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ORIGINAL ARTICLE

Cardinality principle understanding: the role of focusing on the subitizing ability

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Abstract

The *cardinality principle* (CP) is a conceptual basis of counting collections meaningfully and provides a foundation for understanding other key aspects of numeracy, such as the successor principle or counting-on to determine sums. Unfortunately, little research has focused on how best to teach the CP. One suggestion is that modeling the CP should be done with small collections children can subitize—that is, immediately recognize the total without counting. The present study was designed to investigate the following key, not fully resolved, questions: Is subitizing level associated with CP knowledge and is a particular level of subitizing critical for achieving the CP? Does fostering children's subitizing ability improve CP knowledge? Which approach to modeling the CP with subitizable collections is most efficacious in promoting the CP and its transfer? Eighty 2- to 5-year-old participants first received instruction designed to promote the ability to subitize collections from 1 to 5. Subitizing instruction alone resulted in 31 participants learning the CP. The remaining 49 participants were randomly assigned to one of three CP interventions: count-first, label-first, and count-only. All interventions involved collections participants could subitize. Results revealed that the participants who could subitize at least three achieved partial success on the CP task. Those who could subitize four *and* were in the count-first intervention achieved general success on the CP task. The findings underscore the need for early childhood educators and parents to build on subitizing ability to teach the CP.

Keywords Cardinality principle · Subitizing · Early numeracy · Meaningful counting

A key basis for meaningfully counting collections is the *cardinality principle* (CP): The last number-word used in counting process represents the total number of items in a collection. There is broad recognition that this principle is a critically important aspect of numeracy (Council of Chief State School Officers 2010; Frye et al. 2013; Sarnecka and Carey 2008; Sarnecka and Wright 2013; Slusser and Sarnecka 2011). Unfortunately, little has been done on how best to teach the CP. Using subitizable collections (collections that can be recognized without counting) may facilitate learning the CP (Baroody et al. 2006). The present study was designed to investigate the following key, unresolved, pedagogical issues: Is subitizing level associated with CP knowledge? If so, does developing children's subitizing ability promote their understanding of the CP? Is modeling the CP with subitizable collections efficacious and, if so, which approach to modeling works best to promote the CP and its transfer?

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1 Background

1.1 The importance of the CP

Children initially enumerate a collection by subitizing (Benoit et al. 2004; Fischer 1992; Klahr and Wallace 1976; von Glasersfeld 1982). However, such immediate recognition of a collection's cardinality (total) is largely limited to small collections. Counting permits children to enumerate collections beyond the subitizing range. The CP is the conceptual basis for doing so meaningfully.

The CP is a foundation for numeracy. Importantly, children do not connect number words later in the counting sequence to quantity until they have developed the CP (Slusser and Sarnecka 2011). The CP can then serve as a bridge between small numbers and numbers in general by enabling children to generalize insights (patterns and relations) regarding counting, number, and arithmetic gleaned from small collections to a collection of any size (Sarnecka and Wright 2013). Put differently, the CP serves as a developmental prerequisite for other important aspects early numeracy. For example, only children who understood the CP appeared to understand the successor principle: "adding exactly 1 object to a set means moving forward exactly 1 word in the list" (Sarnecka and Carey 2008). A plausible explanation for this relation is that subitizing permits to "see" concretely that one and one more is two and that two can be decomposed into one and one more or that two and one more is three and that three can be decomposed into two and one more (Baroody and Purpura 2017). Once the CP enables children to connect the counting sequence to a mental representation of successively larger collections, they recognize that any two successive numbers in the sequence differ by one. Another example is that only children who understand the CP can classify two collections with same number of objects as equal (Sarnecka and Wright 2013).

Children's informal mathematical knowledge provides a basis for understanding and learning formal mathematics (Frye et al. 2013; Ginsburg 1977). Unfortunately, a lack of learning opportunities can limit this foundational knowledge, interfere with learning school mathematics, and snowball into serious deficiencies in numeracy (Baroody et al. 2006; Dowker 2005). For instance, Starkey and Cooper (1995) found that 2-year-olds already differed significantly in their ability to subitize collections of one to four items. Yun et al. (2011) found that just over 10% of their kindergarten sample of largely minority children from low-income families still could not subitize three and almost 30% could not subitize four. Not surprisingly, then, children's early numeracy knowledge is predictive of school-mathematics achievement (Clements and Sarama 2007, 2008; Frye et al. 2013; Jordan et al. 2012; Sophian 2004). For example, Geary et al. (2018) found that the age children acquired cardinal concepts is more strongly related mathematical development and school readiness than reading achievement, when controlling for intelligence, executive function, and parental education levels.

For these reasons, there is currently considerable interest in early mathematical interventions that focus on key aspects of numeracy in order to "level the playing field"—ensure all children are ready for formal mathematics instruction (Frye et al. 2013). These reasons also explain why the CP is widely cited as a key preschool and kindergarten goal (Council of Chief State School Officers 2010). Common core state standards (Council of Chief State School Officers 2010) for Grade K requires students to be able answer quantitative questions that include quickly recognizing the cardinalities of small sets of objects, counting larger sets to determine their total, and producing sets of given sizes (an ability that builds on knowledge of the CP).

1.2 A common issue in learning to count meaningfully

Many preschoolers learn the procedure for counting collections in a one-to-one fashion but do not understand that the process can serve as a means of enumerating a collection. Specifically, they may not understand the CP—that the last word used in counting in a collection has a special significance: It represents the total number of items in a collection (i.e., its cardinal value). Such children may learn to respond to the how-many question by parroting back the last word used in counting. Fuson (1988) attributed this meaningless response to learning the "last-word rule." Delays in learning a meaningful CP may be due to how parents and teachers teach enumeration (Baroody et al. 2006). In particular, adults may not model the CP in a manner that helps underscore that the last number word in the counting process represents the total as well as label the last item as counted (Mix et al. 2012). This includes overlooking the possible importance of building on children existing knowledge, specifically their subitizing ability, an issue that is discussed next.

1.3 The possible importance of building on subitizing

Subitizing may play an important role in constructing a variety of number, counting, and arithmetic concepts and strategies (Baroody et al. 2006; Starkey and McCandliss 2014). This seems particularly true in the case of the CP, because both subitizing and the application of the CP are a means of identifying the cardinal value (total) of a collection (see Table 1 in Supplemental Materials). Whereas subitizing entails focusing on both the units and whole simultaneously, the process of counting a collection involves first focusing on the units *and then* focusing on the collection's whole or total (Freeman 1912). As noted in Sect. 1.2, many children learn to count one-to-one without connecting the initial focus on units to the final goal of specifying the whole. Relating the process of counting to subitizing may help children make this connection. In other words, modeling the CP with small collections children can already subitize may make more likely they will discover that the last number word used in the counting process has special significance, namely that it matches the total number of items they can literally see (Baroody et al. 2006; Frye et al. 2013). In particular, it may help children to understand why adults—otherwise

Table 1 Study procedures

Week #	Activity
1	Test 1: Subitizing ability and CP understanding (how-many task); $n = 100$
2–3	Stage 1 instruction: Subitizing intervention; $n = 80$
4	Test 2: Subitizing ability and CP understanding (how-many task)
5–7	Stage 2 Instruction: Subitizing-based CP interventions; $n = 49$
9 or 10	Test 3: CP understanding (how-many task) and its transfer (give-*n* task)

mysteriously—sometimes emphasize the last number word by changing the pitch of their voice or repeating the term (e.g., with three items, saying, "One, two, t-h-r-e-e; see three candies"). If the child can subitize the modeled collection, it is far more likely the child recognize that last number word indicates the total and that counting is just another way to determine the total of a collection. Teaching one-to-one counting without first ensuring the ability to subitize small numbers or modeling the CP with collections beyond a child's subitizing range may hamper discovering the CP and the purpose of counting.

1.4 Gaps in the research on teaching the CP

Little research has focused on how best to teach the CP. In a rare exception, Mix et al. (2012) evaluated four CP interventions: (a) counting alone without separately specifying the cardinal value/total (count-only); (b) specifying the total alone; (c) alternating between counting alone and labeling the total alone; and (d) labeling a collection with its cardinal value (total) first and then counting (labeling-first). Only labeling-first was effective in fostering 3-year-olds' learning of the CP. However, Mix et al.'s results are uncertain for three reasons.

Concern 1: type of the interventions Mix et al. (2012) did not evaluate a common approach to teaching the CP: counting a collection, emphasizing the last word counted, and identifying the total. For example, an adult might model counting a collection of three objects by saying, "O*ne, two, t-h-r-e-e—see three.* They reasoned that, unlike a label-first approach, such a count-first approach would be confusing or useless because the counting process and the labeling process are not clearly separated. However, this is an empirical question. Moreover, changing the pitch and stretching out the last count word and then repeating it might underscore that it has special significance.

Concern 2: operational definition of the CP The CP, or what Fuson (1988, 1992) called the count-cardinal concept, entails a quantity-word mapping. Mix et al. (2012) used an indirect measure of the CP, namely the give-*n* task. This task assesses a word-quantity mapping and the relatively advanced cardinality concept Fuson called the "cardinal-count concept." Indeed, Baroody et al. (2017) found that in comparison to the how-many task (a direct measure of the CP), the give-*n* task appears to underestimate 3- and 4-year-olds' CP knowledge. The pretest used by Mix et al. (2012), then, may not have excluded children who already knew the CP, and their posttest may have underestimated CP learning. Might using a direct measure of the CP provide a different picture of the best CP modeling technique than that suggested by Mix et al. (2012)?

Concern 3: size of collection to model the CP Mix et al. (2012) used collections of 2–9 items and did not examine the role of participants' subitizing ability in learning the CP. Might children need to achieve a certain subitizing level for CP modeling to be effective, and might they benefit from seeing the CP modeled with collections they can subitize?

In brief, the results of the Mix et al. (2012) intervention do not clearly indicate on how best to teach the CP.

2 Rationale for the present report

2.1 Addressing the concerns about the Mix et al. (2012) study

Paliwal and Baroody (2018) undertook a study to address the concerns with the Mix et al. (2012) study. Concern 1 above was addressed by including a count-first condition, which involved emphasizing the last count word by changing its pitch and repeating it, as well as a label-first condition (found most effective by Mix et al.) and counting-only condition (not found effective by Mix et al.). Concern 2 above was addressed by using a direct measure of the CP, the how-many task. Although this task may overestimate competence because some children learn a last-word rule by rote (Fuson, 1988), a lack of success on this task more clearly establishes that a child does not know the CP than non-success on the give-*n* task, which is a direct measure of the more advanced cardinal-count concept. The give-*n* task, which entails the meaningful application of the CP, was used to gauge transfer and to ensure CP knowledge was not overestimated. Concern 3 was addressed by assessing participants' ability to subitize 1–5, then providing them with subitizing training as needed, next reassessing their subitizing ability, and only then administering the CP interventions. See Table 1 for an overview of the study's design.

Results regarding the first two concerns were reported previously in Paliwal and Baroody (2018). In brief, the count-first intervention was significantly and (as measured by effect size) substantially more efficacious in promoting a general understanding of the CP than either the label-first or count-only intervention. The label-first intervention was significantly and substantially more efficacious in fostering partial CP knowledge than count-only intervention. Less robust but parallel results were obtained with transfer to the give-*n* task. The data were consistent with the view that subitizing 3 is needed to achieve partial or local CP knowledge and subitizing 4 is needed to achieve general CP knowledge.

However, the previous analysis by Paliwal and Baroody (2018) had several limitations, which are addressed by this report: A clear developmental relationship between subitizing level and achieving partial CP knowledge, in particular, or general CP knowledge were not established. For example, the prior analysis did not distinguish between whether a child achieved partial CP success *as a result* of the Stage-1 (subitizing) training or the CP (Stage-2) intervention.

2.2 Research question

The present report addresses the following four questions.

Question 1: Without intervention that targets subitizing or the CP (i.e., at Test 1), is there a developmental relationship between subitizing ability and CP knowledge? Do children who exhibit CP understanding exhibit higher levels of subitizing ability? Of particular importance, is there a subitizing level that is critical for achieving CP understanding?

Question 2: Does intervention that targets subitizing ability spontaneously promote children's understanding of the CP at Test 2? Might promoting higher levels of subitizing ability in itself promote learning and successful application of the CP? Again of particular interest is whether children need to achieve a critical level of subitizing ability to promote spontaneous discovery of the CP.

Question 3: Which type of CP instruction best builds on subitizing ability to promote CP knowledge at Test 3? Specifically, might controlling for existing partial knowledge of CP change Paliwal and Baroody's (2018) conclusions about which subitizing-based modeling approach is the most efficacious? Again, is modeling success tied to achieving a critical level of subitizing ability?

Question 4: Which approach to CP instruction best promotes transfer at Test 3? Specifically, which subitizing-based CP modeling approach best promotes spontaneous learning of the cardinal-count concept that underlies counting-out larger collections? Yet again, is such learning associated with a particular level of subitizing ability?

Table 2 Participants' characteristics by condition

	Count-first	Label-first	Count-only
Number of participants			
Age 2	0	1	1
Age 3	14	13	12
Age 4	2	3	3
Age			
Mean (SD)	3.6 (0.41)	3.6 (0.49)	3.6 (0.48)
Number of girls/boys	6/10	2/15	9/7
Race			
Caucasian	81.2%	88.2%	81.2%
African–American	12.5%	11.8%	12.5%
Multi-racial, unknown, or other race	6.2%	0%	6.2%

3 Methods

3.1 Participants

Parent permission slips were obtained for 100 children from three prekindergarten schools in the rural (Haralson and Carroll) counties of Georgia. Screening (Test 1) identified 20 4-year-olds as CP knowers, and those children did not participate further in the study. As a result, 80 participants (2, 45, and 33 2-, 3-, and 4-year-olds, respectively) received Stage 1 (subitizing) instruction.[1] Assessment after the subitizing instruction (Test 2) revealed that 6 and 25 of the 3- and 4-year-olds, respectively, demonstrated CP understanding. The remaining participants—2 2-year-olds, 39 3-year-olds, and 8 four-4-year-olds ($n=49$)—received Stage 2 (CP) instruction. See Table 2 for participants' characteristics by intervention condition. As of the 2010 United States Census for Haralson county, about 20.4% of the population of the county were below the poverty line, including 28.8% of those under age 18. The corresponding figures for Carroll county were about 17.3% and 20.3%, respectively. The poverty rate for both counties was above the national average of 14.7%.

[1] Although typically developing US children learn the CP about 3.5-years of age (Wynn, 1990), we included all children who returned a parent-permission letter, because CP development is more closely tied to developmental level than chronological age. In fact, the one 2-year old in the label-first condition was almost 3 (2 years and 11 months), could subitize up to 2 at Test 1 and up to 3 at Test 2 (after the subitizing training), and improved as much as any participant in the label-first condition on the CP task (i.e., progressed from no knowledge of CP at Test 2 to partial knowledge at Test 3). The 2-year old in the counting condition was 2 years and 9 months, could already subitize up to 2 at Test 1 and up to 4 at Test 2, and exhibited partial CP knowledge at Test 2—higher CP achievement at pretest than most participants in the study.

Table 3 Stage 1 (Subitizing) Intervention Games

Fast Fish: Game 1 of Stage 1 sessions 1 (*n* = 1–3) and 3 (*n* = 2–4)
Aim: Subitizing 1–4 items
Materials: Four index cards with pictures of 1–4 fishes pasted on them
Procedure: Say: Let's play the Fast Fish Hiding game. I will briefly show you a picture of some fish that swim fast and hide quickly. After I hide the fish, tell me how many fish you saw
Trials: Show each card for 3 s (One Mississippi, two Mississippi, three Mississippi), cover the card by flipping over the next (blank) card, and ask, "How many fish did you see?"

Number—Not the Number (Palmer and Baroody 2011): Game 2 of Stage 1 sessions 1 (*n* = 1 and 2) and 3 (*n* = 3 and 4)
Aim: Subitizing 1–4 items by contrasting examples and non-examples
Materials: Twelve index cards with 1–4 dots (3 of each kind) pasted on them
Procedure: For example, instruct a child to point to all the collections of two s/he can see and then to all the nonexamples of two s/he see ("Point to something that is not two"). For example, show 3 examples and 3 nonexamples of two. The children will point out an example and a non-example of a number. In either case, the game can be made more challenging by putting a time limit (20 s) on the pointing out process

Can You Find? (Palmer and Baroody 2011): Game 3 of Stage 1 sessions 1 (*n* = 1 and 2) and 3 (*n* = 3 and 4)
Aims: Subitizing and set production of 1–3 (Give-*n*)
Materials: A large and a small blue, red, and yellow block and point to the two red blocks and announce "two red blocks
Procedure: Put out the blocks. Questions might include: "Can you find and give me two blue blocks?" "Can you find and give me two big blocks?"

Slap It: Game 1 of Stage 1 sessions 2 (*n* = 1–4) and 4 (*n* = 1–4)
Aim: Use examples and non-examples to help a child to construct a concept of a small number and foster subitizing to 4
Materials: Four index cards with 1 to 4 dots (one of each kind) pasted on them
Procedure: Put out cards with 1–4 dots. For Slap It 2, for example, the child who slaps a 2 card first gets it. If a player slaps a non-2 card (first), the opponent gets the card. Player with most cards wins

Dominoes Same Number: Game 2 of Stage 1 sessions 2 (*n* = 1 and 2) and 4 (*n* = 3 and 4)
Aim: Subitizing 1 to 4 items by contrasting examples and non-examples
Materials: Set of dominoes with numbers from 1 to 4 on them
Procedure: Place dominos face down in a pile. Have each child take 5 dominos, then place all dominos face up. Place one starter domino face up between the players. Each player will take turns placing a domino with the same number of dots as an open end to begin a domino train. The game ends when either a player run out of dominos or no more dominoes can be played

Is It? Recognizing n: Game 3 of Stage 1 sessions 2 (*n* = 1 and 2) and 4 (*n* = 3 and 4)
Aim: Use examples and non-examples to help a child to construct a concept of a small number and foster subitizing to 4
Materials: A green "Yes" and a red "No" index card for each contestant, and 12 pictures cards with a collection of 1 to 4 dots (3 collections per number)
Procedure: A child is presented with 6 picture cards (3 examples and 3 non-examples of a number) one at a time and asked if it is an example of a number (for "Is It Two?", e.g., a trainer asks: "Is this two birds?"). A child can hold up or point to the green card to indicate "yes"; hold or point to the red card to indicate "no."

3.2 Instruction

3.2.1 Stage 1: Subitizing instruction

Stage 1 instruction focused on promoting the immediate recognition of collections 1–5. Each of the four Stage 1 sessions involved 25–30 min and playing each of three different games twice (see Table 3). Sessions 1 and 2 entailed one-on-one training, and participant played the games individually. Session 3 and 4 involved one-on-two training, and two children played the game together. With an exception to the Slap It game, a larger collection was used the second time a game was played again in the sessions so that participants could apply their knowledge to a larger collection.

Three games (Number—Not a number, Slap It, and Is It?) involved explicitly identifying different examples and non-examples of a number. The former promoted generalization of a number concept such as "three"; the latter, defined its limits to avoid overgeneralization of a concept (e.g., oooo is not "three") understanding the difference between various number representations by looking at various examples. Before starting each session, the trainer informed the participants that they are going to play some games and asked them to respond as fast as they can (this was often done by imposing a time limit to answer) so as to encourage participants to use their subitizing ability.

The games involved different objects/materials to further promote generalization of a number concept. For instance, the Fish game was played using picture cards of fish; Number—Not a Number, Slap It, and Is It games were played using index cards with dots; and Can You Find? involved using blocks of different kinds and colors. So as to promote rapid recognition of small collections, a trainer explained to children that they respond as fast as they can in the games they were going to play. The game Fast Fish also impose a time limit to answer and other games such as Slap It rewarded a quick response. The game Can You Find? entailed applying using subitizing ability to produce a set.

Table 4 Size of the collection used in the CP intervention sessions 1–4 based on subitizing ability

Knower level	Size of collections used
Subitizes up to 2	1, 2, 3
Subitizes up to 3	2, 3, 4, 5
Subitizes up to 4	2, 3, 4, 5, 6

3.2.2 Stage 2: CP instruction

Each CP intervention consisted of 25–30 min one-on-one sessions twice a week for 3 weeks. In each of six CP sessions, the participants practiced counting with 12 examples consisting of counting 1–6 items animal pictures (sessions 1 and 5), concrete objects (sessions 3 and 6), or dots on an index card (sessions 2 and 4). Different items (e.g., toys of various kinds, pictures, or shapes) were used during the intervention so that the participants might better generalize the CP. Sessions 1–4 involved counting collections based on a child's subitizing ability (see Table 4). This was done to ensure that at least half of the counted collections were within a participant's subitizing range. Sessions 5 and 6 entailed counting 3–6 objects. Including larger collection beyond a child's subitizing was intended to foster transfer CP understanding to larger collections.

The CP instruction was administered in three different ways:

1. Label and then count (Label-first). Labeling the set first with its cardinality (total number of items) and then counting. For example, on a page with 3 elephants, the experimenter said, "Look there are 3 elephants. Let's count them." And counted them as, "one, two, three."
2. Count, emphasize, and repeat the last word (Count-first). Counting the set followed by emphasizing the last word counted, and then repeating the last word. For example, on a page with 3 elephants, the experimenter said, "One, two, t-h-r-e-e. There are three elephants."
3. Counting only (Count-only). Counting a given set without emphasizing the total number of items. For example, on a page with 3 elephants, the experimenter said, "One, two, three." And, then moved to the next example to be counted.

To make interventions developmentally appropriate and maximize their impact, each participant's subitizing level (as determined after the Test 2) was used to choose the examples for counting during Sessions 1–4 of the CP intervention interventions (see Table 4). This ensured that counting was done with collections a child could subitize at least 50% of the time. Sessions 5 and 6 involved counting 3–6 objects. The experimenter started each session by telling participants that they are going to count a collection. The modeling counting method was dependent on the participant's assigned intervention as well as analogous to the number they can subitize from the test 2 (in sessions 1–4). A trainer encouraged a participant to count the collections with him/her and assisted a child in counting whenever needed.

3.3 Measures

3.3.1 Subitizing task

The aim of the subitizing task was to assess participants' subitizing ability, that is, the numbers they could recognize without counting. Task involved a tester asking the participant whether a given collection was a certain number. For each number 1–5, there were three examples and three non-examples of the numbers. Testing started with assessing the ability to subitize two and proceeded to one if the child was unsuccessful or to three if successful. Larger numbers were assessed until the upper limit of a child's subitizing ability was found. A child was deemed successful in subitizing a number if correct (responded "yes") on the three examples of the number and correct (responded "no") on at least two non-examples of the number. A child who, for example, could successfully subitized up to three but not larger numbers (four) is referred to as a "three-knower." Similarly, a child who could successfully subitized up to four but not larger numbers is referred to as a "four-knower."

3.3.2 Main CP (or count-cardinal) task: how many

As give-*n* task can seriously underestimate CP knowledge, the relatively direct how-many task was used in the present study as the measure of the CP. The task took the form of the Hidden Stars game (Baroody 1987; Item 7 of the Test of Early Mathematics Ability—Third Edition, Ginsburg and Baroody, 2003, but with trials of 4 and 6 items). The task entailed presenting a linear array of stars on an index card, asking a child to count the stars, covering the stars, and a finally asking how many stars were hidden. One point was awarded for each correct response. A correct response entailed answering with the last number word used in the counting process regardless of counting accuracy (range 0–2 points). Partial knowledge of the CP was defined as 1 point—responding correctly to the trial involving four items but not the trial involving six items. (There were no cases of the reverse case.) Knowledge of the CP (CP knower) was defined as scoring 2 points (i.e., correct on trials involving both four and six).

3.3.3 CP transfer (cardinal-count) task: give-*n*

A concern with the how-many task is that it is an indirect measure of the CP as it measures relatively advanced cardinal-count concept. To avoid overestimating competence, the give-*n* task, which entails the meaningful application of the CP, was used as a transfer task. The task entailed presenting children with a pile of 10–12 plastic frogs and asking them to produce a set of 4 or 6 frogs. One point was awarded for each correct response (range 0–2 points). Partial and full knowledge of cardinal-count concept were defined the same manner as that for the CP.

3.4 Research designs and procedures

The procedures are summarized in Table 1. In week 1, Test 1 was conducted to assess all participants' subitizing level and CP understanding. In weeks 2 and 3, all participants who were not successful (scored 0 or 1) on the main CP task at Test 1 received the same Stage-1 subitizing instruction (four sessions total). In week 4, Test 2 was then administered to assess progress in subitizing ability and CP knowledge. Children who were still not CP-knowers (i.e., scored 0 or 1 on the main CP task) received one of three types of CP intervention. The Stage-2 CP interventions were conducted in weeks 5–7 (six sessions total). Two to three weeks after Stage 2 CP intervention, the participants were administered Test 3, consisting of the main CP (how-many) task and the CP transfer (give-*n*) task to assess the impact of the Stage 2 CP interventions.

Children were individually tested and trained by project staff in a project space outside a child's classroom. Positive assent of the teachers and participants were obtained each time a child was taken for project instruction or testing. To encourage participation in the intervention, children were rewarded with a small appreciation token (e.g., sticker, small toy) after each intervention session. Testers were blind to a participant's intervention assignment. During the testing, two separate trainers assessed each participant's performance, and any conflicts (which happened less than 1% of the time) were resolved by mutual consensus.

A non-experimental design was used to examine the relationship between subitizing level and CP knowledge at Test 1. A pre-experimental design served to evaluate the impact of the Stage 1 subitizing instruction on CP knowledge. All the participants were treated as one group and were tested on the subitizing and main CP tasks at the pretest (Test 1) and the posttest (Test 2) to assess growth in subitizing level and CP understanding. An experimental design (RCT) was used to evaluate the impact of the Stage 2 CP instruction. Specifically, eligible participants were randomly assigned within class/school to count-first, label-first, and count-only conditions. The last effectively served as an active-control condition, which represented regular classroom instruction with extra numeracy intervention. An active-control condition controls for various threats to internal validity such as a history effect (e.g., regular classroom instruction and practice), a maturation effect, and regression to the mean. Unlike a business-as-usual (passive) control, an active control effectively also controls for the impact of a novelty, "special treatment," or researcher familiarity effect. None of the three schools provided any kind of instruction that fosters participants' subitizing ability or CP understanding.

3.5 Analyses

An analysis after Tests 1, 2 and 3 involved examining the relationship between subitizing (*n*-knower) level and the CP treated as dichotomous variables (e.g., achieved a subitizing level of four or not and achieved general CP knowledge). As subitizing level was hypothesized to develop before and to facilitate CP learning (or even serve as a necessary condition for it), discordant relations (successful performance on one but not the other task), but not concordant relations (unsuccessful or successful performance on both subitizing and CP tasks), were of interest. (Concordant pairs of data points provide no information on which competence develops first.) More specifically, a statistic was needed to examine whether confirmatory discordant pairs (successful achievement of a subitizing level but not CP knowledge) significantly outnumbered disconfirming discordant pairs (successful CP achievement but subitizing). A McNemar Chi-squared can effectively serve to test whether confirmatory discordant pairs exceed discomfirmatory ones beyond a chance level. However, if the number confirmatory or discomfirmatory discordant pairs is small or the number of discordant pairs is less than 25, then the McNemar X^2 is not well approximated by the chi-square distribution (Bearden et al. 1982). As the ideal case for hypothesized developmental relations is zero disconfirming discordant pairs and each analysis had a total of 25 discordant pairs or less, (unlike Paliwal and Baroody 2018) an exact binomial test was used. This test is conservative, but this only serve to stack the deck against corroborating the hypothesized relation. A one-tailed significance level (with an alpha of 0.05) was used to test the directional hypotheses (e.g., that success subitizing 4 developmentally precedes general CP knowledge).

4 Results

Question 1: without intervention that targets subitizing or the CP (i.e., at Test 1), is there a developmental relationship between subitizing ability and CP knowledge? Among the 100 children assessed at Test 1, a Pearson correlation indicated that there was a significant and strong positive

Table 5 Subitizing progress × progress in CP knowledge at Test 2 (n = 80)

Test 1–Test 2 change in subitizing level	Test 1–Test 2 change in subitizing level					
	No knowledge (0 correct)		Partial knowledge (1 correct)		General knowledge (2 correct)	
0 → 2	A	2	B	0	C	0
0–2 → 3	D	17	E	0	F	0
3 → 3	G	1	H	0	I	0
0–2 → 4	J	7	K	12	L	24
3 → 4	M	3	N	(5 of) 7	O	(7 of) 7

Of the 80 participants listed, none had general CP knowledge at Test 1, and 12 children who moved from subitizing 3 to subitizing 4 between Tests 1 and 2 had partial CP knowledge at Test 1. The numbers in parentheses indicate the CP scores of these 12 children at Test 2

association between subitizing ability and CP success, [$r(100) = 0.85$, $p < 0.001$]. Specifically, prior to any instruction, all 20 participants who had already achieved general CP knowledge (i.e., scored 2 on the how-many task) could subitize up to four, and all 12 children who exhibited partial knowledge of the principle (i.e., were correct on the trial involving four items but not on the one involving six items) could subitize three (but not four). Among the 68 participants who were not CP knowers (scored 0 on the main CP task) at Test 1, 6 could subitize up to three, 23 could subitize up to two, 32 could subitize only one, and. 7 could not subitize even one.

In terms of what subitizing level is critical for achieving general CP understanding, an Exact Binomial test could not be calculated for the relationship between subitizing 4 and general CP knowledge, because all cases were concordant (Cell *a* [successful on both] = 20, Cell *b* [successful at subitizing 4 only] = 0, Cell *c* [successful CP knowledge only] = 0, and Cell *d* [unsuccessful at both] = 80), but a Fischer Exact Test was highly significant ($p < 0.001$). The breakdown for subitizing 4 and partial CP knowledge was Cell $a = 0$, Cell $b = 0$, Cell $c = 12$ and Cell $d = 68$ (Exact Binomial Test, $p < 0.001$, one-tailed). The results for subitizing 3 and general CP knowledge was Cell $a = 0$, Cell $b = 18$, Cell $c = 0$, and Cell $d = 62$ (Exact Binomial Test, $p < 0.001$, one-tailed). The relationship between subitizing 3 and achieving partial CP knowledge was significant (Cell $a = 12$, Cell $b = 6$, Cell $c = 0$, and Cell $d = 62$; Exact Binomial Test, $p = 0.016$, one-tailed).

Question 2: does intervention that targets subitizing ability spontaneously promote children's understanding of the CP at Test 2? A comparison of 80 participants' subitizing progress from Test 1 to Test 2 scores resulting from Stage 1 (subitizing) instruction, and their performance on the main CP task at Test 2 is summarized in Table 5. A total of 31 (in Cells L and O in Table 5) of the 60 participants who achieved the 4-knower level at Test 2 (Cells J–O) achieved general CP success, whereas none (in Cells C, F and I) of the 20 who did not achieve the 4-knower level (Cells A to I) did so. For the 2 × 2 analysis comparing 4-knower level and general CP knowledge (2 × 2 cell *a* [success on both] =

Cells L and O in Table 5 = 31, 2 × 2 Cell *b* [successful at subitizing 4 only] = Cells J, K, M, & N in Table 5 = 29, 2 × 2 Cell *c* [successful general CP knowledge only] = Cells C, F, and I in Table 5 = 0, and 2 × 2 Cell *d* [unsuccessful at both] = Cells A, B, D, E, G, and H = 20), the Exact Binomial test was significant ($p < 0.001$, one-tailed). Excluding the five children who had already achieved partial CP success at Test 1, 14 (in Cells K and N) of the 44 participants who achieved the 4-knower level at Test 2 (Cells J, K, L, M, N, and O) achieved partial CP success, whereas none (in Cells B, E, and H) of the 20 who did not achieve the 4-knower level (Cells A to I) did so. For the 2 × 2 analysis comparing 4-knower level and partial CP knowledge (2 × 2 Cell *a* [success on both] = Cells K and N in Table 5 = 14, 2 × 2 Cell *b* [successful at subitizing 4 only] = Cells J and M = 10, 2 × 2 Cell *c* [success at partial CP knowledge only] = Cells B, E, and H in Table 5 = 0, and 2 × 2 Cell *d* [unsuccessful on both] = Cells A, D, and G = 20), the Exact Binomial test was significant ($p = 0.001$, one-tailed). None of the 18 children who achieved or stayed at 3-knower status at Test 2 showed any CP improvement at Test 2. For the 2 × 2 analysis comparing 3-knower level and general CP understanding (2 × 2 Cell *a* [success on both] = Cells E, F, H, and I = 0, 2 × 2 Cell *b* [successful at subitizing 3 only] = Cells D and G = 18, 2 × 2 Cell *c* [successful at general CP understanding only] = cells B and C = 0, and Cell *d* [unsuccessful at both] = Cell A = 2), the Exact Binomial test was significant ($p < 0.001$, one-tailed). For the 2 × 2 analysis comparing 3-knower level and partial CP knowledge (2 × 2 Cell *a* [success on both] = Cells E and H = 0, 2 × 2 Cell *b* [successful at subitizing 3 only] = Cells D and G = 18, 2 × 2 Cell *c* [successful at partial CP knowledge only] = cell B = 0, and Cell *d* [unsuccessful at both] = Cell A = 2), the Exact Binomial test was significant ($p < 0.001$, one-tailed, for partial CP knowledge).

Question 3: which type of CP instruction best builds on subitizing ability to promote CP knowledge at Test 3? A comparison of the 49 participants' subitizing level at Test 2

Table 6 Subitizing level at Test 2 × CP-knowledge level at Test 3 × intervention condition (n = 49)

Subitizing level at Test 2	CP knowledge level at Test 3					
	{Regressed to no knowledge level} or [remained at no knowledge] (0 correct) CF, LF, CO		Gained partial knowledge or <remained at partial knowledge level> (1 correct) CF, LF, CO		Gained general knowledge (2 correct) CF, LF, CO	
0–2	A	[**1**, *0*, 1]	B	***0***, *0*, 0	C	***0***, *0*, 0
3	D	[***3***, *4*, 4]	E	***3***, *4,* 0	F	***0***, *0*, 0
4	G	{***0***, *0*, 2} **[0, 0,** 5]	H	***1***, *1*, 0 <***0, 8,*** 4>	I	***8, 0,*** 0

CF indicates counts-first intervention (bold and italicized numbers in the table); LF, label-first intervention (italicized numbers in the table); and CO, counts-only intervention (numbers in regular type)

Of the 49 participants listed, none had general CP knowledge at Test 2, and 19 children who were 4-knowers at Test 2 were scored as partial CP-knowers at Test 2. Of the 19 partial CP knowers at Test 2, 5 CF children gained general CP knowledge at Test 3; 8 LF and 4 CO children made no progress; and two CO performed more poorly on the how-many task at Test 3. Numbers enclosed in [] indicate those who exhibited no CP knowledge at Test 2 and exhibited no progress at Test 3; numbers enclosed in { } indicate those who regressed; numbers enclosed in < > indicate a child who exhibited partial CP knowledge at both Test 2 and Test 3

Among the 3-year-olds, 36% (14 of 39) gained CP knowledge at the Test 3, and 37.5% (3 of 8; 1 LF, 1LF, and 1 CF child in Cell E, H, and I, respectively) 4-year-olds did so

and whether or not their performance on the main CP task improved at Test 3 (after the CP instruction) is summarized in Table 6. Note that, whereas 75% of the children in the count-first intervention showed at least some improvement, only 29% of those in the label-first intervention and 0% of those in count-only intervention did so. More specifically, of the 17 children who showed CP improvement at Test 3, all 8 who achieved general CP success and almost half (4 of the 9) who achieved partial CP success were in the counting-first condition. All five of the children in the label-first condition who made progress only achieved partial CP success.

Eight (Cell I of Table 6) of 29 who could subitize 4 at Test 2 (in Cells G to I) achieved general success on the CP (how-many) task at Test 3, whereas none (in Cells C and F) of the 20 who could not subitize 4 did so. For the 2 × 2 analysis comparing 4-knower level and general CP success at Test 3 (2 × 2 Cell *a* [success at both] = Cell I in Table 6 = 8, 2 × 2 Cell *b* [subitizing 4 only] = Cells G and H = 21, 2 × 2 Cell *c* [general CP success only] = Cells C and F = 0, and 2 × 2 Cell *d* [unsuccessful at both] = Cells A, B, D, and E = 20), the Exact Binomial test was significant ($p < 0.001$, one-tailed). Excluding the 12 children who achieved partial CP before the CP intervention (numbers enclosed in < > in Cell H), 2 (non-enclosed entries in Cell H) of the 9 who could subitize 4 at Test 2 (Cell G and non-enclosed entries of H) achieved partial CP success after receiving CP instruction, whereas 7 (Cell E) of 20 who could not subitize 4 (Cells A, B, D, and E) did so. For the 2 × 2 analysis comparing 4-knower level and partial CP success at Test 3 (2 × 2 Cell *a* [success on both] = Cell H = 2, 2 × 2 Cell *b* [subitzing 4 only] = Cell G = 7, 2 × 2 Cell *c* [partial CP success only] = Cells B and E = 7, and 2 × 2 Cell *d* [unsuccessful on both] = Cells A and D = 13), the Exact Binomial test was significant ($p = 0.50$, one-tailed). None of 18 three-knowers at Test 2 achieved general CP knowledge at Test 3. Seven (Cell E) of the 18 such children (Cells D and E) achieved partial CP success after receiving CP instruction, whereas none (in Cells B) of two who could not subitize 3 (Cells A and B) did so. For the 2 × 2 analyses comparing 3-knower level and partial CP success at Test 3 (2 × 2 Cell *a* [successful on both] = Cell E = 7, 2 × 2 Cell *b* [subitizing 3 only] = Cell D = 11, 2 × 2 Cell *c* [partial CP success only] = Cell B = 0, and 2 × 2 Cell *d* [unsuccessful on both] = Cell A = 2), the Exact Binomial test was significant ($p < 0.001$, one-tailed).

Question 4: which approach to CP instruction best promotes transfer at Test 3? Participants' subitizing levels and their performance at Test 3 on the transfer task is reported in Table 7. Note that, transfer success was achieved by 7 participants (44%) who were all from the count-first intervention. Also, 4 participants (25%) from the count-first intervention achieved partial success, 10 participants (59%) from the label-first intervention and 1 participant (6%) from count-only intervention achieved transfer.

Seven (Cell I of Table 7) of 29 who could subitize 4 at Test 2 (Cells G to I) achieved general success on the transfer (give-*n*) task at Test 3, whereas none (in Cells C and F) of the 20 who could not subitize 4 (Cells A to F) did so. For the 2 × 2 analysis comparing 4-knower level and general CP success (2 × 2 Cell *a* [success on both] = Cell I = 7, 2 × 2 Cell *b* [subitizing 4 only] = Cells G and H = 22, 2 × 2 Cell *c* [general CP success only] = Cells C and F = 0, and 2 × 2 Cell *d* [unsuccessful on both] = Cells A, B, D, and E = 20), the

Table 7 Subitizing level at Test 2 × performance on the transfer task at Test 3 (n = 49)

Subitizing level at Test 2	Transfer to the cardinal-count concept at Test 3					
	No knowledge (0 correct) CF, LF, CO		Partial knowledge (1 correct) CF, LF, CO		General knowledge (2 correct) CF, LF, CO	
0–2	A	***1***, *0*, 1	B	***0***, *0*, 0	C	***0***, *0*, 0
3	D	***4***, *7*, 4	E	***2***, ***1***, 0	F	***0***, *0*, 0
4	G	***0***, *0*, 10	H	***2***, 9, 1	I	***7***, *0*, 0

Among the participants in Cell I with general knowledge, 86% (6 of 7) were 3-year-olds, 14% (1 of 7) were among the participants with partial knowledge, 80% (12 of 15; 3 CF and 9 LF in Cells E and H) were 3-year-olds and 20% (3 of 15; 1 LF and 1 CO in Cell H) were 4-year-olds

Exact Binomial test was significant ($p < 0.001$, one-tailed). Twelve (Cell H) of the 22 who could subitize 4 at Test 2 (Cells G and H) achieved partial success on the transfer task at Test 3 after receiving CP instruction, whereas only 3 (Cell E) of the 20 who could not subitize 4 (Cells A, B, D, and E) did so. For purposes of 2 × 2 analysis comparing 4-knower level and partial CP success (2 × 2 Cell *a* [successful on both] = Cell H = 12, 2 × 2 Cell *b* [subitizing 4 only] = Cell G = 10, 2 × 2 Cell *c* [partial CP success only] = Cells B and E = 3, and 2 × 2 Cell *d* [unsuccessful on both] = Cells A and D = 17), the Exact Binomial test was significant ($p = 0.046$, one-tailed). Three (Cell E) of 18 who could subitize 3 at Test 2 (Cells D and E) achieved partial success on the transfer task at Test 3, whereas none (Cell B) of the 2 who could not subitize 3 (Cells A and B) did so. For the 2 × 2 analysis comparing 3-knower level and partial CP success (2 × 2 Cell *a* = Cell E = 3, 2 × 2 Cell *b* [subitizing 3 only] = Cell D = 15; 2 × 2 Cell *c* [partial success only] = Cell B = 0, and 2 × 2 Cell *d* [unsuccessful on both] = Cell A = 2), the Exact Binomial test was significant ($p < 0.001$, one-tailed).

5 Discussion

The findings of this study clarify how developing subitizing ability facilitates an understanding the cardinality principle (CP) or what Fuson (1988) called the count-cardinal concept.

Question 1: without intervention that targets subitizing or the CP (i.e., at Test 1), is there a developmental relationship between subitizing ability and the CP knowledge? Initial testing indicated that subitizing level positively and strongly associated developmental readiness to learn the CP. Specifically, 20% of the 100 Test 1 participants were identified as a 4-knower, and all and only these children were identified as general CP knowers. This result and the absence of children who were 4-knowers but not CP knowers seem to indicate that the two competencies emerged together and that achieving the 4-knower level is sufficient for constructing an understanding the CP. The results regarding Questions 2 and 3, though, indicate that these strong claims may be true in some but not all cases.

The results clearly indicated that partial knowledge of the CP can emerge before the 4-knower level but not the 3-knower level. Put differently, the 3-knower level clearly develops before partial CP knowledge and appears to be a prerequisite for such knowledge.

Question 2: does intervention that targets subitizing ability spontaneously promote children's understanding of the CP at Test 2? As Table 5 indicates, all but one of the 80 participants who received Stage 1 (subitizing) instruction improved in subitizing ability. The exceptional case was a 3-knower at both Test 1 and Test 2. Just over half of the 60 children who achieved the 4-knower level at Test 2 also spontaneously achieved a general understanding of the CP, whereas none of the 20 who did not did so. These results indicate that 4-knower and CP knowledge apparently can develop simultaneously that the former is sufficient for the latter. However, nearly half of those who achieved 4-knower knowledge did not achieve general CP knowledge. The results overall, then, show that the 4-knower level develops prior to general CP knowledge and may be only a necessary condition for the count-cardinal concept. The results further indicate that this is not the case for the 3-knower level. Test 2 results indicate that both the 3-knower and 4-knower levels can develop before partial CP knowledge.

Question 3: which type of CP instruction best builds on subitizing ability to promote CP knowledge at Test 3? The count-first approach to subitizing-based CP instruction was clearly superior to the count-only method and the label-first technique. Half of the 16 count-first participants achieved general CP knowledge, and another fourth achieved at least partial knowledge. In contrast, none of the count-only participants made any CP progress—despite the advantage that a fourth of these children began the instruction with partial CP knowledge. Moreover, contrary to Mix et al.'s (2012) theorizing and findings, none of the label-first participants achieved general CP knowledge—despite the advantage that almost half (8) of these 17 preschoolers began the instruction with partial CP knowledge. The label-first intervention did have some success in promoting partial CP knowledge in that 55.6% of the remaining 9 achieved this level as result of the intervention.

Achieving general CP knowledge was clearly related to achieving the 4-knower level. Somewhat more than a fourth of 4-knowers achieved general CP knowledge with (count-first) CP training. None of the eighteen 3-knowers (including six who had the count-first training) or the two 2-knowers (including one who had the count-first training) did so. The significant Exact Binomial test is consistent with the view that the 4-knower develops before and is a necessary condition for general CP knowledge. Excluding participants who had already achieved partial CP knowledge, achieving partial CP knowledge was clearly and significantly related to achieving the 3-knower, but not 4-knower, level.

Question 4: which approach to CP instruction best promotes transfer at Test 3? In stark contrast to Mix et al.'s (2012) results, the present results clearly indicate that the count-first approach, but not the label-first (or the count-only) approach, enabled participants to construct a general cardinal-count concept (a cardinal number indicates with what number word the count a collection should end). This more advanced cardinality concept that underlies the ability to count-out a specified number of items beyond the subitizing range. Moreover, all seven of the count-first participants who achieved success with the transfer task could subitize 4. This indicates that subitizing 4, but not subitizing 3, may be critical in constructing a general cardinal-count concept also. The label-first intervention was somewhat efficacious in that 10 of the 17 participants administered this training achieved partial transfer, 9 of whom could subitize 4. Only 1 of the 16 participants from the count-only intervention, who could subitize 4, did so.

The results with give-*n* task, an indirect and conservative measure of CP knowledge, parallel and confirm those based on the how-many task, which (if used alone) may overestimate CP knowledge. Transfer to the give-*n* task is an important indication that nearly all participants who learned the CP or count-cardinal concept actually *understood* it (as opposed to simply applying a last-word rule by rote). Understanding makes it more likely that children will apply newly learned ideas to a new context or problem (Hatano, 2003).

6 Conclusions

6.1 Theoretical implications

The present results are consistent with the proposition that new instruction should build on what children already know, because it makes reflection and assimilation more likely (Piaget, 1964). Specifically, the results indicate that achieving the 4-knower level appears to be critical for achieving general CP knowledge, whereas achieving a 3-knower level appears critical for achieving partial CP knowledge (i.e., success applying the CP to collections of 4 but not 6). The process of counting involves first labeling each item once and only once with a number word and squarely focuses attention on the units (Freeman, 1912). Then the last number word, which to that point served only to label the last item in the collection as counted (an ordinal meaning), suddenly takes a new meaning of indicating the total of the collection (a cardinal meaning)—for those who already understand the CP. For children just learning to count in a one-to-one manner and who do not understand the CP, this latter meaning is not at all obvious. Indeed, in mimicking one-to-one counting, its whole purpose (to determine the total) may not be at all clear to children. By modeling with counting with subitizable collections, a process that involves attention to both units and the goal of specifying the total, a child is more likely to literally see that the process of counting also results in the same total.

Four-knowers are more likely to recognize the purpose of counting and that the last number word has special significance (the CP) than even 3-knowers because they encounter different examples (with collections of 2, 3, and 4) where the connection can be made and, thus, increasing the chance of constructing a general (count-cardinal) concept. Three-knowers may construct only a local understanding of the CP because 4 is just beyond their subitizing range. Thus, with collections of 4, the attention-consuming process of keeping track of counted and uncounted items is minimal, and a child may already be in the process of recognizing this number without counting.

Unlike the count-only intervention, both the count-first and label-first interventions were successful in promoting at least partial, if not general, understanding of CP because both of these approaches to modeling the CP are consistent with Gentner's (2005) structure-mapping theory. Specifically, the overlap or commonality between counting and the cardinal label signals a connection between the two and initiates a process of reflection that reveals the nature of the connection, namely that both processes involve determining the total number of a collection, and the recognition of special status of the last number word in the counting process (i.e., CP). Contrary to Mix et al.'s (2012) hypothesis, the count-first modeling was significantly more effective than the label-first modeling in promoting general CP knowledge because there is a closer temporal connection between the last number word counted and the cardinal label.

6.2 Educational implications

The results of the present study highlight the importance of building children's subitizing ability as basis for promoting the CP and the related cardinal-count concept. A small amount of the subitizing instruction (4 sessions of

25–30 min, each over 2 weeks) enabled nearly all of the 80 preschool participants from largely rural poor communities to improve their subitizing ability. This, in turn, enabled about three-eighths of the participants (largely 4-year-olds) to spontaneously construct the CP. In addition, subitizing instruction provided what appears to be a critically important foundation for CP instruction. The present results indicate that instruction on meaningful one–one object counting should be introduced after children achieve at least the 3-knower level. Indeed, the evidence indicated that achieving the 4-knower levels seems necessary for learning a general CP (count-cardinal concept) and its transfer—the cardinal-count concept (which underlies counting-out a specified number of items).

The importance of early, subitizing-based CP intervention is underscored by several recent studies. Sarnecka et al. (2018) observed that a growing proportion of US students are dual-language learner (DLL) many of whom live in or near poverty. Their sample of DLL preschoolers from low-income families revealed serious delays in acquiring basic number skills. If the results of the present are applicable to Yun et al.'s (2011) sample of largely minority kindergartners from low-income families, then the almost 30% who could not subitize 4 could be expected to lack general CP knowledge. These two studies indicate that a sizeable minority of preschool do not have the opportunity to develop subitizing skills, which—if the results of the present study hold true—may limit the learning of cardinality concepts. This is unsettling particularly unsettling in light of the implication of Geary et al. (2018) research that promoting the CP (count-cardinal concept) and cardinal-count concept early provides a stronger basis for learning formal mathematics.

6.3 Limitations

Although it is important to examine the development and learning potential of a largely rural poor preschoolers, future research that explores the relation between subitizing level and level of CP knowledge or other aspects of the present research (e.g., the relative value of different approaches to modeling the CP) needs to be conducted with a broader sample, including typically developing middle-class children and those with learning difficulties (see, e.g., Geary et al. 2018). A future study of the impact on subitizing instruction on CP learning could also benefit from adding a control group. The present RCT study was limited to examining the benefits of using subitizable collection when modeling the CP in various ways. A future RCT could look at a comparison of CP interventions that used different collection sizes (i.e., compared using small collections only, large collections only, and a combination of both). Additional research is also needed to explore whether counting first by itself or in conjunction with changing the pitch of the last counting word, repeating the last counting word, or both is efficacious in promoting the CP and its transfer.

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