole-group discussions, about addition and subtraction of fractions. Our records indicate that some students often added numerators and denominators when adding fractions and that they applied procedures algorithmically, without being able to explain what they were doing (Lozano and Trigueros 2007).

After these initial explorations, the balance was introduced to the group. First it was used to compare weights with pairs of numbers such as the following: 2 and 1, 0.04 and 0.4, 0.04 and 0.040, 0.4 and 4/10. Later on, students were asked to build more complicated models and to reproduce the problems posed in their textbooks, which involved addition, subtraction and even multiplication or division of fractions.

As students used the balance, they often encountered situations in which they did not know, immediately, how to balance the mobile. In this case, it was observed that students started out by answering with trial and error and then gradually refining their strategies. The following extracts are examples of how students faced a situation they could not solve immediately and how the group dealt with it as a whole:

While students S1 and S2 were working together the following dialogue took place:

> S1: *You have to add this and this, see, 1/3 add 1/3, that is 2/6* (They try their
> answers in the balance) … *No, it doesn't work…*
> S2: *No, it is 2/3!! 1/3 add 1/3 is 2/3, not 2/6.*

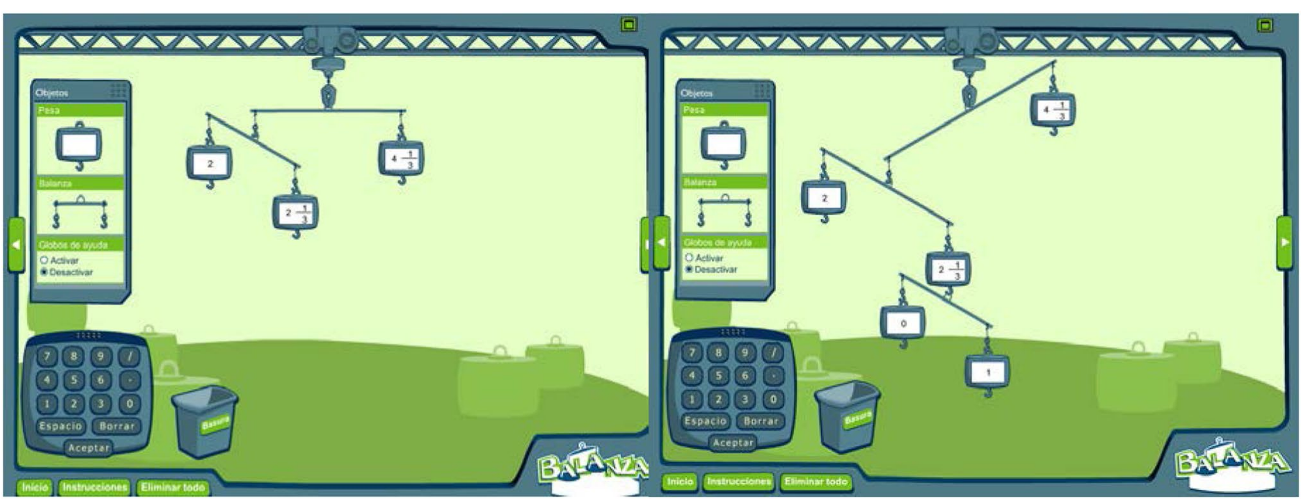

Fig. 2 Interactive program the balance

Mrs. S: *So can you find an explanation of why that works while 2/6 doesn't?*
S1: *When you add fractions you don't add the numbers just like that, these and these. It's thirds, so it's two thirds.*

Another pair of students S3 and S4 were working together when an unexpected event happened:

S3: *Something weird is happening. Mrs. S!*
Mrs. S: *Okay everyone, let's look at this one S3, tell us what's happening.*
S3: *Well we have to get 2 1/4 from these two, and we are using 1 and 1 1/4, it should work!*
Mrs. S.: *So why do you think that's still slanted? Anyone?*
S5: *Well because those two should be the same, and they are not.*
Mrs. S. *What do you think, S3?*
S3: *Well if you add them you get 2 1/4, so it should be fine, but I see what you're saying, they're not the same*.
Mrs. S: *They're not the same, so now what?*
S3: *Well I'm really not sure because how can I get two numbers that are the same there? It's not possible*.
Mrs. S: *What do you think? What can you do to get numbers that are the same? Maybe you and S4 can work together?*

In small groups students started discussing how to get two equal numbers which add up to 2 1/4. In the end, the class figured out that they had to divide 2 1/4 by two, and we observed them using different kinds of drawings and testing some hypotheses with the program. Some of them found systematic ways of dividing a fraction by 2, such as: *if the numerator is even, you just divide by 2, if it's odd you have to find a fraction that is equivalent and then divide*.

Our observations indicated that students refined their strategies as a result of both the feedback provided by the program and Mrs. S's actions when facing situations that were unexpected for the students. We observed that Mrs. S often asked students the question *why?* whether their answer was correct or not. This often created a need for students to find justifications and to generalize results. In addition, in Mrs. S's class, whole-group discussions were organized in every session where students were asked to explain their strategies for balancing the scales. Contingencies such as the one described here, in which students did not immediately know why the balance was not fully in equilibrium if the overall sum of the fractions on both sides was the same, were addressed in plenary sessions in which different ideas could be contrasted. During these whole group discussions, different representations were used by students when dealing with fractions and we observed that teachers and students, together with the program in itself, determined the adequacy of the answers and explanations.

On the one hand, this example shows how technology made evident something that the work with paper and pencil did not. The automatic feedback the balance provides the users with, showed students that for each level of the balance they had to find equivalent amounts and in this way the program created a need for mathematical actions that might not have arisen otherwise and which resulted in, for example, having to find a procedure for dividing fractions. On the other hand, by asking questions herself, and by providing a space for students to explain their answers during whole group discussions, these sessions demonstrated how Mrs. S played a role, which also contributed in triggering mathematical actions and changes in behavior, by refining strategies and finding general procedures.

4.2.2 Mrs. L: the correctness of the solution

Mrs. L started the lesson telling students that they would use the balance to work on similar problems to those on their workbooks. She then drew an example on the blackboard proceeded to use the same numbers on the program (4 1/3 on the scale on one side and on the other side two scales, one with a weight of 4 and the other one of 1/3) which showed that only the main bar of the balance was straight while the others were not.

A student called the teacher's attention to the following:

S1: *But the balance has to be in equilibrium and down there to the right it's slanted. The right part weighs more than the left part.*
Mrs. L.: *But what is important is that the main bar is in equilibrium.*
S2: *At home there is a mobile toy, it has little animals hanging. It looks somehow like this one, but all the bars have to be even, it is not like this, here some bars are slanted.*
Mrs. L.: … *What do you suggest? Then?*
S3: *Can we try again?*

The teacher called up two students, they tried with different numbers (for example 2 and 2 1/3, 3 and 1 1/3) but the balance was still unbalanced. They mentioned that they considered the problem too difficult. The teacher then concluded: *What happens is that the example I did is correct, one side weights 4* 1/3 *and the other one too, that's why it is balanced*. She then asked students to work in groups and said: *Try to do it your way or do it as I did, it's balanced anyway*.

This example is in the beginning similar to the previous one: a contingency triggered by the use of the program which showed that the balance was not fully in equilibrium. Students did not find a way of balancing all the arms of the mobile in that particular moment, Mrs. L decided that her way of solving the problem was correct. The need for finding equivalent fractions or for dividing a fraction by 2 arose in students, but since they were not able to solve the problem, partitions which are not equivalent or equal ended up being accepted.

Students solved the problems in different ways, but there was no whole discussion in which the different procedures could be contrasted and analyzed. The correctness of the solution is in the end determined mainly by Mrs. L.

In this case, the contingency was triggered on the one hand, by the technology, and on the other, by the students' who questioned the solutions in which the balance was not in equilibrium in all the different levels. Mrs. L's immediate coping included asking the students for alternative solutions, and she even invited them to follow their own procedures but in the end maintained her perspective regarding the correctness of her answer.

4.3 Proper fractions: is it greater or smaller than a half?

This program allows users to compare proper fractions using numerical and pictorial (circular) representations. A pictorial representation of a fraction appears on the screen (Fig. 3) and users are asked to select, amongst five options, the numerical representation that corresponds to the given fraction. They also need to determine whether the fraction is greater or smaller than one half. It is possible to select one of three levels of difficulty, defined in terms of the number of parts in the fraction and the spatial organization of the shaded parts. The program gives immediate feedback by telling the users, in written words, whether their answer is correct or not. In enactivist terms, the program is meant to foster students' visual recognition of fractions and mathematical effective actions such as identifying and comparing rational numbers.

4.3.1 Mrs. A: towards a conceptual understanding of fractions

Mrs. A started the lesson by stating that she wanted students to compare fractions with respect to 1/2, in order for them to "understand fractions as numbers" (Mrs. A). She used the program on the basic level, asked a student to solve each activity and wrote each fraction on the blackboard. After 10 examples, she asked students to order those numbers (1/3, 2/3, 4/6, 4/9, 4/7, 2/9, 3/5, 7/16, 8/14, and 5/8) from the smallest to the greatest.

One of the restrictions of the interactive program is that it is not possible to enter any two fractions and compare them either using the numerical or pictorial representations. However, it allows other kinds of mathematical actions, to which Mrs. A directed her students' attention. She guided them so that they could establish two types of relations: one, between the pictorial and the numerical representation of a fraction, and, other between the number of shaded parts and the total number of parts. Students explained: *When the whole is being divided into more and more parts, each part becomes smaller and smaller, and the denominator, that number becomes bigger*.

After ordering the fractions, as different answers were proposed (8/14 and 1/3 as the smallest), Mrs. A coped by opening a space for discussion:

> Mrs. A: *Is 8/14 the smallest? What do you say S3?* [...] *Are you considering equivalent fractions?* [...] *S1 and S2 say that they only looked at the denominator, and they stayed with the idea that the greater the number the smaller the parts. But if I don't take into account the numerator also… What do you say?*
> S3: *8/14 is greater.*
> Mrs. A: *Let's see. Is 8/14 greater or smaller than 1/2.*

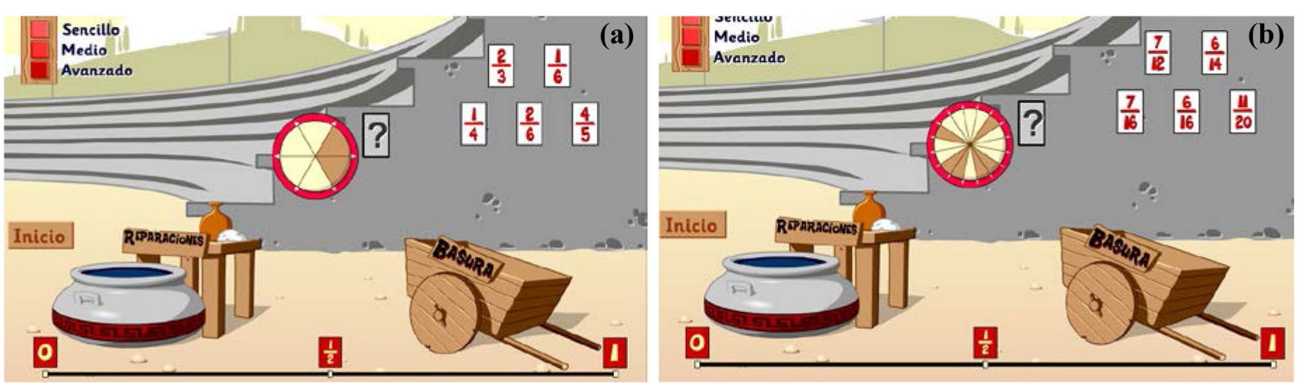

Fig. 3 Interactive program proper fractions: **a** simple level; **b** intermediate level

All: *Greater.*

Mrs. A: *It's greater. What about 1/3?*

All: *Smaller.*

Mrs. A: *Which one is smaller then?*

All: 1/3.

[...]

Mrs. A: 1/3 *is smaller... This is a whole number* [sic] *and I cannot focus on the denominator or the numerator only. One of the mistakes (S1 and S2's) helped us exemplify the fact that I should not focus only on one of the parts of the fraction. The unit is as important as the part. And together ... That relationship tells us something, it's talking about something and it is representing a quantity. I can't separate them* [...].

Mrs. A recognized in the answer given by the first group a conceptual misconception, as she corroborated in the interview. She then used the idea, stemming from the program, of comparing the fractions with 1/2, which created an opportunity for learning. She helped students to 'see' fractions as numbers through the pictorial representation, and by asking whether each of those fractions was greater or smaller than one half.

This showed Mrs. A's ability to invite students to use the strategy suggested by the program (comparing with 1/2) that might help when comparing fractions. Mrs. A's strategy triggered new actions that allow students to notice new relations and reach a consensus.

The discussion continued including all the different strategies which students used for comparing fractions, such as finding equivalent fractions, using the number line, pictorial representations and turning fractions into decimal numbers (using the calculator).

Effective behavior in this case means to open a reflection space for students to recognize and find a relationship between the different representations of the fraction.

Later, when Mrs. A decided to use an intermediate level of the program (Fig. 3b), some students exclaimed: *Now the shaded parts are not together, how can we compare?* Facing this situation, Mrs. A opened a new space for discussion in which, once again, fractions were compared to ½ and different representations were highlighted.

This example shows one the one hand, the way in which Mrs. A used students' mistakes to open a space for discussion through which she guided them to consider not only one aspect of the fraction but to 'see more' and take into account the relationship between the different parts. On the other hand, it shows how the teacher, during the discussions, went back to a strategy proposed by the interactive program which is not commonly used in Mexico (the comparison with 1/2), while she also took into account a variety of strategies that included different representations. This directed students' mathematical actions towards focusing on fractions as numbers.

Mrs. A's teaching strategies made use of questioning as a means to favor students' discussion and reflection, thus creating a rich context in which students can learn.

5 Discussion

The purpose of this paper is twofold, on the one hand, it is related to the exploration of the way in which technology contributes to the emergence of contingencies and of particular teaching situations, and on the other, it investigates how teachers modify their behaviors as a result of those events, creating contexts in which different kinds of mathematics learning are promoted. Here we look back at the results obtained in order to discuss both the role of technology and the different characteristics we observed in teachers' responses, focusing in particular on how those responses are related to mathematics learning.

5.1 The role of technology

The possibilities offered by the different resources used by teachers exemplified in this study show how the use of technology plays an important role in shaping the development of the lesson. In all cases, unanticipated events emerged which triggered certain reactions in teachers and students that would not have occurred if the technology was not used. Moreover, what we found is that those specific events arose as a result of certain features of the programs.

We observed that there are certain aspects of the technological devices themselves that allow users to focus their attention on certain particularities that are not evident without the technology.

There are some characteristics of technology that have been discussed in the literature and which contribute in promoting mathematics learning. One of them is related to the feedback that the programs provide (e.g., Sinclair et al 2016). Another feature is related to the program's capacity to dynamically present a certain situation (e.g., Trigueros, Lozano and Sandoval 2014). A third characteristic is the possibility of showing simultaneously different representations on the screen (e.g., Drijvers et al. 2010).

What we found in this investigation is that these characteristics can be closely related to the emergence of contingencies. For example, the discrepancy between the representation of the track and the position-time graphs in the Cycle Track immediately attracts students' attention and highlights a conceptual misunderstanding. This event may lead to a need for a deeper discussion about constant movement properties and their relation to proportionality. In the case of the balance, a deeper investigation of equivalent fractions and of operations with fractions arose as a result of students having to balance all the different levels of the mobile.

In relation to the emergence of contingencies in the classroom, authors have stated that pedagogical implementations depend on teachers' mathematical knowledge (Rowland and Zazkis 2013). However, in this investigation we noticed that the integration of interactive resources in the classroom created new demands and the emergence of a new range of possible actions from students and teachers, which would not be required if technology were not being used. In other words, when used these digital resources reorganize the type of possible actions that may be done in order to accomplish the goal of a mathematical task.

The use of technology in the classroom can shape teachers' actions. In turn, the way digital resources are used modifies those interactions that may arise and the type of questions that may be asked. The three interactives used by teachers in the examples shown were designed to teach mathematics. They all are limited in terms of what can be done with them. These restrictions conditioned the possible actions teachers could do and, at the same time, allowed them to create new strategies to face the problematic mathematical situations they encountered.

5.2 The teacher's role

The cases in this study provide some examples of unexpected events that appear when rich technological resources are used in the classroom. They show how some teachers' immediate coping with a situation can result in effective actions that can create a stimulating interactive and rich environment where mathematical discussion and learning is possible. It also presents some examples in which teachers' actions and the way they used their available resources may limit opportunities for learning.

When facing unexpected events, teachers' capability for immediately coping and changing their course of action and the environment in the classroom can be related to their previous history of interactions (Maturana and Varela 1992), which might allow for the possibility of using contingencies as opportunities to open up to new awareness.

Teachers need to use their mathematical knowledge, their teaching resources, motivation and creativity to offer responses and justifications to questions posed as a result of the emerging event (Smit and Van Eerde 2011; Mason 2002; Arafeh, et al. 2001). These actions thus provide opportunities to modify the current activity to create a rich environment where both teacher and students can be aware of the mathematics involved in the unforeseen event so that they all learn. We also found that teachers' openness to inquiry and their capability of integrating technological resources appropriately play an important role in the way they cope with new situations.

When comparing the observed teachers who faced similar contingencies, we noticed that their actions were different in terms of the kind of mathematical learning they promoted. While some of them created rich contexts where students could explore and reflect on what they were doing, in others the opportunities to use the programs to deepen the understanding of mathematical concepts were lost.

When looking at the differences between teacher's actions, we noticed that there are three different domains to which they pertain: social domain, mathematical domain and technological domain.

In the social domain, the results analyzed highlight that those teachers who succeeded in creating conditions that favored students' involvement and in-depth learning were those who listened to students and responded to their questions by asking new mathematical questions, leading them to reflect on what was being discussed. They also gave opportunities for students to compare and validate their responses Moreover, in some cases, these teachers also encouraged students to ask new pertinent questions. They showed confidence in the possibility of their students making sense of what they needed to do by themselves, and gave students autonomy and open spaces for exploring, reflecting, discussing and collaborating. In doing so, they created an environment where students could confront their beliefs and become aware of their misconceptions.

Also regarding social interactions, deciding who determines whether an answer is correct or not is relevant. When the correctness of an answer is mainly determined by the teacher, and students do not discuss and interact, participate in decision making, and are not asked to explain and justify their points of view, opportunities for learning might be missed.

Regarding the mathematical domain some differences were also identified. In those contexts in which deeper mathematics learning was promoted, teachers created opportunities to explore and use different representations; compared different procedures and solutions and fostered the emergence of new mathematical ideas and the construction of mathematical relations, which were not previously planned.

In relation to the technology these teachers used the technology flexibly, together with their mathematical knowledge, to design new tasks and devise new strategies to solve the emerging problems. They intermingled different resources to handle new situations and asked students to reflect on questions that could not be illustrated with the program they were using. They also made use of the technology to validate or reject students' conjectures, when possible.

Teachers' actions that arise when a contingency emerges shape mathematics learning in different ways. Whenever actions related to the social, mathematical and technological aspects described above transform the classroom environment into a place where opportunities for reflection and interaction are created, deeper mathematics learning can be promoted.

In enactivist terms, for teachers to be able to cope immediately with contingencies in such a way that their actions promote conceptually deep mathematics learning, they need to construct a history of interactions in which they have opportunities for facing unexpected events while reflecting on them.

6 Conclusions

This study contributes to showing that technological resources that offer possibilities for exploration and interaction can create a classroom context where new questions and interactions arise, but it is clear that they cannot by themselves generate an environment where learning opportunities can be fostered. Teachers' actions in reacting to those events are determinant in creating a rich environment where interaction, discussion and the need for new ways of mathematical thinking and explaining can make students' learning possible.

Regarding the social, mathematical and technological domains, through the analysis of the selected examples, some specific characteristics of teachers' responses that promote in-depth mathematics learning were identified.

The role the teacher plays is significant. It shapes the influence of technological devices. Their ability to deal with contingencies, at times opening possibilities for learning, depends upon their flexibility in the use of resources and their ability to deal with new possibilities that might emerge from listening to students' arguments, allowing for others (students or interactive programs) to determine the correctness of an answer, changing their course of action and even opening spaces for different mathematical topics or procedures to be addressed during the lesson. This flexibility is, in its turn, a result of the teacher's previous history of interactions. A direction for future research would be to investigate teachers' histories in order to identify those factors that might be relevant in shaping that flexibility.

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